Thermodynamics
The Ideal Gas Equation

Lana Sheridan

De Anza College

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Last time

- thermal expansion

Overview

- the ideal gas equation
- moles and molecules
- applying the ideal gas equation

Ideal Gases

An ideal gas is a gas

- at low pressure
- at a temperature much higher than its condensation point
- at a low density

Ideal Gas Equation

The equation of state for an ideal gas:

$$PV = nRT$$

where

- P is pressure
- V is volume
- n is the number of moles (amount of gas)
- $R = 8.314$ J mol^{-1} K^{-1} is the universal gas constant
- T is temperature **measured in Kelvin**

The LHS and RHS of this equation both have units of Joules (energy).

Moles?

[1]Photo from http://thelazybfarm.com.

Moles?

1 mole or 1 mol. of a substance is $N_A = 6.022 \times 10^{23}$ **molecules** of that substance. (N_A is Avogadro's number.)

[1] Photo from http://thelazybfarm.com.

Moles

Why is this even a unit?

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Why is this even a unit?

Not such a long time ago scientists really did not have any idea of how much mass an individual molecule would have, or how many molecules would be present in a cubic meter of gas. Even the existence of atoms and molecules was controversial.

1 mole was a macroscopic unit they could work with.

1 mole of a substance is the amount that has the same number of molecules as there are atoms in 12 grams of a pure Carbon-12 sample. (That is $N_A = 6.022 \times 10^{23}$.)

Some History of Atoms and Molecules

Many ancient Indian, Greek, and Roman philosophers argued for a basic unit of matter: the atom.

Later thinkers in the Middle East were influenced by Greek and Indian ideas.

In Europe, philosophers were exposed to Greek and Roman ideas, with Aristotle (who thought matter was instead continous) very popular for a long time.

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Nevertheless, ideas about "corpuscles" (small particles) were important for Newton and his contemporaries.

A chemist, Robert Boyle (1627-1692) speculated that if atoms / corpuscles made up matter, that would resolve many problems arising in chemistry.

He proved correct.

Understanding from Chemical Reactions

John Dalton, a physicist and chemist in ~1803 started trying to understand the patterns of chemical reactions.

Electrolysis can dissociate water

$$\text{water} \longrightarrow \text{oxygen} + \text{hydrogen}$$

and always the same proportions are produced: twice as much hydrogen as oxygen (by volume, at equal pressure, temperature).

This lead him to suppose that

- matter was composed of atoms
- chemical substances that could not be dissociated were **elements**
- chemical substances that could be dissociated were formed from combinations of atoms

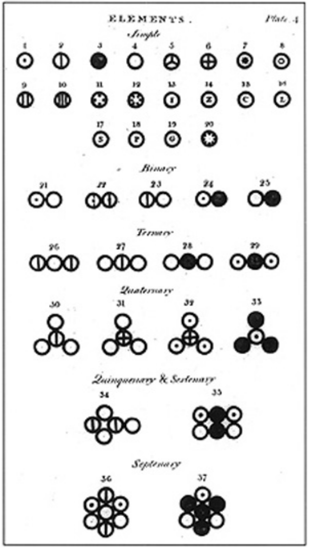
Understanding from Chemical Reactions

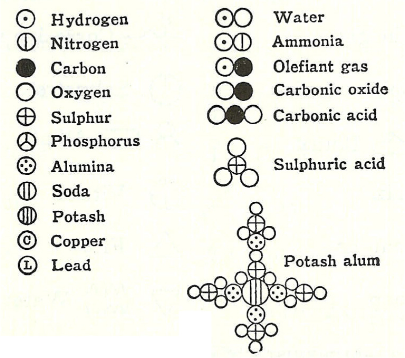
Rules of Dalton's theory[1]:

- Elements are made of extremely small particles called atoms.
- Atoms of a given element are identical in size, mass, and other properties; atoms of different elements differ in size, mass, and other properties.
- Atoms cannot be subdivided, created, or destroyed.
- Atoms of different elements combine in simple whole-number ratios to form chemical compounds.
- In chemical reactions, atoms are combined, separated, or rearranged.

[1] Wikipedia, Dalton, "A New System of Chemical Philosophy" (1808)

Understanding from Chemical Reactions





[1]Images from Dalton, "A New System of Chemical Philosophy" (1808) and Wikimedia.

Additional Evidence for atoms: Brownian Motion

In 1827 Robert Brown, a botanist, observed pollen grains suspended in water through a microscope.

He expected to see them suspended at rest, but did not.

Instead the grains of pollen seemed to jump and wiggle about for no discernible reason.

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Instead the grains of pollen seemed to jump and wiggle about for no discernible reason.

He wondered if it was something peculiar that pollen did so he tried again with dust and soot – and saw the same thing!

This motion is called **Brownian Motion**.

Brownian Motion

Brownian motion remained a mystery until 1905.

Einstein, building on tools he had just developed in his doctoral thesis, developed a theory describing Brownian motion.

It is the result of fast-moving water molecules (too small to see) colliding with the pollen molecules, and jostling them.

Periodic Table

Symbol — Ca 20 — Atomic number
Atomic mass† — 40.078
$4s^{2}$ — Electron configuration

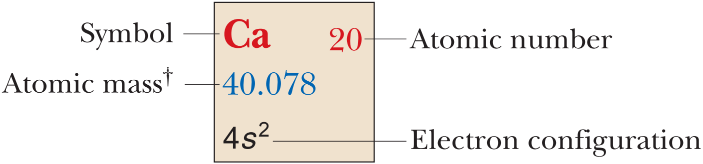
Group I	Group II	Transition elements										Group III	Group IV	Group V	Group VI	Group VII	Group 0
H 1 1.007 9 $1s$																H 1 1.007 9 $1s^{1}$	He 2 4.002 6 $1s^{2}$
Li 3 6.941 $2s^{1}$	Be 4 9.0122 $2s^{2}$											B 5 10.811 $2p^{1}$	C 6 12.011 $2p^{2}$	N 7 14.007 $2p^{3}$	O 8 15.999 $2p^{4}$	F 9 18.998 $2p^{5}$	Ne 10 20.180 $2p^{6}$
Na 11 22.990 $3s^{1}$	Mg 12 24.305 $3s^{2}$											Al 13 26.982 $3p^{1}$	Si 14 28.086 $3p^{2}$	P 15 30.974 $3p^{3}$	S 16 32.066 $3p^{4}$	Cl 17 35.453 $3p^{5}$	Ar 18 39.948 $3p^{6}$
K 19 39.098 $4s^{1}$	Ca 20 40.078 $4s^{2}$	Sc 21 44.956 $3d^{1}4s^{2}$	Ti 22 47.867 $3d^{2}4s^{2}$	V 23 50.942 $3d^{3}4s^{2}$	Cr 24 51.996 $3d^{5}4s^{1}$	Mn 25 54.938 $3d^{5}4s^{2}$	Fe 26 55.845 $3d^{6}4s^{2}$	Co 27 58.933 $3d^{7}4s^{2}$	Ni 28 58.693 $3d^{8}4s^{2}$	Cu 29 63.546 $3d^{10}4s^{1}$	Zn 30 65.41 $3d^{10}4s^{2}$	Ga 31 69.723 $4p^{1}$	Ge 32 72.64 $4p^{2}$	As 33 74.922 $4p^{3}$	Se 34 78.96 $4p^{4}$	Br 35 79.904 $4p^{5}$	Kr 36 83.80 $4p^{6}$
Rb 37 85.468 $5s^{1}$	Sr 38 87.62 $5s^{2}$	Y 39 88.906 $4d^{1}5s^{2}$	Zr 40 91.224 $4d^{2}5s^{2}$	Nb 41 92.906 $4d^{4}5s^{1}$	Mo 42 95.94 $4d^{5}5s^{1}$	Tc 43 (98) $4d^{5}5s^{2}$	Ru 44 101.07 $4d^{7}5s^{1}$	Rh 45 102.91 $4d^{8}5s^{1}$	Pd 46 106.42 $4d^{10}$	Ag 47 107.87 $4d^{10}5s^{1}$	Cd 48 112.41 $4d^{10}5s^{2}$	In 49 114.82 $5p^{1}$	Sn 50 118.71 $5p^{2}$	Sb 51 121.76 $5p^{3}$	Te 52 127.60 $5p^{4}$	I 53 126.90 $5p^{5}$	Xe 54 131.29 $5p^{6}$
Cs 55 132.91 $6s^{1}$	Ba 56 137.33 $6s^{2}$	57–71*	Hf 72 178.49 $5d^{2}6s^{2}$	Ta 73 180.95 $5d^{3}6s^{2}$	W 74 183.84 $5d^{4}6s^{2}$	Re 75 186.21 $5d^{5}6s^{2}$	Os 76 190.23 $5d^{6}6s^{2}$	Ir 77 192.2 $5d^{7}6s^{2}$	Pt 78 195.08 $5d^{9}6s^{1}$	Au 79 196.97 $5d^{10}6s^{1}$	Hg 80 200.59 $5d^{10}6s^{2}$	Tl 81 204.38 $6p^{1}$	Pb 82 207.2 $6p^{2}$	Bi 83 208.98 $6p^{3}$	Po 84 (209) $6p^{4}$	At 85 (210) $6p^{5}$	Rn 86 (222) $6p^{6}$
Fr 87 (223) $7s^{1}$	Ra 88 (226) $7s^{2}$	89–103**	Rf 104 (261) $6d^{2}7s^{2}$	Db 105 (262) $6d^{3}7s^{2}$	Sg 106 (266)	Bh 107 (264)	Hs 108 (277)	Mt 109 (268)	Ds 110 (271)	Rg 111 (272)	Cn 112 (285)	113†† (284)	Fl 114 (289)	115†† (288)	Lv 116 (293)	117†† (294)	118†† (294)

*Lanthanide series	La 57 138.91 $5d^{1}6s^{2}$	Ce 58 140.12 $5d^{1}4f^{1}6s^{2}$	Pr 59 140.91 $4f^{3}6s^{2}$	Nd 60 144.24 $4f^{4}6s^{2}$	Pm 61 (145) $4f^{5}6s^{2}$	Sm 62 150.36 $4f^{6}6s^{2}$	Eu 63 151.96 $4f^{7}6s^{2}$	Gd 64 157.25 $4f^{7}5d^{1}6s^{2}$	Tb 65 158.93 $4f^{8}5d^{1}6s^{2}$	Dy 66 162.50 $4f^{10}6s^{2}$	Ho 67 164.93 $4f^{11}6s^{2}$	Er 68 167.26 $4f^{12}6s^{2}$	Tm 69 168.93 $4f^{13}6s^{2}$	Yb 70 173.04 $4f^{14}6s^{2}$	Lu 71 174.97 $4f^{14}5d^{1}6s^{2}$
**Actinide series	Ac 89 (227) $6d^{1}7s^{2}$	Th 90 232.04 $6d^{2}7s^{2}$	Pa 91 231.04 $5f^{2}6d^{1}7s^{2}$	U 92 238.03 $5f^{3}6d^{1}7s^{2}$	Np 93 (237) $5f^{4}6d^{1}7s^{2}$	Pu 94 (244) $5f^{6}7s^{2}$	Am 95 (243) $5f^{7}7s^{2}$	Cm 96 (247) $5f^{7}6d^{1}7s^{2}$	Bk 97 (247) $5f^{8}6d^{1}7s^{2}$	Cf 98 (251) $5f^{10}7s^{2}$	Es 99 (252) $5f^{11}7s^{2}$	Fm 100 (257) $5f^{12}7s^{2}$	Md 101 (258) $5f^{13}7s^{2}$	No 102 (259) $5f^{14}7s^{2}$	Lr 103 (262) $5f^{14}6d^{1}7s^{2}$

[1]Serway & Jewett, Appendix C.

Periodic Table

The atomic number is the number of protons in the nucleus of an atom for that element.



The atomic mass number is average mass of all isotopes of an element weighted by abundance in nature.

(Isotopes of an element have different numbers of neutrons.)

Back to Moles

Now that we know the make up of various molecules in terms of their constituent atoms, we can determine the mass of each molecule.

We can then work out how many moles are in a certain mass of a substance, m.

$$n = \frac{m}{M}$$

where M is the *molar mass*, or mass of 1 mole of the substance.

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For example, water is H_2O. Hydrogen has atomic mass 1, Oxygen has atomic mass 8.

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The molar mass of water is 18 g/mol.

Question

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How many moles is that?

He: 4.00 amu $\Rightarrow M = 4.00$ g/mol

$$n = \frac{m}{M} = 1.50 \text{ mol}$$

Ideal Gas Equation

The equation of state for an ideal gas:

$$PV = nRT$$

Can also be written:

$$PV = Nk_B T$$

where

- P is pressure
- V is volume
- N is the number of molecules
- $k_B = 1.38 \times 10^{-23}$ J K^{-1} is Boltzmann's constant
- T is temperature

Question

Quick Quiz 19.6[2] On a winter day, you turn on your furnace and the temperature of the air inside your home increases. Assume your home has the normal amount of leakage between inside air and outside air. Is the number of moles of air in your room at the higher temperature

(A) larger than before,

(B) smaller than before, or

(C) the same as before?

[2] Serway & Jewett, pg 579.

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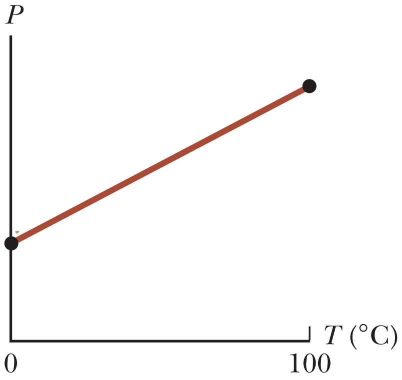
Ideal Gas Equation and the Gas Thermometer

In a constant-volume gas thermometer, the pressure varies linearly with the temperature: a consequence of the ideal gas equation!

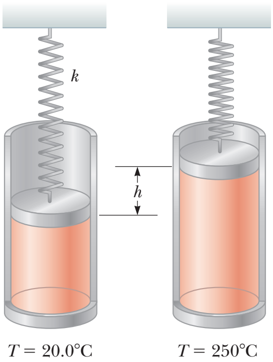


$$P = \left(\frac{nR}{V}\right) T$$

Problem #74

A cylinder is closed by a piston connected to a spring of constant 2.00×10^3 N/m. With the spring relaxed, the cylinder is filled with 5.00 L of gas at a pressure of 1.00 atm and a temperature of 20.0°C.

(a) If the piston has a cross-sectional area of 0.0100 m^2 and negligible mass, how high will it rise when the temperature is raised to 250°C?



[1]Serway & Jewett, page 588.

Problem #74

(a) Find h.

$$PV = nRT$$

We have enough info to know the number of moles, n, or we can work around that because the amount of gas does not change as it is heated.

$$\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}$$

Also, $P_f A = kh + P_0 A$

and $P_i = P_0$ and $V_f = V_i + Ah$.

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$$\begin{aligned} P_f V_f &= P_i V_i \frac{T_f}{T_i} \\ \left(\frac{kh}{A} + P_0\right)(V_i + Ah) &= P_0 V_i \frac{T_f}{T_i} \\ k h^2 + \left(\frac{kV_i}{A} + AP_0\right) h + P_0 V_i \left(1 - \frac{T_f}{T_i}\right) &= 0 \end{aligned}$$

Problem #74

(a) Find h.

Solving quadratic:

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Remember: $T_f = 250 + 273$ K, $T_i = 20 + 273$ K

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positive solution:

$$h = 0.169 \text{ m}$$

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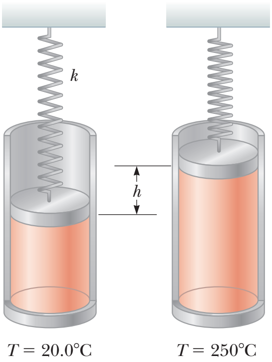
(b) What is the pressure of the gas at 250°C?



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(b) Find P_f.

We already have the expression we need:

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$$P_f = 1.35 \times 10^5 \text{ Pa}$$

Summary

- thermal expansion
- atoms
- the ideal gas equation

(Uncollected) Homework

Serway & Jewett:

- Ch 19, onward from page 581. OQs: 3, 9; CQs: 3, 10; Probs: 27, 29, 37, 39, 44, 45, 69, 77