

Building a Bayesian Network

1. Chain Rule for Joint Distributions

Set of Random Variables $\{X_1, X_2, \dots, X_n\}$

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1}^n P(X_i | X_{i-1}, X_{i-2}, \dots, X_1)$$

2. Conditional Independence

In a Bayesian network, each variable X_i is conditionally independent of its non-descendants, given its parents.

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1}^n P(X_i | \text{Parents}(X_i))$$

3. Construct the network

(a) Nodes

Order the set of variables of the domain in a way that causes precede effects. $\{X_1, X_2, \dots, X_n\}$

(b) Choose a minimal set of parents $C_i \subset \{X_1, \dots, X_{i-1}\}$ for each variable X_i .

3. Insert links for each parent of X_i

4. Conditional Probability Tables (CPT)

$$P(X_i | \text{Parents}(X_i))$$

For each variable X_i .

Example: Alarm Network

- B: Burglary
- E: Earthquake
- A: Alarm
- J: JohnCalls
- M: MaryCalls

Step 1: Node ordering

Possible Order $\{B, E, A, J, M\}$

Step 2: Links

For A: Parents are B and E

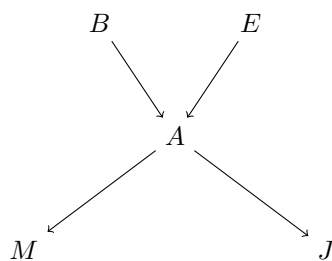
$$B \rightarrow A \quad \text{and} \quad E \rightarrow A$$

For J :

$$A \rightarrow J$$

For M :

$$A \rightarrow M$$



Step 3: CPT

$P(B)$: Prob. of burglary

$P(E)$: Prob. of Earthquake

$P(A|B, E)$: Prob. of alarm given burglary and earthquake

$P(J|A)$ and $P(M|A)$

Joint Dist.

$$P(B, E, A, J, M) = P(B) \cdot P(E) \cdot P(A|B, E) \cdot P(J|A) \cdot P(M|A)$$

$$P(M|J, A, E, B) = P(M|A)$$

Compactness

- A CPT for Boolean X_i with k Boolean parents has 2^k rows for the combinations of parent values
- Each row requires one number p for $X_i = \text{true}$ (the number for $X_i = \text{false}$ is just $1-p$)

If each variable has no more than k parents, the complete network requires $O(n \cdot 2^k)$ numbers

i.e., **grows linearly with n** , vs. $O(2^n)$ for the full joint distribution

For burglary net,
 $1 + 1 + 4 + 2 + 2 = 10$ numbers (vs. $2^5 - 1 = 31$)

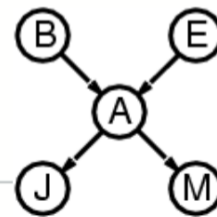


Figure 1: Enter Caption

Global Semantics

The full joint distribution is defined as the product of the local conditional distributions:

$$P(x_1, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

e.g.,

$$P(j \wedge m \wedge a \wedge \neg b \wedge \neg e)$$

$$= P(j|a)P(m|a)P(a|\neg b \wedge \neg e)P(\neg b)P(\neg e)$$

$$= 0.90 \times 0.70 \times 0.01 \times 0.999 \times 0.998 = 0.00628.$$

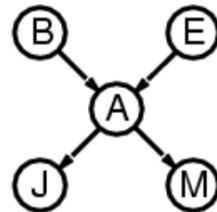


Figure 2: Enter Caption