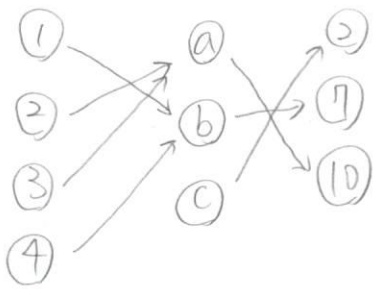


19. a) $f \circ g$

$$A \xrightarrow{g} B \xrightarrow{f} C$$

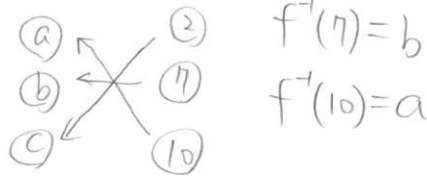


$$f \circ g: A \rightarrow C$$

$$\{(1, 2), (2, 7), (3, 10), (4, 2)\}$$

b) f^{-1}

$$B \leftarrow C \quad f^{-1}(2) = c$$



$$f^{-1}(7) = b$$

$$f^{-1}(10) = a$$

20. $[P \wedge (P \rightarrow Q)] \rightarrow P \equiv \neg[P \wedge (P \rightarrow Q)] \vee P$ by the conditional-disjunction equivalence

$$\equiv [\neg P \vee \neg(P \rightarrow Q)] \vee P \text{ by a De Morgan's law}$$

$$\equiv (\neg P \vee Q) \vee \neg(P \rightarrow Q) \text{ by commutativity and associativity}$$

$$\equiv (\neg P \vee Q) \vee \neg(\neg P \vee Q) \text{ by the conditional-disjunction equivalence}$$

$$\equiv \top \text{ by a negation law}$$

21.

| Step | | Reason |
|------|---|-------------------------------------|
| 1 | $\neg t$ | Hypothesis |
| 2 | $s \rightarrow t$ | Hypothesis |
| 3 | $\neg s$ | Modus tollens using (1) and (2) |
| 4 | $(\neg r \vee \neg f) \rightarrow (s \wedge l)$ | Hypothesis |
| 5 | $(\neg(s \wedge l) \rightarrow \neg(\neg r \vee \neg f))$ | contrapositive of (4) |
| 6 | $(\neg s \vee \neg l) \rightarrow (r \wedge f)$ | De Morgan's law and double negative |
| 7 | $\neg s \vee \neg l$ | Addition, using (3) |
| 8 | $r \wedge f$ | Modus ponens using (6) and (7) |
| 9 | r | simplification using (8) |