離散數學 HW09

易頡 110054809
隨班附讀

1 QUESTION

- page 608, chapter 9.1 Exercise 6
- page 619, chapter 9.2 Exercise 2
- page 627, chapter 9.3 Exercise 14
- page 638, chapter 9.4 Exercise 20
- page 647, chapter 9.5 Exercise 24
- page 662, chapter 9.6 Exercise 8

2 ANSWER

2.1 page 608, chapter 9.1 Exercise 6

Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

(a) $x + y = 0$
(b) $x = \pm y$
(c) $x - y$ is a rational number.
(d) $x = 2y$
(e) $xy \geq 0$
(f) $xy = 0$
(g) $x = 1$
(h) $x = 1$ or $y = 1$

(a)
- not reflexive. Since $1 + 1 \neq 0$.
- symmetric. Since $x + y = y + x$, it follows that $x + y = 0$ if and only if $y + x = 0$.
- not antisymmetric. Since $(1, -1)$ and $(-1, 1)$ are both in R.
- not transitive. For example, $(1, -1) \in R$ and $(-1, 1) \in R$, but $(1, 1) \notin R$.

(b)
- reflexive. Since $x \pm x$
- symmetric. Since $x = \pm y$ if and only if $y = \pm x$
- not antisymmetric. Since (1,-1) and (-1,1) are both in R
- transitive. Since (4,-4) and (-4,4) are both in R and (4,4) also in R

(c)
- reflexive. Since $x - x = 0 is a rational number$
- symmetric. Since if $x - y$ is rational $y - x$ is also rational
- not antisymmetric. Since (1,-1) and (-1,1) are both in R
- transitive. Since that if $(x, y) \in R$ and $(y, z) \in R$ then x-y and y-z are rational numbers. Therefore $x - z$ is rational, means that $(x, z) \in R$

(d)
- not reflexive. Since $a \neq 2a$
- not symmetric. Since $(2, 1) \in R, but \notin (1, 2)$
- antisymmetric. Since $x = 2y$ and $y = 2x$ Then $y = 4y$, the only time that (x, y) and (y, z) are both $\in$ R, is when $x = y$ (and both are 0)
- not transitive. Since $(4, 2) \in R$ and $(2, 1) \in R$, but $(4, 1) \notin R$

(e)
- reflexive. Since R is always nonnegative
- symmetric. Since x, y are interchangeable
- not antisymmetric. Since (1,2) and (2,1) are both in R
- not transitive. Since $(1, 0) \in R$ and $(0, -2) \in R$, but $(1, -2) \notin R$

(f)
- not reflexive. Since $(1, 1) \notin R$
- symmetric. Since x, y are interchangeable
- not antisymmetric. Since (0,1) and (1,0) are both in R
- not transitive. Since (1,0) and (0,1) are both in R, but $(1, 1) \notin R$

(g)
- not reflexive. Since $(2, 2) \notin R$
- not symmetric. Since $(1, 2) \in R$, but $(2, 1) \notin R$
- antisymmetric. Since if both (x,y) and (y,x) are in R, then x=1 and y=1, so x=y
- transitive. Since if both $(x, y) \in R$ and $(y, z) \in R$, then x=1, so $(x, z) \in R$

(h)
- not reflexive. Since $(2, 2) \notin R$
- symmetric. Since x, y are interchangeable
- not antisymmetric. Since (1,2) and (2,1) are both in R
- not transitive. Since (2,1) and (1,3) are both in R, but $(2, 3) \notin R$

2.2 page 619, chapter 9.2 Exercise 2

Which 4-tuples are in the relation $\{(a, b, c, d) \mid a, b, c,$ and d are positive integers with $abcd = 6\}$?

(6, 1, 1, 1),(1, 6, 1, 1),(1, 1, 6, 1),(1, 1, 1, 6),
(1,1,2,3),(1,1,3,2),(1,2,1,3),(1,2,3,1)
(1,3,1,2),(1,3,2,1),(2,1,1,3),(2,1,3,1)
(2,3,1,1),(3,1,1,2),(3,2,1,1),(3,1,2,1)

2.3 page 627, chapter 9.3 Exercise 14

Let R_1 and R_2 be relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent

(a) $R_1 \cup R_2$
(b) $R_1 \cap R_2$
(c) $R_2 \circ R_1$
(d) $R_1 \circ R_1$
(e) $R_1 \oplus R_2$

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

2.4 page 638, chapter 9.4 Exercise 20

Let R be the relation that contains the pair (a, b) if a and b are cities such that there is a direct nonstop airline flight from a to b. When is (a, b) in

(a) R^2 ?
(b) R^3 ?
(c) R^* ?

(a) The pair (a, b) in R^2 precisely when there is a city c such that there is a direct flight from a to c and a direct flight from c to b – in other words,when it is possible to fly from a to b with a scheduled stop (and possibly a plane change) in some intermediate city.
(b) The pair (a, b) in R^3 precisely when there is a city c and d such that there is a direct flight from a to c, a direct flight from c to d and a direct flight from d to b – in other words, when it is possible to fly from a to b with two scheduled stop (and possibly a plane change at one or both) in some intermediate city
(c) The pair (a, b) in R^* precisely when it is possible to fly from a to b

2.5 page 647, chapter 9.5 Exercise 24

Determine whether the relations represented by these zero–one matrices are equivalence relations.

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(a) This is not an equivalence relation, since it is not symmetric.
(b) This is an equivalence relation, since it is reflexive, symmetric and transitive
(c) This is an equivalence relation, since it is reflexive, symmetric and transitive

2.6 page 662, chapter 9.6 Exercise 8

Determine whether the relations represented by these zero–one matrices are partial orders.

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

(a) not a partial order. This relation is $(1, 1), (1, 3), (2, 1), (2, 3), (3, 3)$. It is clearly reflexive and antisymmetric. The only one pairs that might present problems with transitivity are the non-diagonal pairs, $(2, 1)$ and $(1, 3)$. If the relation were to be transitive, then we would also need the pair $(2, 3)$ in the relation.
(b) a partial order, since it is reflexive and antisymmetric, also can cause no problem with transitivity.
(c) not a partial order, since it is not transitivity, (1,3) and (3,4) are present, but not (1,4), so it is not partial ordering