Waves
Standing Waves and Sound
Beats
Nonsinusoidal Wave Patterns

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De Anza College

May 24, 2018

Last time

- interference and sound
- standing waves and sound
- musical instruments

Reminder: Speed of Sound in Air

The speed of sound in air at 20°C

$$v = 343 \text{ m/s}$$

Since the density of air varies a lot with temperature, the speed of sound varies also.

For temperatures near room temperature:

$$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{Cel}}}{273}}$$

where T_{Cel} is the temperature in **Celsius**.

Warm Up Question

Quick Quiz 18.5[1] Balboa Park in San Diego has an outdoor organ. When the air temperature increases, the fundamental frequency of one of the organ pipes

(A) stays the same,

(B) goes down,

(C) goes up,

(D) is impossible to determine.

[1]Serway & Jewett, page 548.

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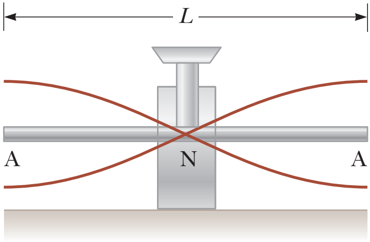
Overview

- standing waves in rods and membranes
- beats
- nonsinusoidal waves

Standing waves in rods

Both longitudinal and transverse standing waves can be created in rods.

Illustration of a longitudinal standing oscillation in a rod, free at the ends, and clamped in the middle:



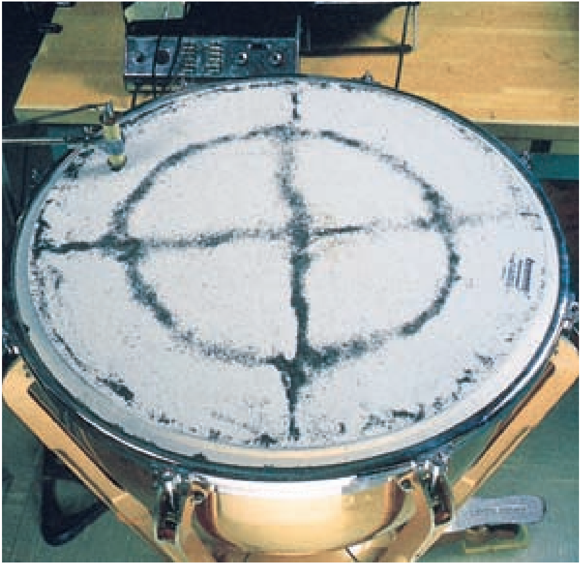
$$\lambda_1 = 2L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

The brown curve represents left-right displacement of the particles in the rod.

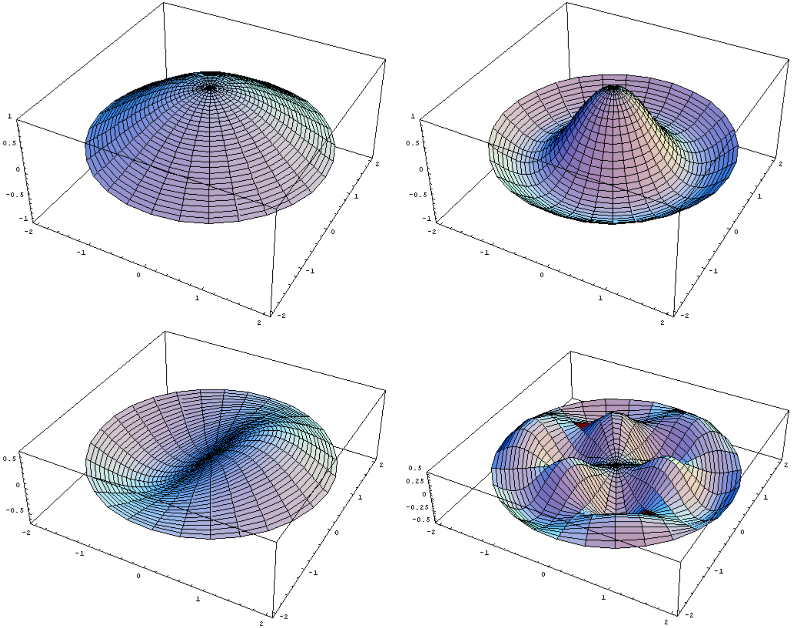
Some musical instruments make use of transverse standing waves, *eg.* triangle, glockenspiel, chimes.

Standing waves in membranes

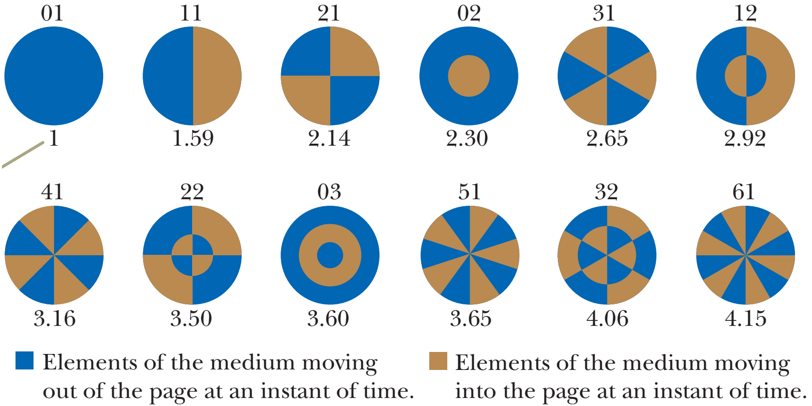


[1]Dust on a kettledrum, Halliday, Resnick, Walker, page 434.

Standing waves in membranes



Standing waves in membranes



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[1] Serway & Jewett, page 550.

Beats

We already considered interference of sine waves when both waves had the same frequency. But what if they do not?

Consider two waves with the same amplitude but different frequencies, f, and therefore different angular frequencies, ω:

$$\begin{aligned} y_1(x,t) &= A\sin(k_1 x - \omega_1 t + \phi_1) \\ y_2(x,t) &= A\sin(k_2 x - \omega_2 t + \phi_2) \end{aligned}$$

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Now let's consider the effect of these waves at the point $x = 0$, and suppose $\phi_1 = \phi_2 = \frac{\pi}{2}$.

(This choice is arbitrary, but we must pick a point in space.)

Beats

The wave functions at this point are:

$$
\begin{aligned}
y_1(0,t) &= A\sin(\frac{\pi}{2} - \omega_1 t) = A\cos(2\pi f_1 t) \\
y_2(0,t) &= A\sin(\frac{\pi}{2} - \omega_2 t) = A\cos(2\pi f_2 t)
\end{aligned}
$$

Using the trig identity:

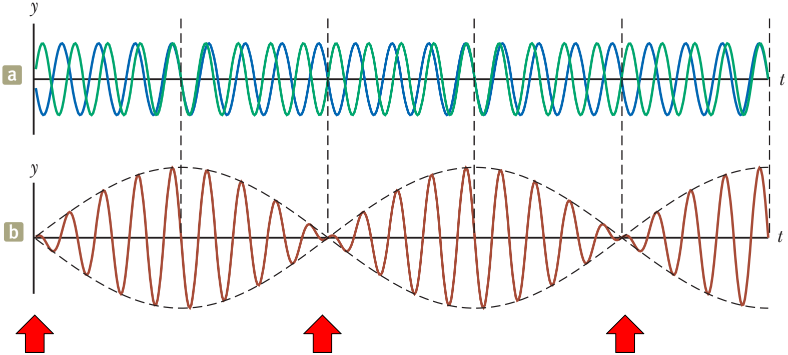
$$
\cos\theta + \cos\psi = 2\cos\left(\frac{\theta-\psi}{2}\right)\cos\left(\frac{\theta+\psi}{2}\right)
$$

$$
\begin{aligned}
y(x,t) &= y_1 + y_2 \\
&= \underbrace{\left[2A\cos\left(2\pi\,\frac{f_1-f_2}{2}\,t\right)\right]}_{\textbf{time-varying amplitude}}\ \underbrace{\cos\left(2\pi\frac{f_1+f_2}{2}\,t\right)}_{\textbf{fast oscillation}}
\end{aligned}
$$

Beats

$$y(x,t) = \left[2A\cos\left(2\pi\,\frac{f_1 - f_2}{2}\,t\right)\right]\cos\left(2\pi\frac{f_1 + f_2}{2}t\right)$$

y vs t (position, x fixed):



$t = \frac{1}{2(f_1 - f_2)}$ $t = \frac{3}{2(f_1 - f_2)}$ $t = \frac{5}{2(f_1 - f_2)}$

Beats

The time difference between minima is $\Delta t = \frac{1}{|f_1 - f_2|}$.

Thus the frequency of the beats is

$$f_{\text{beat}} = |f_1 - f_2|$$

If f_1 and f_2 are similar the beat frequency is much smaller than either f_1 or f_2.

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Humans cannot hear beats if $f_{\text{beat}} \gtrsim 30$ Hz.

If the two frequencies are very different we hear a chord.

If the two frequencies are very close, we hear periodic variations in the sound level.

This is used to tune musical instruments. When instruments are coming into tune with each other the beats get less and less frequent, and vanish entirely when they are perfectly in tune.

Question

A tuning fork is known to vibrate with frequency 262 Hz. When it is sounded along with a mandolin string, four beats are heard every second. Next, a bit of tape is put onto each tine of the tuning fork, and the tuning fork now produces five beats per second with the same mandolin string. What is the frequency of the string?[1]

(A) 257 Hz
(B) 258 Hz
(C) 266 Hz
(D) 267 Hz

[1]Serway & Jewett, objective question 7.

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Nonsinusoidal Periodic Waves

Not all periodic wave functions are pure, single-frequency sinusoidal functions.

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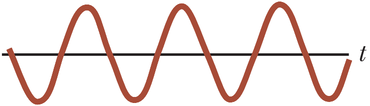
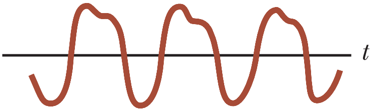
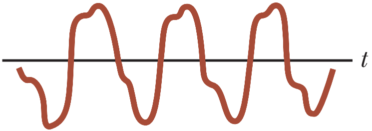


Tuning fork



Flute



Clarinet

For example this is why a flute and a clarinet playing the same note still sound a bit different.

Other harmonics in addition to the fundamental are sounded.

Summary, Announcements, and HW

- standing waves in rods and membranes
- beats
- introducing non-sine oscillations

Collected Homework, due Tuesday, May 29.

Drop Deadline Friday, June 1.

3rd Test Friday, June 1.

No Class on Monday May 28. (Memorial day)

Homework Serway & Jewett:

- Ch 18, onward from page 555. OQs: 3, 5; CQs: 1, 3; Probs: 55, 57, 69