Optics
Nature of Light

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June 4, 2018

Last time

- the Doppler effect
- bow and shock waves

Overview

- the nature of light
- the wave equation for light
- the speed of light

Light

We are now moving on to chapters 35-38.

Light is also a wave.

What is Light?

Physicists have long been interested in the nature and uses of light.

Egyptians and Mesopotamians developed lenses. Later Greeks and Indians began to develop a theory of geometric optics

Geometric optics was greatly advanced in 800-1000 by Arab philosophers, especially Ibn al-Haytham (called Alhazen).

Newton developed a particle model of light, which explained reflection and refraction.

Christian Huygens proposed a wave model of light (1678) and pointed out that it could also explain reflection and refraction, but it was less popular.

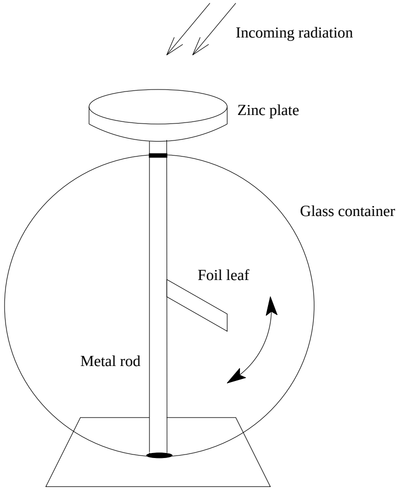
What is Light?

Thomas Young experimentally demonstrated the interference of light, which confirmed that it needed to be considered as being wave-like.

This fit with the understanding of Maxwell's equations.

Hertz then discovered the **Photoelectric effect** and was unable to explain it with a wave model of light.

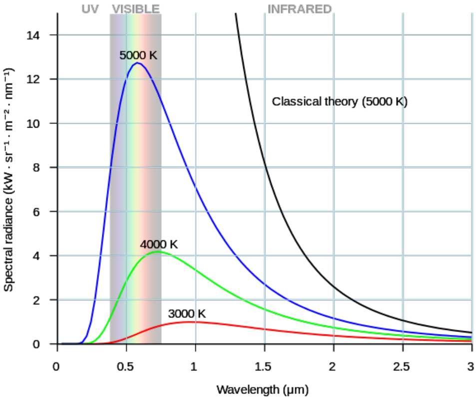
Photoelectric Effect



Even very intense light at a low frequency will not allow the plate to discharge. As soon as just a little light at a high frequency falls on the plate it begins discharging.

What is Light?

Recall the blackbody radiation distribution of wavelengths.



Classical theory, with light as a wave, could not explain the shape of the distribution.

Max Planck suggested a model that imagined light energy came in discrete units.

[1]Graph from Wikipedia, created by user Darth Kule.

What is Light?

Einstein resolved the issue of the photoelectric effect by taking literally Planck's quantization model and showing that light behaves like a wave, but also like a particle.

The "particles" of light are called **photons**.

The energy of a photon depends on its frequency:

$$E = hf$$

where $h = 6.63 \times 10^{-34}$ J s is **Planck's constant**.

Speed of Light

Light travels very fast.

We can figure out how fast it goes from Maxwell's laws, by deriving a wave equation from them.

Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I_{\text{enc}}$$

In free space (a vacuum) with no charges $q_{\text{enc}} = 0$ and $I_{\text{enc}} = 0$.

[1] Strictly, these are Maxwell's equations in a vacuum.

Maxwell's Equations Differential Form

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}
\end{aligned}
$$

In free space with no charges $\rho = 0$ and $\mathbf{J} = 0$.

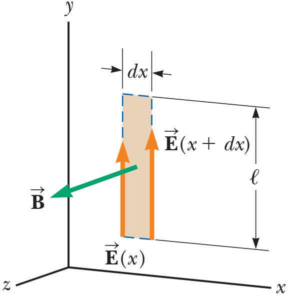
Maxwell's Equations and the Wave Equation

(See the Appendix to these slides for a more complete derivation.)

Let's assume there is an electric field in the y-direction, and it changes magnitude, perhaps because of a plane of moving charges.

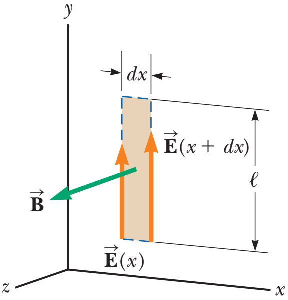
The field in the free space around the charges must update.

There will be a variation in the field along the x-direction.



Maxwell's Equations and the Wave Equation

$\mathbf{E} = E_y \mathbf{j}$



rectangle has area S

There is a curl of the E-field around the z-direction only:

$$\lim_{S \to 0} \oint_S \mathbf{E} \cdot d\mathbf{s} = \boldsymbol{\nabla} \times \mathbf{E} = \frac{\partial E}{\partial x} \mathbf{k}$$

Maxwell's Equations and the Wave Equation

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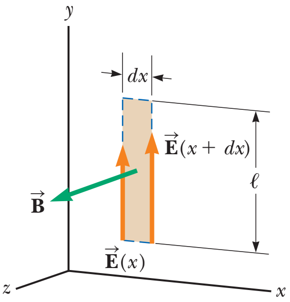

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$$\lim_{S \to 0} \oint_S \mathbf{E} \cdot d\mathbf{s} = \boldsymbol{\nabla} \times \mathbf{E} = \frac{\partial E}{\partial x} \mathbf{k}$$

Consulting the 3rd Maxwell equation:

$$\frac{\partial E}{\partial x} \mathbf{k} = -\frac{\partial B}{\partial t} \mathbf{k}$$

So there will be a time-varying B-field pointing in the z-direction.

Maxwell's Equations and the Wave Equation

Taking the partial derivative of this expression with respect to x:

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right)$$

Maxwell's Equations and the Wave Equation

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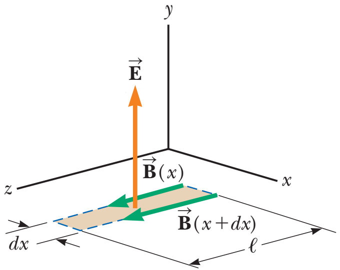
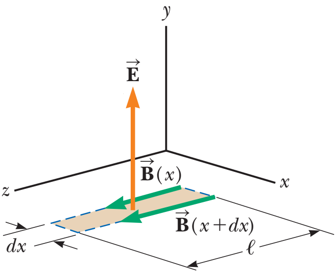
We now need to think about how **B** varies with x.



As the B-field changes in time, the field updates in through the space around it.

The field varies along the x-direction.

Maxwell's Equations and the Wave Equation



There is a curl of the B-field around the y-direction only:

$$\lim_{S \to 0} \oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{s} = \boldsymbol{\nabla} \times \mathbf{B} = -\frac{\partial B}{\partial x}\mathbf{j}$$

Using the 4th Maxwell equation:

$$-\frac{\partial B}{\partial x}\mathbf{j} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}\mathbf{j}$$

And this perpetuates the time-varying E-field in the y-direction.

Maxwell's Equations and the Wave Equation

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) \qquad (1)$$

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \qquad (2)$$

(2) into (1):

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

The wave equation!

Another Implication of Maxwell's Equations

For a wave propogating in x direction:

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial \mathrm{t}^2}$$

The constant c appears as the wave speed and

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$c = 3.00 \times 10^8$ m/s, is the speed of light.

The values of ϵ_0 and μ_0 together predict the speed of light!

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2\,N^{-1}m^{-2}}$ and $\mu_0 = 4\pi \times 10^{-7}$ kg m $\mathrm{C^{-2}}$

Another Implication of Maxwell's Equations

The same process gives the same wave equation for the magnetic field: $\frac{\partial^2 B}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$

Wave solutions:

$$\begin{aligned} \mathbf{E} &= \mathbf{E_0} \sin(kx - \omega t) \\ \mathbf{B} &= \mathbf{B_0} \sin(kx - \omega t) \end{aligned}$$

where $c = \frac{\omega}{k}$.

These two solutions are in phase. There is no offset in the angles inside the sine functions.

The two fields peak at the same point in space and time.

At all times:

$$\frac{E}{B} = c$$

Maxwell's Equations and the Wave Equation

Wave solutions for the wave equation in for E and B:

$$\begin{aligned} \mathbf{E} &= \mathbf{E_0} \sin(kx - \omega t) \\ \mathbf{B} &= \mathbf{B_0} \sin(kx - \omega t) \end{aligned}$$

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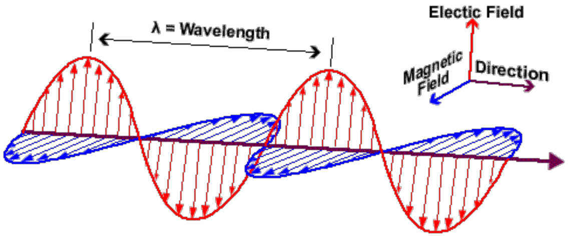


Measurements of the Speed of Light

Since light propagates so quickly it is difficult to measure its speed in practice.

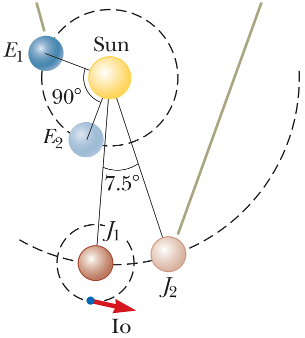
Galileo and others tried to measure it with procedures that relied on human reactions.

Human reactions are way too slow! This error dominates the data.

Many clever alternative methods were developed.

Roemer's Method

Ole Roemer observed the orbit of Io, a moon of Jupiter.



If light travels infinitely fast, the orbit should always be observed to have the same period. Instead it appears to have a slightly shorter period as Earth approaches Jupiter and longer when Earth moves away.

Roemer's 1675 lower limit for c: 2.3×10^8 m/s.

Wheatstone’s Rotating Mirror

Charles Wheatstone created a rotating mirror arrangement to study fast phenomena in electricity.

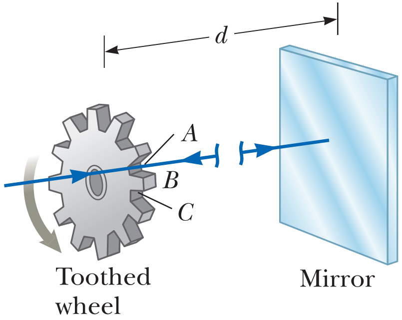
He told Françios Arago that he thought this could be used to measure the speed of light.

Arago passed on the suggestion to Armand Fizeau and Léon Foucault who were collaborating on various optical studies. He suggested it might be useful to measure the speed of light in *water* as well as air to compare them.

Fizeau and Foucault then fell out and stopped working together. Both pursued their investigation separately.

Fizeau's Method

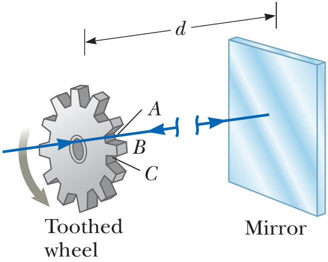
Fizeau sent a beam of light through a gear-tooth wheel toward a mirror 5 miles (8 km) away.



The teeth broke up the beam into pulses as it rotated.

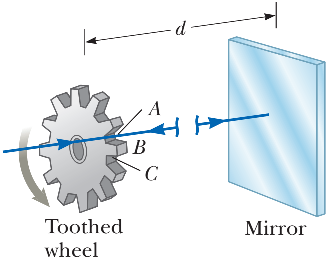
When the wheel rotates fast enough, the light passing through gap A is blocked by tooth B on it's return from the mirror.

Fizeau's Wheel Example



Assume Fizeau's wheel has 360 teeth and rotates at 27.5 rev/s when a pulse of light passing through opening A is blocked by tooth B on its return. If the distance to the mirror is 7500 m, what is the speed of light?

Fizeau's Wheel Example



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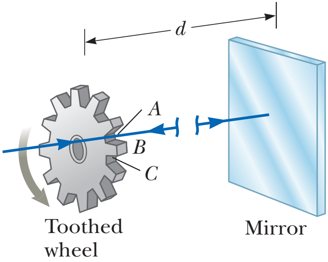
$$\Delta t = \frac{\Delta\theta}{\omega} \qquad\qquad c = \frac{2d}{\Delta t}$$

Fizeau's Wheel Example



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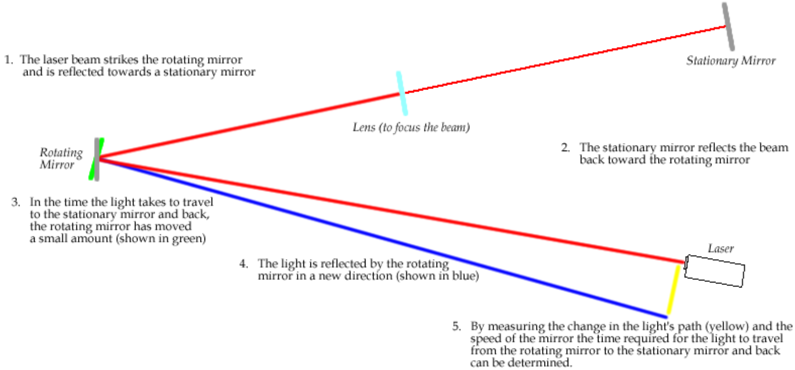
$$\Delta t = \frac{\Delta\theta}{\omega} \qquad c = \frac{2d}{\Delta t}$$

$$c = 2.97 \times 10^8 \text{ m/s}$$

Foucault's Method

Foucault used a rotating mirror, to send light from a source to a stationary mirror and back again.

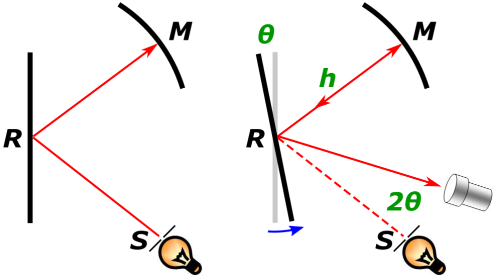


(Foucault did not use a laser, obviously.)

[1] Figure from Wikipedia, by user Rhodesl.

Foucault's Method

The angle formed between the source and the returning light beam allowed him to figure out how much the mirror had rotated (therefore how much time had passed) while the light traveled from R to M and back.



Foucault could only separate the mirrors by a distance of 20m, due to limitations on his mirrors and lenses.

[1] Figure from Wikipedia, by user Stigmatella aurantiaca.

Michelson's Refinement

Albert Michelson adapted Foucault's apparatus to increase path length of the light to 22 miles!

He used two observatories on adjacent mountains.

In spite of a forest fire and an earthquake, he got the value of

$$299,796 \pm 4 \text{ km/s}$$

This is only 4 km/s faster that the current accepted value.

He later worked on the famous Michelson-Morley experiment which showed that light needs no medium.

Summary

- light and the wave equation

Homework

- Ch 35, onward from page 1077. CQs: 15; Probs: 1, 3

Appendix: Notation

The differential operators in 3 dimensions.

Gradient of a scalar field at a point, f:

$$\nabla f = \frac{\partial f}{\partial x}\,\mathbf{i} + \frac{\partial f}{\partial y}\,\mathbf{j} + \frac{\partial f}{\partial z}\,\mathbf{k}$$

Divergence of a vector field at a point $\mathbf{v} = [v_x, v_y, v_z]$:

$$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl of a vector field at a point $\mathbf{v}$:

$$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\mathbf{k}$$

Appendix: Maxwell's Equations and the Wave Equation (More complete derivation)

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= 0 \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}
$$

Take the curl of both sides of equation 3:

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B})$$

Using the vector triple product rule, $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$

$$\nabla(\nabla \cdot \mathbf{E}) - (\nabla \cdot \nabla)\mathbf{E} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B})$$

Maxwell's Equations and the Wave Equation

$$\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E}) - \nabla^2\mathbf{E} = -\frac{\partial}{\partial t}(\boldsymbol{\nabla}\times\mathbf{B})$$

Using the 1st equation:

$$\nabla^2\mathbf{E} = \frac{\partial}{\partial t}(\boldsymbol{\nabla}\times\mathbf{B})$$

Using the 4th equation,

$$\nabla^2\mathbf{E} = \mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}$$

Another Implication of Maxwell's Equations

Starting from the curl of the 4th equation a similar equation can be found for $\mathbf{B}$, givings a pair of wave equations for the electric and magnetic fields:

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

with wave solutions:

$$\mathbf{E} = \mathbf{E_0} \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

$$\mathbf{B} = \mathbf{B_0} \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

where $c = \frac{\omega}{k}$.