

# Building a Bayesian Network

## 1. Chain Rule for Joint Distributions

Set of Random Variables  $\{X_1, X_2, \dots, X_n\}$

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1}^n P(X_i | X_{i-1}, X_{i-2}, \dots, X_1)$$

## 2. Conditional Independence

In a Bayesian network, each variable  $X_i$  is conditionally independent of its non-descendants, given its parents.

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1}^n P(X_i | \text{Parents}(X_i))$$

## 3. Construct the network

### (a) Nodes

Order the set of variables of the domain in a way that causes precede effects.  $\{X_1, X_2, \dots, X_n\}$

### (b) Choose a minimal set of parents $C_i \subset \{X_1, \dots, X_{i-1}\}$ for each variable $X_i$ .

## 3. Insert links for each parent of $X_i$

## 4. Conditional Probability Tables (CPT)

$$P(X_i | \text{Parents}(X_i))$$

For each variable  $X_i$ .

### Example: Alarm Network

- B: Burglary
- E: Earthquake
- A: Alarm
- J: JohnCalls
- M: MaryCalls

### Step 1: Node ordering

Possible Order  $\{B, E, A, J, M\}$

### Step 2: Links

For A: Parents are B and E

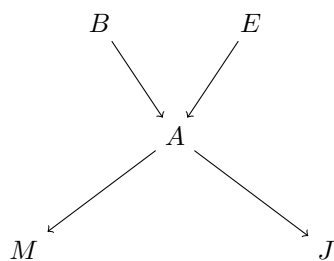
$$B \rightarrow A \quad \text{and} \quad E \rightarrow A$$

For  $J$ :

$$A \rightarrow J$$

For  $M$ :

$$A \rightarrow M$$



**Step 3: CPT**

$P(B)$  : Prob. of burglary

$P(E)$  : Prob. of Earthquake

$P(A|B, E)$  : Prob. of alarm given burglary and earthquake

$P(J|A)$  and  $P(M|A)$

**Joint Dist.**

$$P(B, E, A, J, M) = P(B) \cdot P(E) \cdot P(A|B, E) \cdot P(J|A) \cdot P(M|A)$$

$$P(M|J, A, E, B) = P(M|A)$$