Optics
Image formation from Mirrors
Refracting Surfaces

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June 8, 2018

Last time

- ray diagrams and terminology
- image formation from mirrors

Overview

- image formation from mirrors
- refracting surfaces

Ray Diagrams for Spherical Mirrors

For a ray diagram: draw at least two rays that you know the path of accurately.

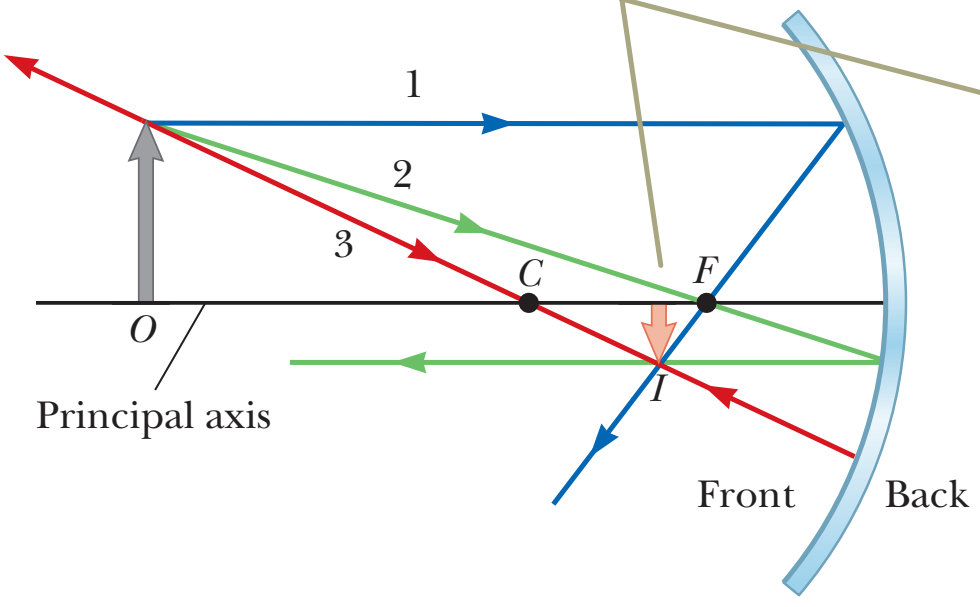
For Spherical mirrors:

1. Draw a ray from the top of the object parallel to the principle axis reflected through the focal point F.
2. Draw a ray from the top of the object through the focal point and reflected parallel to the principal axis.
3. Draw a ray from the top of the object through the center of curvature C and reflected back on itself.

Where the lines meet, an image is formed.

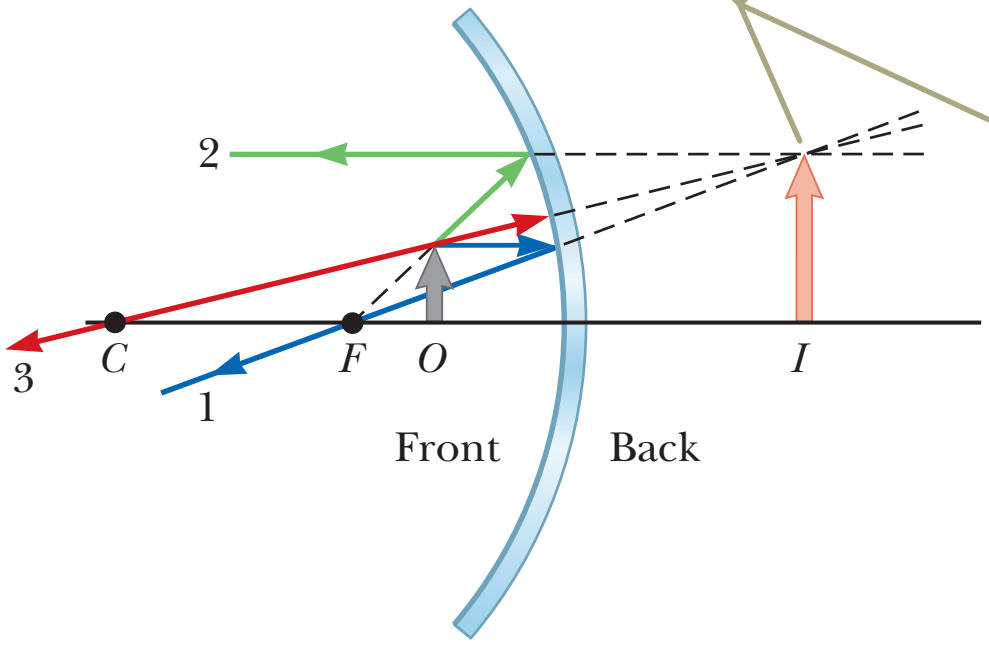
Examples of Ray Diagrams

When the object is located so that the center of curvature lies between the object and a concave mirror surface, the image is real, inverted, and reduced in size.



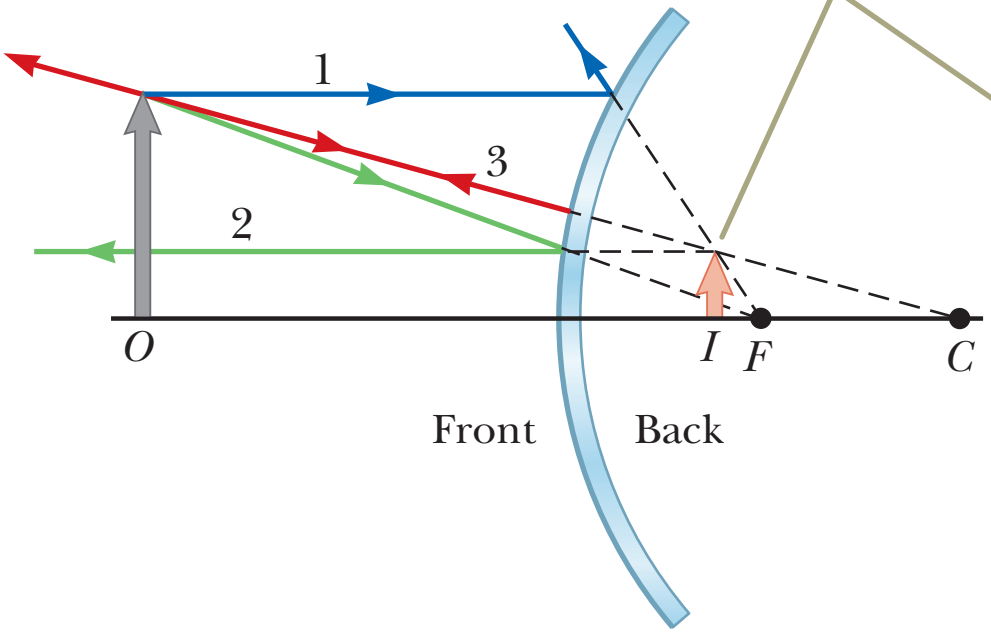
Examples of Ray Diagrams

When the object is located between the focal point and a concave mirror surface, the image is virtual, upright, and enlarged.



Examples of Ray Diagrams (Convex Mirror)

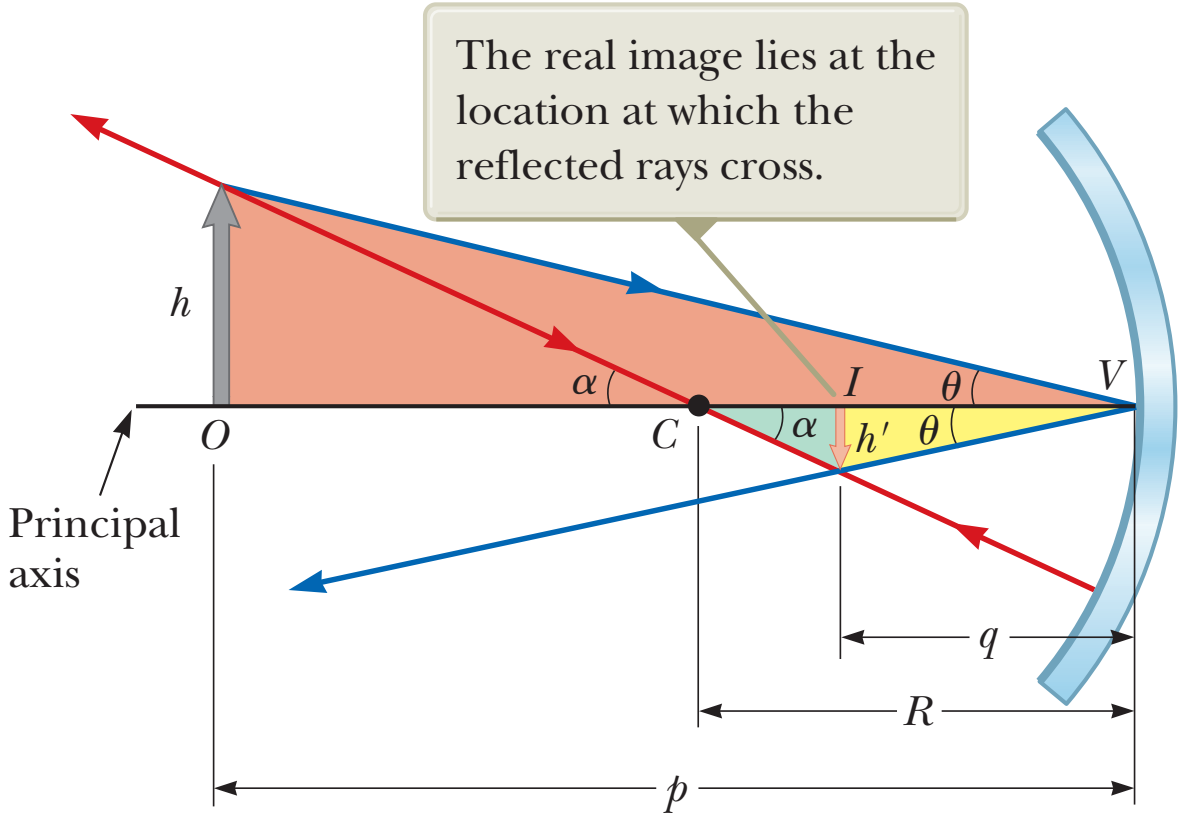
When the object is in front of a convex mirror, the image is virtual, upright, and reduced in size.



Cases for Spherical Mirrors

$f > 0$, converging, concave	$p > 2f$	real	inverted	diminished
	$f < p < 2f$	real	inverted	enlarged
	$p < f$	virtual	upright	enlarged
$f < 0$, diverging, convex	any p	virtual	upright	diminished

Concave Mirrors and the Mirror Equation



The angles of incidence and reflection are the same magnitude, θ.

So, $\frac{h'}{h} = -\frac{q}{p}$, and

$$M = -\frac{q}{p}$$

Concave Mirrors and the Mirror Equation

Looking at the green triangle and the (small) red triangle with angle α:

$$\frac{-h'}{R-q} = \frac{h}{p-R}$$

which rearranges to

$$\frac{-h'}{h} = -\frac{q-R}{p-R}$$

Using our magnification expression:

$$\frac{q}{p} = -\frac{q-R}{p-R}$$

Concave Mirrors and the Mirror Equation

$$\frac{q}{p} = -\frac{q-R}{p-R}$$

Cross-multiplying and rearranging gives

$$\frac{2}{R} = \frac{1}{p} + \frac{1}{q}$$

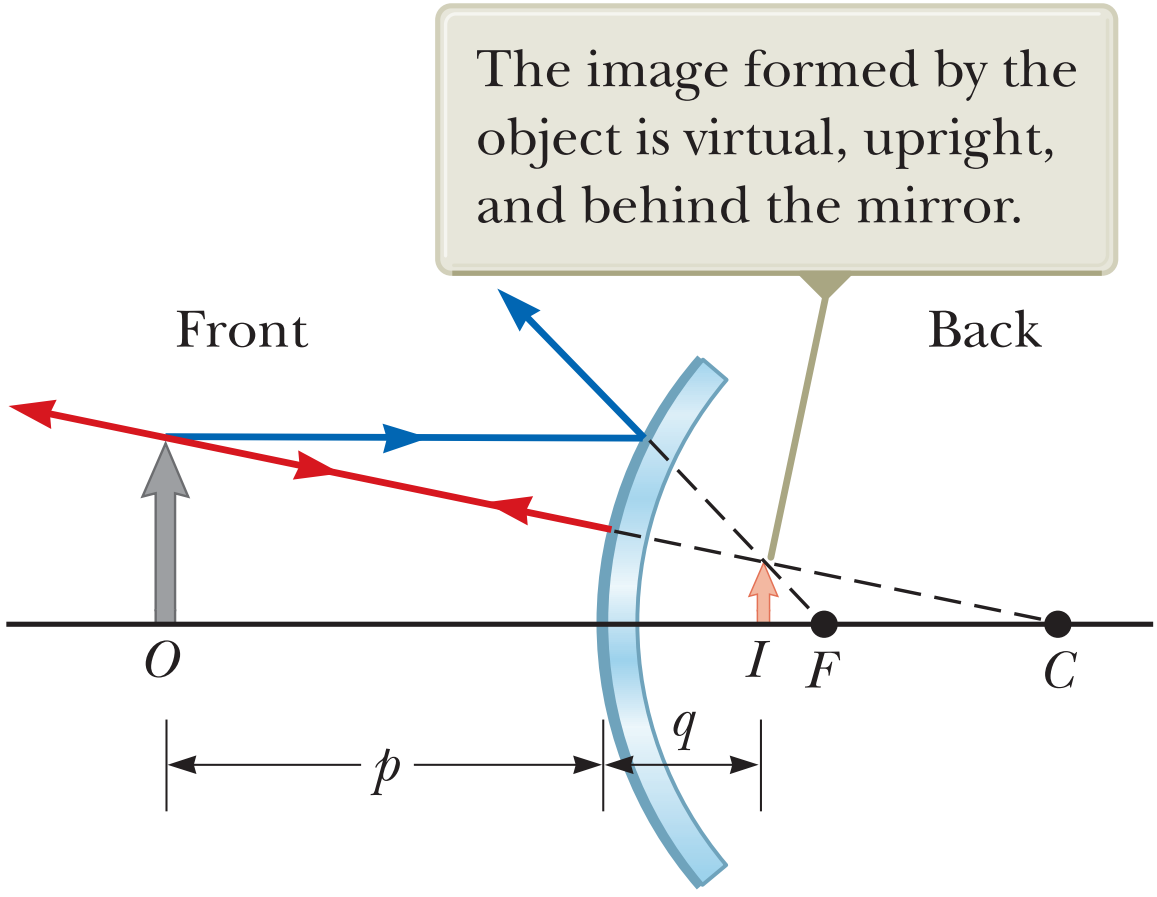
However, we already concluded that $f = R/2$, so

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

We have confirmed the mirror equation for spherical concave mirrors, and it follows from simple geometry.

Convex Mirrors

We can draw the ray diagrams for convex mirrors in the same way.



Mirror Question

Quick Quiz 36.3[1] Consider the image in the mirror in shown. Based on the appearance of this image, which of the following should you conclude?

(A) the mirror is concave and the image is real

(B) the mirror is concave and the image is virtual

(C) the mirror is convex and the image is real

(D) the mirror is convex and the image is virtual

NASA

[1] Serway & Jewett, page 1098.

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NASA

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Sign Conventions for Mirrors!

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Variable	is Positive	is Negative
p	object in front of mirror	—
q	image in front of mirror (real)	image behind mirror (virtual)
h' and M	image upright	image inverted
f and R	concave mirror	convex mirror

Example 36.4: Convex Mirror Image

An automobile rearview mirror shows an image of a truck located 10.0 m from the mirror. The focal length of the mirror is −0.60 m.

Find the position of the image of the truck and the magnification of the image.

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$$q = \underline{-0.57 \text{ m}}$$

Find the magnification of the image.

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Find the magnification of the image.

$$M = -\frac{q}{p} = \underline{+0.057}$$

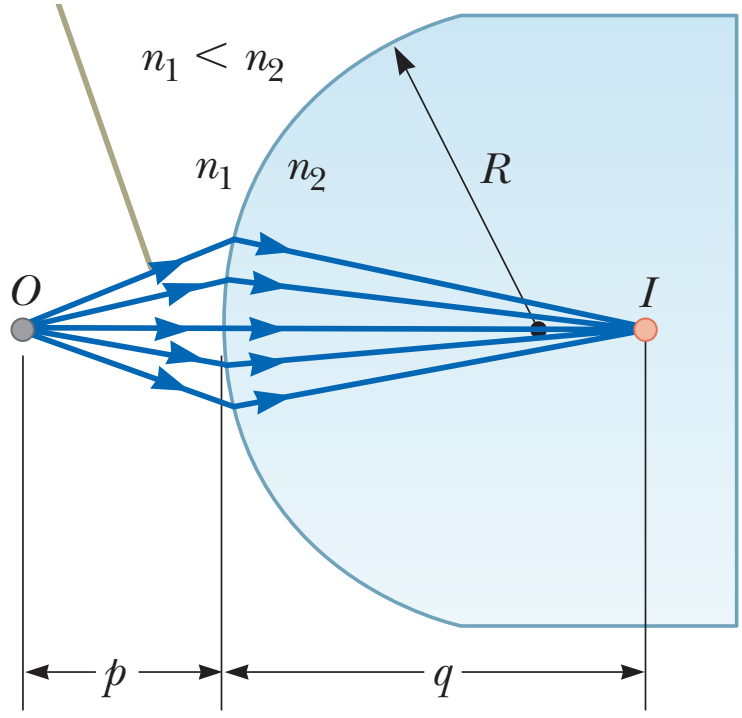
Images Formed by Refraction

When light rays change media they are bent.

This also can form images.

Images Formed by Refraction

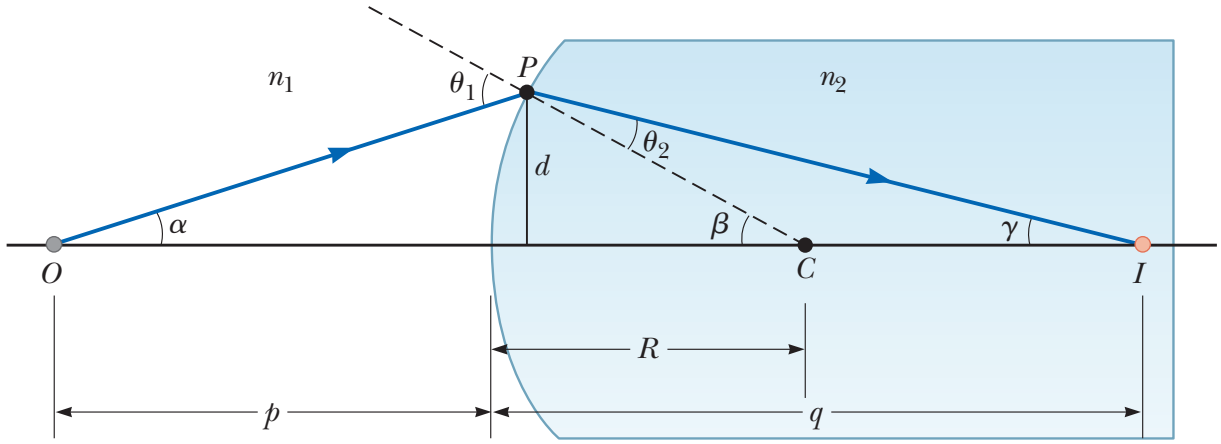
We can find the location and size of the image formed by considering paraxial rays.



For paraxial rays, Snell's law becomes:

$$n_1\theta_1 = n_2\theta_2 \tag{1}$$

Images Formed by Refraction



Looking at the diagram:

$$180^\circ - \theta_1 = 180^\circ - \alpha - \beta \quad \Rightarrow \quad \theta_1 = \alpha + \beta \qquad (2)$$

likewise:

$$\beta = \gamma + \theta_2 \qquad (3)$$

Multiply (2) by n_1 and (3) by n_2, add them and subtract (1):

$$n_2\beta = n_1\alpha + n_1\beta + n_2\gamma$$

Rearranging:

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$

Images Formed by Refraction

For small angles (paraxial approx):

$$\alpha \approx \tan\alpha = \frac{d}{p}$$

Similarly,

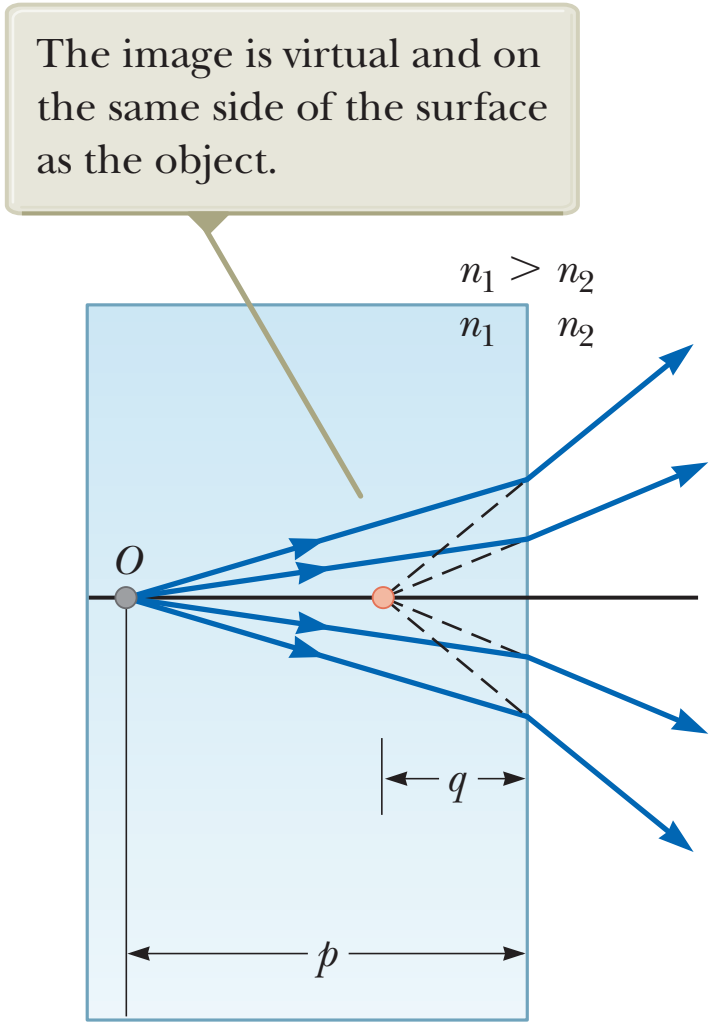
$$\beta \approx \frac{d}{R} \text{ and } \gamma \approx \frac{d}{q}$$

So,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

Flat Refracting Surfaces

(Like a rectangular fish tank.)



In this case $R \to \infty$.

$$\frac{n_1}{p} + \frac{n_2}{q} = 0$$

And so

$$q = -\frac{n_2}{n_1} p$$

Flat Refracting Surfaces Example (Problem 30)

A cubical block of ice 50.0 cm on a side is placed over a speck of dust on a level floor. Find the location of the image of the speck as viewed from above. The index of refraction of ice is 1.309.

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$$\Rightarrow q = -\frac{n_2}{n_1} p$$

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$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \ , \ R \to \infty$$

$$\begin{aligned} \Rightarrow q &= -\frac{n_2}{n_1} p \\ &= -\frac{1}{1.309} (50.0 \text{ cm}) \\ &= \underline{38.2 \text{ cm}} \end{aligned}$$

Sign Conventions for Refracting Surfaces!

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

Variable	is Positive	is Negative
p	object in front of surface	[virtual object][2]
q	image behind surface (real)	image in front of surface (virtual)
h' (and M)	image upright	image inverted
R	object faces convex surf. (C behind surface)	object faces concave surf. (C in front of surface)

C is the center of curvature.

$M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p}$

[2] Will be useful in derivations.

Summary

- image formation from mirrors
- refracting surfaces

4th Collected Homework due Monday June 18.

Quiz Monday.

Homework Serway & Jewett:

- Carefully read *all* of Chapter 36. (over the next few days)
- Ch 36, onward from page 1123. OQs: 13; Probs: 29, 34, 37
- (to be set in future, will not be on Monday's quiz) Ch 36. OQs: 1, 3, 5, 11; CQs: 5, 9, 11; Probs: 39, 43, 53, 55, 71, 73, 87, 89