Building a Bayesian Network

1. **Chain Rule for Joint Distributions**

   Set of Random Variables $\{X_1, X_2, \ldots, X_n\}$

   $$P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | X_{i-1}, X_{i-2}, \ldots, X_1)$$

2. **Conditional Independence**

   In a Bayesian network, each variable X_i is conditionally independent of its non-descendants, given its parents.

   $$P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | \text{Parents}(X_i))$$

3. **Construct the network**

   (a) **Nodes**
   Order the set of variables of the domain in a way that causes precede effects. $\{X_1, X_2, \ldots, X_n\}$

   (b) Choose a minimal set of parents $C_i \subset \{X_1, \ldots, X_{i-1}\}$ for each variable X_i.

3. **Insert links for each parent of X_i**

4. **Conditional Probability Tables (CPT)**

   $$P(X_i | \text{Parents}(X_i))$$

   For each variable X_i.

   Example: Alarm Network

   - B: Burglary
   - E: Earthquake
   - A: Alarm
   - J: JohnCalls
   - M: MaryCalls

   Step 1: Node ordering

   Possible Order $\{B, E, A, J, M\}$

   Step 2: Links

   For A: Parents are B and E

$$B \to A \quad \text{and} \quad E \to A$$

For J:

$$A \to J$$

For M:

$$A \to M$$

$$\begin{tikzcd}
B \arrow[dr] & & E \arrow[dl] \\
& A \arrow[dl] \arrow[dr] & \\
M & & J
\end{tikzcd}$$

Step 3: CPT

$$P(B) : \text{Prob. of burglary}$$
$$P(E) : \text{Prob. of Earthquake}$$
$$P(A|B, E) : \text{Prob. of alarm given burglary and earthquake}$$
$$P(J|A) \quad \text{and} \quad P(M|A)$$

Joint Dist.

$$P(B, E, A, J, M) = P(B) \cdot P(E) \cdot P(A|B, E) \cdot P(J|A) \cdot P(M|A)$$

$$P(M|J, A, E, B) = P(M|A)$$