Waves
Standing Waves
Sound Waves

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De Anza College

May 22, 2018

Last time

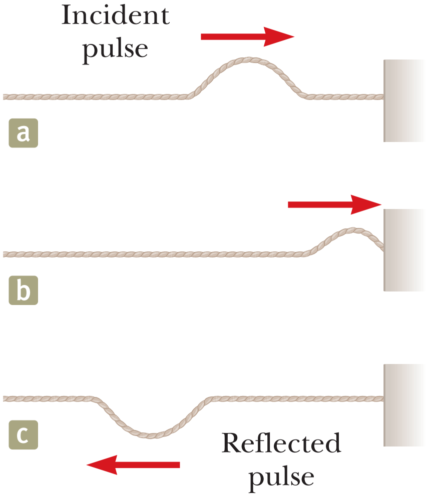
- interference
- boundary conditions
- reflection

Overview

- reflection and transmission
- standing waves
- sound

Wave Reflection from a fixed end point

The reflected pulse is inverted.



Boundaries and Wave Reflection and Transmission

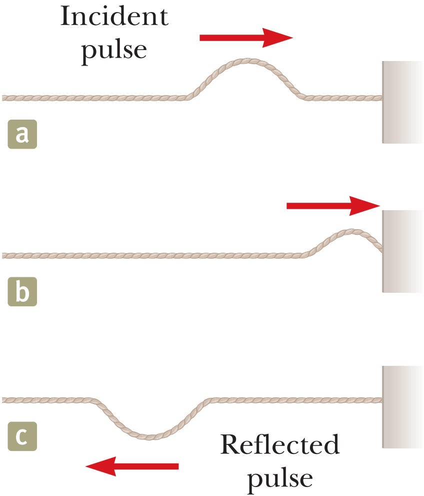
When waves reach the end of their medium, or move from one medium to another, they can be reflected.

The behavior is different in difference circumstances.

We can describe the different circumstances mathematically using **boundary conditions** on our wave function.

These will help us to correctly predict how a wave will reflect or be transmitted.

Wave Reflection from a fixed end point



The reflected pulse is inverted. How does this happen?

Wave Reflection from a fixed end point

The boundary condition for a fixed end point at position $x = 0$ is:

$$y(x = 0, t) = 0$$

At any time, the point of the string at $x = 0$ cannot have any vertical displacement. It is tied to a wall!

The wave function for single pulse on the string does not satisfy this boundary condition.

$$y_1(x, t) = f(x - vt)$$

This pulse will continue in the $+x$ direction forever, past the end of the string. Makes no sense.

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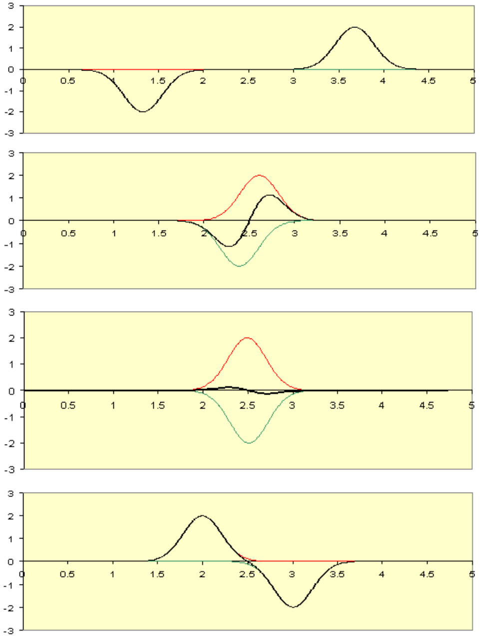
What if we imagine the string continues inside the wall, and there is a pulse traveling behind the wall in the $-x$ direction?



[1] Wall at $x = 2.5$. Digrams by Michal Fowler http://galileo.phys.virginia.edu

Wave Reflection from a fixed end point

If we allow another wave function:

$$y_2(x, t) = -f(-x - vt)$$

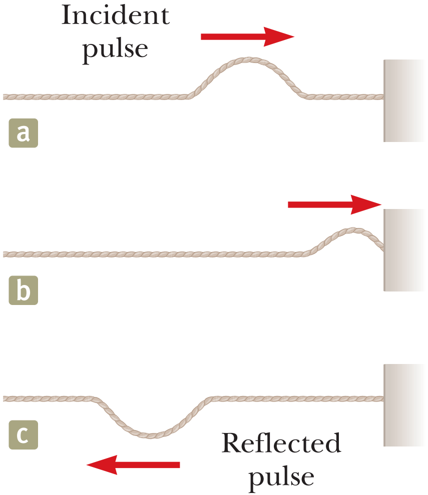
the total wave function will satisfy the boundary condition!

$$\begin{aligned} y(x, t) &= y_1(x, t) + y_2(x, t) \\ y(x, t) &= f(x - vt) + [-f(-x - vt)] \\ y(x = 0, t) &= 0 \end{aligned}$$

However, $-f(-x - vt)$ corresponds to an inverted wave pulse.
The reflected pulse is inverted.

Wave Reflection from a fixed end point

The reflected pulse is inverted.



Wave Reflection from a freely movable end point

Now we have a different boundary condition.

The *slope* of the string at the boundary must be zero.

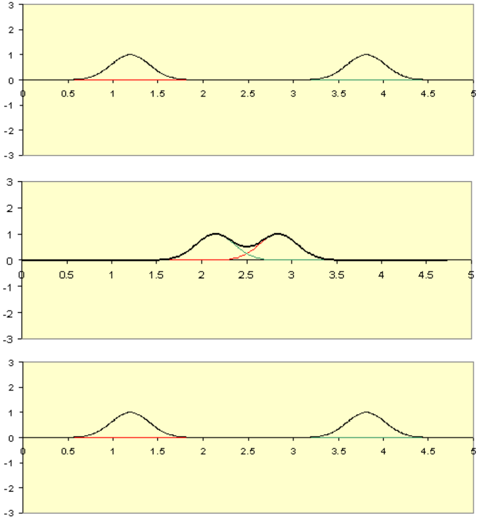
$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0$$

This ensures that the string will stay attached to the wall and there will not be an infinite force on the last tiny bit of string.

To satisfy this boundary condition, imagine there is another pulse that is upright but moving in the $-x$ direction.

Wave Reflection from a freely movable end point

Imagine the free end of the string at $x = 2.5$. The slope there is zero at all times.



Wave Reflection from a freely movable end point

The new boundary condition is satisfied if $y_2 = f(-x - vt)$:

Let $u_1 = x - vt$ and $u_2 = -x - vt$.

$$\begin{aligned} y(x,t) &= f(x - vt) + f(-x - vt) \\ \frac{\partial y(x,t)}{\partial x} &= \frac{\partial f(u_1)}{\partial x} + \frac{\partial f(u_2)}{\partial x} \\ &= f'(u_1) + (-1) f'(u_2) \end{aligned}$$

The terms cancel when $u_1 = u_2$, that is, at $x = 0$.

$$\left. \frac{\partial y}{\partial x} \right|_{x=0} = 0$$

The pulse $f(-x - vt)$ is not inverted.

Transmitted and Reflected Waves at a Boundary

If two ropes of different linear mass densities, μ_1 and μ_2 are attached together (under the same tension), an incoming pulse will be partially transmitted and partially reflected.

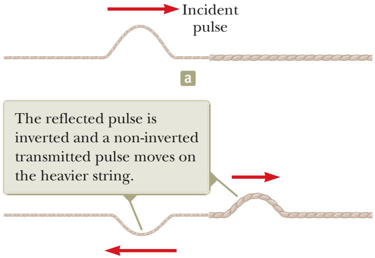
The boundary conditions here are different again:

Now the slope of the string at the boundary should be zero and the displacements at the boundary must be the same (otherwise the string breaks).

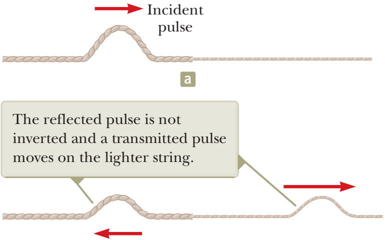
Transmitted and Reflected Waves at a Boundary

From those boundary conditions it is possible to deduce the behavior:

$\mu_1 < \mu_2$



$\mu_1 > \mu_2$



[1] Serway & Jewett, page 495.

Transmitted and Reflected Waves at a Boundary

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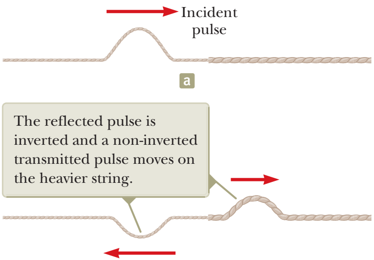

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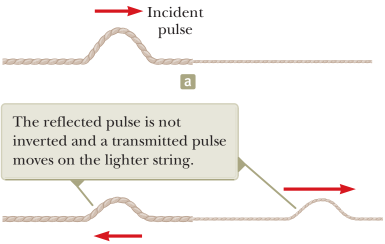
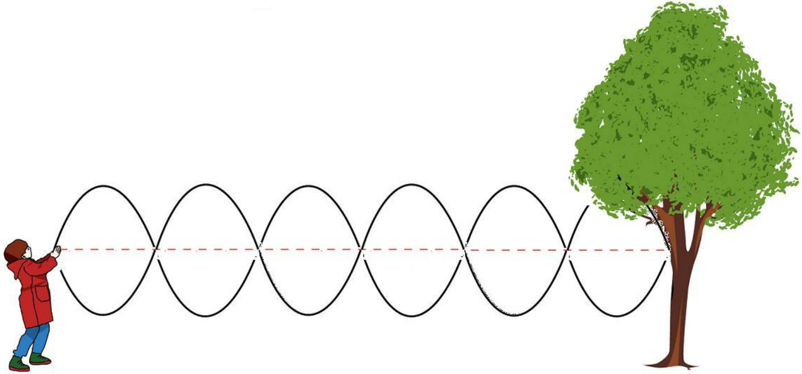


[1]Serway & Jewett, page 495.

Standing Waves

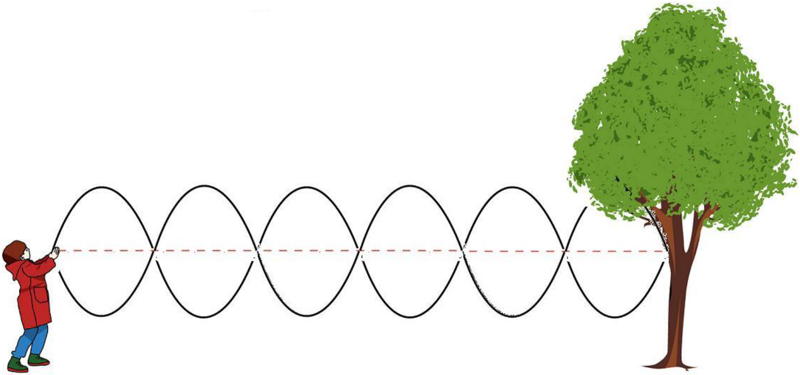
It is possible to create waves that do not seem to propagate.



They are produced by a wave moving to the left interfering with the wave reflected back the right.

Standing Waves

Standing waves are formed from sine waves that are traveling in opposite directions.



Notice that there are a whole number of half wavelengths between the child and the tree.

Standing Waves

The incoming wave:

$$y_1(x,t) = A\sin(kx - \omega t)$$

Reflected wave:

$$y_2(x,t) = A\sin(kx + \omega t)$$

Using the trig identity:

$$\sin(\theta \pm \psi) = \sin\theta\cos\psi \pm \cos\theta\sin\psi$$

The resultant wave is:

$$y = \underset{\substack{\uparrow \\ \textbf{Amplitude at } x}}{[2A\sin(kx)]} \;\; \underset{\substack{\uparrow \\ \textbf{SHM oscillation}}}{\cos(\omega t)}$$

Standing Waves

$$y = [2A \sin(kx)] \ \cos(\omega t)$$

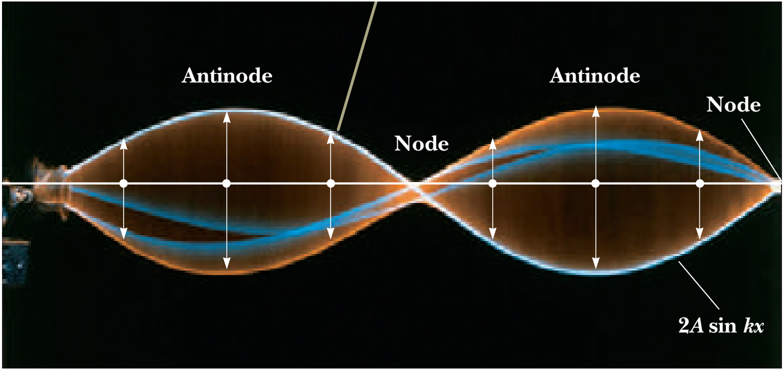
This does not correspond to a traveling wave!

It is a **standing wave**.

Points where $\sin kx = 0$ are called **nodes**. At these points the medium does not move.

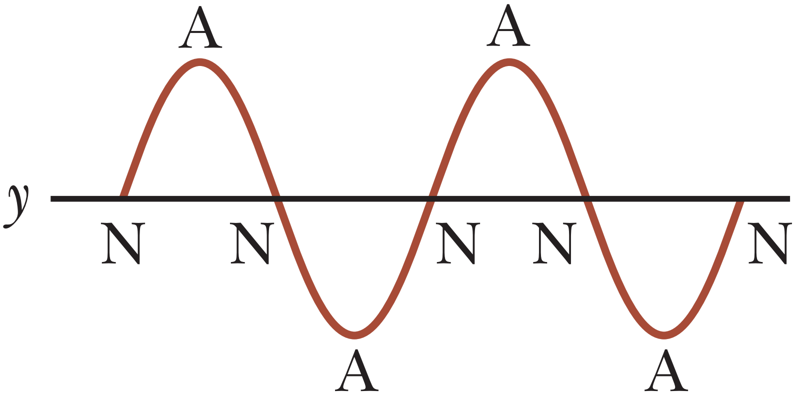
Points where $\sin kx = \pm 1$ are called **antinodes**. At these points particles in the medium undergo their largest displacement.

Nodes and Antinodes



© 1991 Richard Megna/Fundamental Photographs

Nodes and Antinodes

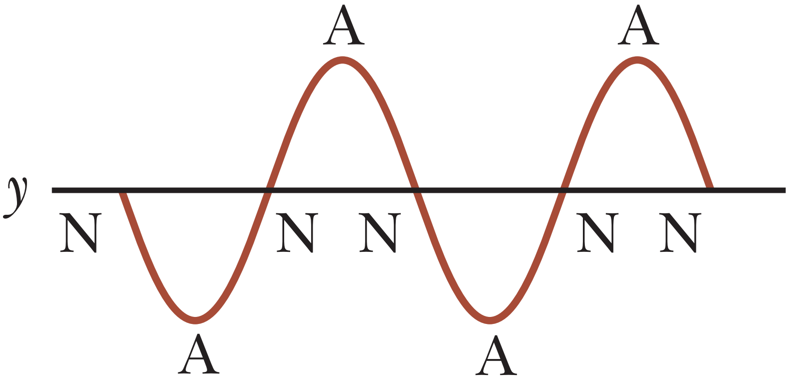


$t = 0$

Nodes and Antinodes

$$t = T/4$$

Nodes and Antinodes



$t = T/2$

Nodes and Antinodes

(Remember that $k = 2\pi/\lambda$)

Assuming $x = 0$ corresponds to a fixed point:

Nodes occur at

$$x = \frac{n\lambda}{2}$$

where n is an integer.

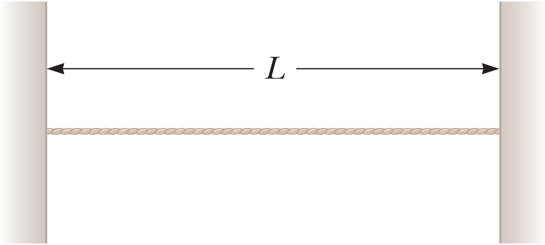
Antinodes occur at

$$x = \frac{(2n+1)\lambda}{4}$$

where again n is an integer.

Standing Waves and Resonance on a String

For a given string, fixed at both ends, only some wavelengths can correspond to standing waves.

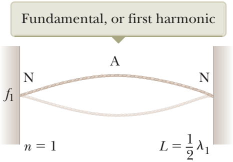


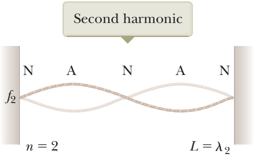
The boundary conditions are now

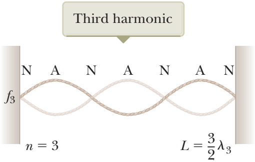
$$y(x = 0, t) = y(x = L, t) = 0$$

$x = 0$ and $x = L$ must be the positions of nodes.

Standing Waves and Resonance on a String







$$\lambda_1 = 2L \qquad \lambda_2 = L \qquad \lambda_3 = \frac{2L}{3}$$

Standing Waves and Resonance

These types of standing wave motions are called **normal modes**.

normal mode

A pattern of motion in a physical system where all parts of the system move sinusoidally with the same frequency and in phase.

Standing Waves and Resonance on a String

The wavelengths of these normal modes are given by the constraint $\sin(0) = \sin(kL) = 0$:

$$\lambda_n = \frac{2L}{n}$$

where n is a positive natural number (1, 2, 3...).

The frequencies that correspond to these wavelengths are called the **natural frequencies**:

$$f_n = \frac{nv}{2L} = n\,f_1$$

where n is a positive natural number.

For a string of density μ under tension T, the wave speed is constant $v = \sqrt{\frac{T}{\mu}}$.

Standing Waves and Resonance on a String

When a string is plucked, resonant (natural) frequencies tend to persist, while other waves at other frequencies are quickly dissipated.

Stringed instruments like guitars can be tuned by adjusting the tension in the strings.

While playing, pressing a string against a particular fret will change the string length or promote a specific harmonic.

Standing Waves and Resonance Question

Quick Quiz 18.3[1] When a standing wave is set up on a string fixed at both ends, which of the following statements is true?

(A) The number of nodes is equal to the number of antinodes.

(B) The wavelength is equal to the length of the string divided by an integer.

(C) The frequency is equal to the number of nodes times the fundamental frequency.

(D) The shape of the string at any instant shows a symmetry about the midpoint of the string.

[1] Serway & Jewett, page 543.

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Summary and Announcements and HW

- reflection and transmission
- standing waves
- sound

Collected Homework, due Tuesday, May 29.

Drop Deadline Friday, June 1.

3rd Test Friday, June 1.

Quiz this Friday.

No Class on Monday May 28. (Memorial day)

Homework Serway & Jewett:

- Ch 18, onward from page 555. OQs: 11; Probs: 15, 17, 21, 25, 33, 81, 83, 85, 87