WARSAW UNIVERSITY OF TECHNOLOGY
FACULTY OF MATHEMATICS
AND INFORMATION SCIENCE

Neural Networks

Lecture 10

The Hopfield Model

The Hopfield Model

W 1992 J. J. Hopfield
Neural Networks and Physical Systems with Emergent Collective Computational Abilities

Model similar to the perceptron – but with many differences.

It is not only the model – it is the ideology.

The Hopfield Model

Hopfield exploited an analogy to energy states in physics and introduced the *computational energy function*. Like a physical system, the network seeks its lowest energy state and with the iteration procedure converges to the stable state.

The Hopfield network is able to *memorize* and next *reproduce* the information on the base of an incomplete or noisy input signal.

The Hopfield Model

The system associates the input information with this stored which is the ”*closest*” in accordance to the measure of similarity.

The algorithm realized by the network is called

nearest neighbour algorithm

The Hopfield model has a shortage of precise mathematical description and precise convergence conditions.

The Hopfield Model

Network description

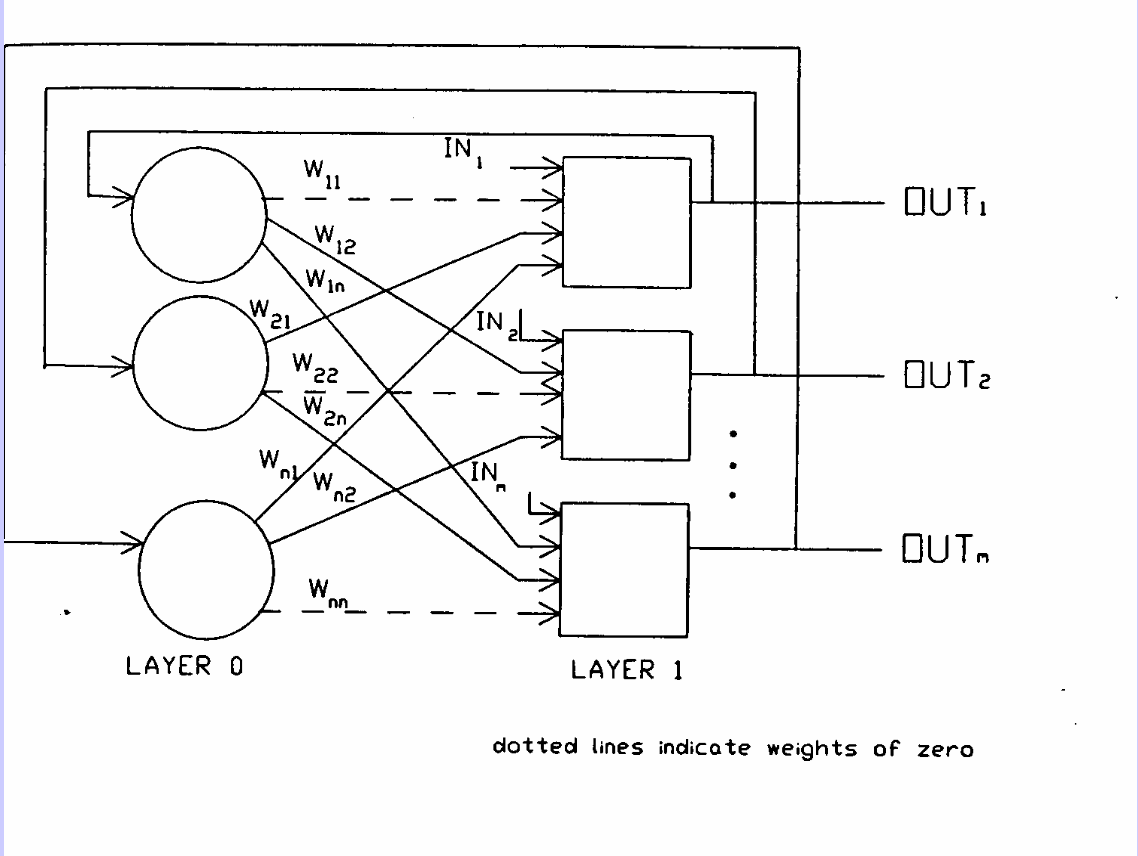
The Hopfield net consists of a number of elements, each connected to every other element - it is fully connected network (but no self feedback loops).

It is also symmetrically-weighted network, since the weights on the connections from one element to another are the same in both directions.

Each element has, like the single-layer perceptron, a threshold and each element calculates the weighted sum of their inputs minus the threshold value.

The Hopfield Model

The system



The Hopfield Model

Network operation:

The input and output signals can be binary e.g. $x \in \{-1,+1\}$ (the bipolar case) or $x \in \{0,1\}$ (the unipolar case) or continuous valued.

Next an unknown object is input to the network which proceeds to cycle (the first network output is taken as the new input, which produces an output and so on.) through a succession of states, until it converges on a stable solution, which happens when the output values of elements no longer alter.

The Hopfield Model

The network is prepared during the initialization (or learning) phase when the interconnecting matrix is calculated.

The interconnection weights w_{ij}, *i,j =1,2,...,n* form the *n* x *n* symmetric interconnection matrix ***W***, which is defined by the outer - product learning rule

$$w_{ij} = \begin{cases} \sum_{s=1}^{N} x_i^s x_j^s & \text{dla} \quad \mathrm{i} \neq \mathrm{j} \\ 0 & \text{dla} \quad \mathrm{i} = \mathrm{j} \end{cases}$$

N is the number of stored objects,

x_j^s is the *j* element of object *s*.

The Hopfield Model

Comparison Perceptron - Hopfield

- in a perceptron network is learned through the repeated adjustment of weights
- in a Hopfield model network is prepared during the initialization (or learning) phase when the interconnecting matrix is calculated.

The Hopfield Model

Comparison Perceptron - Hopfield

- in a perceptron network is addressed by the input signal – and generates the appropriate output signal
- in a Hopfield model the first output signal is used as a new input signal etc. (until it converges to the stable state).

The Hopfield Model

Analysis of system energy:

Network ”calculates” an error (calculate energy)

$$E = -\frac{1}{2}\sum_{i}(y_i - y_j^*)^2$$

E determines the value the actual network output signal **Y** differs from required signal **Y***

Big difference – big energy. Small difference – small energy

The Hopfield Model

The network output signal is a function of the weights values and an input signal.

Assuming the network with two weights only – the geometrical interpretation is a surface in 3D

Each next weight increase the problem dimension.

Generally – all weights are the subject of correction which lead to multidimensional energy function

The Hopfield Model

Learning rule.

The network updates its weights such that the euclidean (?) distance of the output vector and the target vector is minimized minimizing the Energy E.

Learning method – a gradient descent method

A knowledge of Y^* and Y are necessary. In the Hopfield's model we do not have such a knowledge – in the consecutive steps – an algorithm has to be changed.

The Hopfield Model

For the Hopfield network the energy has the form:

$$E = -\frac{1}{2}\sum_i \sum_{j \neq i} w_{ij} x_i x_j + \sum_i x_i T_i$$

where

w_{ij} is the weight between the i^{th} and j^{th} element,

x_{i} is the input signal of element i^{th},

T_i is the threshold value of the element i^{th}.

and $w_{ij} = w_{ji}$ and $w_{ii} = 0$