Milestone Withdrawal Management

Queueing System Data Analysis

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I. Introduction

Milestone Recovery is a 501(C)(3) nonprofit organization providing services to individuals with substance use and behavioral health disorders in southern Maine. The detoxification program offers Maine residents a medically supervised transition through the acute stages of withdrawal from addictive substances. The detox facility currently has a maximum capacity of thirteen beds for men and three beds for women and operates with a first come first serve (FIFO) queueing discipline. That is, if at least one bed is open in the gender-specific detox center, then the client enters the system. Otherwise, the system is full, and the client must reapply at another time. In managing a queueing system, it is desirable to determine an appropriate number of servers for the system. We model the Milestone Recovery detox queueing system as two parallel-server birth-death queues with a limit on the number of entities allowed in the system at any one time. These queues are assumed to operate independently. Priority of pregnant women in the woman's detox center was not considered. We assess current performance then explore the impact of adding additional beds to each system using Erlang's loss formula.

II. Background

Each year, an average of 152 men and 120 women seeking treatment for drug addiction at the Milestone facility are turned away due to a lack of available bed space. With the current system, approximately 41 percent of females and 11 percent of males looking to enter the system are turned away. Milestone Recovery is currently able to serve an average of 94 males and 34 females per month, turning away an approximate average of 13 females and 10 males per month.

III. Methods

III.I. Acquisition of Data

Admittance data recorded over the years 2016-2019 was provided by Milestone Recovery. This data supplies information on the date and time of arrival and departure, gender, and a categorical variable for whether or not the client was accepted into the system for each entity. This data was reformatted using regular expressions and roughly 50 entries with missing values or errors were not considered in the analysis.

First, we use the *POSIXct* function in R to manipulate the calendar dates and times of entities which were accepted into the system. For each entry, we use the arrival date and time to find the arrival time in days–assuming that the earliest entry arrives at day zero. We note the importance of the distribution of interarrival times now, which is defined as the time between each arrival, but will go into further detail when discussing the model. Similar to the arrival time, we find the departure time in days for each entry. Next, we define service time in days in the system as departure time minus arrival time. Note that this new variable, service time, describes the continuous distribution of the time in days it takes a client to move through the detox system. This distribution has an average which is currently 3.578 days, resulting in a mean service rate, denoted by μ, of on average 0.279 clients serviced per day. This value is assumed to be the same for both queueing systems.

Finally we find the discrete distribution for the number of arrivals for both the female and male detox facilities. Using the group by function in *TidyVerse*, we find how many females and males arrived to the facility per month. These are entities that were either accepted into the system or were turned away due to the system being full only. Once again, these distributions have averages, denoted by λ, which is known as the average arrival rate to the system. After an approximate conversion from average arrivals per month to average arrivals per day, we found that an average of 1.006 females and 3.036 males arrive to the facility per day.

III.II. Queueing Model and Validation

In an M/G/c/c queueing system, while the service time may have a general distribution, it is necessary that arrivals have a Poisson distribution. According to statistical theory, if the distribution of interarrival times have an exponential distribution then we may assume that arrivals have a Poisson distribution. To do this, we first must only consider time periods when the facility is open and entities are allowed to arrive. Additionally, we must reduce the time unit so that at most there is one arrival per unit. We found that the mean interarrival time is 139 minutes. Using this, we may perform the Kolmogorov-Smirnov test. This is a nonparametric test of the equality of continuous probability distributions. The test showed that the distribution of interarrival times is not statistically different from an exponential distribution.

To further validate the choice to model this system as an M/G/c/c queueing system, we compare the actual and theoretical effective arrival rates. The effective arrival rate, denoted λ_{eff}, is defined as the average arrival rate of entities which are admitted into the system. The actual effective arrival rate is found similarly to the average arrival rate discussed above, except that we limit the data to only those entities which were admitted. The theoretical effective arrival rate is defined as the estimated arrival rate times the probability that an entity will be admitted upon arrival. The probability that an entity will be admitted upon arrival is equal to one minus the probability that an entity will be blocked from the system upon arrival–otherwise known as the blocking probability calculated by Erlang's loss formula. We found that the actual effective arrival rate for males was 2.7 per day and the theoretical was 2.69 per day. Moreover, we found that the actual effective arrival rate for females was 0.6514 per day and the theoretical was 0.5911 per day. We note that it is likely the average service rate is different for the female detox facility and the data is skewed towards men. However, since the actual and theoretical values are close, we may assume that this model is a good fit and the service time for females can be easily considered in the future.

IV. Results and Conclusion

Increasing the number of beds in the detox center by three for men and four for women, would have a significant impact on the population. The number of individuals requesting and not receiving treatment could be reduced to less than 5 percent. Alternatively, increasing the number of beds in the detox center by six for men and seven for women means that the number of individuals requesting and not receiving treatment could be reduced to less than 1 percent.

V. References

[1] Shortle, J. F. (2018). Fundamentals of queueing theory. Hoboken: Wiley.