7.

a) finite, $X = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ ⇒ size = 9

b) finite, 因為集合有四個元素, size = $2^4 = 16$

c) infinite

d) finite, $|A| = 5$, $|B| = 3$ size = $5 \times 3 = 15$

e) finite, $9x^2 = 1$, 沒有正整數解 size = 0

f) finite, 因為集合有三個元素, size = $2^3 = 8$

g) finite, $|A| = 3$ $|B| = 0$ $A \times B = \varnothing$ size = 0

h) finite, $4x^2 = 8$, $x^2 = 2$, 沒有正整數解 size = 0

i) finite, $x^2 = 2$ 沒有整數解, size = 0

j) finite, 因為集合有兩個元素, size = $2^2 = 4$

k) infinite

l) finite, $|S| = 3$, $|T| = 5$ size = $3 \times 5 = 15$

m) finite, $X = \{-2, -1, 0, 1, 2\}$, size = 5

8.

$$23^{1002} = (23^{40})^{25} \cdot 23^2 \equiv 1^{25} \cdot 529 = 529 \equiv 37 \pmod{41}$$

$23^2 \bmod 41 = 529 \bmod 41 = 37$

$23^4 \bmod 41 = (37 \bmod 41)^2 \bmod 41 = 16$

$23^8 \bmod 41 = (16 \bmod 41)^2 \bmod 41 = 10$

$23^{16} \bmod 41 = (10 \bmod 41)^2 \bmod 41 = 18$

$23^{32} \bmod 41 = (18 \bmod 41)^2 \bmod 41 = 37$

$23^{40} \bmod 41 = (23^{32} \bmod 41) \cdot (23^8 \bmod 41) = (37 \cdot 10) \bmod 41 = 1$