Graph Signal Processing-Based Network Health Estimation for Next Generation Wireless Systems

Faisal Yaseen, Usama Masood, Ahmad Nayyar Hassan, and Ijaz Haider Naqvi

***Abstract*— In this letter, we propose a novel network health estimation technique for wireless cellular networks. The proposed scheme makes use of graph signal processing techniques to estimate network health over the entire coverage area with sparse availability of measured data. To achieve this objective, we solve an optimization problem on graph using proximal splitting method. The results show that the proposed technique outperforms existing methods such as kriging for network health estimation with an improved accuracy of more than 60% along with a reduced time complexity of $\mathcal{O}(n)$ compared with $\mathcal{O}(n^4)$ for kriging. Thereafter, coverage holes present in the network are found with an extremely high detection rate and extremely low false positive rate. The results, unlike kriging, are devoid of any spatial bias present in the training data.**

***Index Terms*— Graph signal processing (GSP), proximal splitting, network health estimation, coverage optimization.**

I. Introduction

In RECENT times, an exponential growth in the number of mobile users has been accomplished due to advanced digital and RF circuit fabrication, large scale circuit integration and progress in miniaturization technologies. We have a ubiquitous network of wireless technologies and their role has been extended to a range of diverse applications such as home automation, smart grids, health care, education etc. Therefore, network operators face an increasingly large number of challenges as they not only have to provide satisfactory coverage but also have to cater for, often conflicting, demands of assorted applications.

In cellular networks, the received signal power varies over space, because of the random propagation channel, and results in outage (or very poor service) at different locations. One of the goals for 5G networks is to meet the Quality of Service (QoS) requirements regardless of the locations of the mobile users. The revenue of the mobile operators is directly linked to the QoS provided across the entire coverage area. The base station antennas are generally placed to ensure highest coverage at the time of greenfield deployment. However, factors such as the availability of real estate and later geographical changes become an impediment to optimum coverage. Thus the task at hand is to find out ways to improve coverage in a continuously changing wireless channel.

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F. Yaseen and I. H. Naqvi are with the Lahore University of Management Sciences, Lahore 54792, Pakistan (e-mail: ijaznaqvi@lums.edu.pk).

U. Masood was with the Lahore University of Management Sciences, Lahore 54792, Pakistan. He is now with the University of Oklahoma, Tulsa, OK 74135 USA.

A. N. Hassan was with the Lahore University of Management Sciences, Lahore 54792, Pakistan. He is now with the University of Waterloo, Waterloo, ON N2L 3G1, Canada.

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There have been numerous approaches to wireless network management; almost all of which require measurements of network performance indicators across the coverage area. Since the measurements can only be acquired over finite locations, interpolation techniques are typically employed to estimate network health at unknown locations. These techniques are used for creating a radio environmental map (REM) [1] that represents the geo-located RF conditions in the area. One of the most commonly used schemes for spatial interpolation is kriging that is an exact interpolation technique and is highly dependent on spatial correlation. Kriging has been used to estimate the coverage map using received signal strength measurements collected through drive tests [2]. Furthermore, Bayesian kriging is used for the predication of received signal code power (RSCP) [3] and for coverage analysis of received signal received power (RSRP) [4], [5].

Recently, wireless network management has been carried out in a smart manner, for example in self organizing networks (SON), where machine learning tools are used to enable the system to organize itself in an autonomous manner. In the context of 5G networks, artificial intelligence based network management seems to be the only way forward [6]. It typically involves dimensionality reduction over massive and complex data to explore hidden structures where traditional techniques like principal component analysis (PCA) and singular value decomposition (SVD) etc. do not perform very well in terms of computational complexity and robustness [7]. Alternatively, dimensionality reduction has been performed through graphs [8], [9], where the processed data with reduced dimensions gets represented using the graph Laplacian eigenbasis. Such techniques have better computational efficiency and are insensitive to outliers and noise.

Graph Signal Processing (GSP) is an emerging field that has recently been used in diverse applications. However, to the best of our knowledge, GSP has never been used for network health estimation in cellular networks. In this letter, GSP technique has been used to estimate the network health of a wireless network. To achieve this objective, we solve an optimization problem on graph by minimizing the total variation (TV) norm with different types of regularizations namely graph TV and Tikhonov regularization. Graph TV regularization promotes sparsity while Tikhonov regularization promotes smoothness on the graph. The problem is solved by proximal splitting methods using the Douglas-Rachford algorithm. The proposed technique gives better results for coverage hole detection and significantly outperforms other spatial interpolation techniques like kriging. In addition, the complexity of the proposed scheme is $\mathcal{O}(n)$ as compared to $\mathcal{O}(n^4)$ for kriging.

II. System Model and Problem Formulation

Graph nodes are uniformly placed where the graph signal for each node consists of geographical coordinates and the signal strength (RSRP). For few of these nodes, data has been acquired through a custom developed mobile application by carrying out campus wide measurement campaigns. The acquired data was transferred to a central database server where it was filtered and cleaned before being used for coverage optimization. In order to minimize small scale fading effects, spatial binning was carried out by making use of military grade reference system (MGRS). Each grid having a dimension of 10×10 m^2 uniquely identifies spatial bins across the entire coverage space. The average of all the measurements in one MGRS grid is mapped to the top left vertex of the grid, since they depict similar RF conditions. This ensures that all the readings within a single grid are considered as one training example; otherwise it may result in over-fitting with a strong spatial bias in the interpolation. Before proceeding to the problem formulation, we briefly review fundamentals of graph signal processing [7], [10].

A. Introduction to Graph

A graph contains two sets $\mathcal{V}$, $\mathcal{E}$ and a weight matrix $\mathcal{W}$ where $\mathcal{V}$, the vertex set, represents the nodes on the graph, $\mathcal{E}$, the edge set, represents edges between two nodes and weight matrix $\mathcal{W}$ contains weight information associated between nodes. Thus, a graph is a triplet which is defined as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$. If $|\mathcal{V}| = N$, then the constructed weight matrix associated with graph $\mathcal{G}$ is $\mathcal{W} \in \mathcal{R}^{N \times N}$. Weight connecting the nodes v_j and v_k is represented by $W_{jk} = \mathcal{W}(v_j, v_k)$. If there is no edge between the nodes then $W_{jk} = 0$. The degree of the node $d(v_j)$ is a non-negative number and represents number of edges connecting a vertex. For a degree matrix $\mathcal{D}$, its Laplacian is written as $\mathcal{L} = \mathcal{D} - \mathcal{W}$.

A signal $\{\mathbf{x} : \mathcal{V} \to \mathcal{R}\}$ on graph $\mathcal{G}$ is a function that assigns a value to each vertex on the graph and can be considered as a vector $\mathbf{x} \in \mathcal{R}^N$, where the i^{th} element of the vector represents the function value at the i^{th} vertex of the graph. In our problem, the sample on each node represents spatial coordinates and RSRP value at each node. Gradient $\nabla_{\mathcal{G}}$ of a graph signal measures the change in signal $\mathbf{x}$ along the edges of the graph and is given as

$$\nabla_{\mathcal{G}} \tilde{\mathbf{x}}(j,k) = \sqrt{W_{jk}} \left(\frac{\mathbf{x}(k)}{\sqrt{d(k)}} - \frac{\mathbf{x}(j)}{\sqrt{d(j)}} \right), \quad (1)$$

where $\tilde{\mathbf{x}}(j,k)$ represents the flow into the node k from node j. In our context, it represents difference in the received signal strength when we move from node j to k. The adjoint of the gradient is known as the divergence of the signal $\mathbf{x}$ defined as

$$div(\mathcal{G}, \mathbf{x}) = \sqrt{W_{jk}} \left(\frac{1}{\sqrt{d(j)}} \tilde{\mathbf{x}}(j,j) - \frac{1}{\sqrt{d(k)}} \tilde{\mathbf{x}}(j,k) \right). \quad (2)$$

The graph Laplacian is represented in terms of its divergence and gradient, which is the second order derivative of the graph signal given as

$$\mathcal{L} = div(\mathcal{G}) \times \nabla_{\mathcal{G}} \ .$$

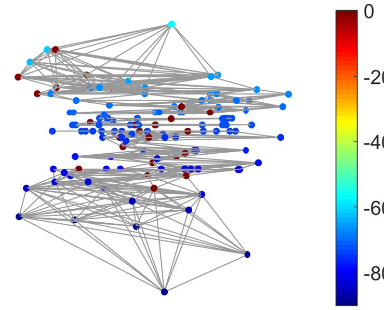

Fig. 1. Missing value graph representing RSRP values with known (blue) and unknown (red) points.

B. Graph Construction and Problem Formulation

We construct a root mean squared error (RMSE) missing value graph, where the points on each node are connected to other nodes using the k-nearest neighbors, as shown in Figure 1. These nodes represent the signal strength (RSRP) on MGRS grid points. The red points are grid locations with no coverage information (missing values). For the construction of a normalized graph Laplacian $\mathcal{L}_n$, we first calculate the distance between samples of each pair of nodes (x_j, x_k),

$$H_{jk} = \sqrt{\frac{\|M_{jk} \circ (x_j - x_k)\|_2^2}{\|M_{jk}\|_1}}, \quad (3)$$

where $\|M_{jk}\|_1$ is the ℓ_1 norm of masking function and is used to normalize the distances. H_{jk} represents a pairwise distance between nodes j and k, $\circ$ is a Hadamard operator. The masking function, M_{jk}, contains information of both known and unknown MGRS grids, i.e.,

$$M_{jk} = \begin{cases} 1, & \text{for known MGRS grid}\ , \\ 0, & \text{otherwise.} \end{cases}$$

Let $\mathcal{H}$ be the matrix of all pairwise distances with $(j,k)^{th}$ entry given as H_{jk} and the minimum entry of h_{min}. Hence, the $(j,k)^{th}$ element of the weight matrix $\mathcal{W}$ for graph $\mathcal{G}$ is constructed as follows

$$W_{jk} = \exp\left(-\frac{(H_{jk} - h_{min})^2}{\sigma^2} \right). \quad (4)$$

Finally, the normalized graph Laplacian is calculated as

$$\mathcal{L}_n = \mathcal{D}^{-\frac{1}{2}} (\mathcal{D} - \mathcal{W}) \mathcal{D}^{-\frac{1}{2}} = \mathcal{I} - \mathcal{D}^{-\frac{1}{2}} \mathcal{W} \mathcal{D}^{-\frac{1}{2}}, \quad (5)$$

where $\mathcal{D}$ is the diagonal degree matrix and $\mathcal{I}$ is the identity matrix.

It is assumed that we have a complete knowledge of the masking function M_{jk}. Moreover, since the graph signal does not vary abruptly along the edges locally, it is considered to be smooth. Furthermore, measurements are only available over finite locations. Hence, the gradient of the graph signal $\mathbf{x}$ will always be sparse. In order to find the missing values of a signal with sparse gradient, the problem can be reduced to minimization of total variation (TV) norm on the graph.

This reduced problem is given as

$$\underset{\mathbf{x}}{\operatorname{argmin}} \ \|\nabla_G(\mathbf{x})\|_1, \quad \text{subject to } \mathcal{M}\mathrm{x} = \mathrm{y}, \tag{6}$$

where $\mathcal{M}$ is the masking transformation matrix with its $(j,k)^{th}$ entry of M_{jk} and $\mathbf{y}$ contains RSRP of the locations. Since, the gradient of the objective function $\|\nabla_G(\mathbf{x})\|_1$ does not exist at $\mathbf{x} = \mathbf{0}$, we solve the problem by using proximal splitting method. If $\mathbf{g}(\mathbf{x}) = \|\nabla_G(\mathbf{x})\|_1$, its proximal can be written as

$$\mathbf{Prox}_{g\gamma}(\mathbf{y}) = \underset{\mathbf{x}}{\operatorname{argmin}} \ \frac{1}{2}\|\mathbf{x}-\mathbf{y}\|_2^2 + \gamma\|\nabla_G(\mathbf{y})\|_1, \tag{7}$$

where the first term represents data fidelity and the second term is the regularization term parameterized by γ, that controls the trade-off between these two terms. Smaller the regularization parameter, the more trust we place on the prior values. For an optimal solution to be considered in the feasible set $\mathcal{C}$ defined by the constraint $\mathbf{y} = \mathcal{M}\mathbf{x}$, we use the projection operator instead of proximal operator as the proximal operator of a convex function can be viewed as an extension of projection onto a convex set [11]. Let $h_{\mathcal{C}}(x)$ be an indicator function such that

$$h_{\mathcal{C}}(x) = \begin{cases} 0, & \text{if } x \in \mathcal{C}, \\ +\infty, & \text{otherwise} . \end{cases} \tag{8}$$

then the proximal operator of $h_{\mathcal{C}}(x)$ onto the convex set $\mathcal{C}$ can be written as

$$\mathbf{Prox}_{h\gamma}(\mathbf{y}) = \underset{\mathbf{x}}{\operatorname{argmin}} \ \frac{1}{2}\|\mathbf{x}-\mathbf{y}\|_2^2 + h_{\mathcal{C}}(x) \ , \tag{9}$$

Note that if we take into account our constraint $\mathcal{M}\mathbf{x} = \mathbf{y}$, the search space for the above proximal operator reduces to convex set $\mathcal{C}$ instead of $\mathcal{R}^N$. After applying graph TV regularization, our optimization problem changes to

$$\underset{\mathbf{x}}{\operatorname{argmin}} \ \|\mathbf{y} - \mathcal{M}\mathbf{x}\|_2^2 + \gamma\|\nabla_G(\mathbf{x})\|_1 \ . \tag{10}$$

Again, the first term represents data fidelity and γ is the regularization parameter. When Tikhonov regularization is applied, the problem has the following form

$$\underset{\mathbf{x}}{\operatorname{argmin}} \ \|\mathbf{y} - \mathcal{M}\mathbf{x}\|_2^2 + \gamma\|\nabla_G(\mathbf{x})\|_2^2 \ . \tag{11}$$

Solving the above problems (equation (10) and equation (11)), using proximal splitting method, achieves computational complexity of $\mathcal{O}(n)$ [12] which is significantly lower than other algorithms e.g., solving through the normal equations has a complexity of $\mathcal{O}(n^3)$ [12] and kriging has a complexity of $\mathcal{O}(n^4)$ [13].

C. Optimization Algorithm

One of the methods for solving optimization problems using proximal splitting is the Douglas-Rachford algorithm. It does not require smooth and continuously differentiable objective functions that may be required in other algorithms such as Forward-Backward algorithm. The problem in hand also has a similar form where the first function in equation (10) is differentiable and the other is not. For equation (11), since we are using squared norm, both of the terms are differentiable. Douglas-Rachford method is capable of solving all such cases. For the above optimization problems, assuming $\gamma > 0$, there exists at least one solution $\mathbf{x}^*$ defined in terms of proximal operator, given by the following two conditions [14]

$$\begin{cases} \mathrm{x}^* = \mathbf{Prox}_{\mathrm{g}\gamma}(\mathrm{y}*) \\ \mathbf{Prox}_{\mathrm{g}\gamma}(\mathrm{y}*) = \mathbf{Prox}_{\mathrm{h}\gamma}(\mathrm{y})(2\mathbf{Prox}_{\mathrm{g}\gamma}(\mathrm{y}*) - \mathrm{y}^*), \end{cases} \tag{12}$$

The optimal x^* and y^* are computed in an iterative manner

$$\begin{cases} \mathrm{x_k} = \mathbf{Prox}_{\mathrm{g}\gamma}(\mathrm{y_k}) \\ \mathrm{y_{k+1}} = \mathrm{y_k} + \tau(\mathbf{Prox}_{\mathrm{h}\gamma}(2\ \mathrm{x_k} - \mathrm{y_k}) - \mathrm{x_k}). \end{cases} \tag{13}$$

where $\tau \in (0,2)$ is the step size. The algorithm converges when $x_{k+1} - x_k < \epsilon$.

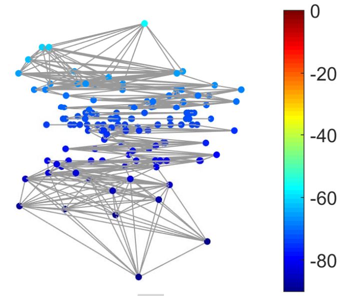

Fig. 2. Network Health Estimation on graph with graph TV regularization.

III. Results

Multiple measurement campaigns have been carried out to acquire data. The results of the proposed technique are compared with kriging, which is regarded as a benchmark for spatial interpolation [15], [16]. In order to find the error associated with prediction, the original data set is partitioned into training and testing sets using K-Means clustering. Selecting any one of the clusters as a test set at random alleviates spatial bias in the prediction. The error in interpolation and proposed scheme is calculated by comparing predicted values from the ground truth.

Figure 2 is the constructed graph after solving the optimization problem with graph TV regularization. Note that the missing values (red points) in Fig. 1 have now been replaced by the predicted values (blue points); constructed by minimization of total variation norm on the graph. We get a similar graph after applying Tikhonov regularization with an improved accuracy in terms of root mean squared error (RMSE). Kriging gives an RMSE value of 3.1 dB while GSP, with graph TV and Tikhonov regularization, give RMSE values of 1.3 dB and 1.1 dB respectively i.e., the accuracy with the proposed approach improves by more than 60% as compared to kriging. Moreover, the time complexity gets significantly reduced; $\mathcal{O}(n)$ as compared to $\mathcal{O}(n^4)$ for kriging.

Coverage analysis has also been carried to find out the coverage holes present in the region of interest. The coverage

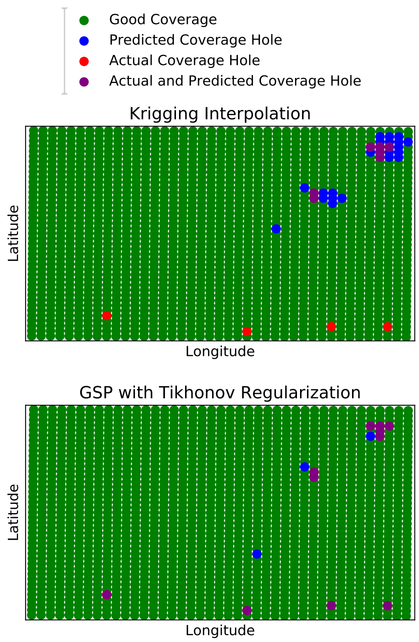

Fig. 3. Comparison of Predicted Coverage Holes with Ground Truth.

holes are classified by applying five point summary in which the locations with signal strength lying in the lower outlier range are labeled as coverage holes. Fig. 3 shows the comparison of the coverage estimation with kriging interpolation and the proposed approach (with Tikhonov regularization). It shows good coverage points, predicted coverage holes and actual coverage holes and clearly demonstrates that while kriging interpolation fails to predict a number of coverage holes, the proposed scheme detects all of the coverage holes. For kriging, the coverage holes are clustered together and are significantly greater than the actual coverage holes, indicating an enhanced false positive rate and a spatial bias in interpolation. In addition, few other coverage holes are missed pointing towards a poor detection rate. In contrast, the proposed approach has much better detection accuracy and is void of any spatial bias. For the measurements reported in this letter, the proposed approach correctly predicts almost all the coverage holes present in the region.

IV. Conclusions

In this letter, we propose a novel technique for network health estimation. The proposed scheme makes use of graph signal processing on measured data at known locations to predict network health at unknown locations. The problem is formulated as a convex optimization problem and has been solved through proximal splitting method after applying two different regularization schemes. We show that the proposed approach outperforms existing methods like kriging for network health estimation with an improved accuracy of more than 60% in terms of RMSE under similar conditions. Furthermore, the time complexity with the proposed approach is of $\mathcal{O}(n)$, which is significantly smaller as compared to kriging which is $\mathcal{O}(n^4)$. We also show the superiority of GSP with regards to coverage hole detection, both in terms of detection rate and false positive rate.

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