

離散數學 HW03

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1 QUESTION

- page 228, chapter 3.2 Exercises 2

2 ANSWER

2.1 page 228, chapter 3.2 Exercises 2

Determine whether each of these functions is $O(x^2)$.

(a) $f(x) = 17x + 11$

(b) $f(x) = x^2 + 1000$

(c) $f(x) = x \log x$

(d) $f(x) = x^4/2$

(e) $f(x) = 2^x$

(f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

(a) Yes, $C = 18, k = 11$.

(b) Yes, $x^2 + 1000 \leq x^2 + x^2 = 2x^2$, for all $x > \sqrt{1000}, C = 2, k = \sqrt{1000}$

(c) Yes, $x \log x \leq x \cdot x = x^2$, for all $x, C = 1, k = 0$

(d) No, if there were a constant C such that $x^4/2 \leq Cx^2$ for sufficiently large x , then we would have $C \geq x^2$

(e) No, if 2^x were $O(x^2)$, then the fraction $2^x/x^2$ would have to be bounded above by some constant C

(f) Yes, since $\lfloor x \rfloor \lceil x \rceil \leq x(x+1) \leq x \cdot 2x = 2x^2$, for all $x > 1, C = 2, k = 1$