Waves
Quick Review of Oscillations and SHM
Introducing Waves
Wave Speed on a String

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Last time

- heat engines
- wrapped up thermodynamics

Overview

- oscillations and quantities
- simple harmonic motion (SHM)
- spring systems
- energy in SHM
- introducing waves
- kinds of waves

Oscillations and Periodic Motion

Many physical systems exhibit cycles of repetitive behavior.

After some time, they return to their initial configuration.

Examples:

- clocks
- rolling wheels
- a pendulum
- bobs on springs

Oscillations

oscillation

motion that repeats over a period of time

amplitude

the magnitude of the vibration; how far does the object move from its average (equilibrium) position.

period, T

the time for one complete oscillation.

After 1 period, the motion repeats itself.

Oscillations

frequency

The number of complete oscillations in some amount of time. Usually, oscillations per second.

$$f = \frac{1}{T}$$

Units of frequency: Hertz. $1 \text{ Hz} = 1 \text{ s}^{-1}$

If one oscillation takes a quarter of a second (0.25 s), then there are 4 oscillations per second. The frequency is $4 \text{ s}^{-1} = 4 \text{ Hz}$.

Oscillations

angular frequency

angular displacement per unit time in rotation, or the rate of change of the phase of a sinusoidal waveform

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Simple Harmonic Motion

The oscillations of bobs on springs and pendula are very regular and simple to describe.

It is called simple harmonic motion.

simple harmonic motion (SHM)

any motion in which the acceleration is proportional to the displacement from equilibrium, but opposite in direction

The force causing the acceleration is called the "restoring force".

SHM and Springs

If a mass is attached to a spring, the force on the mass depends on its displacement from the spring's natural length.

Hooke's Law:

$$\mathbf{F} = -k\mathbf{x}$$

where k is the spring constant and x is the displacement (position) of the mass.

Hooke's law gives the force on the bob $\Rightarrow$ SHM.

The spring force is the *restoring force*.

SHM and Springs

How can we find an equation of motion for the block?

Newton's second law:

$$\mathbf{F}_{\text{net}} = \mathbf{F}_s = m\mathbf{a}$$

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Using the definition of acceleration: $a = \frac{\mathrm{d}^2 x}{\mathrm{dt}^2}$

$$\frac{\mathrm{d}^2 x}{\mathrm{dt}^2} = -\frac{k}{m}x$$

Define

$$\omega = \sqrt{\frac{k}{m}}$$

and we can write this equation as:

$$\frac{\mathrm{d}^2 x}{\mathrm{dt}^2} = -\omega^2 x$$

SHM and Springs

To solve:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$$

notice that it is a second order linear differential equation.

We can actually find the solutions just be inspection.

A solution $x(t)$ to this equation has the property that if we take its derivative twice, we get the same form of the function back again, but with an additional factor of $-\omega^2$.

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A solution $x(t)$ to this equation has the property that if we take its derivative twice, we get the same form of the function back again, but with an additional factor of $-\omega^2$.

Candidate: $x(t) = A\cos(\omega t)$, where A is a constant.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 \left(A \;\cos(\omega t)\right) = -\omega^2 x \qquad \checkmark$$

SHM and Springs

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$$

In fact, any solutions of the form:

$$x = B_1 \cos(\omega t + \phi_1) + B_2 \sin(\omega t + \phi_2)$$

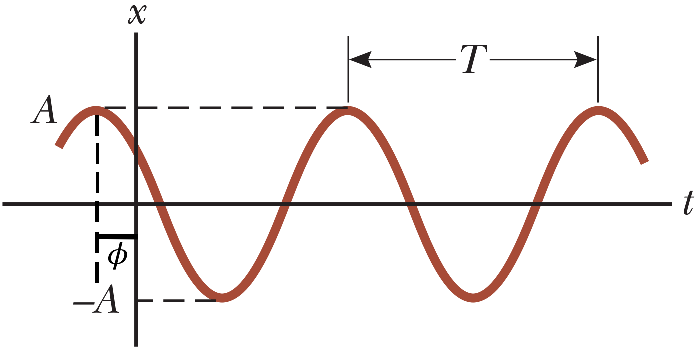
where B_1, B_2, ϕ_1, and ϕ_2 are constants.

However, since $\sin(\theta) = \cos(\theta + \pi/2)$, in general any solution can be written in the form:

$$x = A\cos(\omega t + \phi)$$

Waveform

$$x = A\cos(\omega t + \phi)$$



$$f = \frac{1}{T}$$

[1]Figure from Serway & Jewett, 9th ed, pg 453.

Oscillations and Waveforms

Any oscillation can be plotted against time. *eg.* the position of a vibrating object against time.

The result is a *waveform*.

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From this wave description of the motion, a lot of parameters can be specified.

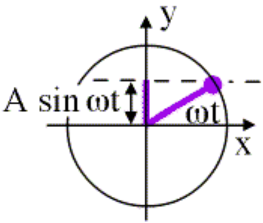
This allows us to quantitatively compare one oscillation to another.

Examples of quantities: period, amplitude, frequency.

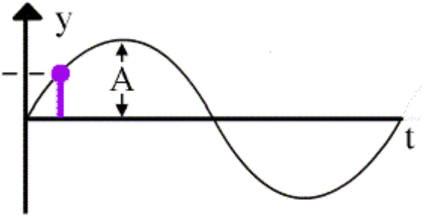
Oscillating Solutions

$$y = A\sin(\omega t)$$

$x = A\cos(\omega t)$



Phasor diagram



y plotted against t

Here, ωt is the *phase* at time t.

In general, if $y = A\sin(\omega t + \phi)$, the phase at time t is $\omega t + \phi$.

[1]Figure from School of Physics webpage, University of New South Wales.

SHM and Springs

$$x = A\cos(\omega t + \phi)$$

ω is the angular frequency of the oscillation.

When $t = \frac{2\pi}{\omega}$ the block has returned to the position it had at $t = 0$. That is one complete cycle.

SHM and Springs

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Recalling that $\omega = \sqrt{k/m}$:

$$\text{Period, } T = 2\pi\sqrt{\frac{m}{k}}$$

Only depends on the mass of the bob and the spring constant.

SHM and Springs Question

A mass-spring system has a period, T. If the amplitude of the motion is quadrupled (and everything else is unchanged), what happens to the period of the motion?

(A) halves, $T/2$

(B) remains unchanged, T

(C) doubles, $2T$

(D) quadruples, $4T$

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(B) remains unchanged, T $\leftarrow$

(C) doubles, $2T$

(D) quadruples, $4T$

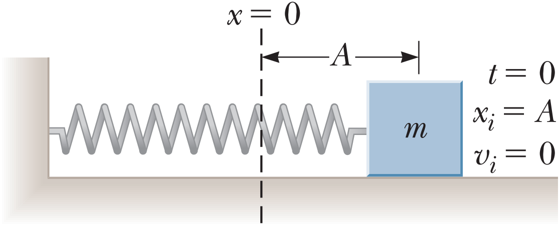
T does not depend on the amplitude.

SHM and Springs

The position of the bob at a given time is given by:

$$x = A\cos(\omega t + \phi)$$

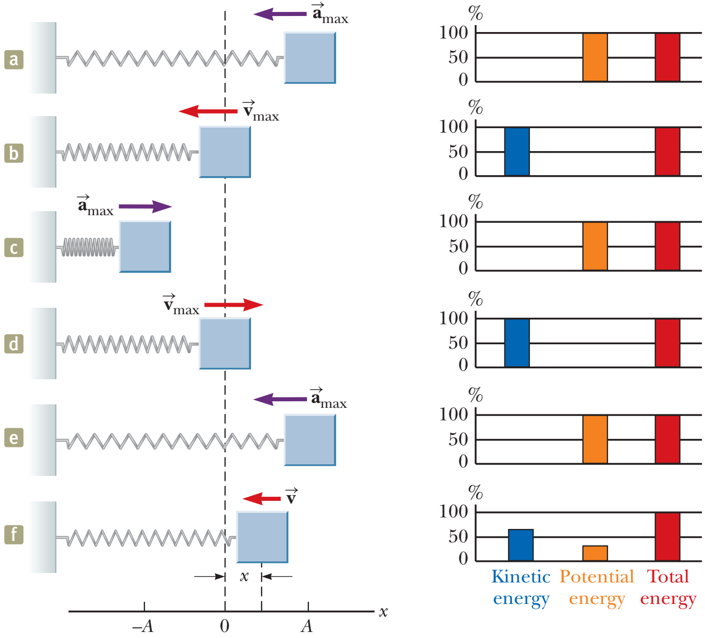
A is the amplitude of the oscillation. We could also write $x_{\text{max}} = A$.



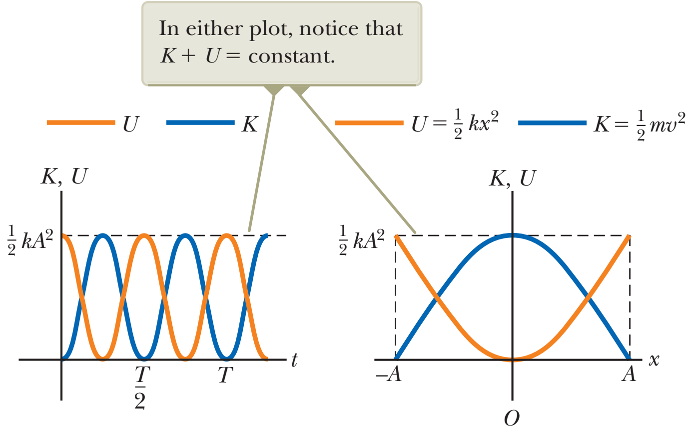
The speed of the particle at any point in time is:

$$v = \frac{\mathrm{dx}}{\mathrm{dt}} = -A\omega\sin(\omega t + \phi)$$

Energy in SHM



Energy in SHM



$$K + U = \frac{1}{2} kA^2$$

[1]Figure from Serway & Jewett, 9th ed.

Waves

Very often an oscillation or one-time disturbance can be detected far away.

Plucking one end of a stretched string will eventually result in the far end of the string vibrating.

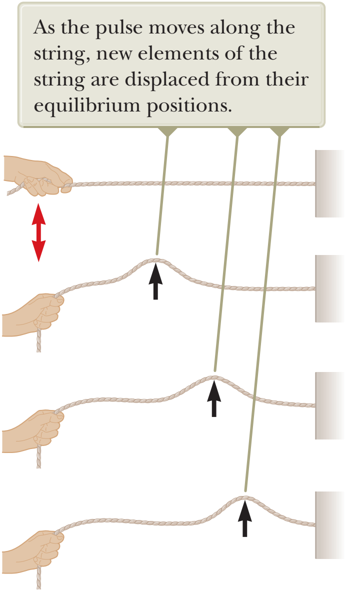
The string is a medium along which the vibration travels.

It carries energy from on part of the string to another.

Wave

a disturbance or oscillation that transfers energy through matter or space.

Wave Pulses


As the pulse moves along the string, new elements of the string are displaced from their equilibrium positions.

Wave Motion

Wave

a disturbance or oscillation that transfers energy through matter or space.

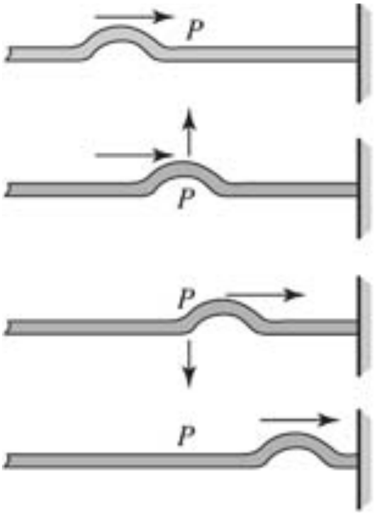
The waveform moves along the medium and energy is carried with it.

The particles in the medium *do not* move along with the wave.

The particles in the medium are briefly shifted from their equilibrium positions, and then return to them.

Wave pulses

A point P in the middle of the string moves up and down, just as the hand did.



Kinds of Waves

medium

a material substance that carries waves. The constituent particles are temporarily displaced as the wave passes, but they return to their original position.

Kinds of waves:

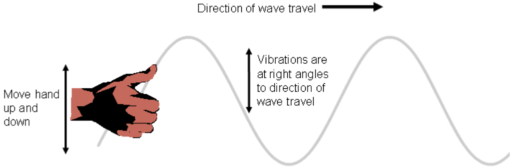
- mechanical waves – waves that travel on a medium, *eg.* sound waves, waves on string, water waves
- electromagnetic waves – light, in all its various wavelengths, *eg.* x-rays, uv, infrared, radio waves
- matter waves – wait for Phys4D!

Kinds of Waves

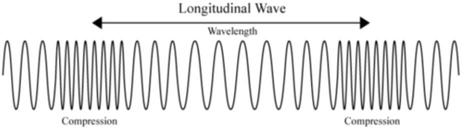
Kinds of waves:

- transverse – displacement perpendicular to direction of wave travel
- longitudinal – displacement parallel to direction of wave travel

Transverse



Longitudinal



Transverse vs. Longitudinal

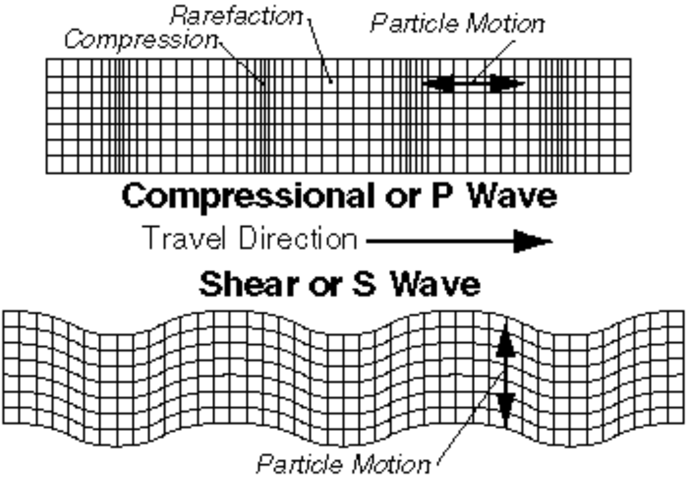
Examples of transverse waves:

- vibrations on a guitar string
- ripples in water
- light
- S-waves in an earthquake (more destructive)

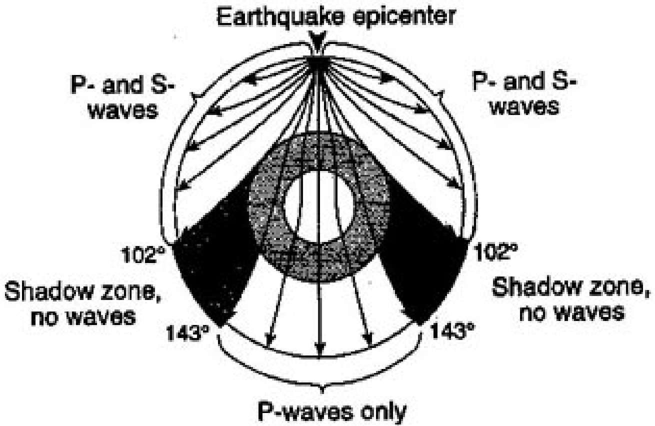
Examples of longitudinal waves:

- sound
- P-waves in an earthquake (initial shockwave, faster moving)

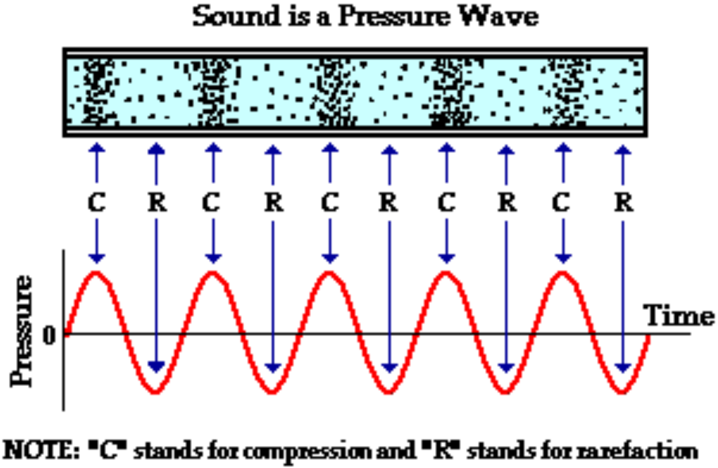
Earthquakes



Earthquakes

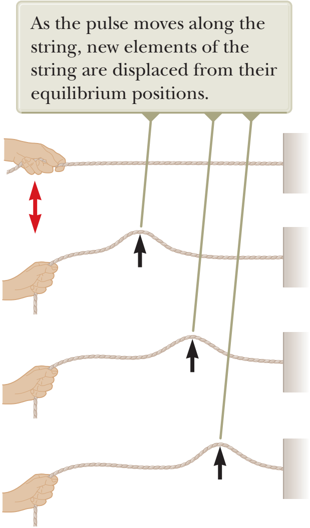


Sound waves


Sound is a Pressure Wave
C
R
C
R
C
R
C
R
C
R
Pressure
0
Time
NOTE: "C" stands for compression and "R" stands for rarefaction

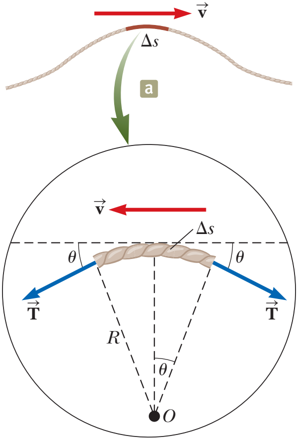
Wave Speed on a String

How fast does a disturbance propagate on a string under tension?



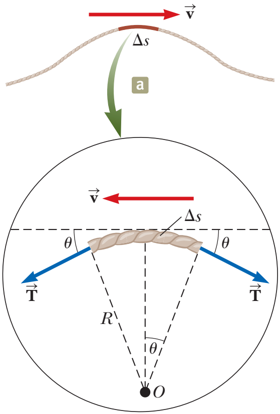
Wave Speed on a String

Imagine traveling with the pulse at speed v to the right. Each small section of the rope travels to the left along a circular arc from your point of view.



We will find find how fast a point on the string *moves backwards relative to the wave pulse*.

Wave Speed on a String



We can use the force diagram to find the force on a *small* length of string Δs:

$$F_{net} = 2T \sin\theta \approx 2T\theta \tag{1}$$

Wave Speed on a String

Consider the centripetal force on the piece of string.

If R is the radius of curvature and m is the mass of the small piece of string:

$$F_{\text{net}} = \frac{mv^2}{R}$$

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Suppose the string has mass density μ (units: kg m^{-1})

$$m = \mu\,\Delta s = \mu R(2\theta)$$

Put this into our expression for centripetal force:

$$F_{\text{net}} = \frac{2\mu R\theta v^2}{R}$$

Wave Speed on a String

Put this into our expression for centripetal force:

$$F_{\text{net}} = 2\mu\theta v^2$$

And using eq. (1), $F_{\text{net}} = 2T\theta$:

$$2T\theta = 2\mu\theta v^2$$

The wave speed is:

$$v = \sqrt{\frac{T}{\mu}}$$

For a given string under a given tension, all waves travel with the same speed!

Summary

- oscillations
- simple harmonic motion (SHM)
- spring systems
- intro to waves

Homework Serway & Jewett:

- (for review) Ch 15, onward from page 472. OQs: 13; CQs: 5, 7; Probs: 1, 3, 9, 35, 41, 86
- Ch 16, onward from page 499. CQs: 1, 9; Probs: 1