

Variable Elimination:

$$P(b \mid j, m) = \alpha P(b) \sum_e P(e) \sum_a P(a \mid b, e) P(j \mid a) P(m \mid a)$$

$$f_1(b) = P(b)$$

$$f_2(e) = P(e)$$

$$f_3(a, b, e) = P(a \mid b, e)$$

$$f_4(a) = P(j \mid a)$$

$$f_5(a) = P(m \mid a)$$

$$f_4(a) = P(j \mid a) = \begin{bmatrix} 0.90 \\ 0.05 \end{bmatrix}$$

$$f_5(a) = P(m \mid a) = \begin{bmatrix} 0.70 \\ 0.01 \end{bmatrix}$$

The matrix $f_3(a, b, e)$ is a $2 \times 2 \times 2$ matrix representing the probability $P(a \mid b, e)$, where:

$$P(a \mid b, e) = 0.95 \quad \text{when } (a = \text{true}, b = \text{true}, e = \text{true})$$

$$P(b \mid j, m) = \alpha f_1(b) \times \sum_e f_2(e) \times \sum_a f_3(a, b, e) \times f_4(a) \times f_5(a)$$

Element-wise product:

$$f_6(b, e) = \sum_a f_3(a, b, e) \times f_4(a) \times f_5(a)$$

$$= f_3(a, b, e) \times f_4(a) \times f_5(a) + f_3(\neg a, b, e) \times f_4(\neg a) \times f_5(\neg a)$$

$$P(b \mid j, m) = \alpha f_1(b) \times \sum_e f_2(e) \times f_6(b, e)$$

$$f_7(b) = \sum_e f_2(e) \times f_6(b, e)$$

$$= f_2(e) \times f_6(b, e) + f_2(\neg e) \times f_6(b, \neg e)$$

$$P(b \mid j, m) = \alpha f_1(b) \times f_7(b)$$