# QinR: Queueing Models in R

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STA 564 Queueing Networks
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## How To Install

- To install a package from Github for the first time, first install the devtools package. This can be downloaded from the CRAN.

```
install.packages("devtools", repos = "http://cran.us.r-project.org")
```

- Load the devtools package.

```
library(devtools)
```

- Download the package from GitHub.

```
install_github("angyalkavalcsics/QueueingNetworks/QinR")
```

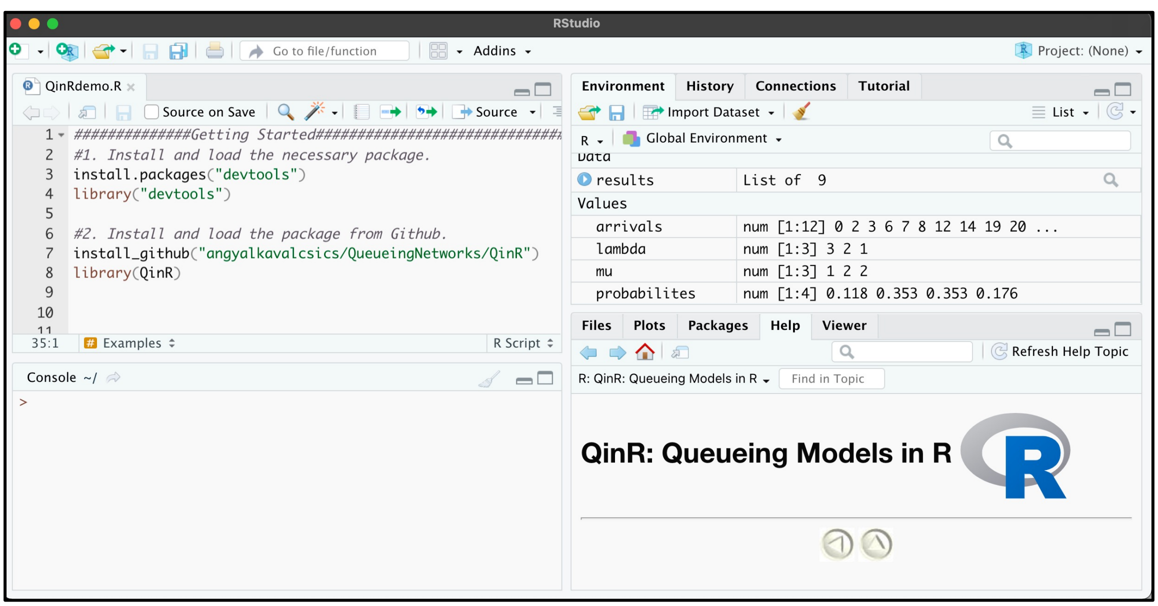
- Load the package to use the functions.

```
library (QinR)
```

- Help files can be accessed using “?function name” and contain examples of how to use the functions.

```
?mmc.summary
```

mmc.summary R Documentation



# Example 1: G/G/1 Queue

**1.17.** Table 1.7 gives observations regarding customers at a single-server FCFS queue.

**(a)** Compute the average time in the queue and the average time in the system.

**(b)** Calculate the average system waiting time of those customers who had to wait for service (i.e., exclude those who were immediately taken into service). Calculate the average length of the queue, the average number in the system, and the fraction of idle time of the server.

# Example 1: Textbook Solution

**1.17** We use the Delay Analysis for Sample Single-Server Queue model in the Basic Model category in QtsPlus:

**DELAY ANALYSIS FOR SAMPLE SINGLE-SERVER QUEUE**

This is a basic line waiting-time analysis for a sample G/G/1 queue constructed from an input sequence of interarrival and service times.

**Output:**

| | |
|---|---|
| Number of Observations | 20 |
| Total time horizon | 147 |
| Mean interarrival time | 7.35 |
| Arrival rate ($\lambda$) | 0.136054422 |
| Mean service time | 6.2 |
| Service rate ($\mu$) | 0.161290323 |
| Empirical traffic intensity ($\rho$) | 84.35% |
| Average line delay (Wq) | 3.95 |
| Average system wait (W) | 10.15 |

Clear Old Data

Put data below into two columns of equal length.
Enter data and then press "Solve" button.

Solve

| Customer n | Line Delays Wq(n) | System Waits W(n) | Service Time S(n) | Inter-arrival Time T(n) |
|---|---|---|---|---|
| 0 | *N/A* | *N/A* | *N/A* | 1. |
| 1 | 0.0 | 3.0 | 3. | 9. |
| 2 | 0.0 | 7.0 | 7. | 6. |
| 3 | 1.0 | 10.0 | 9. | 4. |
| 4 | 6.0 | 15.0 | 9. | 7. |
| 5 | 8.0 | 18.0 | 10. | 9. |
| 6 | 9.0 | 13.0 | 4. | 5. |
| 7 | 8.0 | 16.0 | 8. | 8. |
| 8 | 8.0 | 13.0 | 5. | 4. |
| 9 | 9.0 | 14.0 | 5. | 10. |
| 10 | 4.0 | 7.0 | 3. | 6. |
| 11 | 1.0 | 7.0 | 6. | 12. |
| 12 | 0.0 | 3.0 | 3. | 6. |
| 13 | 0.0 | 5.0 | 5. | 8. |
| 14 | 0.0 | 4.0 | 4. | 9. |
| 15 | 0.0 | 9.0 | 9. | 5. |
| 16 | 4.0 | 13.0 | 9. | 7. |
| 17 | 6.0 | 14.0 | 8. | 8. |
| 18 | 6.0 | 12.0 | 6. | 8. |
| 19 | 4.0 | 12.0 | 8. | 7. |
| 20 | 5.0 | 8.0 | 3. | |

# Example 2: Birth-Death Process

**3.1.** You are told that a small single-server, birth–death-type queue with finite capacity cannot hold more than three customers. The three arrival or birth rates are $(\lambda_0, \lambda_1, \lambda_2) = (3, 2, 1)$, while the service or death rates are $(\mu_1, \mu_2, \mu_3) = (1, 2, 2)$. Find the steady-state probabilities $\{p_i, i = 0, 1, 2, 3\}$

# Example 2: Textbook Solution

The QtsPlus solution:

**FINITE BIRTH-DEATH PROCESS**

To start a new problem, enter number of states and press button to setup new problem.

**Number of Rates:** 3

Enter birth and death rates below, then press "Solve" button.

Solve

| | Solution |
|---|---|
| **Mean** | 1.588235 |
| **lam_eff** | 1.411765 |

| State | $p_n$ |
|---|---|
| 0 | 0.117647 |
| 1 | 0.352941 |
| 2 | 0.352941 |
| 3 | 0.176471 |

| Birth Rates | Death Rates |
|---|---|
| 3. | ***N/A*** |
| 2. | 1. |
| 1. | 2. |
| ***N/A*** | 2. |

# Example 3: M/M/1 and M/M/C

**3.27.** A small branch bank has two tellers, one for receipts and one for withdrawals. Customers arrive to each teller's cage according to a Poisson distribution with a mean of 20/h. (The total mean arrival rate at the bank is 40/h.) The service time of each teller is exponential with a mean of 2 min. The bank manager is considering changing the setup to allow each teller to handle both withdrawals and deposits to avoid the situations that arise from time to time when the queue is sizable in front of one teller while the other is idle. However, since the tellers would have to handle both receipts and withdrawals, their efficiency would decrease to a mean service time of 2.4 min. Compare the present system with the proposed system with respect to the total expected number of people in the bank, the expected time a customer would have to spend in the bank, the probability of a customer having to wait more than 5 min, and the average idle time of the tellers.

# Example 3: Textbook Solution

**3.27** **(a)** The current organization is two $M/M/1$ queues. Using QtsPlus $M/M/1$ module, we have

**M/M/1: POISSON ARRIVALS TO A SINGLE EXPONENTIAL SERVER**

| | |
|---|---|
| **Input Parameters:** | |
| Arrival rate ($\lambda$) | 0.333333 |
| Mean service time ($1/\mu$) | 2. |
| **Plot Parameters:** | |
| Maximum size for probability chart | 20 |
| Total time horizon for probability plotting | 6. |
| **Results:** | |
| Mean interarrival time ($1/\lambda$) | 3 |
| Service rate ($\mu$) | 0.5 |
| Server utilization ($\rho$) | 66.67% |
| Mean number of customers in the system (L) | 2 |
| Mean number of customers in the queue (Lq) | 1.333333333 |
| Expected non-empty queue size (Lq') | 3 |
| Mean waiting time (W) | 6 |
| Mean waiting time in the queue (Wq) | 4 |
| Mean length of busy period (B) | 6 |

**Customer Size Distribution**

| Size | prob(n) | CDF(n) |
|---|---|---|
| 0 | 0.333333 | 0.333333 |
| 1 | 0.222222 | 0.555556 |
| 2 | 0.148148 | 0.703704 |
| 3 | 0.098765 | 0.802469 |
| 4 | 0.065844 | 0.868313 |
| 5 | 0.043896 | 0.912209 |

**Waiting Time Distributions**

| t | W(t) | Wq(t) |
|---|---|---|
| 0.00 | 0.000000 | 0.333333 |
| 0.06 | 0.009950 | 0.339967 |
| 0.12 | 0.019801 | 0.346534 |
| 0.18 | 0.029554 | 0.353036 |
| 0.24 | 0.039211 | 0.359474 |
| 0.30 | 0.048771 | 0.365847 |
| . | . | . |
| 4.98 | 0.563951 | 0.709300 |
| 5.04 | 0.568289 | 0.712193 |
| 5.10 | 0.572585 | 0.715057 |

$L = 2$ for one teller, so $L = 4$ for entire system. Idle time $= 1 - \rho = 33.33\%$. From the plot data chart of the QtsPlus model run, we see that $\Pr\{\text{wait} > 5 \text{ min}\} = .29$.

**(b)** The proposed system is $M/M/2$. From the QtsPlus $M/M/2$ module, we get:

**M/M/c: POISSON ARRIVALS TO MULTIPLE EXPONENTIAL SERVERS**

| | |
|---|---|
| **Input Parameters:** | |
| Arrival rate ($\lambda$) | 0.666667 |
| Mean service time ($1/\mu$) | 2.4 |
| Number of servers in the system (c) | 2 |
| **Plot Parameters:** | |
| Maximum size for probability chart | 15 |
| Total time horizon for probability plotting | 6. |
| **Results:** | |
| Mean interarrival time ($1/\lambda$) | 1.5 |
| Service rate ($\mu$) | 0.416667 |
| Average # arrivals in mean service time (r) | 1.6 |
| Server utilization ($\rho$) | 80.00% |
| Fraction of time all servers are idle ($p_0$) | 0.111111 |
| Mean number of customers in the system (L) | 4.444444 |
| Mean number of customers in the queue (Lq) | 2.844444 |
| Mean wait time (W) | 6.666667 |
| Mean wait time in the queue (Wq) | 4.266667 |

**Customer Size Distribution**

| n | prob(n) | CDF(n) |
|---|---|---|
| 0 | 0.111111 | 0.111111 |
| 1 | 0.177778 | 0.288889 |
| 2 | 0.142222 | 0.431111 |
| 3 | 0.113778 | 0.544889 |
| 4 | 0.091022 | 0.635911 |
| 5 | 0.072818 | 0.708729 |

**Waiting Time Distributions**

| t | Wq(t) |
|---|---|
| 0.00 | 0.288889 |
| 0.06 | 0.295965 |
| 0.12 | 0.302970 |
| 0.18 | 0.309905 |
| 0.24 | 0.316772 |
| 0.30 | 0.323570 |
| . | . |
| 4.92 | 0.686804 |
| 4.98 | 0.689921 |
| 5.04 | 0.693006 |

and we see that $L = 4.4$, idle time $= p_0 + .5p_1 = .2$ and $\Pr\{\text{wait} > 5\} = 1 - W_q(5) = 1 - .69 = .31$.

# Example 4: M/M/1/N Queues

**Example.** An OR analyst believes he can model the word processing and electronic mail activities in his executive office as an $M/M/1//4$ queueing system. He generates so many letters, memos, and email messages each day that four secretaries ceaselessly type away at their workstations that are connected to a large computer system over an $LAN$. Each secretary works on average, for 40 seconds before she makes a request for service to the computer system. A request for service is processed in one second on average (processing times being exponentially distributed). The analyst has measured the mean response time using his electronic watch (he gets 1.05 seconds) and estimates the throughput as 350 requests per hour. The OR analyst has decided to hire two additional secretaries to keep up with his productivity and will connect their workstations to the same $LAN$ if the $M/M/1//6$ model indicates a mean response time of less than 1.5 seconds. Should he hire the two secretaries?

# Example 4: Course Notes Solution

**Solution.**

First examine the current system. Here $1/\lambda = 40$ sec/job, $1/\mu = 1$ sec/job.

With $N = 4$,

$$p_0 = [\sum_{n=0}^{N} \frac{N!a^n}{(N-n)!}]^{-1} = 0.90253$$

$$\begin{aligned} L &:= N - \frac{\mu}{\lambda}(1 - p_0) = 4 - 40(1 - .90253) = .1012 \\ \lambda' &= \lambda(N - L) = 0.09747 \text{ jobs/sec} = 350.88 \text{ jobs/hr} \\ W &= \frac{L}{\lambda'} = 1.038 \end{aligned}$$

With $N = 6$, $p_0 = 0.85390$, $\lambda' = 525.95$ requests/hr and $W = 1.069$ seconds.

The OR analyst can add two workstations without seriously degrading the performance of his office staff.

**Exercise.** Verify all calculations.

# Example 5: Open-Jackson Networks

**5.10.** Consider a seven-node, open single-server Jackson network, where only nodes 2 and 4 get input from the outside (at rate 5/min). Nodes 1 and 2 have service rates of 85, nodes 3 and 4 have rates of 120, node 5 has a rate of 70, and nodes 6 and 7 have rates of 20 (all in units per minute). The routing matrix is given by

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{6} & 0 \\ \frac{1}{3} & \frac{1}{4} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{5} & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} \end{bmatrix}.$$

Find the average system size and average wait at each node (time in queue plus time in service).

# Example 5: Textbook Solution

**5.10** Using the QtsPlus open Jackson network module, we get:

**OPEN JACKSON QUEUEING NETWORK**

To setup new problem, enter number of nodes in queueing network.

**Number of Nodes:** 7

Enter maxium size for marginal probabilities.
**Max Marginal Prob:** 10

Press "Solve" button to compute Jackson performance metrics. **Solve**

| $\gamma$ | $\mu$ | Servers | Node | Routing Table | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 85. | 1. | 1 | 0.333333 | 0.25 | 0. | 0.25 | 0. | 0.166667 | 0. |
| 5. | 85. | 1. | 2 | 0.333333 | 0.25 | 0. | 0.333333 | 0. | 0. | 0. |
| 0. | 120. | 1. | 3 | 0. | 0. | 0.333333 | 0.333333 | 0.333333 | 0. | 0. |
| 5. | 120. | 1. | 4 | 0.333333 | 0. | 0.333333 | 0. | 0.333333 | 0. | 0. |
| 0. | 70. | 1. | 5 | 0. | 0. | 0. | 0.8 | 0. | 0. | 0.166667 |
| 0. | 20. | 1. | 6 | 0.166667 | 0. | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0. |
| 0. | 20. | 1. | 7 | 0. | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0. | 0.166667 |

**System Performance Measures**

| | |
|---|---|
| Total number in the network (L) | 60.940413 |
| Total sojourn time through the network (W) | 6.0940413 |

**Node Performance Measures**

| **Node** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\gamma$ | 0 | 5 | 0 | 5 | 0 | 0 | 0 |
| $\mu$ | 85 | 85 | 120 | 120 | 70 | 20 | 20 |
| **Servers** | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\lambda$ | 79.764706 | 36.141176 | 64.941176 | 115.41176 | 64.941176 | 15.952941 | 12.988235 |
| $\rho$ | 93.84% | 42.52% | 54.12% | 96.18% | 92.77% | 79.76% | 64.94% |
| **L** | 15.235955 | 0.7397062 | 1.1794872 | 25.153846 | 12.837209 | 3.9418605 | 1.852349 |
| **Lq** | 14.297547 | 0.3145159 | 0.6383107 | 24.192081 | 11.909478 | 3.1442134 | 1.2029372 |
| **W** | 0.1910112 | 0.0204671 | 0.0181624 | 0.2179487 | 0.1976744 | 0.247093 | 0.1426174 |
| **Wq** | 0.1792465 | 0.0087024 | 0.0098291 | 0.2096154 | 0.1833887 | 0.197093 | 0.0926174 |

# Example 6: Closed-Jackson Network

**5.23.** Find the average number of customers at each node and the node delay time for a closed network with 35 customers circulating between seven nodes using the switch matrix given by

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{6} & 0 \\ \frac{1}{3} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

and assuming the same service rates as in Problem 5.10.

# Example 6: Textbook Solution

**5.23** Using the QtsPlus closed-network Buzen algorithm, we have the following:

**CLOSED, MULTI-SERVER JACKSON NETS USING BUZEN'S PROCEDURE**

To setup new problem, enter number of nodes in queueing network.
**Number of Nodes:** 7

Enter number of customers
**Number of Customers:** 35

Press "Solve" button to compute Jackson performance metrics. **Solve**

| μ | Servers | Node | Routing Table | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 85. | 1. | 1 | 0.333333 | 0.25 | 0. | 0.25 | 0. | 0.166667 | 0. |
| 85. | 1. | 2 | 0.333333 | 0.25 | 0. | 0.25 | 0.166667 | 0. | 0. |
| 120. | 1. | 3 | 0. | 0. | 0.333333 | 0.333333 | 0.333333 | 0. | 0. |
| 120. | 1. | 4 | 0.333333 | 0. | 0.333333 | 0. | 0.333333 | 0. | 0. |
| 70. | 1. | 5 | 0. | 0. | 0. | 0.833333 | 0. | 0. | 0.166667 |
| 20. | 1. | 6 | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0. |
| 20. | 1. | 7 | 0. | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0.166667 |

**Results**

**Node Performance Measures**

| **Node** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **μ** | 85 | 85 | 120 | 120 | 70 | 20 | 20 |
| **Servers** | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| λ | 74.321135 | 31.656879 | 61.848951 | 108.21078 | 67.125097 | 17.549231 | 13.425019 |
| ρ | 0.8743663 | 0.3724339 | 0.5154079 | 0.9017565 | 0.95893 | 0.8774615 | 0.671251 |
| **L** | 6.0002942 | 0.5919877 | 1.0573237 | 7.3263849 | 11.885878 | 6.1316778 | 2.0064534 |
| **Lq** | 5.1259279 | 0.2195539 | 0.5419157 | 6.4246285 | 10.926948 | 5.2542163 | 1.3352025 |
| **W** | 0.0807347 | 0.0187001 | 0.0170953 | 0.0677048 | 0.1770706 | 0.3493987 | 0.1494563 |
| **Wq** | 0.06897 | 0.0069354 | 0.0087619 | 0.0593714 | 0.1627848 | 0.2993987 | 0.0994563 |