Probability and Propositional Logic CheatSheet

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Basic Probability Rules

Probability of an Event:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Complement Rule:

$$P(\neg A) = 1 - P(A)$$

Addition Rule (for Union of Two Events):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

Conditional Probability:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Chain Rule:

$$P(A \cap B) = P(A \mid B) \cdot P(B)$$

Bayes' Theorem:

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

Naive Bayes Classifier

Posterior Probability:

$$P(C \mid X) = \frac{P(X \mid C) \cdot P(C)}{P(X)}$$

Naive Bayes Assumption (Conditional Independence):

$$P(X_1, X_2, \ldots, X_n \mid C) = \prod_{i=1}^{n} P(X_i \mid C)$$

Propositional Logic Symbols

- $A, B, C, \dots$ - Propositional variables (statements)
- $\neg A$ - Negation (not A)
- $A \wedge B$ - Conjunction (both A and B are true)
- $A \vee B$ - Disjunction (either A or B or both are true)
- $A \rightarrow B$ - Implication (if A is true, then B is true)
- $A \leftrightarrow B$ - Biconditional (if A, then B, and if B, then A)
- $\top$ - True
- $\bot$ - False

Logical Equivalences

Commutative Laws:

$$A \wedge B \equiv B \wedge A$$

$$A \vee B \equiv B \vee A$$

Associative Laws:

$$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$$

$$(A \vee B) \vee C \equiv A \vee (B \vee C)$$

Distributive Laws:

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

De Morgan's Laws:

$$\neg(A \wedge B) \equiv \neg A \vee \neg B$$

$$\neg(A \vee B) \equiv \neg A \wedge \neg B$$

Implication Equivalences:

$$A \rightarrow B \equiv \neg A \vee B$$

Rules of Inference

Modus Ponens:

$$\frac{A, A \to B}{B}$$

If A and $A \to B$ are true, then B must be true.

Modus Tollens:

$$\frac{\neg B, A \to B}{\neg A}$$

If $A \to B$ and $\neg B$ are true, then $\neg A$ must be true.

Disjunctive Syllogism:

$$\frac{A \vee B, \neg A}{B}$$

If $A \vee B$ is true and A is false, then B must be true.

Hypothetical Syllogism:

$$\frac{A \to B, B \to C}{A \to C}$$

If $A \to B$ and $B \to C$ are true, then $A \to C$ must be true.

Conjunction:

$$\frac{A, B}{A \wedge B}$$

If A and B are true, then $A \wedge B$ must be true.

Truth Table Definitions

Negation ($\neg$):

A	$\neg A$
$\top$	$\bot$
$\bot$	$\top$

Conjunction ($\wedge$):

A	B	$A \wedge B$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

Disjunction ($\vee$):

A	B	$A \vee B$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

Implication ($\to$):

A	B	$A \to B$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$

Biconditional ($\leftrightarrow$):

A	B	$A \leftrightarrow B$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\top$