Expected Value for Discrete Random Variable

Given a discrete random variable X with n possible values $x_1, x_2, \ldots, x_n$, the expected value of a function $U(X)$ is:

$$E[U(X)] = \sum_{i=1}^{n} p_i \, U(x_i)$$

where $U(x_i)$ is the utility or value associated with the state x_i.

Example

Suppose X represents the state reached after performing an action A under uncertainty. The utility function U assigns a value $U(x_i)$ to each state x_i.

Illustrative Calculation

Consider the following specific states:

- State x_1 with $p_1 = 0.2$ and $U(x_1) = 10$
- State x_2 with $p_2 = 0.5$ and $U(x_2) = 20$
- State x_3 with $p_3 = 0.3$ and $U(x_3) = 30$

The expected value of $U(X)$ is:

$$E[U(X)] = p_1 U(x_1) + p_2 U(x_2) + p_3 U(x_3)$$

$$= 0.2 \times 10 + 0.5 \times 20 + 0.3 \times 30$$

$$= 2 + 10 + 9$$

$$= 21$$

Continuous Random Variables

For continuous random variables, the expected value is calculated using an integral. Given a continuous random variable X with a probability density function $f_X(x)$ and a utility function U, the expected value is:

$$E[U(X)] = \int_{-\infty}^{\infty} U(x) f_X(x) \, dx$$