離散數學 HW01 解答

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1 QUESTION

- page 16, chapter 1.1 Exercises 34(f)
- page 24, chapter 1.2 Exercises 8
- page 38, chapter 1.3 Exercises 10(c)
- page 58, chapter 1.4 Exercises 28
- page 58, chapter 1.4 Exercises 30
- page 71, chapter 1.5 Exercises 26
- page 71, chapter 1.5 Exercises 32(c)
- page 82, chapter 1.6 Exercises 6
- page 96, chapter 1.7 Exercises 30
- page 113, chapter 1.8 Exercises 6

2 ANSWER

2.1 page 16, chapter 1.1 Exercises 34(f)

Construct a truth table for each of these compound propositions.

$$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$$

Table 1: Truth Table for the Compound Propositions

p	q	$\neg p$	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

2.2 page 24, chapter 1.2 Exercises 8

Express these system specifications using the propositions p: "The user enters a valid password," q: "Access is granted," and r: "The user has paid the subscription fee" and logical connectives (including negations).

(a) "The user has paid the subscription fee, but does not enter a valid password."
(b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
(c) "Access is denied if the user has not paid the subscription fee."
(d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

(a) $r \wedge \neg p$
(b) $(r \wedge p) \rightarrow q$
(c) $\neg r \rightarrow \neg q$
(d) $(\neg p \wedge r) \rightarrow q$

2.3 page 38, chapter 1.3 Exercises 10(c)

For each of these compound propositions, use the conditional-disjunction equivalence (Example 3) to find an equivalent compound proposition that does not involve conditionals.

$$(p \rightarrow \neg q) \rightarrow (\neg p \rightarrow q)$$

$$\begin{aligned}
(p \rightarrow \neg q) \rightarrow (\neg p \rightarrow q) &\equiv \neg(p \rightarrow \neg q) \vee (\neg p \rightarrow q) \\
&\qquad \text{by the condition-disjunction equivalence} \\
&\equiv \underline{\neg(\neg p \vee \neg q) \vee (\neg\neg p \vee q)} \\
&\qquad \text{by the condition-disjunction equivalence} \\
&\equiv \underline{(p \wedge q) \vee (p \vee q)} \\
&\qquad \text{by the double negation and DeMorgan's laws} \\
&\equiv \underline{(p \wedge q) \vee p \vee q} \qquad \text{by the associative law} \\
&\equiv \underline{p \vee q} \qquad \text{by the absorbtion laws}
\end{aligned}$$

2.4 page 58, chapter 1.4 Exercises 28

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

(a) Something is not in the correct place.
(b) All tools are in the correct place and are in excellent condition.
(c) Everything is in the correct place and in excellent condition.
(d) Nothing is in the correct place and is in excellent condition.
(e) One of your tools is not in the correct place, but it is in excellent condition.

let $R(x)$ be "x is in the correct place,"
let $E(x)$ be "x is in excellent condition,"
let $T(x)$ be "x is a [or your] tool,"
and let the domain of discourse be all things.

(a) $\exists x\, \neg R(x)$
(b) $\forall x(T(x) \rightarrow (R(x) \wedge E(x))$
(c) $\forall x(R(x) \wedge E(x))$
(d) $\forall x \neg(R(x) \wedge E(x))$
(e) $\exists x(T(x) \wedge \neg R(x) \wedge E(x))$

2.5 page 58, chapter 1.4 Exercises 30

Suppose the domain of the propositional function P(x, y) consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

(a) $\exists x\, P(x, 3)$
(b) $\forall y\, P(1, y)$
(c) $\exists y\, \neg P(2, y)$

(d) $\forall x\ \neg P(x,2)$

(a) $P(1,3) \vee P(2,3) \vee P(3,3)$
(b) $P(1,1) \wedge P(1,2) \wedge P(1,3)$
(c) $\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$
(d) $\neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

2.6 page 71, chapter 1.5 Exercises 26

Let $Q(x,y)$ be the statement $x+y=x-y$. If the domain for both variables consists of all integers, what are the truth values?

(a) $Q(1,1)$
(b) $Q(2,0)$
(c) $\forall y Q(1,y)$
(d) $\exists x Q(x,2)$
(e) $\exists x \exists y Q(x,y)$
(f) $\forall x \exists y Q(x,y)$
(g) $\exists y \forall x Q(x,y)$
(h) $\forall y \exists x Q(x,y)$
(i) $\forall x \forall y Q(x,y)$

(a) F
(b) T
(c) F
(d) F
(e) T
(f) T
(g) T
(h) F
(i) F

2.7 page 71, chapter 1.5 Exercises 32(c)

Express the negations of each of these statements so that all negation symbols immediately precede predicates.

$$\exists x \exists y (Q(x,y) \leftrightarrow Q(y,x))$$

$$\begin{aligned} \neg\exists x \exists y (Q(x,y) \leftrightarrow Q(y,x)) &\equiv \forall x \neg \exists y (Q(x,y) \leftrightarrow Q(y,x)) \\ &\equiv \forall x \forall y \neg (Q(x,y) \leftrightarrow Q(y,x)) \\ &\equiv \forall x \forall y (\neg Q(x,y) \leftrightarrow Q(y,x)) \end{aligned}$$

2.8 page 82, chapter 1.6 Exercises 6

Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

let r be the proposition "It rains,"
let f be the proposition "It is foggy,"
let s be the proposition "The sailing race will be held,"
let l be the proposition "The life saving demonstration will go on,"
let t be the proposition "The trophy will be awarded."

Step	推導	Reason
1	$\neg t$	Hypothesis
2	$s \rightarrow t$	Hypothesis
3	$\neg s$	Modus tollens using (1) and (2)
4	$(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Hypothesis
5	$(\neg(s \wedge l)) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6	$(\neg s \vee \neg l) \rightarrow (r \wedge f)$	De Morgan's law and double negative
7	$\neg s \vee \neg l$	Addition, using (3)
8	$r \wedge f$	Modus ponens using (6) and (7)
9	r	Simplification using (8)

2.9 page 96, chapter 1.7 Exercises 30

Prove that $m^2 = n^2$ if and only if $m = n$ or $m = n$.

For the "if" part, there are two cases.
If $m = n$, then $m^2 = n^2$
If $m = -n$, $m^2 = (-n)^2 = (-1)^2 n^2 = n^2$
For the "only if" part, we suppose that $m^2 = n^2$.
$m^2 - n^2 = (m+n)(m-n) = 0,\ m+n = 0\ or\ m-n = 0, m = n\ or\ m = -n$

2.10 page 113, chapter 1.8 Exercises 6

Use a proof by cases to show that min(a, min(b, c)) = min(min(a, b), c) whenever a, b, and c are real numbers.

Case 1: If a is smallest.
Then clearly $a \leq min(b,c)$, and so the left-hand side equals.
On the other hand, for the right-hand side we have min(a,c) = a as well
Case 2: If b is smallest.
Left side equals to min(a,b) = b, and right side equals to min(b,c) = b
Case 3: If c is smallest.
Left side is min(a,c) = c, right side clearly $c \leq min(a,b)$ so two sides are equal.
Since one of the three has to be the smallest we have taken care of all the cases.