

17.

- a) finite, $X = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\} \Rightarrow \text{size} = 9$
 b) finite, 因為集合有四個元素, $\text{size} = 2^4 = 16$
 c) infinite
 d) finite, $|A| = 5, |B| = 3 \Rightarrow \text{size} = 5 \times 3 = 15$
 e) finite, $9x^2 = 1$, 沒有正整數解 $\text{size} = 0$
 f) finite, 因為集合有三個元素, $\text{size} = 2^3 = 8$
 g) finite, $|A| = 3, |B| = 0, A \times B = \emptyset \Rightarrow \text{size} = 0$
 h) finite, $4x^2 = 8, x^2 = 2$, 沒有正整數解 $\text{size} = 0$
 i) finite, $x^2 = 2$ 沒有整數解, $\text{size} = 0$
 j) finite, 因為集合有兩個元素, $\text{size} = 2^2 = 4$
 k) infinite
 l) finite, $|S| = 3, |T| = 5 \Rightarrow \text{size} = 3 \times 5 = 15$
 m) finite, $X = \{-2, -1, 0, 1, 2\} \Rightarrow \text{size} = 5$

8.

$$23^{1002} = (23^{40})^{25} \cdot 23^2 = 1^{25} \cdot 529 = 529 \equiv 37 \pmod{41}$$

$$23^2 \pmod{41} = 529 \pmod{41} = 37$$

$$23^4 \pmod{41} = (37 \pmod{41})^2 \pmod{41} = 16$$

$$23^8 \pmod{41} = (16 \pmod{41})^2 \pmod{41} = 10$$

$$23^{16} \pmod{41} = (10 \pmod{41})^2 \pmod{41} = 18$$

$$23^{32} \pmod{41} = (18 \pmod{41})^2 \pmod{41} = 37$$

$$23^{40} \pmod{41} = (23^{32} \pmod{41}) \cdot (23^8 \pmod{41}) = (37 \cdot 10) \pmod{41} = 1$$

$\begin{array}{r} 7 \\ 4 \overline{) 324} \\ \underline{287} \\ 37 \end{array}$	$\begin{array}{r} 6 \\ 18 \\ 18 \\ \underline{144} \\ 18 \\ \underline{324} \\ 0 \end{array}$	$\begin{array}{r} 16 \\ 16 \\ \underline{16} \\ 0 \end{array}$	$\begin{array}{r} 23 \\ 23 \\ \underline{69} \\ 46 \\ \underline{46} \\ 0 \end{array}$	$\begin{array}{r} 12 \\ 41 \overline{) 529} \\ \underline{41} \\ 119 \\ \underline{119} \\ 0 \end{array}$
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