Introduction

The Earthquake example is a classic problem in Bayesian networks. In this scenario, you are at work, and your neighbor John calls to inform you that your alarm is ringing, but another neighbor, Mary, does not call. Sometimes the alarm is triggered by minor earthquakes. The key question we are trying to answer is: "Is there a burglar?"

The variables involved are:

- **Burglary** (B): Whether a burglary has occurred.
- **Earthquake** (E): Whether an earthquake has occurred.
- **Alarm** (A): Whether the alarm has gone off.
- **JohnCalls** (J): Whether John calls.
- **MaryCalls** (M): Whether Mary calls.

The network topology reflects the causal relationships:

- A burglary can set the alarm off.
- An earthquake can set the alarm off.
- The alarm can cause Mary to call.
- The alarm can cause John to call.

The following Bayes network represents these relationships:

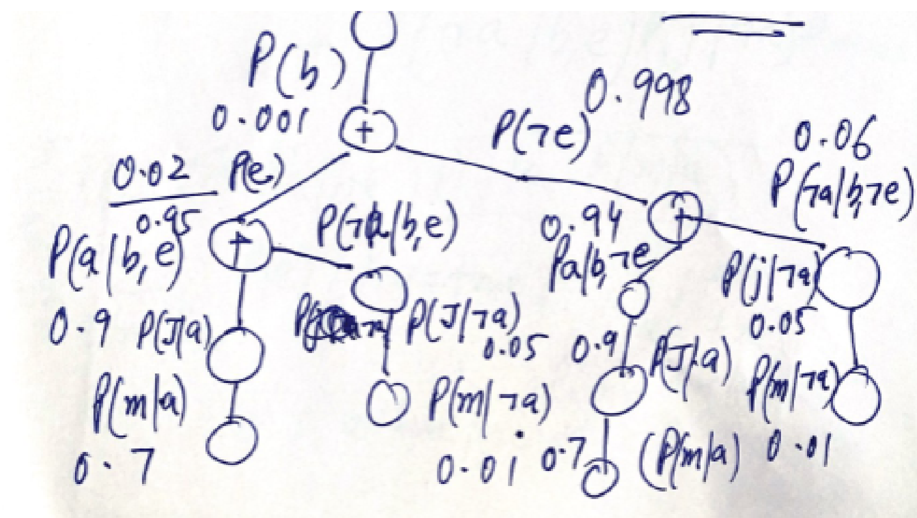

Figure 1: Bayes Net for the Earthquake Example

Query: Inference from Bayes Net

We are interested in finding the probability of a burglary given that both John and Mary have called:

$$P(B \mid J = T, M = T)$$

Hidden Variables: Earthquake & Alarm.

$$P(B \mid J = T, M = T) = \alpha \sum_e \sum_a P(B, J = T, M = T, e, a)$$

$$P(b \mid j, m) = \alpha \sum_e \sum_a P(b)P(e)P(a \mid b, e)P(j \mid a)P(m \mid a)$$

$$P(b \mid j, m) = \alpha P(b) \sum_e P(e) \sum_a P(a \mid b, e)P(j \mid a)P(m \mid a)$$

The prior probability of a burglary is given as:

$$P(b) = 0.001$$

Therefore:

$$P(b \mid j, m) = \alpha \cdot 0.001 \left(\sum_e P(e) \sum_a P(a \mid b, e)P(j \mid a)P(m \mid a) \right)$$

We calculate the probabilities by summing over all possible values of the hidden variables:

$$P(b \mid j, m) = \alpha \cdot 0.000059224$$

$$+ \ P(\neg a \mid b, e)P(j \mid \neg a)P(m \mid \neg a)$$

Summing over the possible values of the hidden variables:

$$\sum_e P(e) \sum_a P(a \mid b, e)P(j \mid a)P(m \mid a)$$

$$= \sum_{e \in \{\text{true},\text{false}\}} P(e) \left(P(a = \text{true} \mid b, e)P(j = \text{true} \mid a = \text{true})P(m = \text{true} \mid a = \text{true}) \right)$$

$$+ \ P(a = \text{false} \mid b, e)P(j = \text{true} \mid a = \text{false})P(m = \text{true} \mid a = \text{false})$$

Variable Elimination Algorithm

We can use the Variable Elimination algorithm to simplify the calculation of $P(b \mid j, m)$:

$$P(b \mid j, m) = \alpha P(b) \sum_e P(e) \sum_a P(a \mid b, e) P(j \mid a) P(m \mid a)$$

Define the factors:

- $f_1(b) = P(b)$
- $f_2(e) = P(e)$
- $f_3(a, b, e) = P(a \mid b, e)$
- $f_4(a) = P(j \mid a)$
- $f_5(a) = P(m \mid a)$

Then, we have:

$$P(b \mid j, m) = \alpha f_1(b) \sum_e f_2(e) \sum_a f_3(a, b, e) f_4(a) f_5(a)$$

Where:

$$f_4(a) = \begin{pmatrix} P(j \mid a) = 0.90 \\ P(j \mid \neg a) = 0.05 \end{pmatrix}$$

$$f_5(a) = \begin{pmatrix} P(m \mid a) = 0.70 \\ P(m \mid \neg a) = 0.01 \end{pmatrix}$$

The joint factor $f_3(a, b, e)$ is a 2x2x2 matrix representing $P(a \mid b, e)$, where $P(a \mid b, e) = 0.95$ and similar values for other cases.

The next steps involve element-wise product and summation over the hidden variables:

$$f_6(b, e) = \sum_a f_3(a, b, e) f_4(a) f_5(a)$$

$$f_6(b, e) = f_3(a, b, e) f_4(a) f_5(a) + f_3(\neg a, b, e) f_4(\neg a) f_5(\neg a)$$

$$P(b \mid j, m) = \alpha f_1(b) \sum_e f_2(e) f_6(b, e)$$

Finally, summing over e:

$$f_7(b) = \sum_e f_2(e) \times f_6(b, e)$$

$$P(b \mid j, m) = \alpha f_1(b) \times f_7(b)$$