Simulation

1. Problem #1

Choose (b) Simpy discrete event simulation of a bank.

Results for the base model and default inputs:

Average wait for 50 completions was 3.66 min, and the total delay is 183 mins.
Average wait for 50 completions was 2.62 min, and the total delay is 131 mins.
Average wait for 50 completions was 8.97 min, and the total delay is 448 mins.
Average wait for 50 completions was 5.34 min, and the total delay is 267 mins.

Decide the number of runs in the experiment:

(1) compute the mean $\bar{x}$ of the average wait time;

(2) compute the variance of the sample Var(x) = $\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$;

(3) compute the accuracy $r = \frac{t_{[1-\frac{\alpha}{2},n-1]}\frac{\sqrt{Var(x)}}{\sqrt{n}}}{\bar{x}} \times 100$;

(4) if r > desired value, then n = n+1 and go to (1), else stop.

Follow this method, choose α = 0.1, r = 5, need to run 931 times, and Var(x) = 24.135.

2. Problem #2

Choose time in bank, arrival interval and number of counters as factors and each one has two levels, so the total number of experiments is 8. Time in bank, level-1: 6, level-2: 24; arrival interval, level-1: 5, level-2: 20; number of counters, level-1: 1, level-2: 4.

Table 1, Simulation matrix (- for level-1, + for level-2)

Time in bank	Arrival interval	Number of counters	Observation	
-	-	-	444.396258	(1)
+	-	-	3767.521889	a
-	+	-	32.760095	b
+	+	-	1373.376815	ab
-	-	+	1.503347	c
+	-	+	344.240164	ac
-	+	+	0	bc
+	+	+	6.013386	abc

The factors are variables have directly impacts on wait time of the system; the level-1 and level-2 are chosen to try to cover the ranges of the values, and we assume that the responses are linear over the ranges; each simulation use 10 replications.

A = 1/4n[a+ab+ac+abc-(1)-b-c-bc] = 125.3123
B = 1/4n[b+ab+bc+abc-(1)-a-c-ac] = -78.6378
C = 1/4n[c+ac+bc+abc-(1)-a-b-ab] = -131.657
AB = 1/4n[abc-bc+ab-b-ac+c-a+(1)] = -57.9808
AC = 1/4n[(1)-a+b-ab-c+ac-bc+abc] = -107.875
BC = 1/4n[(1)+a-b-ab-c-ac+bc+abc] = 61.65128
ABC = 1/4n[abc-bc-ac+c-ab+b+a-(1)] = 41.14464

We can see that effects of Time in bank, Number of counters and the interaction of these two are large, and they have positive, negative and negative directions with wait time. The influences from big to small are C, A and AC. Since A, C and AC are the largest three, the regression model should be:

$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$

$\hat{y} = 74.62265 + (125.2123/2)\, x_1 + (-131.657/2)\, x_2 + (-107.875/2)\, x_1 x_2$

$= 74.62265 + 62.60615 x_1 - 65.8285 x_2 - 53.9375 x_1 x_2$

3. Problem #3

Table 1, Simulation matrix (- for level-1, + for level-2)

Time in bank	Arrival interval	Number of counters	Observation	
-	-	-	35865.9369218	(1)
+	-	-	356122.545752	a
-	+	-	2255.27022932	b
+	+	-	111231.611746	ab
-	-	+	68.277619933	c
+	-	+	27701.769777	ac
-	+	+	0.1326532	bc
+	+	+	270.1783	abc

Each simulation use 931 replications.

A = 1/4n[a+ab+ac+abc-(1)-b-c-bc] = 122.7542
B = 1/4n[b+ab+bc+abc-(1)-a-c-ac] = -82.1701
C = 1/4n[c+ac+bc+abc-(1)-a-b-ab] = -128.205
AB = 1/4n[abc-bc+ab-b-ac+c-a+(1)] = -64.0826
AC = 1/4n[(1)-a+b-ab-c+ac-bc+abc] = -107.768
BC = 1/4n[(1)+a-b-ab-c-ac+bc+abc] = 67.40115

ABC = 1/4n[abc-bc-ac+c-ab+b+a-(1)] = 49.3869

We can see that A, C and AC are still the largest three, the influences from big to small are still C, A and AC, and the directions didn't change. The regression model should be:

$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$

$\hat{y} = 71.632 + (122.7542/2)\, x_1 + (-128.205/2)\, x_2 + (-107.768/2)\, x_1 x_2$

$= 71.632 + 61.377 x_1 - 64.1024 x_2 - 53.8842 x_1 x_2$

On one hand, we can see that the regression models built are different, which means that the prediction accuracy must be improved by more times of runs; One the other hand, limited runs could also help to gain insight from the data, especially when the cost of each run is large.