

離散數學 HW02

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1 QUESTION

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2 ANSWER

2.1 page 132, chapter 2.1 Exercises 36

Find A^3 if

- (a) $A = \{a\}$
- (b) $A = \{0, a\}$

- (a) $\{(a, a, a)\}$
- (b) $\{(0,0,0), (a,0,0), (0,a,0), (0,0,a), (a,a,0), (a,0,a), (0,a,a), (a,a,a)\}$

2.2 page 144, chapter 2.2 Exercises 4

Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

- (a) $A \cup B$
- (b) $A \cap B$
- (c) $A - B$
- (d) $B - A$

- (a) $\{a, b, c, d, e, f, g, h\}$
- (b) $\{a, b, c, d, e\}$
- (c) \emptyset
- (d) $\{f, g, h\}$

2.3 page 163, chapter 2.3 Exercises 38

Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from \mathbf{R} to \mathbf{R} .

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f(x+2) = (x+2)^2 + 1 = x^2 + 4x + 5 \\(g \circ f)(x) &= g(f(x)) = g(x^2 + 1) = x^2 + 1 + 2 = x^2 + 3\end{aligned}$$

2.4 page 179, chapter 2.4 Exercises 34

Compute each of these double sums.

- (a) $\sum_{i=1}^3 \sum_{j=1}^2 (i - j)$
- (b) $\sum_{i=0}^3 \sum_{j=0}^2 (3i + 2j)$
- (c) $\sum_{i=1}^3 \sum_{j=0}^2 j$

$$(d) \sum_{i=1}^2 \sum_{j=0}^3 (i^2 j^3)$$

- (a) $(1-1) + (1-2) + (2-1) + (2-2) + (3-1) + (3-2) = 3$
- (b) $(0+0) + (0+2) + (0+4) + (3+0) + (3+2) + (3+4) + (6+0) + (6+2) + (6+4) + (9+0) + (9+2) + (9+4) = 78$
- (c) $(1+2) + (1+2) + (1+2) = 9$
- (d) $(1*0) + (1*1) + (1*8) + (1*27) + (4*0) + (4*1) + (4*8) + (4*27) = 180$

2.5 page 186, chapter 2.5 Exercises 6

Suppose that Hilbert's Grand Hotel is fully occupied, but the hotel closes all the even numbered rooms for maintenance. Show that all guests can remain in the hotel.

Suppose the hotel has infinite rooms. Move guests from room n to room $f(n)=2n-1$. That is guest in room 2 move to room $2*2-1=3$, guest in room 3 move to room $3*2-1=5$ and so on.