Waves
Standing Waves and Sound

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Last time

- sound

Overview

- interference and sound
- standing waves and sound
- musical instruments

Speed of Sound waves

$$v = \sqrt{\frac{B}{\rho}}$$

Compare this expression to the speed of a pulse on a string.

$$v = \sqrt{\frac{T}{\mu}}$$

Both of these expressions can be thought of as:

$$v = \sqrt{\frac{\text{elastic quantity}}{\text{inertial quantity}}}$$

These expressions are the same in spirit, but the precise quantities are the ones that represent elasticity and inertia in each case.

Speed of Sound in Air

For air the adiabatic bulk modulus

$$B = 1.42 \times 10^5 \text{ Pa}$$

and

$$\rho = 1.2041 \text{ kg/m}^3$$

at 20°C.

This gives a speed of sound in air at 20°C of

$$v = 343 \text{ m/s}$$

This is approximately 1/3 km/s or 1/5 mi/s.

Speed of Sound in Air

The speed of sound in air at 20°C

$$v = 343 \text{ m/s}$$

Since the density of air vary with temperature, the speed of sound varies also.

For temperatures near room temperature:

$$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{Cel}}}{273}}$$

where T_{Cel} is the temperature in **Celsius**.

[1]The bulk modulus depends on the pressure.

Pressure Waves

$$\Delta P(x,t) = \Delta P_{\text{max}} \sin(kx - \omega t)$$

where

$$\Delta P_{\text{max}} = B\, s_{\text{max}} k$$

It is easier to express the amplitude in terms of the wave speed, since it is usually easier to look up the wave speed than the bulk modulus:

$$\Delta P_{\text{max}} = (\rho v^2)\, s_{\text{max}} \frac{\omega}{v}$$

Then

$$\Delta P_{\text{max}} = \rho v \omega s_{\text{max}}$$

Sound Waves

Displacement:

$$s(x, t) = s_{\max} \cos(kx - \omega t)$$

Pressure:

$$\Delta P(x, t) = \Delta P_{\max} \sin(kx - \omega t)$$

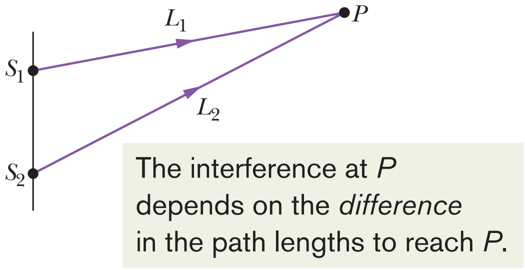
where

$$\Delta P_{\max} = B\, s_{\max} k = \rho v \omega s_{\max}$$

Interference of Sound Waves

Imagine two point sources of sinusoidal sound waves that emit identical signals: same amplitude, wavelength, and phase.

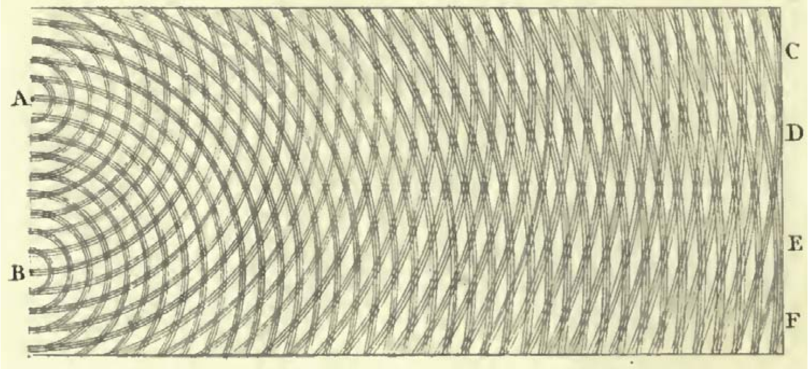
$$\Delta P_1(x,t) = \Delta P_2(x,t) = \Delta P_{\max} \sin(kx - \omega t)$$



The sound will be louder at different points, depending on the difference in the path lengths that the sound waves take.

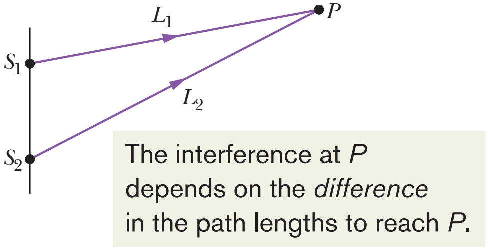
Interference of Sound Waves

Interference pattern from two sources, with equal wavelength and in phase:



[1]Thomas Young, On the Nature of Light and Colours, Lecture 39, Course of Lectures on Natural Philosophy and Mechanical Arts (London, 1897)

Interference of Sound Waves



This is because the path difference will correspond to a phase offset of the arriving waves at P:

$$\begin{aligned} P(x,t) &= P_1 + P_2 \\ &= P_{\max}(\sin(kL_1 - \omega t) + \sin(kL_2 - \omega t)) \\ &= \underbrace{\left[2P_{\max}\cos\left(\frac{k(L_2 - L_1)}{2}\right)\right]}_{\textbf{new amplitude}} \sin\left(\frac{k(L_2 + L_1)}{2} - \omega t\right) \end{aligned}$$

Interference of Sound Waves

The new amplitude could be written as:

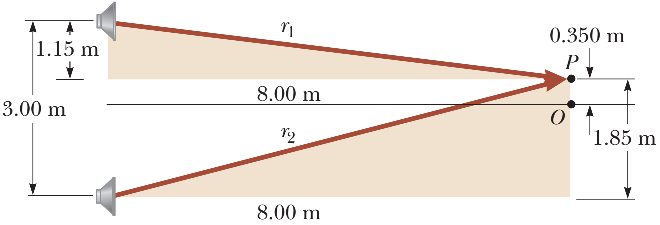
$$2P_{\max} \cos\left(\frac{\pi(L_2 - L_1)}{\lambda}\right)$$

When $|L_2 - L_1| = n\lambda$ and $n = 0, 1, 2, \ldots$ the sound from the two speakers is loudest (a maximum).

When $|L_2 - L_1| = \frac{(2n+1)\lambda}{2}$ and $n = 0, 1, 2, \ldots$ the sound from the two speakers is cancelled out (a minimum).

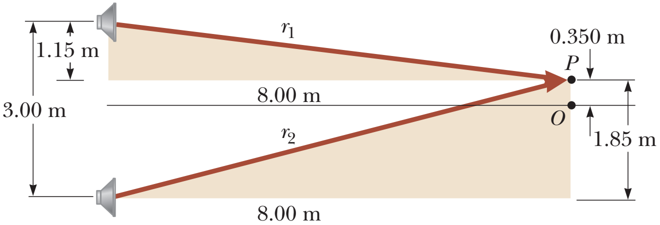
Example 18.1

Two identical loudspeakers placed 3.00 m apart are driven by the same oscillator. A listener is originally at point O, located 8.00 m from the center of the line connecting the two speakers. The listener then moves to point P, which is a perpendicular distance 0.350 m from O, and she experiences the first minimum in sound intensity. What is the frequency of the oscillator?



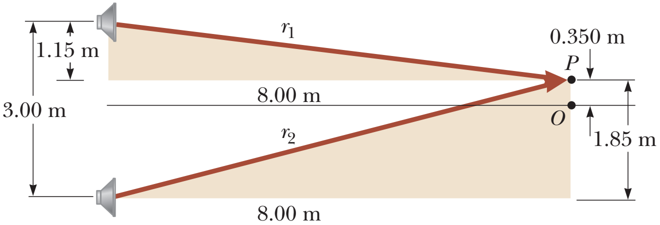
[0]Serway & Jewett, page 537

Example 18.1



P is first minimum.

Example 18.1



P is first minimum.

That means $r_2 - r_1 = \frac{\lambda}{2}$. If we find λ, we can find f, since we know the speed of sound.

$$\lambda = 2(\sqrt{1.85^2 + 8^2} - \sqrt{1.15^2 + 8^2}) = 0.26 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.26 \text{ m}} = \underline{1.3 \text{ kHz}}$$

Standing Sound Waves in air columns

Standing sound waves can be set up in hollow tubes.

This is the idea behind how pipe organs, clarinets, didgeridoos, *etc.* work.

Displacement fluctuation:

$$s(x,t) = [2s_{max}\cos(kx)]\ \cos(\omega t)$$

For a tube with a closed end, the closed end forms a **displacement node**.

This is logical because the air cannot move past the sealed end.

(But that does also mean that the closed end is a pressure antinode. Here we will speak about sound waves in terms of displacement.)

Standing Sound Waves in air columns

The open end or ends of a tube are approximately[1] **displacement antinode**.

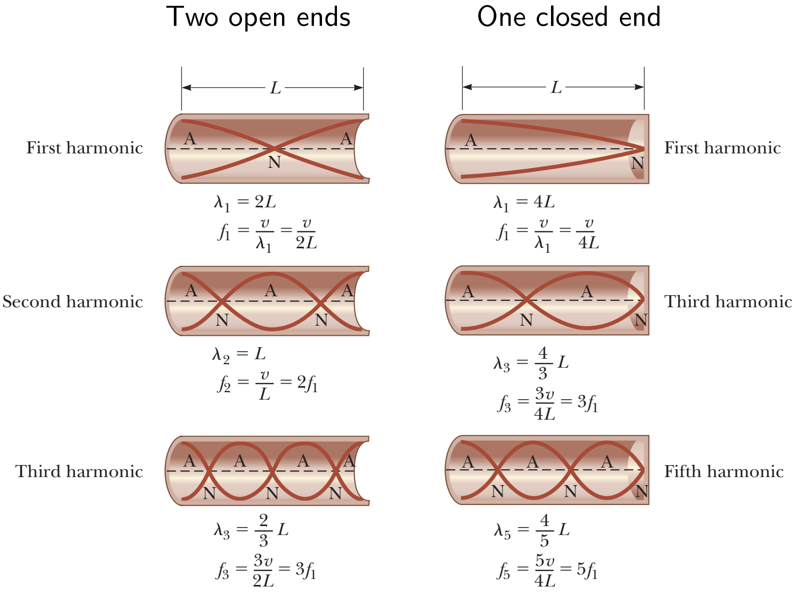
This can be thought of as being because the pressure outside the tube is atmospheric pressure, P_0, so the open end is a pressure node, therefore a displacement antinode.

The waves are partially reflected from the open end because the air outside the tube can expand in 3-dimensions, rather than just one, so it behaves very differently than the air in the tube.

It is effectively a change of medium.

[1] This is not exactly true. Actually the antinode is located just beyond the end of the tube. For this course, we will say the the open end is an antinode.

Standing Sound Waves in air columns



[1]Figure from Serway & Jewett, page 547.

Standing Sound Waves in air columns

For double open ended tubes:

The wavelengths of the normal modes are given by the constraint $|\cos(0)| = |\cos(kL)| = 1$:

$$\lambda_n = \frac{2L}{n}$$

where n is a positive natural number (1, 2, 3...).

The **natural frequencies**:

$$f_n = \frac{nv}{2L} = n\,f_1$$

where n is a positive natural number.

The **fundamental frequency**, also called the **first harmonic** is the lowest frequency sound produced in the column. It is

$$f_1 = \frac{v}{2L}$$

Standing Sound Waves in air columns

For tubes with one closed end: **fundamental frequency** $f_1 = \frac{v}{4L}$

The wavelengths of the normal modes are given by the constraint $\cos(0) = 0$, $|\cos(kL)| = 1$:

$$\lambda_{2n+1} = \frac{4L}{(2n+1)}$$

where n is a natural number including zero (0, 1, 2, 3...).

The **natural frequencies**:

$$f_{2n+1} = \frac{(2n+1)v}{4L} = (2n+1)f_1$$

where n is a natural number including zero (0, 1, 2, 3...).

Musical Instruments

Didgeridoo:

Longer didgeridoos have lower pitch, but tubes that flare outward have higher pitches this can also change the spacing of the resonant frequencies.

[1]Matt Roberts via Getty Images.

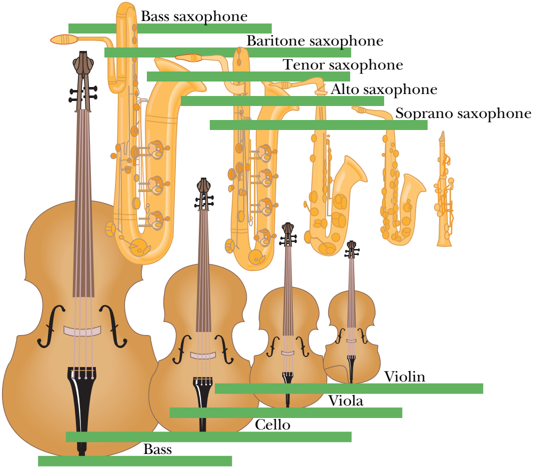
Musical Instruments, Pipe Organ

The longest pipes made for organs are open-ended 64-foot stops (tube is effectively 64 feet+ long). There are two of them in the world. The fundamental frequency associated with such a pipe is 8 Hz.

32' stops give 16 Hz sound, 16' stops give 32 Hz, 8' stops give 64 Hz, *etc*.

[1]Picture of Sydney Town Hall Grand Organ from Wikipedia, user Jason7825.

Musical Instruments



In general, larger instruments can create lower tones, whether string instruments or tube instruments.

[1] Halliday, Resnick, Walker, 9th ed, page 458.

Summary

- standing waves and sound
- musical instruments

Announcements and HW

Collected Homework, due Tuesday, May 29.

Drop Deadline Friday, June 1.

3rd Test Friday, June 1.

Quiz tomorrow.

No Class on Monday May 28. (Memorial day)

Homework Serway & Jewett:

- Ch 18, onward from page 555. OQs: 3, 5; CQs: 1, 3; Probs: 37, 39, 49, 51, 53