Joint Probability Distribution Example

Consider two random variables:

- X_1: Toothache (True & False)
- X_2: Cavity (True & False)

Given probabilities:

$$P(\text{Cavity} \mid X_1 = \text{True}) = 0.04$$
$$P(\neg\text{Cavity} \mid X_2 = \text{False}) = 0.01$$
$$P(\text{Toothache} \wedge \text{Cavity}) = 0.04$$
$$P(\neg\text{Toothache} \wedge \text{Cavity}) = 0.06$$
$$P(\text{Toothache} \wedge \neg\text{Cavity}) = 0.08$$
$$P(\neg\text{Toothache} \wedge \neg\text{Cavity}) = 0.89$$

Properties

- **Non-negativity:**

$$P(X_1 = x_1, X_2 = x_2) \geq 0 \quad \text{for all } x_1, x_2$$

- **Normalization:**

$$\sum_{x_1} \sum_{x_2} P(X_1 = x_1, X_2 = x_2) = 1$$

  Sum of all probabilities in the joint distribution must equal 1.

Marginal Distribution

The marginal distribution is obtained by summing (or integrating for continuous variables) the joint probabilities over other variables.

Consider $X_1 = \text{Toothache}$, the marginal distribution of X_1:

$$P(\text{Toothache}) = P(\text{Toothache} \wedge \text{Cavity}) + P(\text{Toothache} \wedge \neg\text{Cavity})$$

$$= 0.04 + 0.01 = 0.05$$

Similarly,

$$P(\neg\text{Toothache}) = P(\neg\text{Toothache} \wedge \text{Cavity}) + P(\neg\text{Toothache} \wedge \neg\text{Cavity})$$

$$= 0.06 + 0.89 = 0.95$$