Bayes' Rule: Updating the Expected Value Based on the Color of the Bead

Suppose a bead is drawn and it is black. We can use Bayes' rule to update the probabilities.

1. The probabilities given the bead is drawn from a white or a large jar:

$$P(\text{Black} \mid W) = \frac{2}{4} = 0.5$$

$$P(\text{Black} \mid L) = \frac{2}{3} = 0.667$$

The overall probability of drawing a black bead, $P(B)$, can be calculated as:

$$P(B) = P(\text{Black} \mid W) \cdot P(W) + P(\text{Black} \mid L) \cdot P(L)$$

Given that $P(W) = 0.5$ and $P(L) = 0.5$:

$$P(B) = 0.5 \times 0.5 + 0.667 \times 0.5 = 0.5835$$

Now, using Bayes' rule to update the probability of the bead coming from the white jar given that it is black:

$$P(W \mid \text{Black}) = \frac{P(\text{Black} \mid W) \cdot P(W)}{P(\text{Black})}$$

Substituting the values:

$$P(W \mid \text{Black}) = \frac{0.5 \times 0.5}{0.5835} \approx 0.428$$

Updating Probabilities and Expected Value

1. **Updating Probabilities**

Given the posterior probability $P(W \mid \text{Black}) = 0.428$, the probability that the bead came from the large jar given it is black is:

$$P(L \mid \text{Black}) = 1 - P(W \mid \text{Black})$$
$$= 1 - 0.428 = 0.572$$

2. **Expected Value Calculation**

The expected value based on the updated probabilities is:

$$\text{Expected Value} = 1 \cdot P(W \mid \text{Black}) + 0 \cdot P(L \mid \text{Black})$$
$$= 1 \cdot 0.428 + 0.572 \times 0 = 0.428$$

3. **Case 2: Red Bead is Drawn**

If a red bead is drawn, the probabilities are:

$$P(\text{Red} \mid W) = \frac{2}{4} = 0.5$$

$$P(\text{Red} \mid L) = \frac{1}{3} = 0.333$$

The overall probability of drawing a red bead, $P(\text{Red})$, can be calculated as:

$$P(\text{Red}) = P(\text{Red} \mid W) \cdot P(W) + P(\text{Red} \mid L) \cdot P(L)$$

Substituting the values:

$$P(\text{Red}) = 0.5 \times 0.5 + 0.333 \times 0.5 = 0.4165$$

Bayes' Rule: Updating Probabilities Based on Red Bead

Given the observation that a red bead is drawn, we update the probability that it came from the white jar:

$$P(W \mid \text{Red}) = \frac{P(\text{Red} \mid W) \cdot P(W)}{P(\text{Red})}$$

Substituting the values:

$$P(W \mid \text{Red}) = \frac{0.5 \times 0.5}{0.4165} \approx 0.6$$

The probability that the bead came from the large jar given it is red is:

$$\begin{aligned} P(L \mid \text{Red}) &= 1 - P(W \mid \text{Red}) \\ &= 1 - 0.6 = 0.4 \end{aligned}$$

Expected Value Calculation

The expected value based on the updated probabilities is:

$$\begin{aligned} \text{Expected Value} &= 1 \cdot P(W \mid \text{Red}) + 0 \cdot P(L \mid \text{Red}) \\ &= 1 \times 0.6 + 0 \times 0.4 = 0.6 \end{aligned}$$