Waves
Standing Waves
Sound Waves

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Last time

- finish up reflection and transmission
- standing waves

Warm Up Question: Standing Waves and Resonance

In the following series of resonant frequencies, one frequency (lower than 400 Hz) is missing:

150, 225, 300, 375 Hz.

(a) What is the missing frequency?

(b) What is the frequency of the seventh harmonic?

Warm Up Question: Standing Waves and Resonance

In the following series of resonant frequencies, one frequency (lower than 400 Hz) is missing:

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(a) What is the missing frequency?

(b) What is the frequency of the seventh harmonic?

(a) 75 Hz (b) 525 Hz

Overview

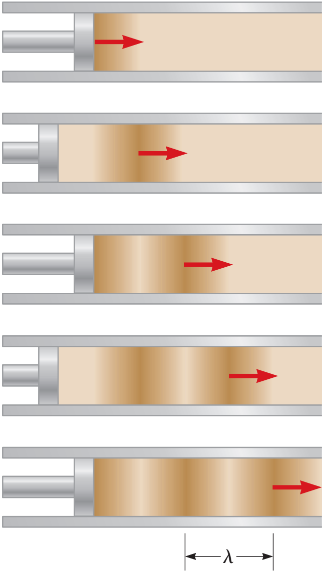
- sound
- displacement and pressure
- speed of sound
- interference and sound (?)

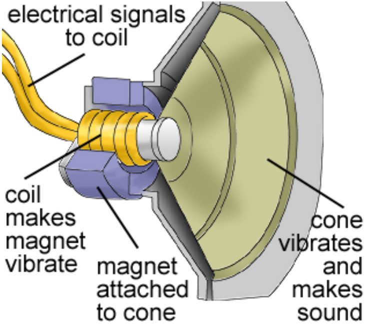
Sound Waves

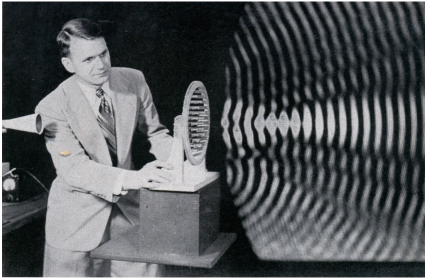
An important application of standing waves is the creation of musical instruments.

Before looking into that, we will understand sound as a longitudinal wave that causes pressure variations in air or other substances. (Ch 17.)

Pressure Variations







Sound Waves

Sound wave are longitudinal, so we imagine thin slices of air being displaced left and right along the direction of propagation of the wave (the x-axis).

This is similar to what we did to derive the wave equation considering a chain of masses connected by springs.

Now let s be the magnitude of the left-right displacement of a thin slice of air from its equilibrium position.

For a pulse wave function:

$$s(x, t) = f(x - vt)$$

For a sine-type wave function:

$$s(x, t) = s_{\text{max}} \cos(kx - \omega t)$$

(Standing) Sound Waves

Sound Waves: Displacement and Pressure Variation

We wish to relate the displacement of slices of air to the pressure variations in the air that they cause.

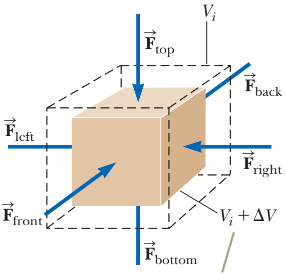
The relation between pressure and volume changes in air (at constant temperature) is characterized by the *bulk modulus*.

Bulk Modulus: Volume Elasticity

Bulk modulus, B (or sometimes K)

The ratio of the pressure change over the outside of a material to its fractional change in volume.

$$B = -\frac{\Delta P}{\Delta V / V_i}$$



The negative sign ensures B will be a positive number. Units are Pascals, Pa.

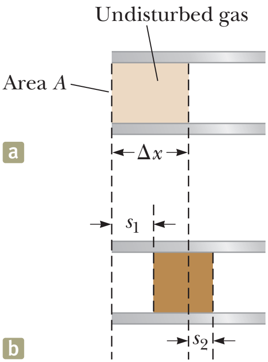
The reciprocal of the bulk modulus, $1/B$, is the **compressibility** of the material.

[1]See Serway & Jewett, Chapter 12 section 4.

Pressure and Sound

From the definition

$$\Delta P = -B\,\frac{\Delta V}{V_i}$$



For a column of air of cross sectional area A:

$$V_i = A\,\Delta x \quad , \quad \Delta V = A\,\Delta s$$

$$\Delta s = s_2 - s_1$$

Then letting $\Delta x \to 0$

$$\Delta P = -B\,\frac{\partial s}{\partial x}$$

Pressure and Sound

$$\Delta P = -B\,\frac{\partial s}{\partial x}$$

Recalling for a sine-type wave: $s(x,t) = s_{\text{max}} \cos(kx - \omega t)$,

$$\Delta P = B\, s_{\text{max}} k\, \sin(kx - \omega t)$$

Look at $B\, s_{\text{max}} k$. The units are:

$$[\text{Pa}]\ [\text{m}]\ [\text{m}^{-1}] = [\text{Pa}]$$

So, $B\, s_{\text{max}} k$ is a pressure.

Let

$$\Delta P_{\text{max}} = B\, s_{\text{max}} k$$

Pressure and Sound

We can now express sound as a pressure wave:

$$\Delta P(x, t) = (\Delta P_{\text{max}}) \sin(kx - \omega t)$$

ΔP is the variation of the pressure from the ambient (background) pressure.

If the sound wave is in air at sea level, the background pressure is $P_0 = 1.013 \times 10^5$ Pa. ΔP will be much smaller than this!

Question

Quick Quiz 17.1[1] If you blow across the top of an empty soft-drink bottle, a pulse of sound travels down through the air in the bottle. At the moment the pulse reaches the bottom of the bottle, what is the correct description of the displacement of elements of air from their equilibrium positions and the pressure of the air at this point?

(A) The displacement and pressure are both at a maximum.
(B) The displacement and pressure are both at a minimum.
(C) The displacement is zero, and the pressure is a maximum.
(D) The displacement is zero, and the pressure is a minimum.

[1] Serway & Jewett, page 510.

Question

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$\leftarrow$

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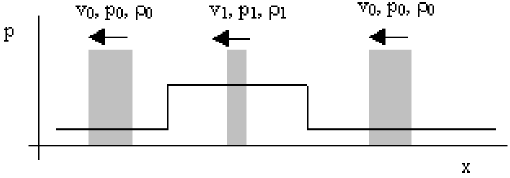
[1] Serway & Jewett, page 510.

Speed of Sound waves

As we did for waves on a string, imagine a pulse moving to the right.

Now let us choose a reference frame where we **move with the pulse** and the air moves back to the left.



call $\leftarrow$ +ve

The speed of the air outside the pulse is v. (This is the speed of sound relative to the air.)

Speed of Sound waves

Think about how the speed of the air thin packet changes as it moves into the higher-pressure pulse and is compressed.

It goes $v \rightarrow v + \Delta v$, where Δv is a *negative* number (it slows).

The relative volume change can be related to the speed change:

$$\frac{\Delta V}{V} = \frac{A \Delta v \, \Delta t}{A v \, \Delta t} = \frac{\Delta v}{v}$$

Now use Newton's 2nd law:

$$\begin{aligned}
F_{net} &= (\Delta m) a \\
PA - (P + \Delta P)A &= (\rho A \, \Delta x) \left(\frac{\Delta v}{\Delta t} \right) \\
\Delta P \cancel{A} &= -\rho \cancel{A} \, v \cancel{\Delta t} \left(\frac{\Delta v}{\cancel{\Delta t}} \right) \\
\Delta P &= -\rho v^2 \, \frac{\Delta v}{v}
\end{aligned}$$

Speed of Sound waves

Rearranging:

$$\rho v^2 = -\frac{\Delta P}{\Delta v / v}$$

Using $\frac{\Delta V}{V} = \frac{\Delta v}{v}$:

$$\rho v^2 = -\frac{\Delta P}{\Delta V / V}$$

And noticing that the LHS is the definition of B:

$$\rho v^2 = B$$

The speed of sound

$$v = \sqrt{\frac{B}{\rho}}$$

Speed of Sound waves

$$v = \sqrt{\frac{B}{\rho}}$$

Compare this expression to the speed of a pulse on a string.

$$v = \sqrt{\frac{T}{\mu}}$$

Both of these expressions can be thought of as:

$$v = \sqrt{\frac{\text{elastic quantity}}{\text{inertial quantity}}}$$

These expressions are the same in spirit, but the precise quantities are the ones that represent elasticity and inertia in each case.

Speed of Sound in Air

For air the adiabatic bulk modulus

$$B = 1.42 \times 10^5 \text{ Pa}$$

and

$$\rho = 1.2041 \text{ kg/m}^3$$

at 20°C.

This gives a speed of sound in air at 20°C of

$$\boxed{v = 343 \text{ m/s}}$$

This is approximately 1/3 km/s or 1/5 mi/s.

Speed of Sound in Air

The speed of sound in air at 20°C

$$v = 343 \text{ m/s}$$

Since the density of air varies a lot with temperature, the speed of sound varies also.

For temperatures near room temperature:

$$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{Cel}}}{273}}$$

where T_{Cel} is the temperature in **Celsius**.

Pressure Waves

$$\Delta P(x,t) = \Delta P_{\max} \sin(kx - \omega t)$$

where

$$\Delta P_{\max} = B\, s_{\max} k$$

It is easier to express the amplitude in terms of the wave speed, since it is usually easier to look up the wave speed than the bulk modulus:

$$\Delta P_{\max} = (\rho v^2)\, s_{\max} \frac{\omega}{v}$$

Then

$$\Delta P_{\max} = \rho v \omega s_{\max}$$

Sound Waves

Displacement:

$$s(x, t) = s_{\text{max}} \cos(kx - \omega t)$$

Pressure:

$$\Delta P(x, t) = \Delta P_{\text{max}} \sin(kx - \omega t)$$

where

$$\Delta P_{\text{max}} = B\, s_{\text{max}} k = \rho v \omega s_{\text{max}}$$

Summary

- sound
- displacement and pressure
- speed of sound
- interference in sound (?)

Announcements and HW

Collected Homework, due Tuesday, May 29.

Drop Deadline Friday, June 1.

3rd Test Friday, June 1.

Quiz this Friday.

No Class on Monday May 28. (Memorial day)

Homework Serway & Jewett:

- Ch 17, onward from page 523. OQs: 1, 7; CQs: 5; Probs: 1, 3, 5, 9, 13, 16, 17
- Ch 21, page 650, problem 56.