Optics
Image Formation by Refracting Surfaces and Lenses

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Last time

- images formed by spherical mirrors
- refracting surfaces

Overview

- lenses
- images formed by lenses
- images formed by lens combinations

Images Formed by Refraction

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

Images Formed by Thin Lenses

We will derive the **thin lens equation**

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

(Notice it is the same as the mirror equation!)

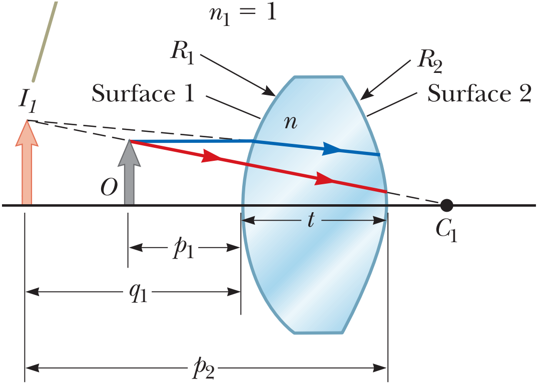
And the **lens maker's equation**

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We do this by considering each side of the lens as a refracting surface.

Images Formed by Thin Lenses

A thick lens:

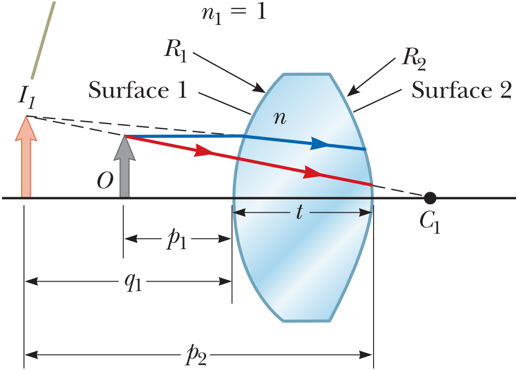


We can imagine that there first surface forms an image I_1, and that image becomes the *object* for the second surface.

The pink arrow shows I_1

First image formed

Let $n_1 = 1$, $n_2 = n$.



$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1}$$

Second image formed

$$\frac{n}{p_2} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

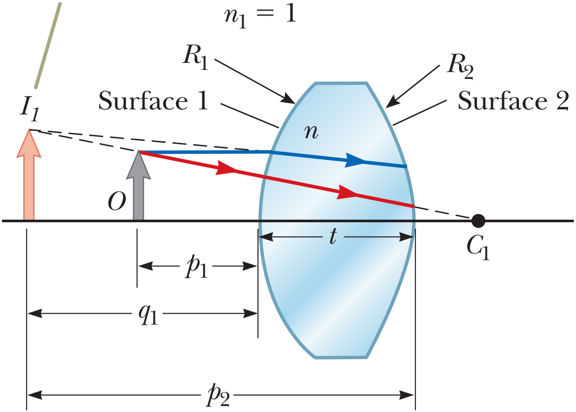
We must relate p_2 to q_1. The new object is the image from the previous surface so:

$$p_2 = -q_1 + t$$

where t is the lens thickness.

p_2 could be positive (real object)...

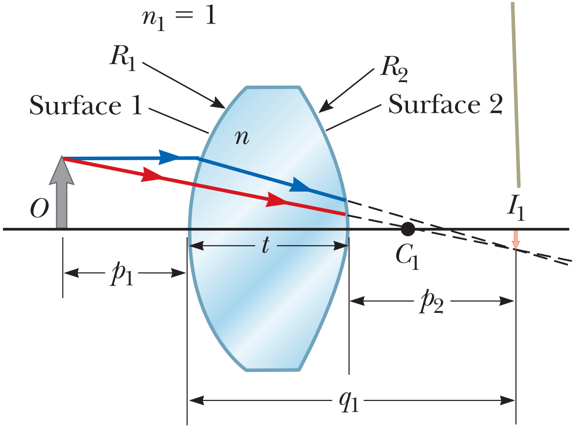
If q_1 is negative (virtual image 1)...



... p_2 is positive (object in front of surface).

p_2 could be negative (virtual object)...

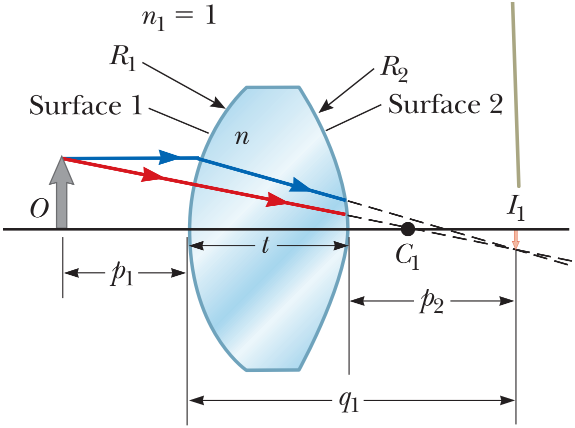
If q_1 is positive (real image 1)...



... p_2 is negative (object behind surface).

p_2 could be negative (virtual object)...

If q_1 is positive (real image 1)...



... p_2 is negative (object behind surface).

Either way, the sign is taken care of in the expression $p_2 = -q_1 + t$.

Relate the original object to the final image

$$\text{surface 1} \qquad \frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1} \tag{1}$$

$$\text{surface 2} \qquad \frac{n}{(-q_1 + t)} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

For a thin lens, t is negligible, so take it to be zero.

$$\text{surface 2} \qquad -\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2} \tag{2}$$

Add equation (1) to equation (2) to get

$$\frac{1}{p_1} + \frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thin Lens Equation

Lastly, measure the object distance to be $p = p_1 + t/2$ and $q = q_2 + t/2$.

Since t is effectively zero, we have:

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

p and q can vary, but since n, R_1, and R_2 are constant for a fixed lens, we know $\frac{1}{p} + \frac{1}{q} = \text{const}$.

Thin Lens Equation

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If $p \to \infty$, $q \to f$, by the definition of the focal length.

This gives the **thin lens equation**

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Lens Maker's Equation

Thin lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

and

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The LHS is the inverse of the focal length, giving the lens maker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Magnification with a Lens

By definition,

$$M = \frac{h'}{h}$$

And it follows from simple trigonometry that

$$M = -\frac{q}{p}$$

Same as for a mirror!

Sign Conventions for Lenses!

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Variable	is Positive	is Negative
p	object in front of surface	[virtual object][1]
q	image behind lens (real)	image in front of lens (virtual)
h' (and M)	image upright	image inverted
R_1 and R_2	object faces convex surf. (C behind surface)	object faces concave surf. (C in front of surface)
f	lens is **converging**	lens is **diverging**

[1]Useful in derivations.

Understanding the Sign of f

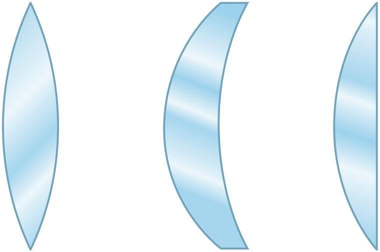
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Always $n \geqslant 1$, so $(n-1)$ is positive (if zero, there's no lens!).

$\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ could be positive or negative, depending on the signs and magnitudes of R_1 and R_2.

Sign Examples: Converging Lenses

Biconvex	Convex-concave	Plano-convex



R_1 R_2	R_1 R_2	R_1 R_2
R_1 +ve	R_1 +ve	R_1 +ve
R_2 −ve	R_2 +ve	R_2 ∞
	$R_1 < R_2$	
f +ve	f +ve	f +ve

Sign Examples: Diverging Lenses

Biconcave	Convex-concave	Plano-concave

Biconcave	Convex-concave	Plano-concave
R_1 $-$ve	R_1 $+$ve	R_1 ∞
R_2 $+$ve	R_2 $+$ve	R_2 $-$ve
	$R_1 > R_2$	
f $-$ve	f $-$ve	f $-$ve

Focal Length Question

Quick Quiz 36.6 What is the focal length of a pane of window glass?

(A) zero
(B) infinity
(C) the thickness of the glass
(D) impossible to determine

Focal Length Question

Quick Quiz 36.6 What is the focal length of a pane of window glass?

(A) zero

(B) infinity $\longleftarrow$

(C) the thickness of the glass

(D) impossible to determine

Ray Diagrams for Converging Lenses

Again, draw rays whose behavior we know.

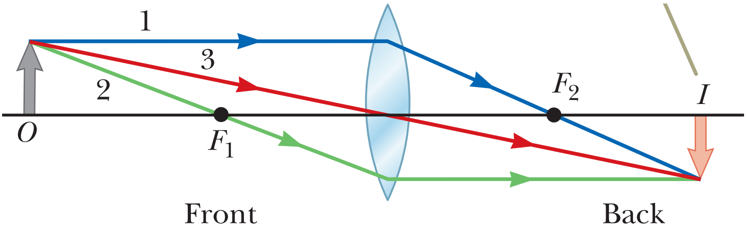
Rays parallel to the principle axis are refracted through the focal point.

Rays that travel through the center of the lens are (effectively) not refracted.

For Converging Lenses:

1. Draw a ray from the top of the object parallel to the principle axis refracted through the focal point F.
2. Draw a ray from the top of the object through the focal point (or back to the focal point) and refracted parallel to the principal axis.
3. Draw a ray from the top of the object through the center of the lens, and continuing in a straight line.

Ray Diagrams for Converging Lenses



Ray Diagrams for Converging Lenses

There are two cases of interest:

Object is beyond focal point

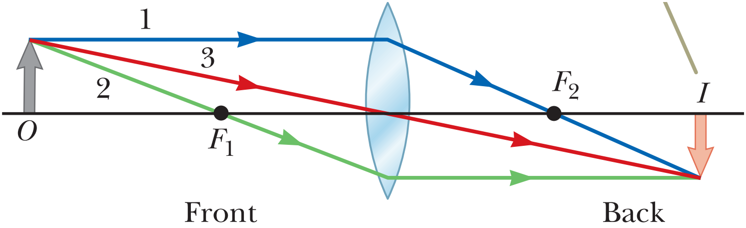


Image is real and inverted.

Object Beyond Focal Point

The object is the Sun.

[1]Photo from
http://www.mahalo.com/how-to-start-a-fire-with-a-magnifying-glass

Ray Diagrams for Converging Lenses

Object is closer than the focal point

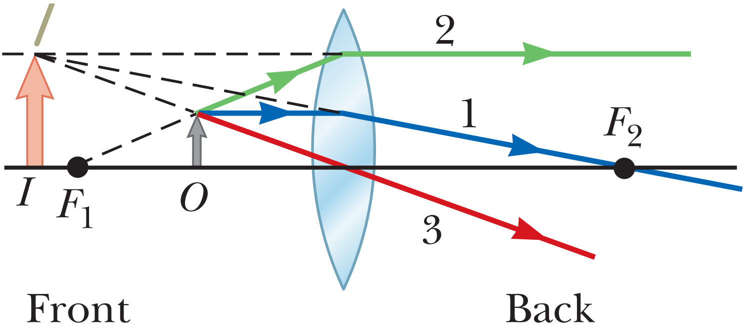


Image is virtual, upright, and magnified.

Object Closer Than the Focal Point

The object is the stamp.

Converging Lens Example

A converging lens has a focal length of 10.0 cm.

(a) An object is placed 30.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

(b) An object is placed 5.00 cm from the lens. Find the image distance and describe the image.

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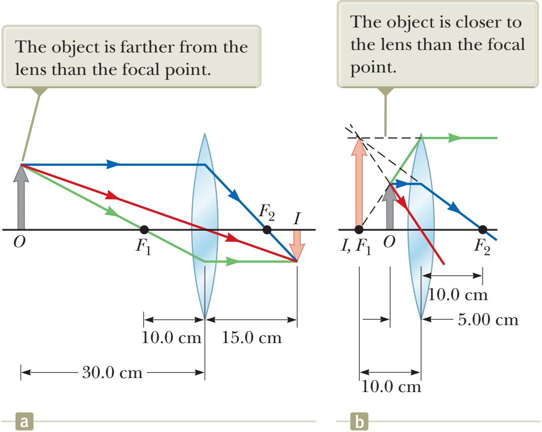

Converging Lens Example

(a) An object is placed 30.0 cm from the lens.

$$q = +15.0\text{cm}$$

$$M = -0.500$$

(b) An object is placed 5.00 cm from the lens.

$$q = -10.0\text{cm}$$

$$M = +2.00$$

(c) An object is placed 10.0 cm from the lens. Find the image distance and describe the image.

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$$M = +2.00$$

(c) An object is placed 10.0 cm from the lens. Find the image distance and describe the image.

$$q = \infty$$

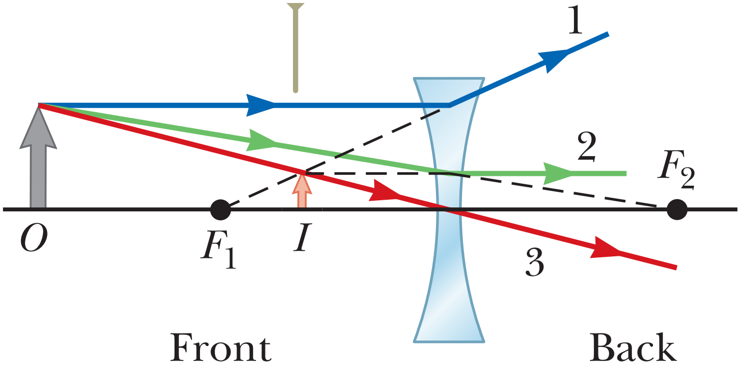
No image is formed!

Ray Diagrams for Diverging Lenses

For Diverging Lenses:

1. Draw a ray from the top of the object parallel to the principle axis refracted outward directly away focal point F on the front side of the lens.
2. Draw a ray from the top of the object toward the focal point on the back side of the lens and refracted parallel to the principal axis.
3. Draw a ray from the top of the object through the center of the lens, and continuing in a straight line.

Ray Diagrams for Diverging Lenses



The image formed is upright, virtual, and smaller in size.

Diverging Lenses

The image formed is upright, virtual, and smaller in size.

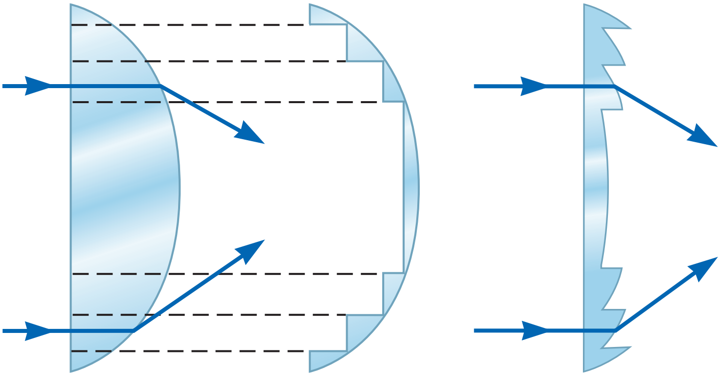
[1]Photo from the Capilano University Physics Dept. webpage.

Cases for Lenses

$f > 0$, converging, convex	$p > 2f$	real	inverted	diminished
	$f < p < 2f$	real	inverted	enlarged
	$p < f$	virtual	upright	enlarged
$f < 0$, diverging, concave	any p	virtual	upright	diminished

[1] Notice that this table is actually identical to the one for mirrors, except that the words "convex" and "concave" have been swapped.

Fresnel Lenses



Combinations of Lenses

Two or more lenses can be used in series to produce and image.

This is used in

- eyeglasses (your eyeball lens is the second lens)
- refracting telescopes
- microscopes
- camera zoom lenses

Lenses can also be used together with curved mirrors, for example in reflecting telescopes.

Combinations of Lenses

Two thin lenses placed right up against each other (touching) will act like one lens with a focal length

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

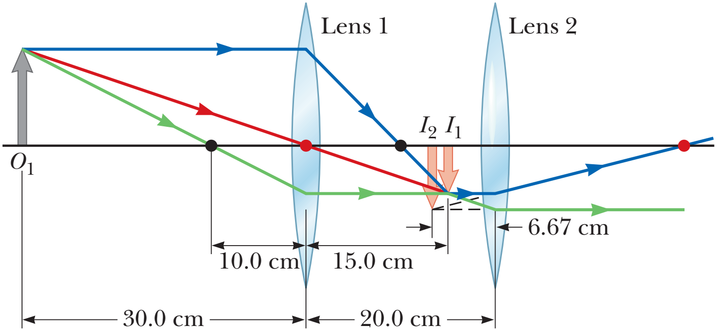
In other cases, the intermediate image formed by the first lens must be found, and imaged again in the second lens to find the final image.

The magnification of a series of lenses is the product of the magnification produced by each one.

Combinations of Lenses Example 36.10

Two thin converging lenses of focal lengths $f_1 = 10.0$ cm and $f_2 = 20.0$ cm are separated by 20.0 cm as illustrated. An object is placed 30.0 cm to the left of lens 1.

Find the position and the magnification of the final image.



Summary

- lenses
- images formed by lenses
- images formed by lens combinations

Collected Homework! due Monday, June 18.

Final Exam 9:15-11:15am, Tuesday, June 26.

Homework Serway & Jewett:

- Carefully read *all* of Chapter 36.
- prev: Ch 36, onward from page 1123. OQs: 1, 3, 5, 11; CQs: 5, 9, 11; Probs: 39, 43, 53, 71
- new: Ch 36, onward from page 1123. Probs: 55, 73, 87, 89, (93)