Importance Sampling with Likelihood Weighting

1 Procedure for Importance Sampling

1. Start by setting the values of the evidence variables.
2. Each variable Z_i is sampled, conditioned on its parents in the Bayesian network.
3. The weight $W(Z)$ for each sample is calculated as:

$$W(Z) = \frac{P(Z \mid e)}{Q_{\text{ws}}(Z)}$$

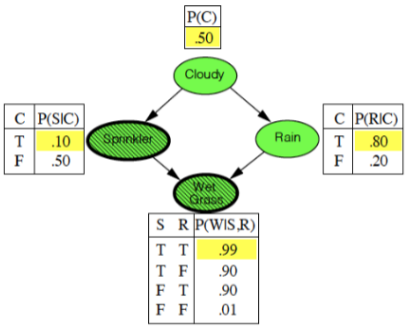
   where:

   - $P(Z \mid e)$ is the true probability of the event given the evidence.
   - $Q_{\text{ws}}(Z)$ is the proposal distribution based on likelihood weighting.

4. Multiply the weight by a normalizing factor α such that the weights sum to 1. This ensures the weights correctly reflect probabilities.

2 Example: Bayesian Network Query

Consider the query $P(\text{Rain} \mid \text{Cloudy} = \text{true}, \text{WetGrass} = \text{true})$ in a Bayesian network. The nodes are in topological order: Cloudy, Sprinkler, Rain, WetGrass.



2.1 Steps:

1. Set the initial weight $W = 1.0$.
2. Generate a sample event by setting the evidence variables and updating the weights as follows:
   - Sample Cloudy: Evidence variable Cloudy = true.
   - Update the weight: $W = W \times P(\text{Cloudy} = \text{true}) = 1.0 \times 0.5 = 0.5$.
   - Sample Sprinkler given Cloudy = true: Sprinkler = false.
   - Sample Rain given Cloudy = true: Rain = true.
3. Now consider the evidence variable WetGrass = true:
   - Update the weight again:

     $$W = W \times P(\text{WetGrass} = \text{true} \mid \text{Sprinkler} = \text{false}, \text{Rain} = \text{true}) = 0.5 \times 0.9 = 0.45$$

2.2 Final Result:

The final event sampled is (Cloudy = true, Sprinkler = false, Rain = true, WetGrass = true), and the associated weight is 0.45.