Uniform Distribution Example Between 18 and 26

Probability Density Function

For a continuous random variable X that is uniformly distributed between 18 and 26, the probability density function is defined as:

$$f_X(x) = \frac{1}{b-a}$$

where $a = 18$, $b = 26$. Therefore,

$$f_X(x) = \frac{1}{8} = 0.125$$

The probability density function (pdf) is constant because the uniform distribution assigns equal probability to any interval of the same length within $[18, 26]$.

Finding the PDF at $x = 20.5$

The probability density function (pdf) at $x = 20.5$ is given by:

$$f_X(20.5) = \frac{1}{8} = 0.125$$

This does not represent the probability of X being exactly 20.5 (that probability is zero). Instead, it represents the density at 20.5.

Density is understood in terms of an infinitesimally small interval around 20.5. Consider an interval $[20.5, 20.5 + dx]$. The probability that X lies within this interval is approximately:

$$P(20.5 \leq X \leq 20.5 + dx) \approx f_X(20.5) \cdot dx = 0.125 \cdot dx$$

Taking the limit as $dx \to 0$:

$$\lim_{dx \to 0} \frac{P(20.5 \leq X \leq 20.5 + dx)}{dx} = f_X(20.5) = 0.125$$

Properties of the Probability Density Function (PDF)

1. **Non-negativity:**
$$f_X(x) \geq 0 \quad \text{for all } x$$

2. **Normalization:** The integral of the PDF over the entire range is 1:
$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

For our uniform distribution:

$$\int_{18}^{26} \frac{1}{8}\, dx = \frac{1}{8} \times (26 - 18) = 1$$

The probability that X lies between 20 and 22 is:

$$P(20 \le X \le 22) = \int_{20}^{22} \frac{1}{8}\, dx = \frac{1}{8} \times (22 - 20) = \frac{2}{8} = 0.25$$