EXAMPLE 1

$x_i \in \{-1, 1\}$ → bipolar vectors

Build Hopfield and Test stability of the vectors

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

Matrix Hopfield:

$$T = \frac{1}{3} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 3 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & 3 & 3 & 0 \end{bmatrix}$$

ANSWER

$x_1 \to x_1$ (stable)

$x_2 \to x_2$ (stable)

$x_3 \to x_2$ (not stable)

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Stability of x_1

$$Tx_1 = \frac{1}{3} \begin{bmatrix} 3 \\ 7 \\ 7 \\ 7 \end{bmatrix} \Rightarrow sgn(Tx_1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = x_1 \Rightarrow x_1 \text{ is stable}$$

Stability of x_2

$$Tx_2 = \frac{1}{3} \begin{bmatrix} -3 \\ -7 \\ -7 \\ -7 \end{bmatrix} \Rightarrow sgn(Tx_2) = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = x_2 \Rightarrow x_2 \text{ is stable}$$

Stability of x_3

$$Tx_3 = \frac{1}{3} \begin{bmatrix} -3 \\ -5 \\ -5 \\ -5 \end{bmatrix} \Rightarrow sgn(Tx_3) = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = x_3' \neq x_3$$

$x_3' \neq x_3$ ⇒ we need to iterate again

$$Tx_3' = \frac{1}{3} \begin{bmatrix} -3 \\ -7 \\ -7 \\ -7 \end{bmatrix} \Rightarrow sgn(Tx_3') = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = x_3' = x_2$$

⇒ x_3 is not stable, it converges to x_2