Waves
Power of a Wave

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May 18, 2018

Last time

- solutions to the wave equation
- sine waves
- transverse speed of an element of the medium

Overview

- energy transfer by a sine wave

Rate of Energy Transfer in Sine Wave

Waves do transmit energy.

A wave pulse causes the mass at each point of the string to displace from its equilibrium point.

At what rate does this transfer happen? (Find $\frac{dE}{dt}$)

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Consider the kinetic and potential energies in a small length of string.

Kinetic:

$$\mathrm{dK} = \frac{1}{2}(\mathrm{dm})v_y^2$$

Replacing v_y:

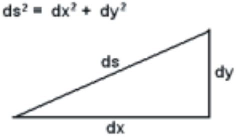
$$\mathrm{dK} = \frac{1}{2}(\mathrm{dm})A^2\omega^2\cos^2(kx - \omega t)$$

Rate of Energy Transfer in Sine Wave

Potential:

$$\mathrm{dU} = F\,\mathrm{d}\ell = T(\mathrm{ds} - \mathrm{dx})$$

where $\mathrm{d}\ell = \mathrm{ds} - \mathrm{dx}$ is the amount by which a small element of the string is stretched, ds is the stretched length and dx is the unstretched length.



$$\mathrm{ds} = \sqrt{\mathrm{dx}^2 + \mathrm{dy}^2} = \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2}\,\mathrm{dx} \approx \left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right]\mathrm{dx}$$

[1]Diagram from solitaryroad .com, James Miller.

Rate of Energy Transfer in Sine Wave

$$\mathrm{ds} - \mathrm{dx} = \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 \mathrm{dx}$$

$$\begin{aligned}
\mathrm{dU} &= \frac{1}{2} T \left(\frac{\partial y}{\partial x}\right)^2 \mathrm{dx} \\
&= \frac{1}{2} T \left(Ak \cos(kx - \omega t)\right)^2 \mathrm{dx} \\
&= \frac{1}{2} \mu \omega^2 A^2 \cos^2(kx - \omega t)\, \mathrm{dx}
\end{aligned}$$

having used $v = \omega / k$ and $v = \sqrt{T/\mu}$ in the last line.

Rate of Energy Transfer in Sine Wave

$$\mathrm{d}K = \frac{1}{2}\mu\,\mathrm{d}x\,A^2\omega^2\cos^2(kx-\omega t)$$

$$\mathrm{d}U = \frac{1}{2}\mu A^2\omega^2\cos^2(kx-\omega t)\,\mathrm{d}x$$

Adding $\mathrm{d}U + \mathrm{d}K$ gives

$$\mathrm{d}E = \mu\omega^2 A^2\cos^2(kx-\omega t)\,\mathrm{d}x$$

Integrating over one wavelength gives the energy per wavelength:

$$\begin{aligned} E_\lambda &= \mu\omega^2 A^2\int_0^\lambda \cos^2(kx-\omega t)\,\mathrm{d}x \\ &= \mu\omega^2 A^2\frac{\lambda}{2} \end{aligned}$$

Rate of Energy Transfer in Sine Wave

For one wavelength:

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

Power averaged over one wavelength:

$$P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T}$$

Average power of a wave on a string:

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

Question

Quick Quiz 16.5[1] Which of the following, taken by itself, would be most effective in increasing the rate at which energy is transferred by a wave traveling along a string?

(A) reducing the linear mass density of the string by one half
(B) doubling the wavelength of the wave
(C) doubling the tension in the string
(D) doubling the amplitude of the wave

[1] Serway & Jewett, page 496.

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Summary

- energy transfer by a sine wave

Homework Serway & Jewett:

- Ch 16, onward from page 499. Probs: 33, 35, 61