Waves
Interference
Wave Reflections

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Last time

- energy transfer by a sine wave

Overview

- interference
- boundary conditions
- reflection and transmission

Reminder: Rate of Energy Transfer in Sine Wave

For one wavelength:

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

Power averaged over one wavelength:

$$P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T}$$

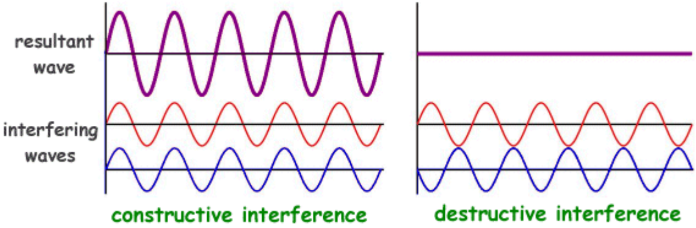
Average power of a wave on a string:

$$\boxed{P = \frac{1}{2}\mu\omega^2 A^2 v}$$

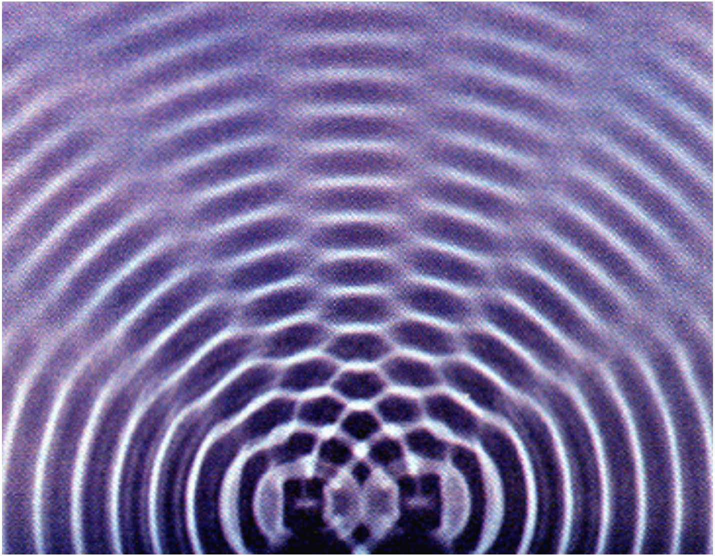
Interference of Waves

When two wave disturbances interact with one another they can amplify or cancel out.

Waves of the same frequency that are “in phase” will reinforce, amplitude will increase; waves that are “out of phase” will cancel out.



Interference of Waves



Interference of Waves

Waves that exist at the same time in the same position in space add together.

superposition principle

If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.

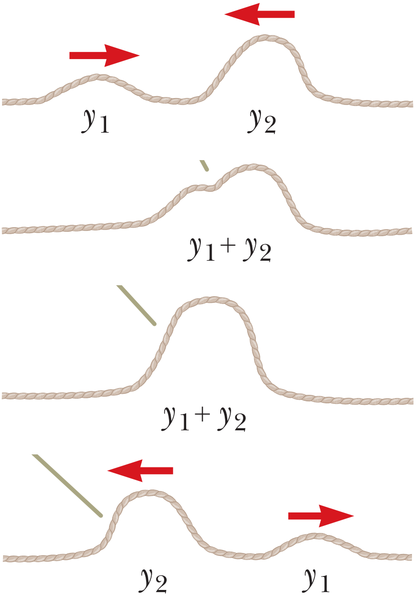
This works because the wave equation we are studying is *linear*.

This means solutions to the wave equations can be added:

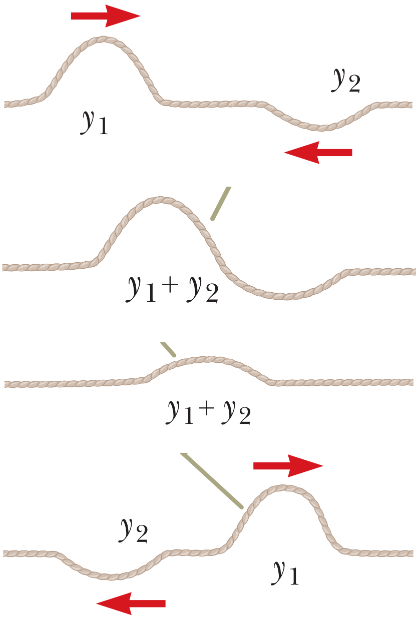
$$y(x,t) = y_1(x,t) + y_2(x,t)$$

y is the resultant wave function.

Interference of Waves: Constructive Interference



Interference of Waves: Destructive Interference



Superposition of Sine Waves

Consider two sine waves with the same wavelength and amplitude, but different phases, that interfere.

$$y_1(x,t) = A\sin(kx - \omega t) \quad y_2(x,t) = A\sin(kx - \omega t + \phi)$$

Add them together to find the resultant wave function, using the identity:

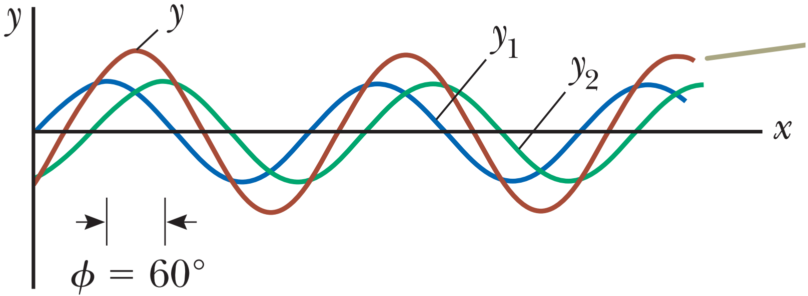
$$\sin\theta + \sin\psi = 2\cos\left(\frac{\theta - \psi}{2}\right)\sin\left(\frac{\theta + \psi}{2}\right)$$

Then

$$y(x,t) = \underbrace{\left[2A\cos\left(\frac{\phi}{2}\right)\right]}_{\textbf{New amplitude}} \underbrace{\sin(kx - \omega t + \frac{\phi}{2})}_{\textbf{Sine oscillation}}$$

Interference of Two Sine Waves (equal wavelength)

$$y(x,t) = \left[2A\cos\left(\frac{\phi}{2}\right)\right]\sin(kx - \omega t + \frac{\phi}{2})$$



Dependence on Phase Difference

The amplitude of the resultant wave is $A' = 2A\cos\left(\frac{\phi}{2}\right)$, where ϕ is the phase difference.

For what value of ϕ is A' maximized?

Dependence on Phase Difference

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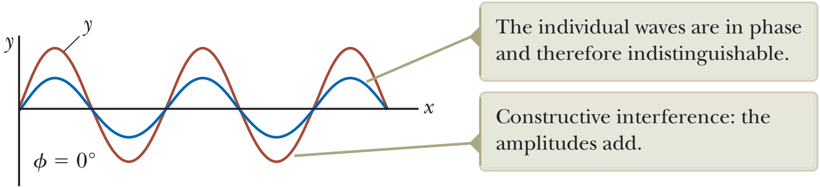
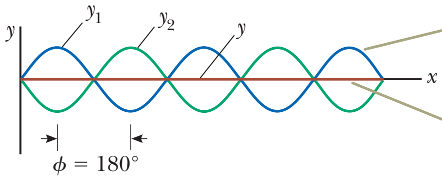
For what value of ϕ is A' maximized? $\phi = 0$ or $\phi = 2\pi, -2\pi,\ 4\pi,$ etc.

The waves are "in phase" and constructively interfere.



Dependence on Phase Difference

If $\phi = \pi, -\pi,\ 3\pi, -3\pi$, etc. $A' = 0$. Destructive interference.



The individual waves are 180° out of phase.

Destructive interference: the waves cancel.

Phase Differences

We can count phase differences in terms of wavelengths also.

If two waves have a phase difference of 1 wavelength then $\phi = 2\pi$. Constructive interference.

If two waves have a phase difference of half a wavelength then $\phi = \pi$. Destructive interference.

Question

Here are four possible phase differences between two identical waves, expressed in wavelengths:

0.20, 0.45, 0.60, and 0.80.

Rank them according to the amplitude of the resultant wave, greatest first.

(A) 0.20, 0.45, 0.60, 0.80

(B) 0.80, 0.60, 0.45, 0.20

(C) (0.20 and 0.80), 0.60, 0.45

(D) 0.45, 0.60, (0.20 and 0.80)

[1] Halliday, Resnick, Walker, page 427.

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(D) 0.45, 0.60, (0.20 and 0.80)

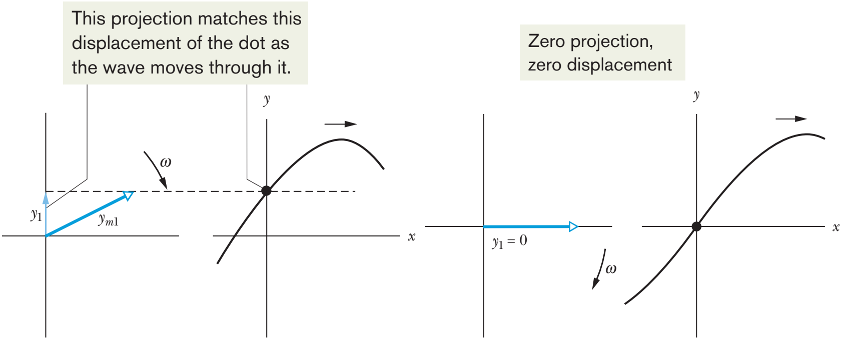
[1] Halliday, Resnick, Walker, page 427.

Phasors

We can represent sine waves and their addition with a **phasor diagram**.

This works for sine waves with equal wavelengths, even if they have different amplitudes.

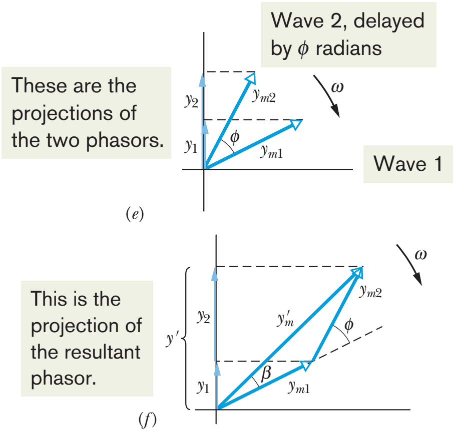
Each wave function at point (x, t) is represented by a vector.



[1]Figures from Halliday, Resnick, & Walker, 9th ed, page 429.

Phasors

Add the vectors to find the sum.



In the diagram $A' = y'_m$ is the amplitude of the resulting wave.

Example

Two sinusoidal waves $y_1(x, t)$ and $y_2(x, t)$ have the same wavelength and travel together in the same direction along a string. Their amplitudes are $A_1 = 4.0$ mm and $A_2 = 3.0$ mm, and their phase constants are 0 and $\pi/3$ rad, respectively.

What are the amplitude A' and phase constant ϕ' of the resultant wave? Also give resultant wave function.

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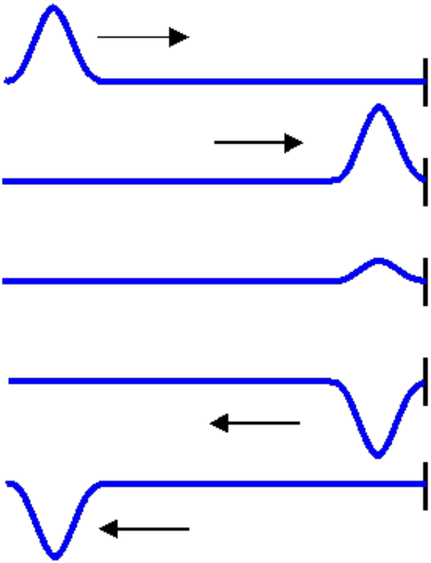
What are the amplitude A' and phase constant ϕ' of the resultant wave? Also give resultant wave function.

$$A' = 6.1 \text{ mm} \; ; \quad \phi' = 0.44 \text{ rad}$$

$$y(x,t) = (6.1 \text{ mm}) \sin(kx - \omega t + 0.44)$$

Wave Reflection



Boundaries and Wave Reflection and Transmission

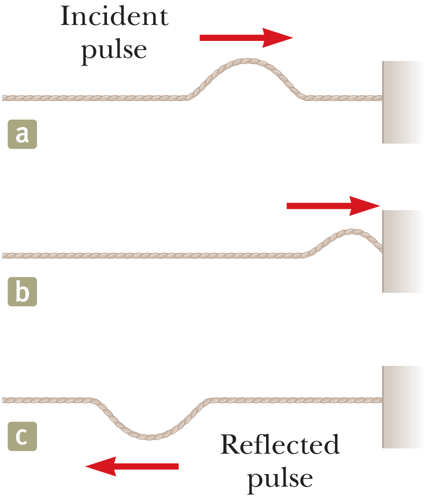
When waves reach the end of their medium, or move from one medium to another, they can be reflected.

The behavior is different in difference circumstances.

We can describe the different circumstances mathematically using **boundary conditions** on our wave function.

These will help us to correctly predict how a wave will reflect or be transmitted.

Wave Reflection from a fixed end point



The reflected pulse is inverted. How does this happen?

Wave Reflection from a fixed end point

The boundary condition for a fixed end point at position $x = 0$ is:

$$y(x = 0, t) = 0$$

At any time, the point of the string at $x = 0$ cannot have any vertical displacement. It is tied to a wall!

The wave function for single pulse on the string does not satisfy this boundary condition.

$$y_1(x, t) = f(x - vt)$$

This pulse will continue in the $+x$ direction forever, past the end of the string. Makes no sense.

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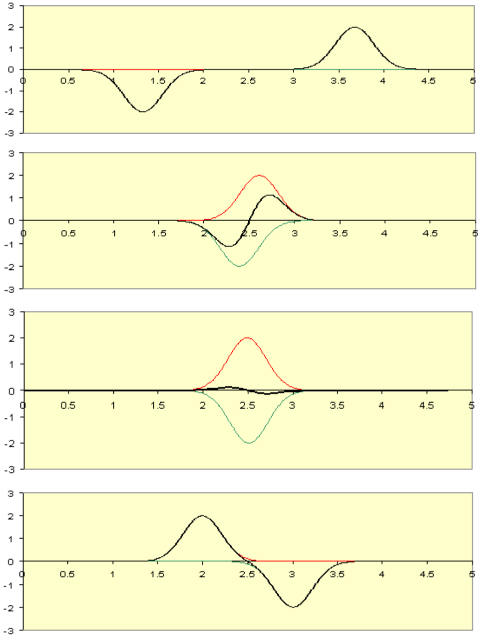
What if we imagine the string continues inside the wall, and there is a pulse traveling behind the wall in the $-x$ direction?



[1]Wall at $x = 2.5$. Digrams by Michal Fowler http://galileo.phys.virginia.edu

Wave Reflection from a fixed end point

If we allow another wave function:

$$y_2(x,t) = -f(-x - vt)$$

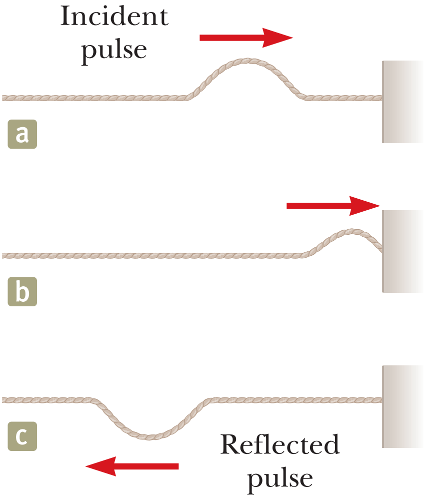
the total wave function will satisfy the boundary condition!

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ y(x,t) &= f(x - vt) + [-f(-x - vt)] \\ y(x = 0, t) &= 0 \end{aligned}$$

However, $-f(-x - vt)$ corresponds to an inverted wave pulse.
The reflected pulse is inverted.

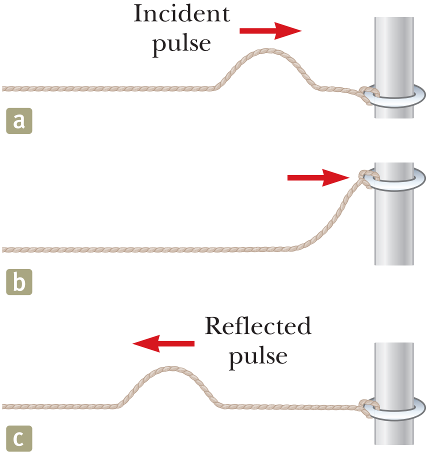
Wave Reflection from a fixed end point

The reflected pulse is inverted.



Wave Reflection from a freely movable end point

In this case, reflected pulse is not inverted.



Wave Reflection from a freely movable end point

Now we have a different boundary condition.

The *slope* of the string at the boundary must be zero.

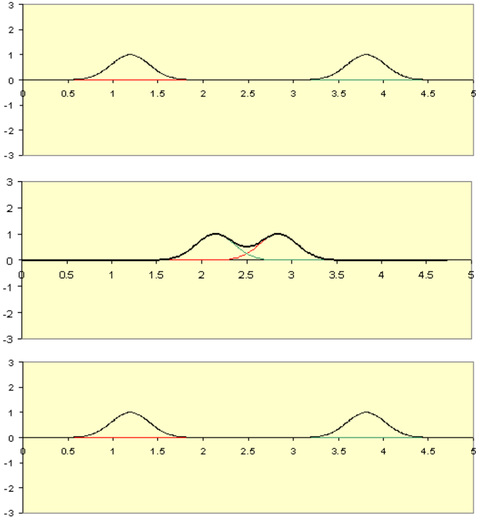
$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0$$

This ensures that the string will stay attached to the wall and there will not be an infinite force on the last tiny bit of string.

To satisfy this boundary condition, imagine there is another pulse that is upright but moving in the $-x$ direction.

Wave Reflection from a freely movable end point

Imagine the free end of the string at $x = 2.5$. The slope there is zero at all times.



Wave Reflection from a freely movable end point

The new boundary condition is satisfied if $y_2 = f(-x - vt)$:

Let $u_1 = x - vt$ and $u_2 = -x - vt$.

$$
\begin{aligned}
y(x,t) &= f(x - vt) + f(-x - vt) \\
\frac{\partial y(x,t)}{\partial x} &= \frac{\partial f(u_1)}{\partial x} + \frac{\partial f(u_2)}{\partial x} \\
&= f'(u_1) + (-1) f'(u_2)
\end{aligned}
$$

The terms cancel when $u_1 = u_2$, that is, at $x = 0$.

$$
\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0
$$

The pulse $f(-x - vt)$ is not inverted.

Transmitted and Reflected Waves at a Boundary

If two ropes of different linear mass densities, μ_1 and μ_2 are attached together (under the same tension), an incoming pulse will be partially transmitted and partially reflected.

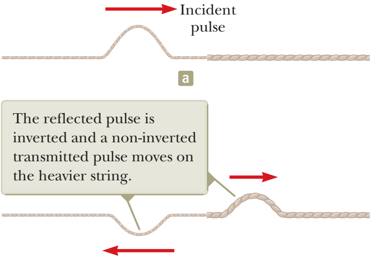
The boundary conditions here are different again:

Now the slope of the string at the boundary should be zero and the displacements at the boundary must be the same (otherwise the string breaks).

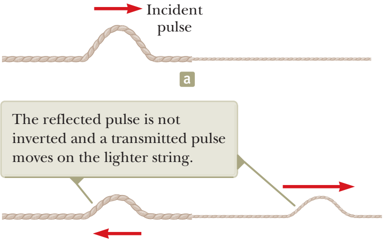
Transmitted and Reflected Waves at a Boundary

From those boundary conditions it is possible to deduce the behavior:

$\mu_1 < \mu_2$



$\mu_1 > \mu_2$



[1] Serway & Jewett, page 495.

Summary

- interference
- reflection

3rd Test Friday, June 1.

3rd Collected Homework due Tuesday, May 29.

Homework Serway & Jewett:

- Ch 18, onward from page 555. OQs: 9; CQs: 9; Probs: 1, 3, 7, 9, 11