

## Belief Networks Exercises

### Exercise 3.1:

Based on the party animal belief network structure, the following factorization is chosen:

$$p(A, U, H, D, P) = p(D)p(P)p(H|P)p(U|P, D)p(A|U)$$

$$p(P=tr|A=tr, H=tr)$$

$$= \frac{\sum_{U,D} p(P=tr, A=tr, H=tr)}{\sum_{U,D,P} p(A=tr, H=tr)}$$

$$= \frac{\sum_{U,D} p(D)p(P=tr)p(H=tr|P=tr)p(U|P=tr, D)p(A=tr|U)}{\sum_{U,D,P} p(D)p(P)p(H=tr|P)p(U|P, D)p(A=tr|U)}$$

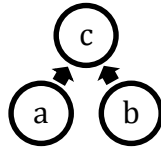
$$= \frac{0.4 \times 0.2 \times 0.9 \times 0.999 \times 0.95 + 0.6 \times 0.2 \times 0.9 \times 0.9 \times 0.95}{0.4 \times 0.2 \times 0.9 \times 0.999 \times 0.95 + 0.6 \times 0.2 \times 0.9 \times 0.9 \times 0.95 + 0.4 \times 0.8 \times 0.2 \times 0.9 \times 0.95 + 0.6 \times 0.8 \times 0.2 \times 0.01 \times 0.95}$$

$$= \frac{0.169128}{0.169128 + 0.05856}$$

$$\approx 0.7428$$

### Exercise 3.2:

(i) True. The only possible graph built is like this:



Since  $c$  is not in the conditioning set,  $a$  and  $b$  are unconditional independence.

(ii) False. The conditioning on  $c$  makes  $a$  and  $b$  graphical dependence.

### Exercise 3.3:

1. False. After the conditioning on  $d$ , the path from  $t$  to  $s$  is no more blocked, so  $t$  and  $s$  are dependent.
2. True. After the conditioning on  $s$ ,  $l$  and  $b$  are made independent because they are the descendants of  $s$  and there's no path between them.
3. True. Although  $l$  is conditioned, the paths from  $a$  to  $s$  are still blocked.
4. False. The conditioning on  $d$  make the path from  $t$  to  $s$  available, so they are dependent.