Introduction

The Wumpus World is a classic artificial intelligence problem that involves navigating a grid-based world populated with pits, a Wumpus (a dangerous monster), and gold. The agent's goal is to find the gold while avoiding the Wumpus and the pits. The world is partially observable, meaning the agent can only sense its immediate surroundings, such as detecting breezes (which indicate nearby pits) or stench (which indicates the proximity of the Wumpus).

In this document, we will explore the probabilistic reasoning involved in determining the presence of pits based on the observed breezes. Specifically, we will use concepts such as joint distributions, conditional probabilities, and conditional independence to infer the likelihood of pits being present in certain cells of the grid.

Wumpus World: Joint Distribution and Breeziness

In the Wumpus World, we define the following variables:

- $P_{i,j}$: True if there is a pit in cell (i, j).
- $B_{i,j}$: True if cell (i, j) is breezy.

Joint Distribution

The joint probability distribution for the presence of pits in all cells is given by:

$$P(P_{1,1}, P_{1,2}, \dots, P_{4,4}) = \prod_{i=1}^{4}\prod_{j=1}^{4} P(P_{i,j})$$

where n is the number of pits, and:

$$P(P_{i,j}) = 0.2 \quad \text{and} \quad P(\neg P_{i,j}) = 0.8$$

Conditional Probability

The conditional probability that certain cells are breezy, given the presence or absence of pits, is:

$$P(B_{1,1}, B_{1,2}, B_{2,1} \mid P_{1,1}, P_{1,2}, \dots, P_{4,4}) = P(B_{1,1} \mid P_{1,1}, P_{1,2}, P_{2,1}) \times \dots$$

Given:

$$B_{1,1} = \text{false}$$
$$B_{1,2} = \text{true}$$

$$B_{2,1} = \text{true}$$

And the fact that there are no pits in $(1,1)$, $(1,2)$, and $(2,1)$:

$$\neg P_{1,1} \wedge \neg P_{1,2} \wedge \neg P_{2,1}$$

Wumpus World: Conditional Probability Query

The query we want to solve is:

$$P(P_{1,3} \mid \neg P_{1,1}, \neg P_{1,2}, \neg P_{2,1}, \neg P_{2,2}, B_{1,1}, B_{1,2}, B_{2,1})$$

Known and Unknown Variables

- **Known:** $\neg P_{1,1}$, $\neg P_{1,2}$, $\neg P_{2,1}$
- **Unknown:** All other cells except $P_{1,3}$

Fringe: Neighboring cells that can influence breezes directly (given pits).
We assume:

$$P(B_{1,1}, B_{1,2}, B_{2,1} \mid P_{1,3}, \text{known}, \text{Unknown}) = P(B_{1,1}, B_{1,2}, B_{2,1} \mid P_{1,3}, \text{known}, \text{Fringe})$$

Here, "fringe" refers to the adjacent cells.

$$P(P_{1,3} \mid \text{known}, B_{1,1}, B_{1,2}, B_{2,1}) \propto \sum_{\text{Unknown}} P(P_{1,3}, \text{Unknown} \mid \text{known}, B_{1,1}, B_{1,2}, B_{2,1})$$

Using Conditional Independence

The probability of a pit being in cell $P_{1,3}$ given the unknown cells and the observed breezes is calculated using conditional independence:

$$P(P_{1,3} \mid \text{Unknown}, B_{1,1}, B_{1,2}, B_{2,1}) \propto P(P_{1,3}) \sum_{\text{Unknown}} P(\text{Unknown} \mid P_{1,3}, \text{known})$$

$$\times P(B_{1,1}, B_{1,2}, B_{2,1} \mid P_{1,3}, \text{known}, \text{Fringe})$$

We also need to calculate:

$$P(P_{2,2} \mid \text{known}, B_{1,1}, B_{1,2}, B_{2,1})$$

Considering the implications of the breezes:

- $\neg P_{1,1} \rightarrow \neg P_{1,2} \wedge \neg P_{2,1}$

- $B_{1,1} \to \neg P_{1,2} \land P_{2,1}$
- $B_{1,2} \to P_{1,3} \lor P_{2,2}$
- $B_{2,1} \to P_{1,2} \lor P_{2,2} \lor P_{3,1}$