WARSAW UNIVERSITY OF TECHNOLOGY
FACULTY OF MATHEMATICS
AND INFORMATION SCIENCE

Neural Networks

Lecture 3

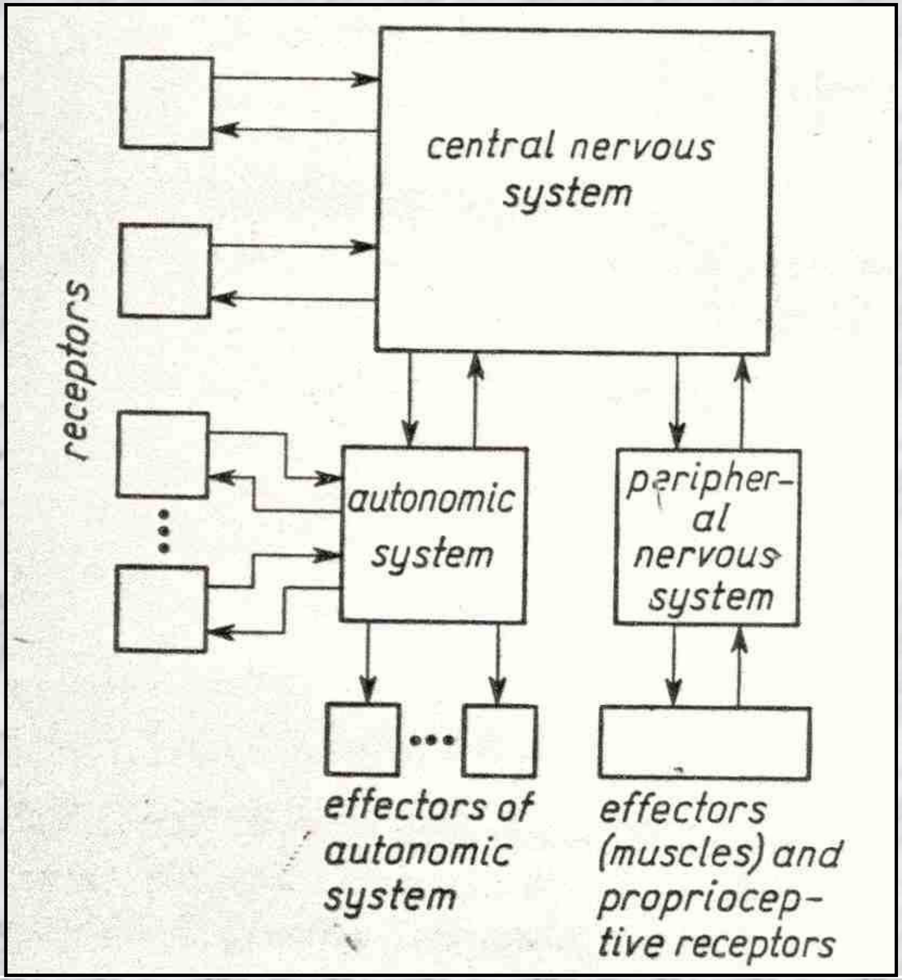
Principles to which the nervous system works

Some biology and neurophysiology

Nervous system

- central nervous system
- peripheral nervous system
- autonomic nervous system

Diagram of the nervous system



Some biology and neurophysiology

Central nervous system has three hierarchical levels:

- the spinal cord level,
- the lower brain level,
- the cortical level.

The **spinal cord** acts as the organ controlling the simplest reaction of the organism (spinal reflexes)

Some biology and neurophysiology

Lower region of the **brain** and regions in the **cerebellum** are coordinating the motor activities, orientation in space, general regulation of body (temperature, blood pressure etc.)

Cerebral cortex establish interrelations between lower regions and coordinating their functions. Decision are taking, information is stored in cerebral cortex,

Some biology and neurophysiology

Peripheral nervous system composed of the nerve processes running out from the brain and spinal cord.

Nerves are the connections for communication between centers and organs.

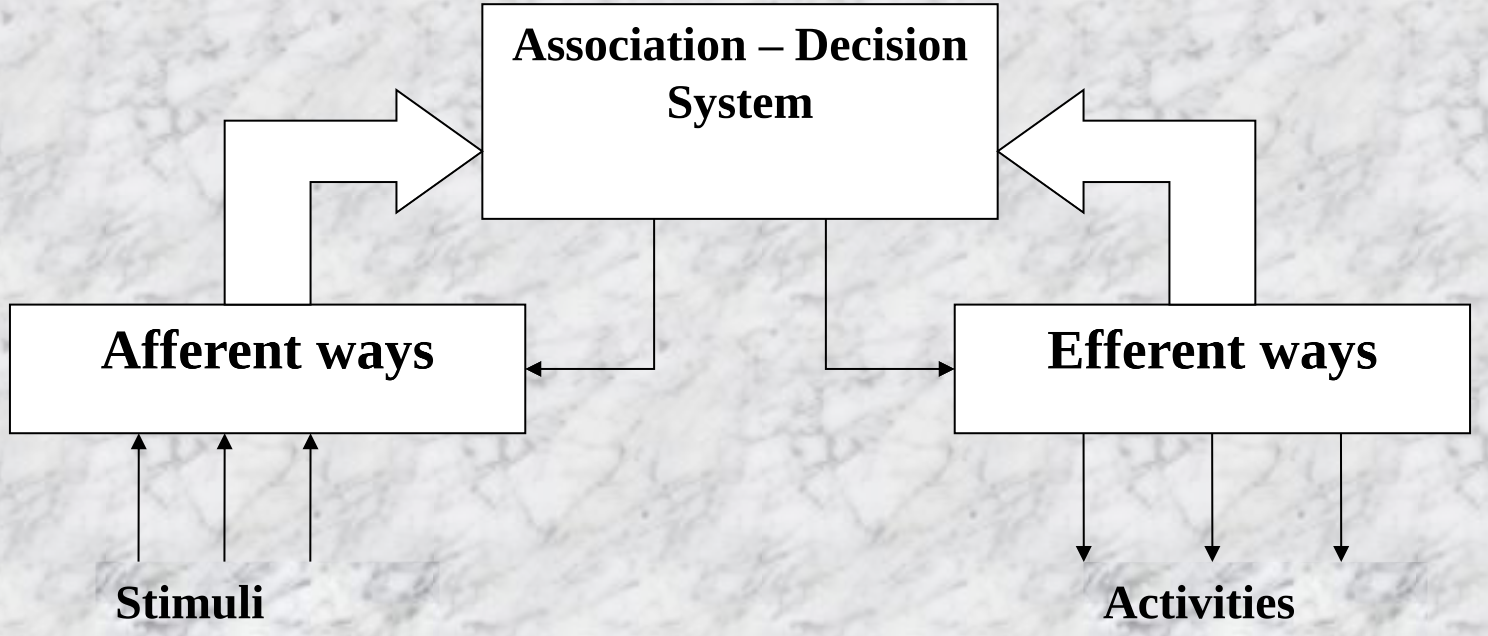
Some biology and neurophysiology

The task of the Autonomous nervous system is to control the most important vital processes such as breathing, blood circulation, concentration of chemicals in the blood etc.

Functional scheme of connections of the nervous system:

1. an afferent system
2. a central association decision making system
3. an efferent system

Some biology and neurophysiology



Some biology and neurophysiology

Afferent ways

an afferent system in which signals arriving from the environment are transmitted and analyzed, the degree and mode of analysis is controlled by superior coordinating and decision making system,

multi level and hierarchical structures supplying the brain with information about external world (environment).

Some biology and neurophysiology

The efferent system

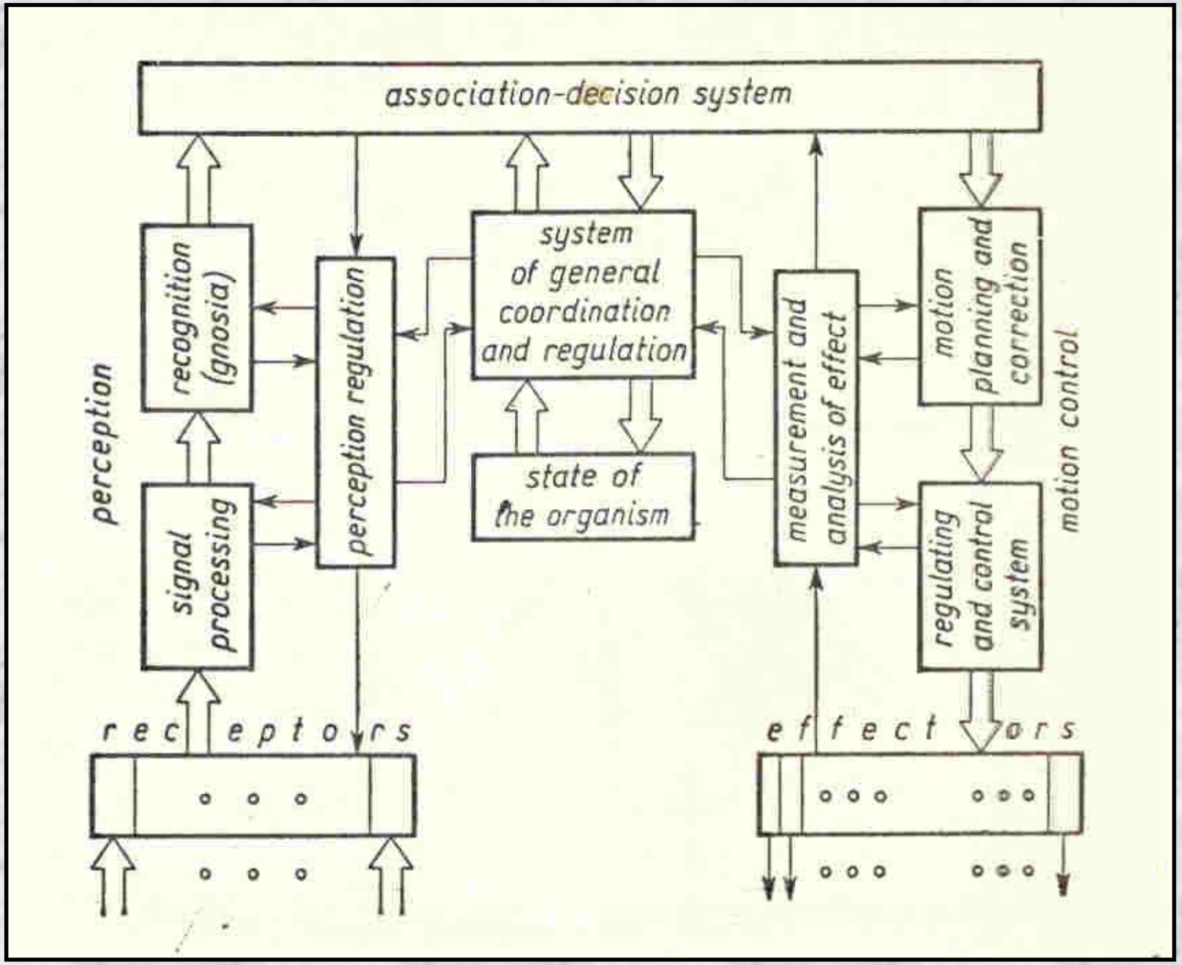
in which, on the basis of the decision taken a plan of reaction of the organism is worked out, on the base of static and dynamic situation, experience and optimization rules, output channels of a nervous system responsible for transmission and processing of signals controlling the effectors

Some biology and neurophysiology

The central association and decision making system

where a decision about the reaction of the organism is worked out on the basis of the state of the environment, the state of the organism, previous experience, and a prediction of effect

Some biology and neurophysiology



Nerve cell models

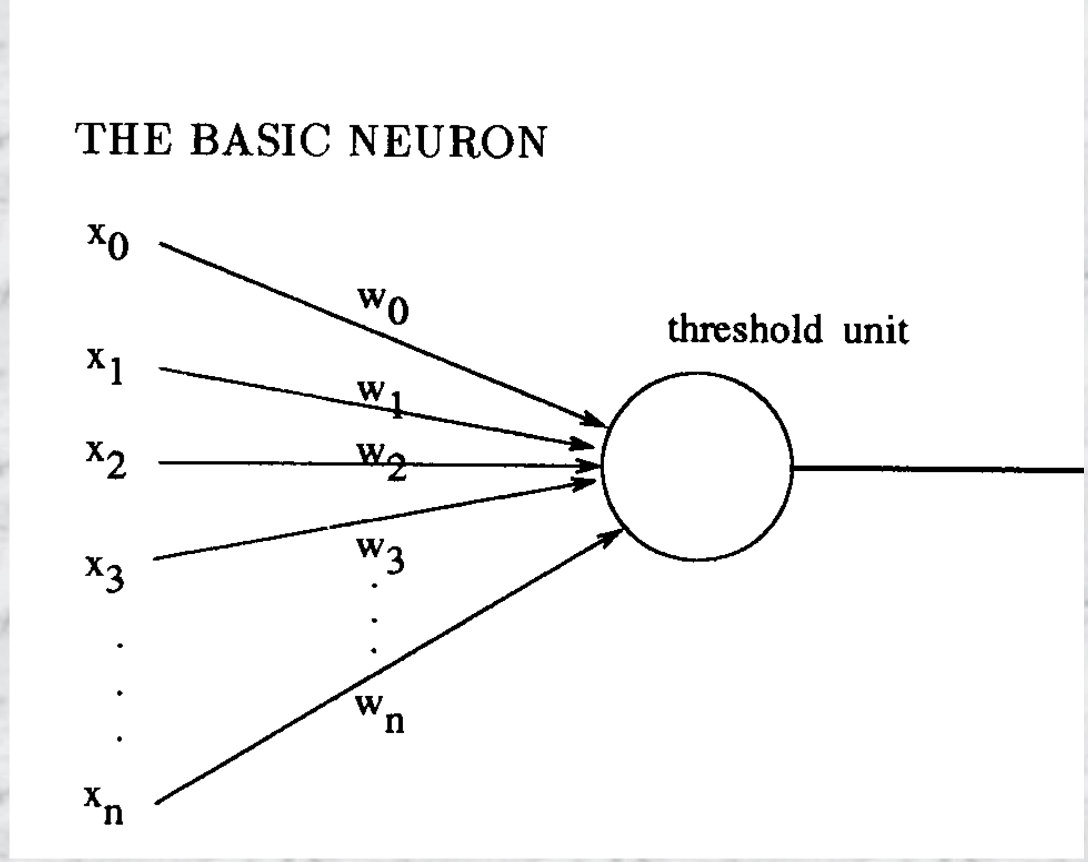
The first model of neuron was proposed in 1943 by W.S. McCulloch and W.Pitts

The model came from the research into behavior of the neurons in the brain. It was very simple unit, thresholding the weighted sum of its inputs to get an output.

It was the result of the actual state of knowledge and used the methods of mathematical and formal logic.

The element was also called *a formal neuron*.

Nerve cell models



Nerve cell models

The formal neuron was characterized by describing its state (or output).

Changing of the state from inactive (0) to active (1) was when the weighted sum of input signals was greater than the threshold; and there was no inhibitory input.

Nerve cell models

Model assumptions:

1. The element activity is based on the „all-or-none" principle.
2. The excitation (state 1) is preceded by a constant delay while accumulating the signals incoming to synapses (independent from the previous activity and localization of synapses).
3. The only neuronal delay between the input simulation and activity at the output, is the synaptic delay.

Nerve cell models

Model assumptions:

3. Stimulation of any inhibitory input excludes a response at the output at the moment under consideration.
4. The net structure and neuron properties do not change with time.

Nerve cell models

The discrete time is logical, because in the real neuron, after the action potential, the membrane is non-excitable, i.e. another impulse cannot be generated (appr. 1 ms). This interval is called the ***absolute refractory period***.

It specifies the maximum impulse repetition rate to about 1000 impulses per second.

Mathematical models of a nerve cell

The methods of selection of the properties of neural element depends not only on previous results, our level of knowledge – but mainly from the phenomena to be modeled.
Another properties will be important while modeling the steady states, another for dynamic processes or for the learning processes.

But always, the model has to be as simple as possible.

Mathematical models of a nerve cell

Functional model of a neuron:

1.input signals

- adding signals (inhibitory and excitatory),
- multiplying signals without memory ,
- multiplying signals with memory.

Physiologically –synapses (basically linear units).

Mathematical models of a nerve cell

$x_i(t)$ – input signals, continuous time functions,

$u_i(t)$ – input to multiplying inputs with memory, weight control $v_{i,}$ - continuous time functions,

$s(t)$ –facilitation hypothesis, (influence of long-lasting signals).

Mathematical models of a nerve cell

weight control

initial weight

weight increment constrain

$$v_i(u_i, s, t) = v_{oi} +$$

$$\gamma \exp(-\alpha_z t) \int_0^t \exp(-\delta_t) u_i(t) s(t) dt$$

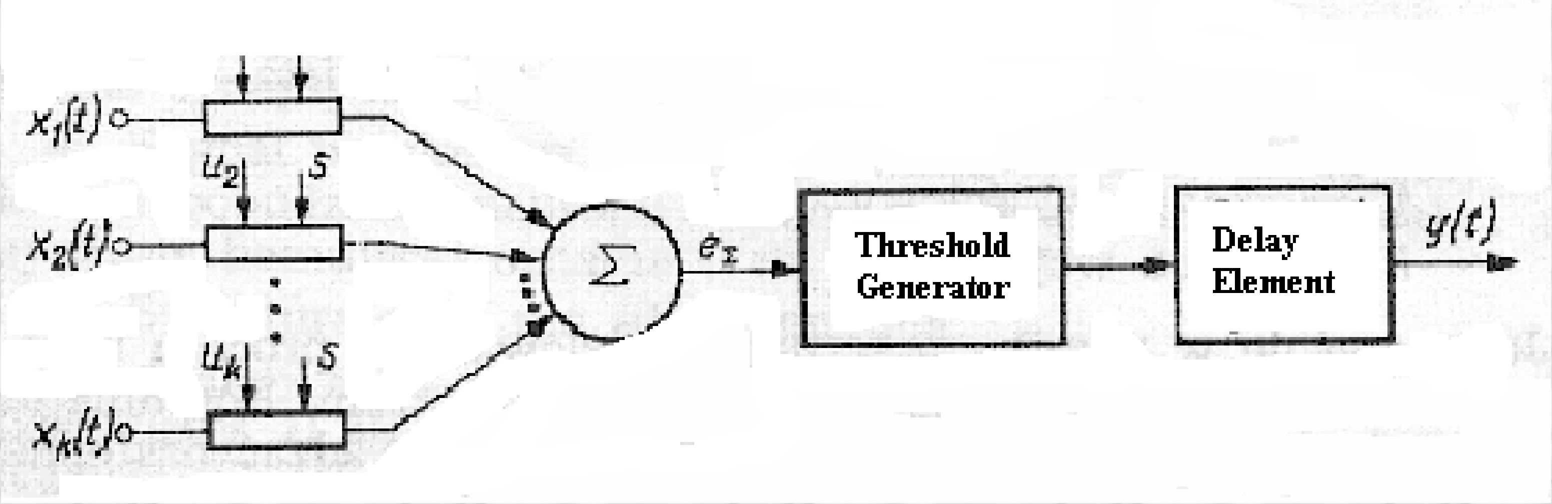
storage coefficient

forgetting time-delay

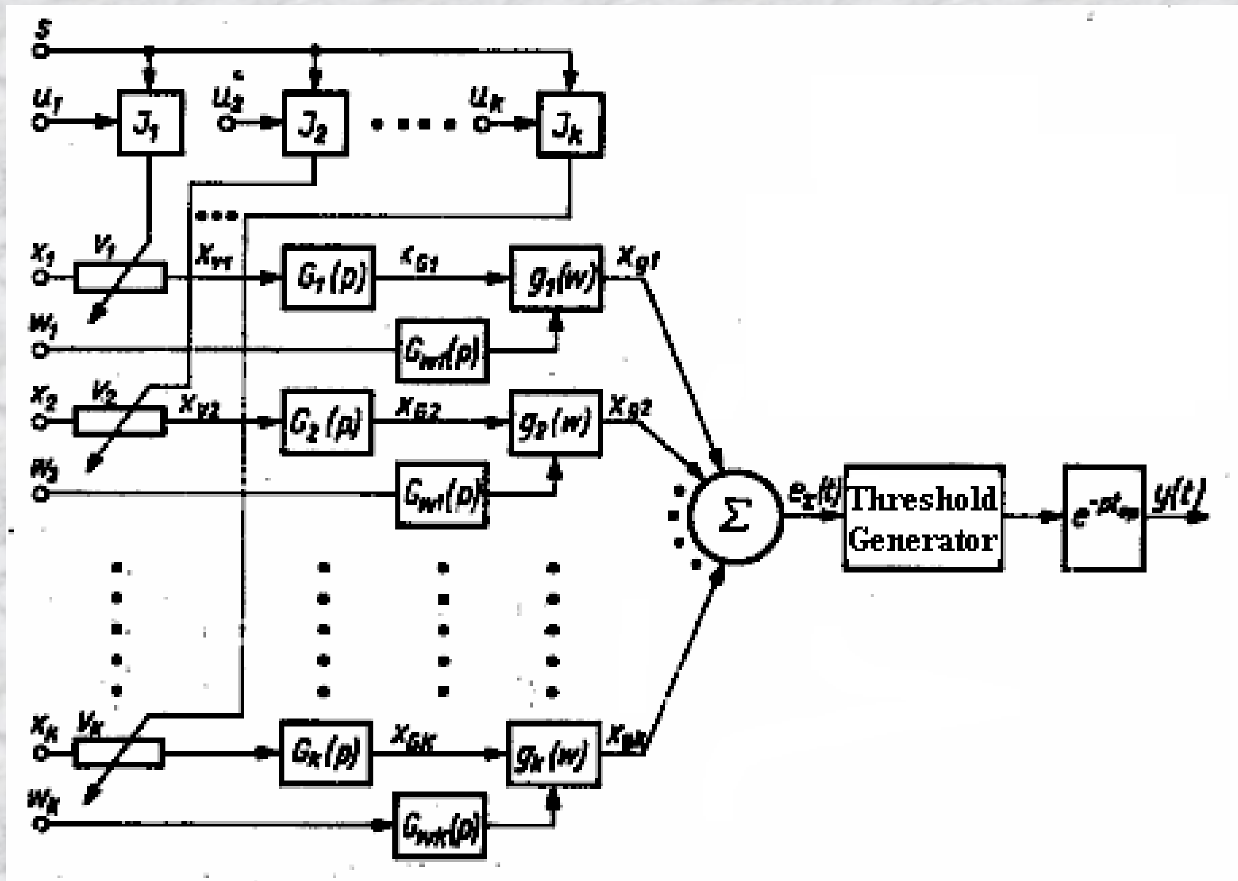
hence: $x_{v_i}(t) = x_i(t) * v_i(u_i, s, t)$

Mathematical models of a nerve cell

2. analog adder
3. threshold impulse generator
4. delay element



Functional model of a neuron

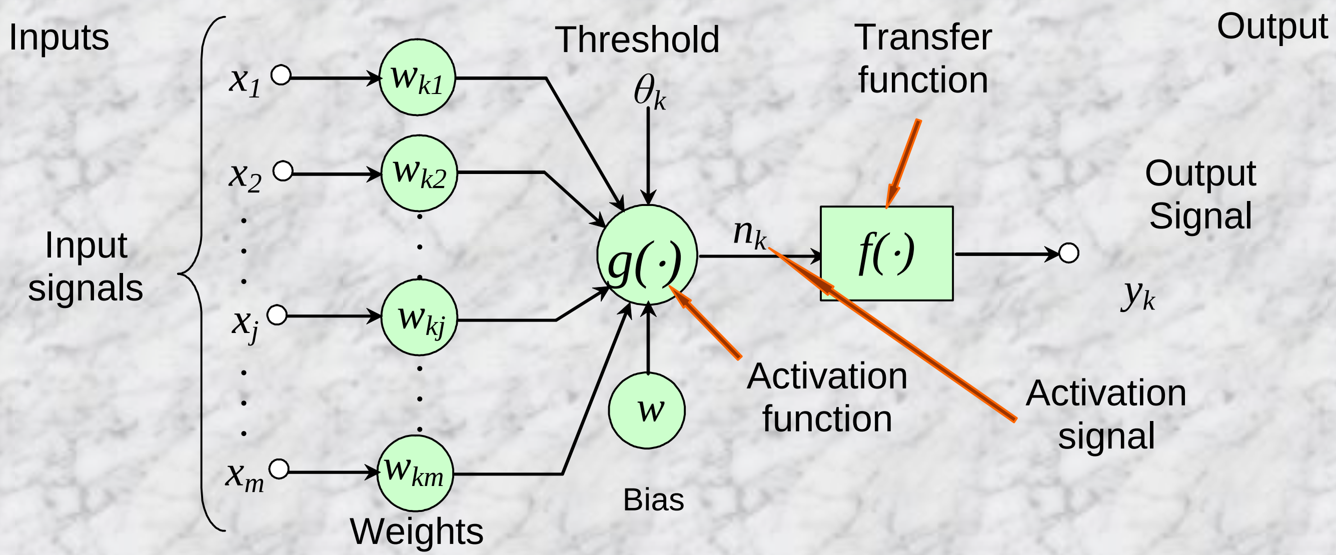


Neural cell models

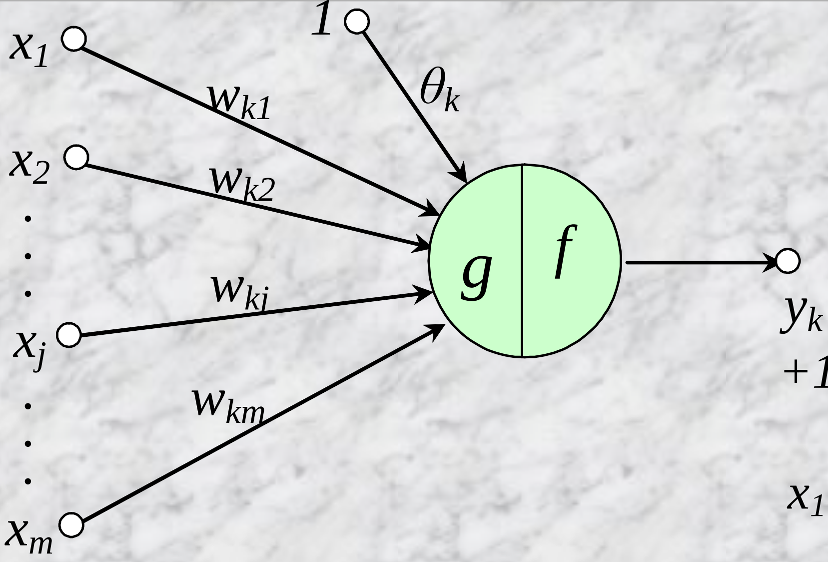
Static model of neural cell

Two kinds of neurons: static and dynamic.
Static model of neuron (used in continuous and discrete static nets and dynamic discrete sets).

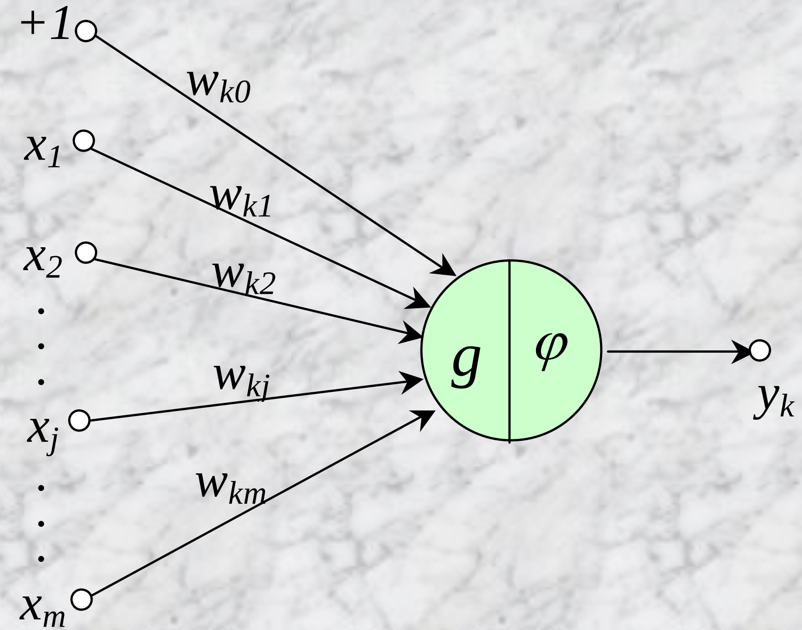
Model of a static neuron k



Simplified models of static neuron

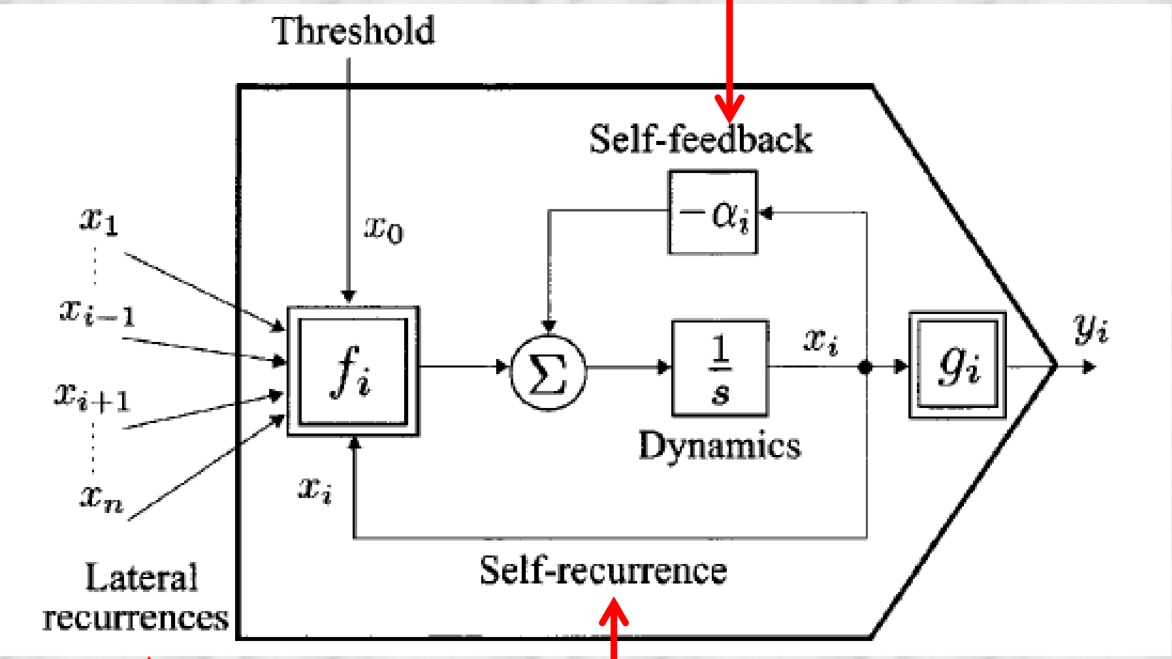


or



Dynamic model of neuron

Internal self-feedback



Self recursion

External recursion

Activation functions

1. *Adder*
2. *Product*
3. *Maximum*
4. *Minimum*
5. *Dominant*
6. *Cumulative sum*

Activation functions

Sum function

$$n_k = \sum_{j=1}^{m} w_{kj} x_j + \theta_k \quad or \quad n_k = \sum_{j=0}^{m} w_{kj} x_j$$

Product function

$$n_k = \prod_{j=1}^{m} w_{kj} x_j$$

Maximum

$$n_k = \max_{j} \left\{ w_{kj} x_j \right\}$$

(k – neuron's number, j - number of neuron's input)

Activation functions

Minimum

$$n_k = \min_{\mathrm{j}} \left\{ w_{kj} x_j \right\}$$

Dominant function

$$n_k = \sum_{j=1}^{m} \mu_j \quad \textit{where} \quad \mu_{\mathrm{j}} = \begin{cases} 1 & \textit{if} \quad w_{\mathrm{i,j}} > 0 \\ 0 & \textit{otherwise} \end{cases}$$

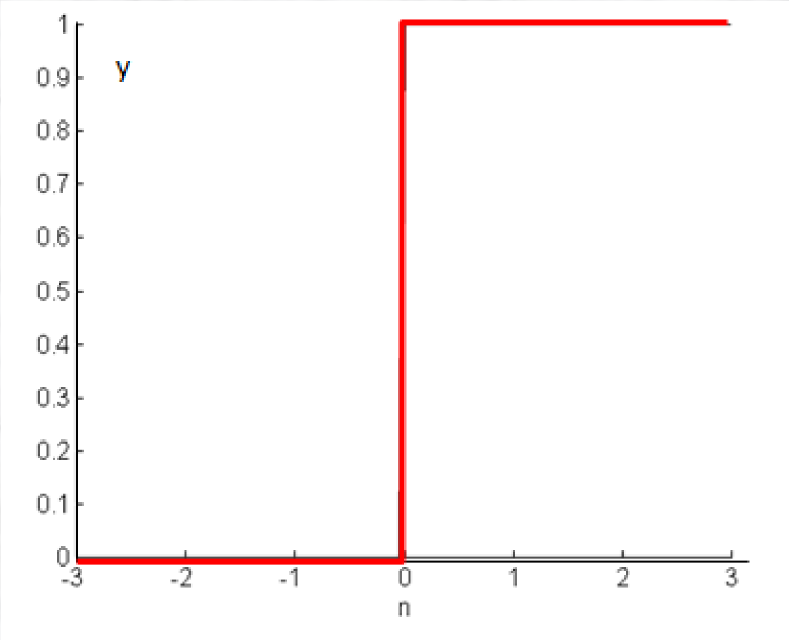
Cumulative sum function

$$n^{(i+1)} = n^{(i)} + \sum_{j=1}^{m} w_{k,j}^{(i)} x_j^{(i)} \quad \textit{where} \quad \textit{i - iteration}$$

(k – neuron's number, j - number of neuron's input)

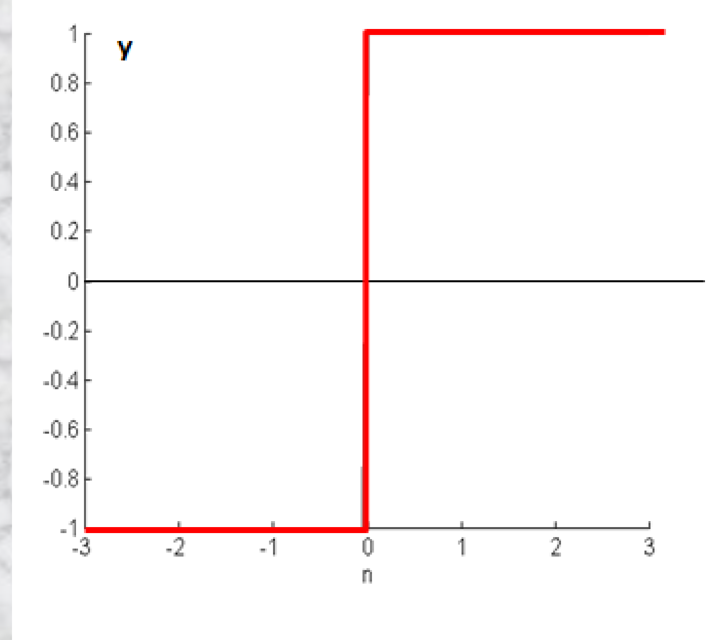
Transfer functions

Unipolar function



$$y = \begin{cases} 0 & if \quad \mathrm{n} < 0 \\ 1 & otherwise \end{cases}$$

Bipolar function

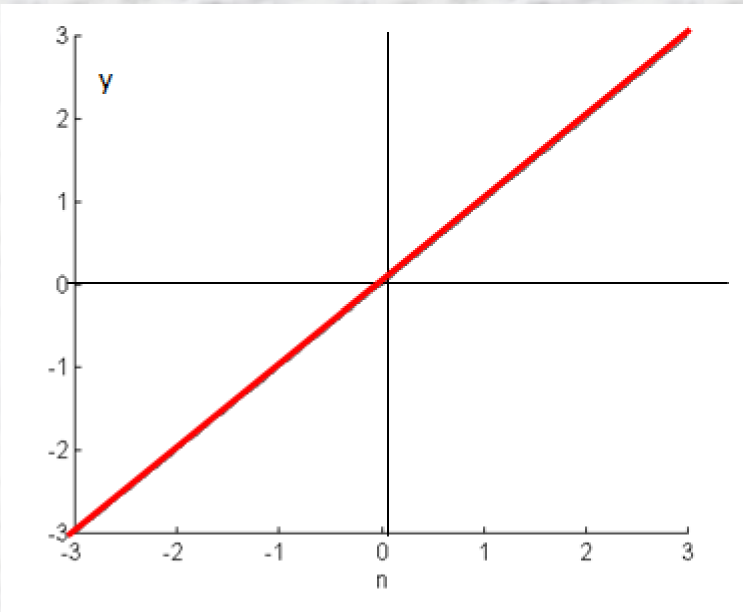


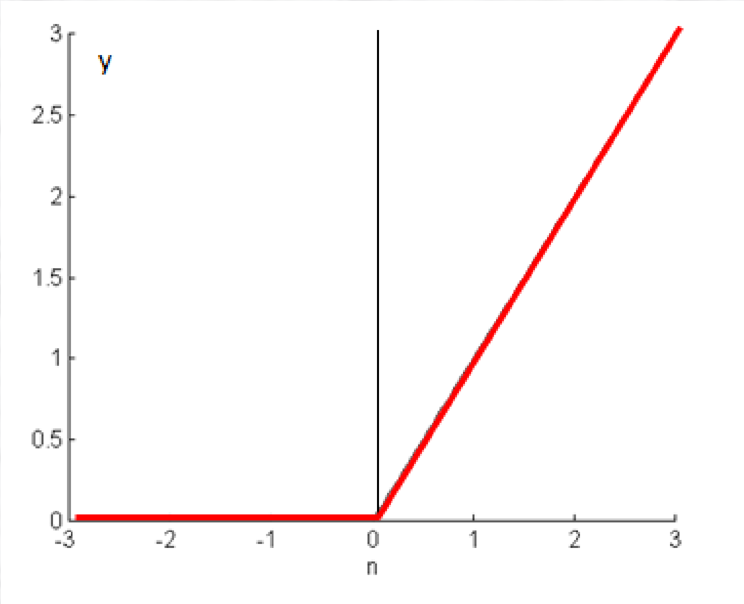
$$y = \begin{cases} -1 & if \quad \mathrm{n} < 0 \\ +1 & otherwise \end{cases}$$

Transfer functions

Linear function

Linear positive function



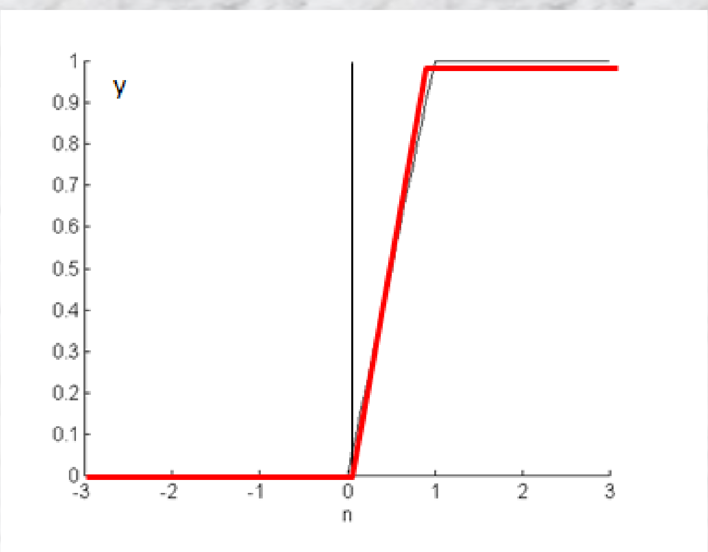


$$y = n$$

$$y = \begin{cases} 0 & \text{if} \quad n < 0 \\ n & \text{otherwise} \end{cases}$$

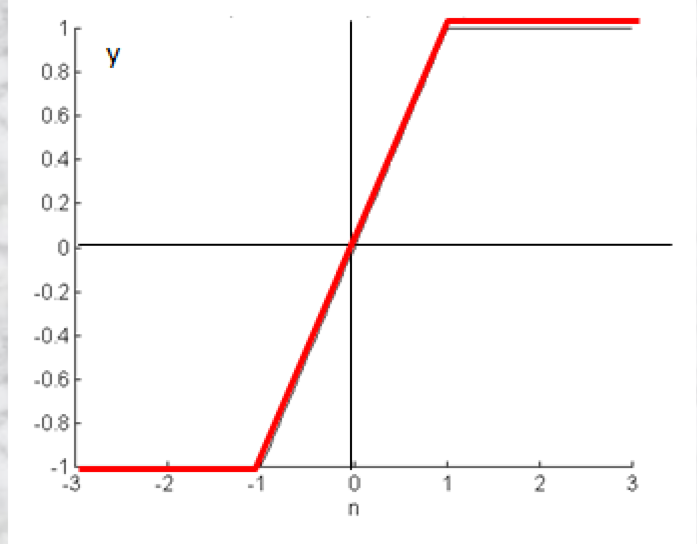
Transfer functions

Linear function with saturation (non symmetric)



$$y=\begin{cases} 0 & if \quad n<0 \\ n & if \quad 0\le n\le\alpha \\ 1 & if \quad n>\alpha \end{cases}$$

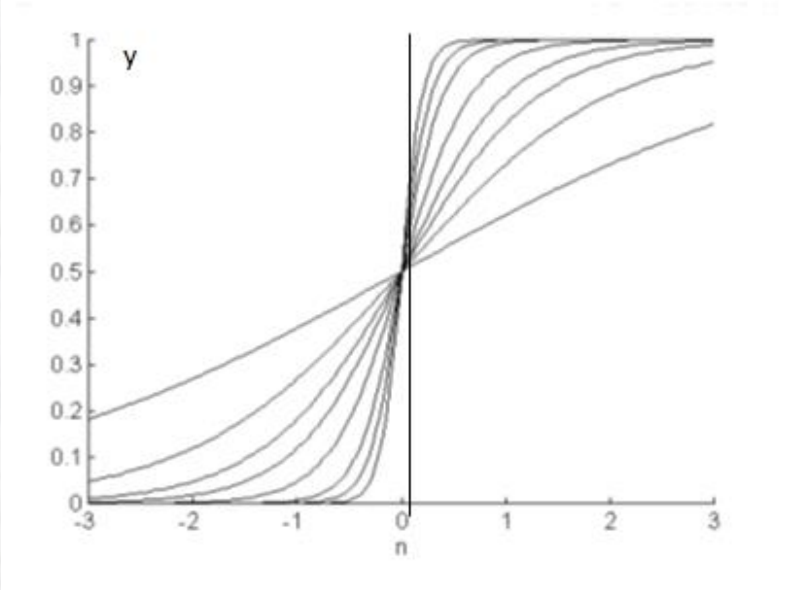
Linear function with saturation (symmetric)



$$y=\begin{cases} -1 & if \quad n<-\alpha \\ n & if \quad -\alpha\le n\le\alpha \\ +1 & if \quad n>\alpha \end{cases}$$

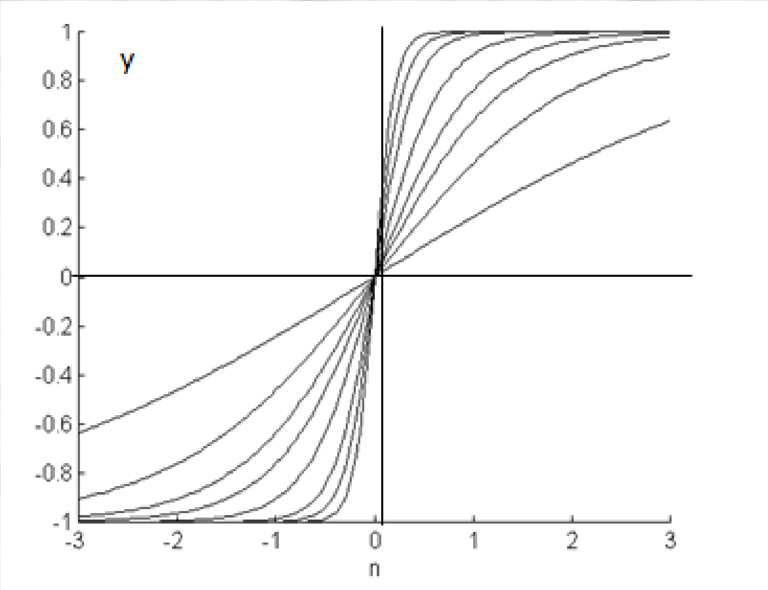
Transfer functions

Unipolar sigmoidal function (log) (non symmetric)



$$y = \frac{1}{1+\exp(-\lambda n)}$$

Bipolar sigmoidal function (log) (symmetric)

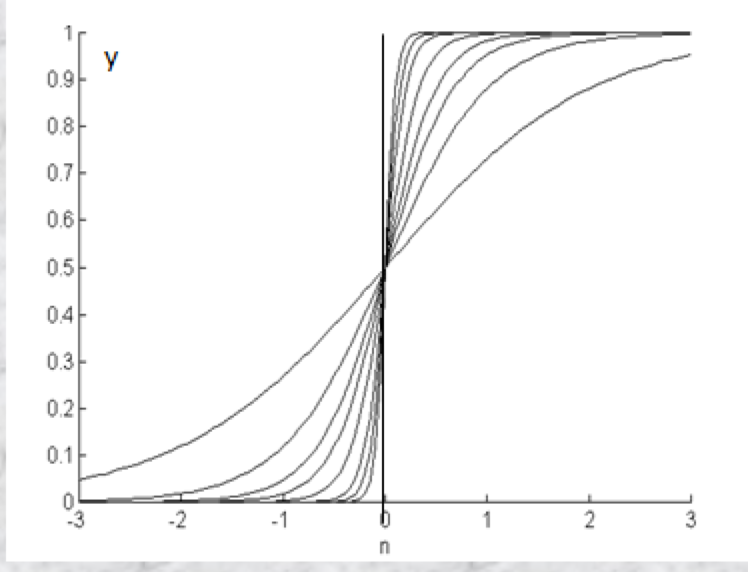


$$y = \frac{2}{1+\exp(-\lambda n)} - 1$$

$$\lambda \in \{0.5; 1; 1.5; 2; 3; 5; 7; 10\}$$

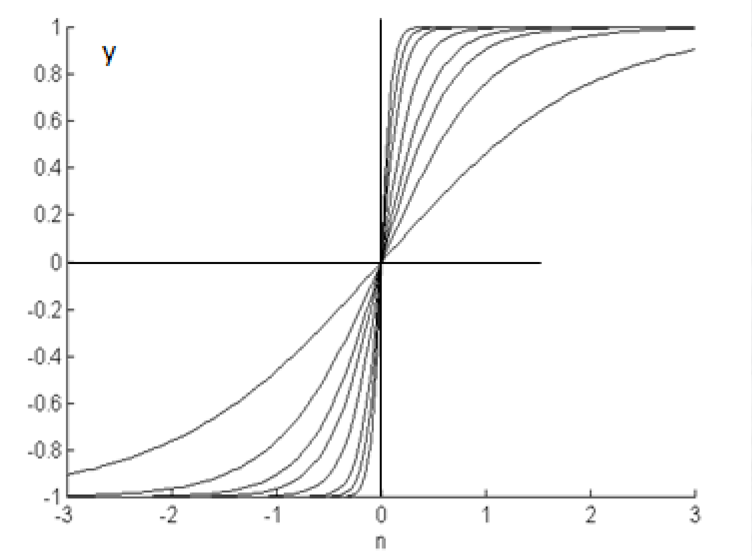
Transfer functions

Unipolar sigmoidal function (th) (non symmetric)



$$y = \frac{1}{2}\tanh(\lambda n) + \frac{1}{2} = \frac{1}{2}\frac{\exp(\lambda n) - \exp(\lambda n)}{\exp(\lambda n) + \exp(\lambda n)} + \frac{1}{2}$$

Bipolar sigmoidal function (th) (symmetric)

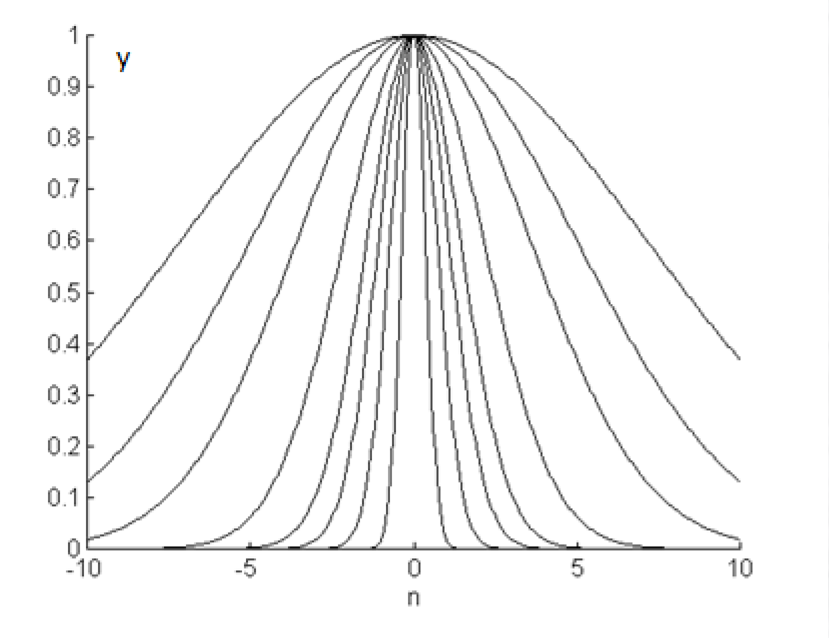


$$y = \tanh(\lambda n) = \frac{\exp(\lambda n) - \exp(\lambda n)}{\exp(\lambda n) + \exp(\lambda n)}$$

$$\lambda \in \{0.5; 1; 1.5; 2; 3; 5; 7; 10\}$$

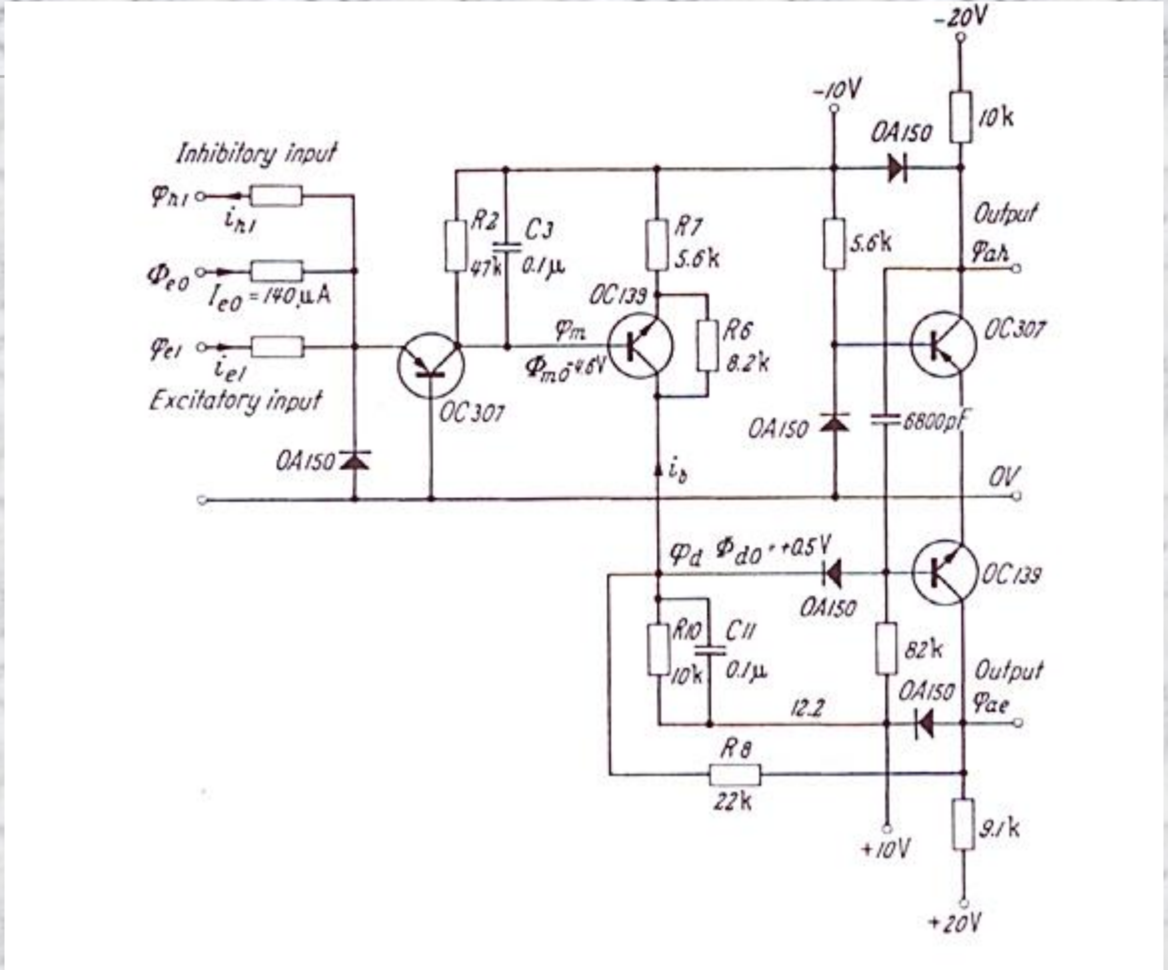
Transfer functions

Gauss function
(Radial Basis)



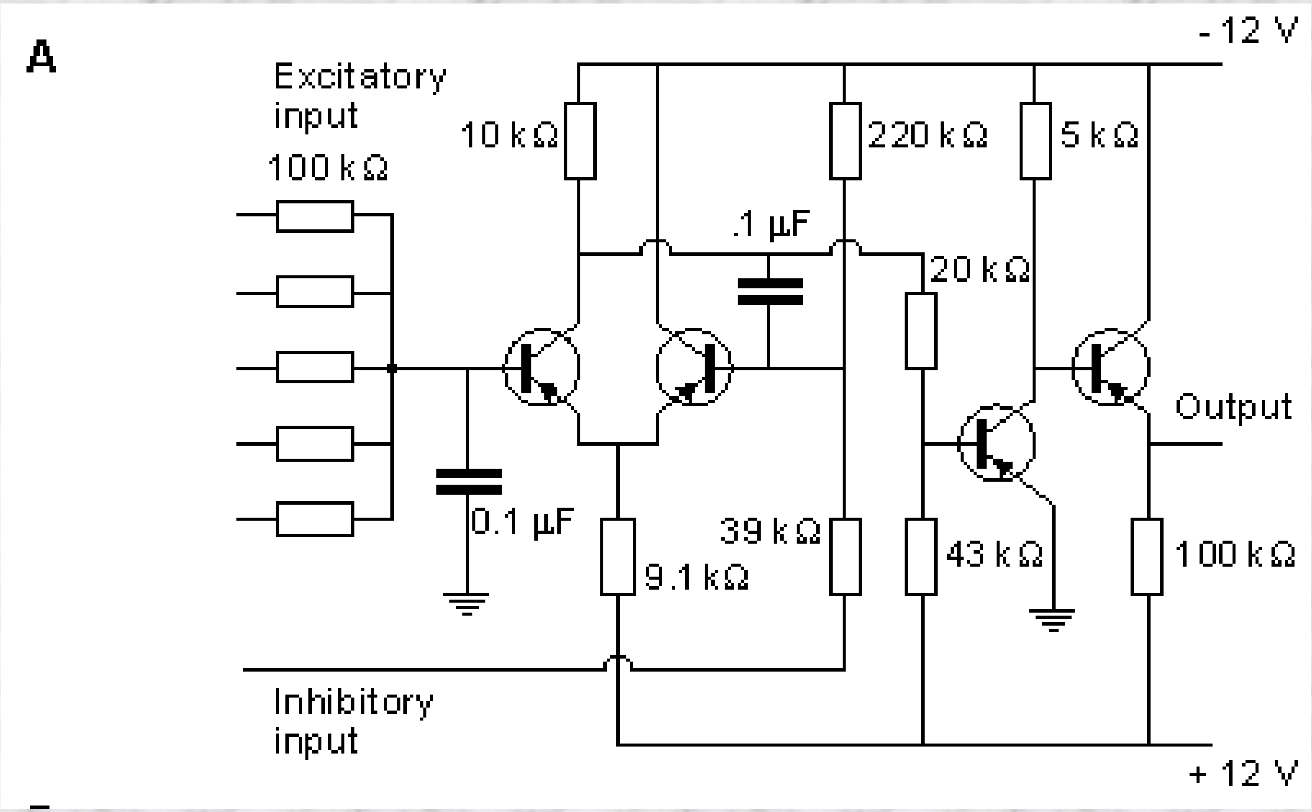
$$y = exp\left(-\frac{n^2}{\sigma^2}\right) \qquad \sigma \in \{0.5; 1; 1.5; 2; 3; 5; 7; 10\}$$

Electronic models



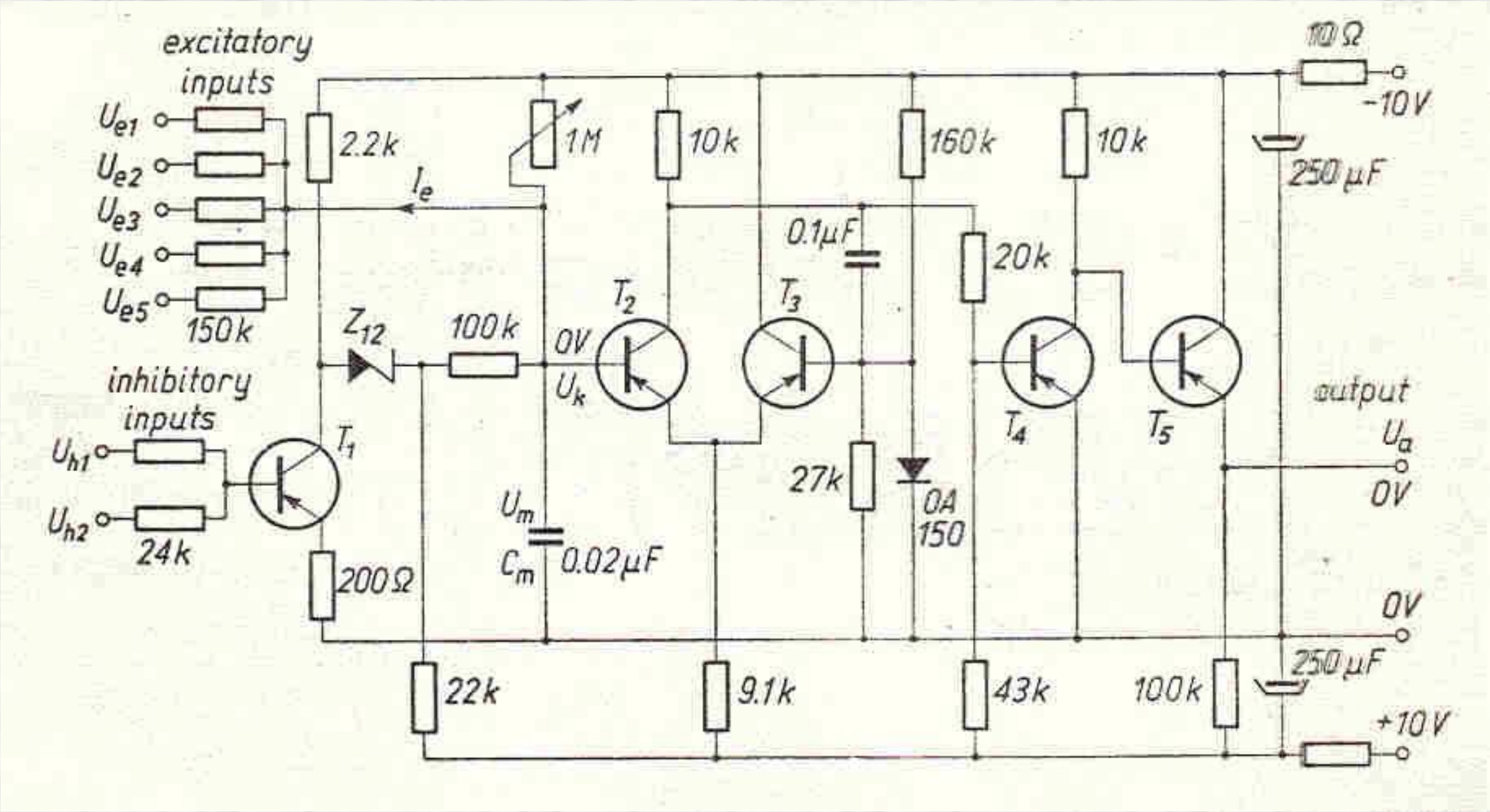
Electronic neural cell model due to McGrogan

Electronic models



Electronic neural cell model due to Harmon.

Electronic models



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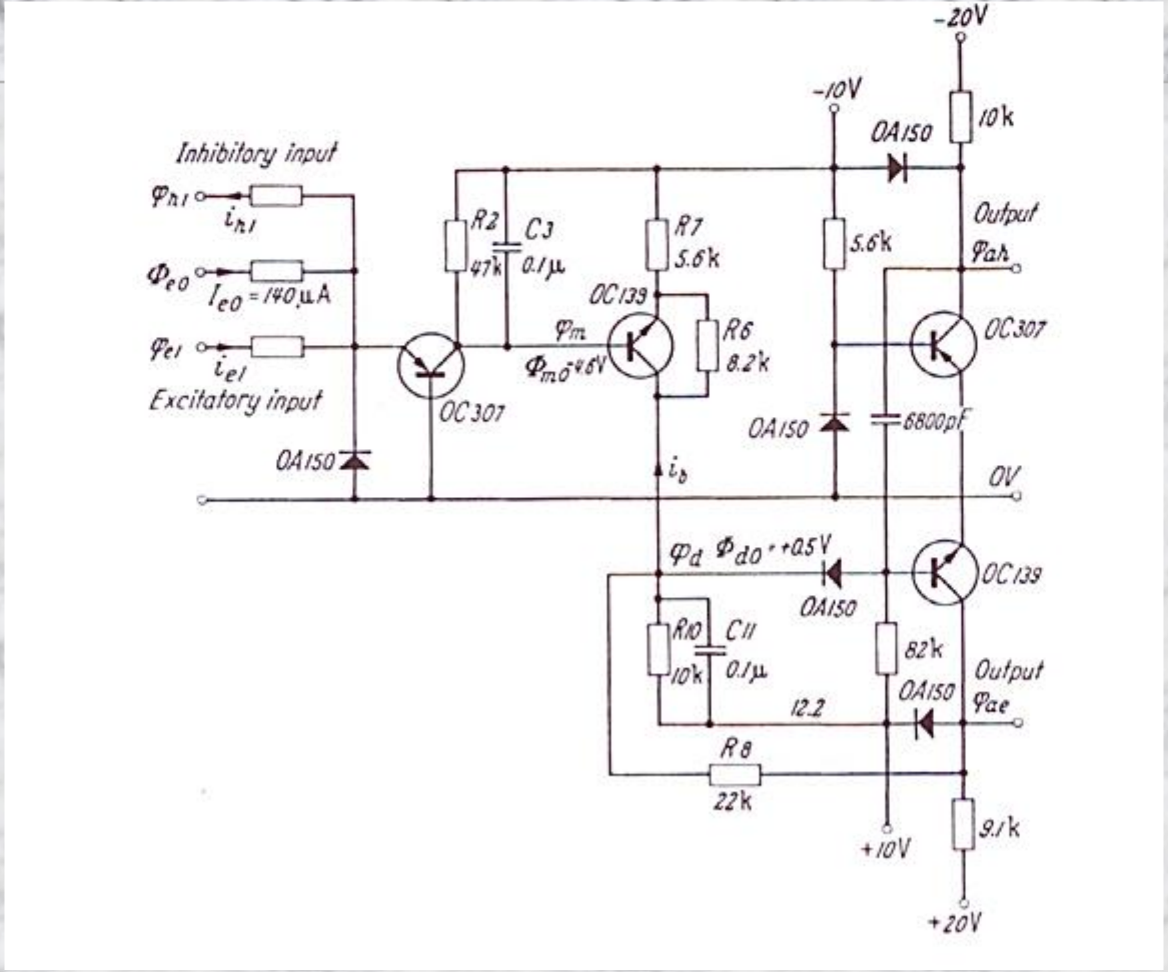
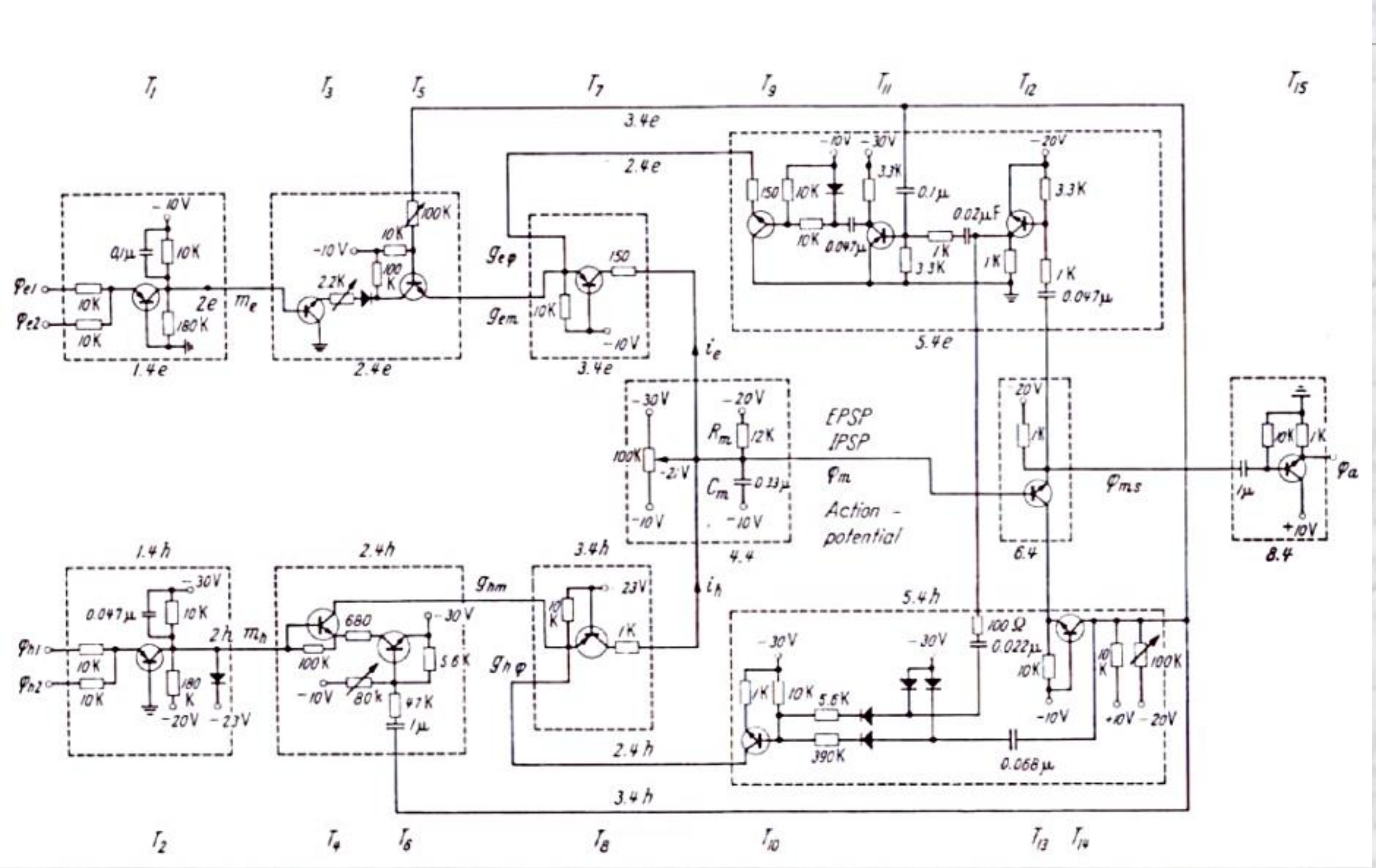
Electronic models



Electronic neural cell model due to Taylor.

Electronic models

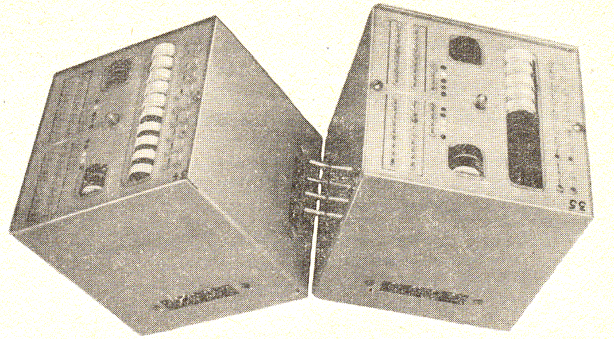


Electronic neural cell model due to Küpfmüller and Janik

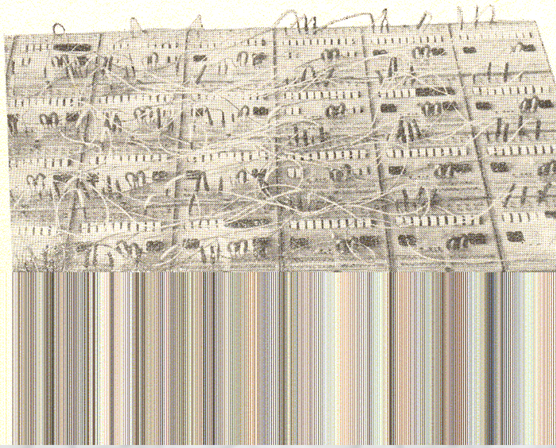
Models built in the Bionics Laboratory, PAS

Neuron model built in the Bionics Laboratory IA PAS, in 1969

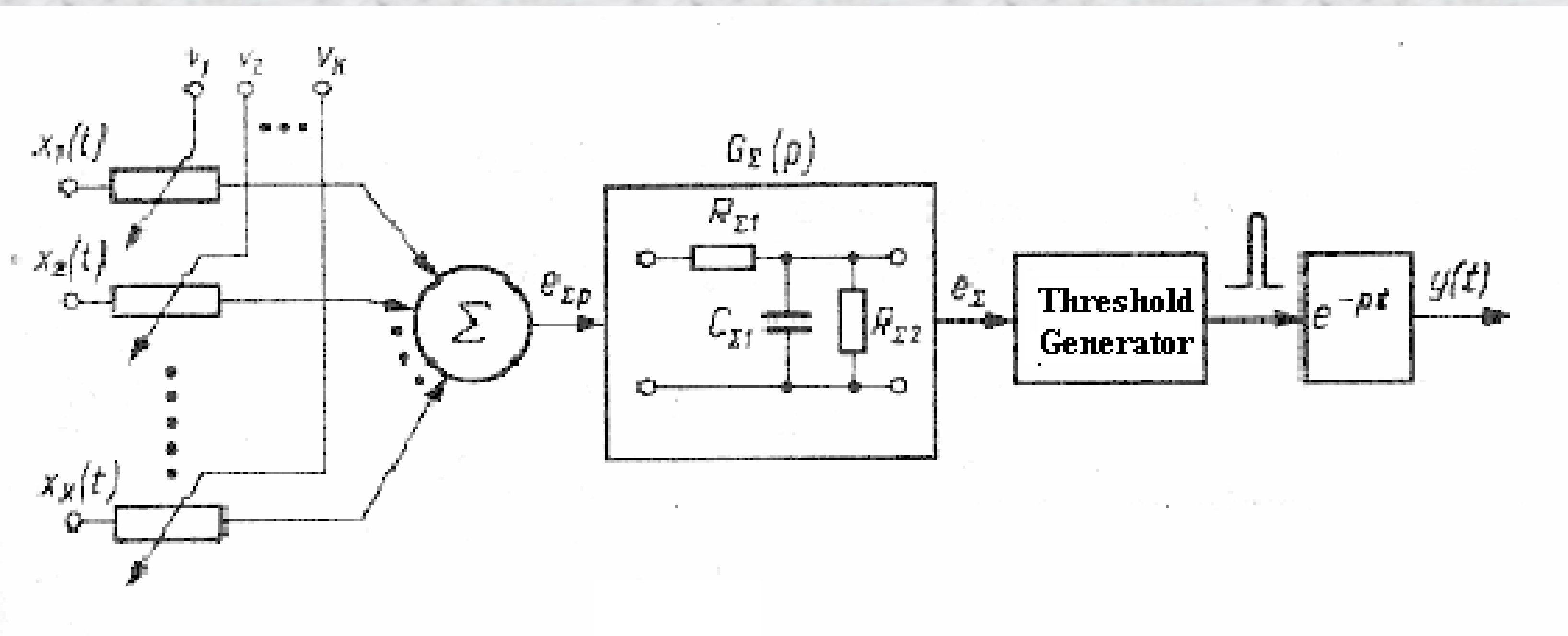
Neural network model built in the Bionics Laboratory IA PAS, in 1969



Rys. 2.44. Fotografia modelu neuronu skonstruowanego w Samodzielnej Pracowni Bioniki IA PAN



Simplified model of a neural cell



McCulloch Symbolism

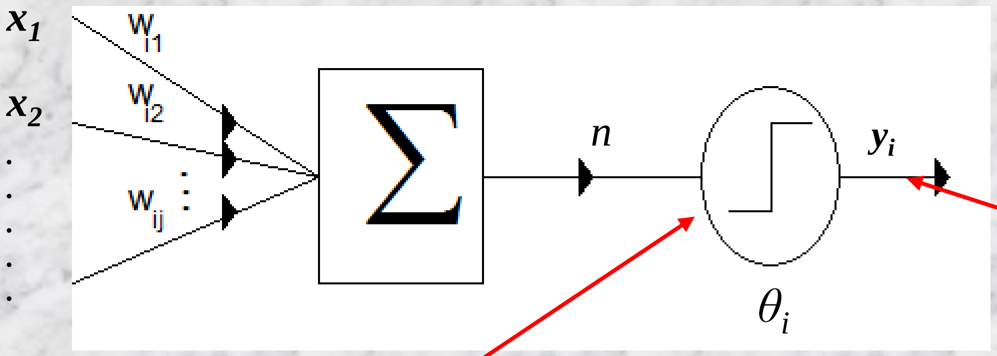
McCulloch-Pitts Model

In 1943 Warren McCulloch and Walter Pitts proposed the first simple mathematics model of a neuron as a two-values threshold element.

The McCulloch-Pitts neuron calculates the weighted sum of input signals incoming from other neurons and produce at the output value 1 (on) or 0 (off) depending the sum is greater or smaller from the threshold value.

McCulloch-Pitts Model

McCulloch and Pitts model of a single neuron



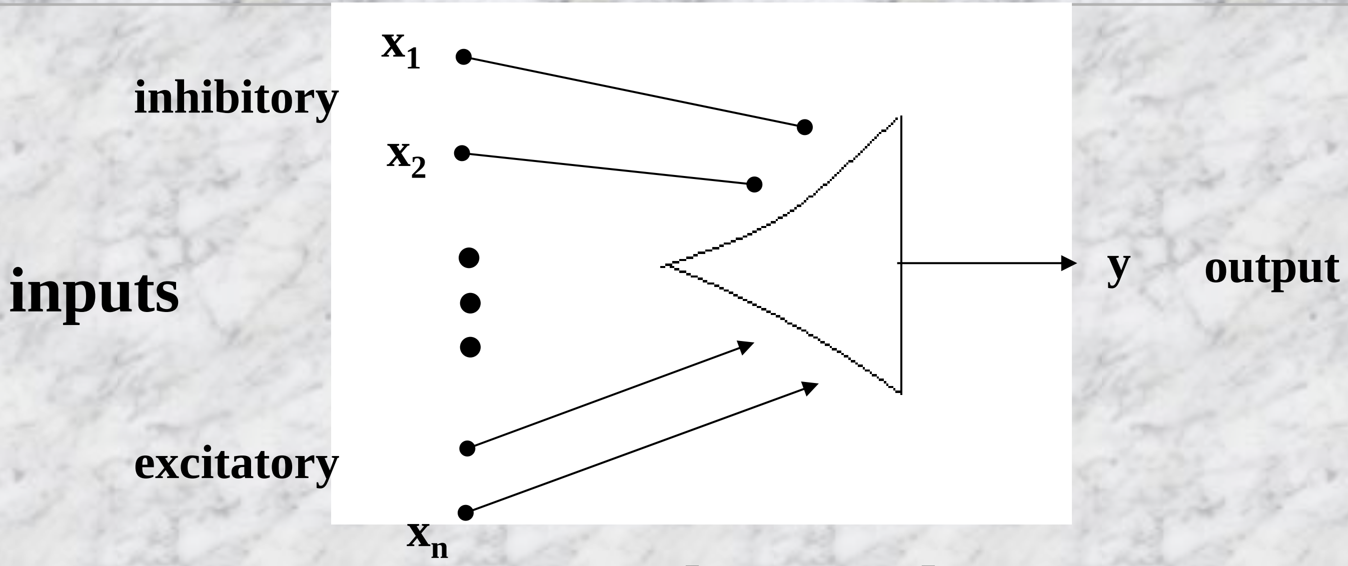
$$y_i(t+1) = f\left(\sum_j w_{ij} x_j(t) - \theta_i\right)$$

Transfer function

$$f(x) = \begin{cases} 1 & if \;\; n \geq 0 \\ 0 & otherwise \end{cases}$$

Neural cell models

McCulloch and Pitts Model



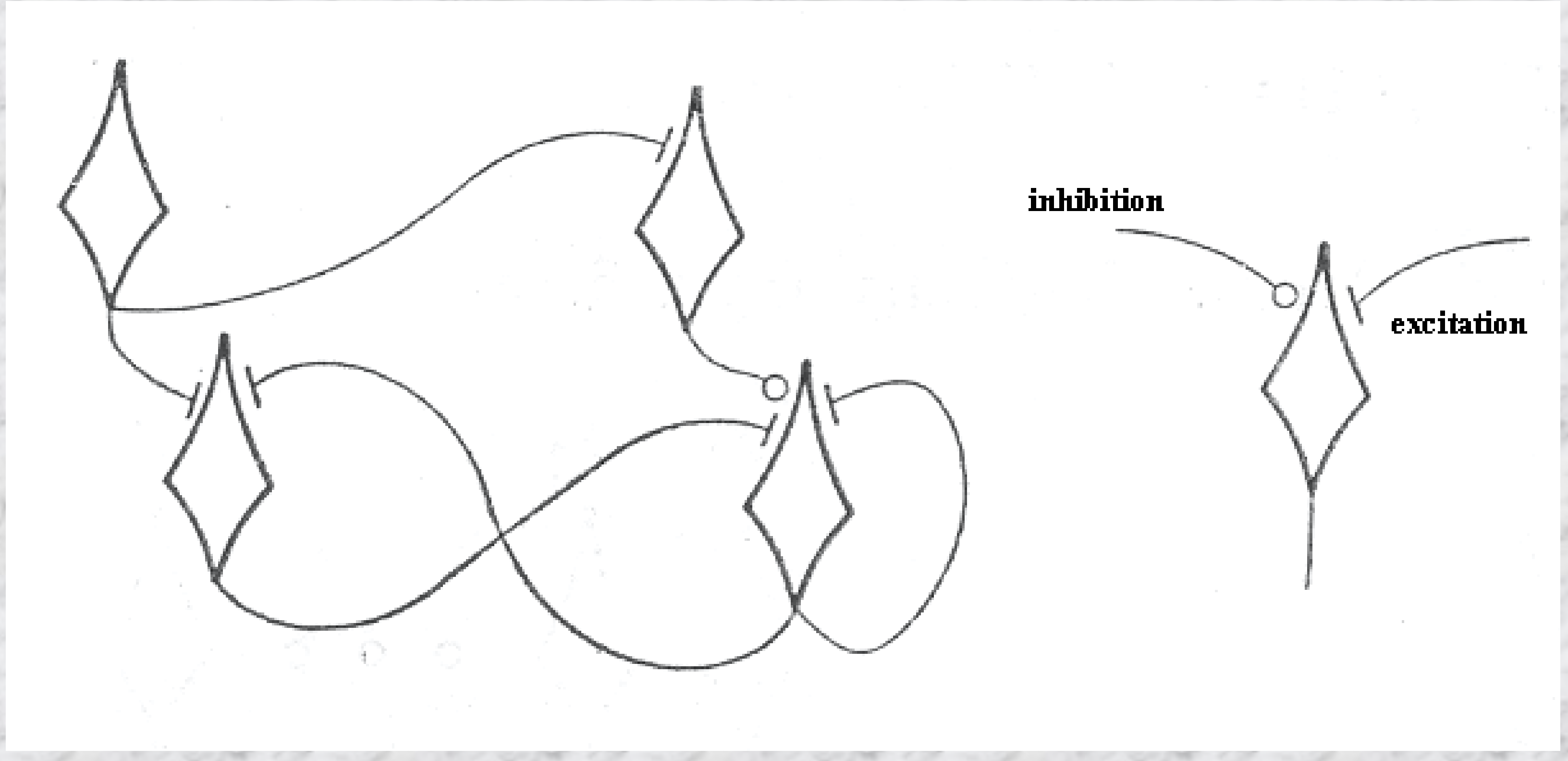
$$y(t+1) = \frac{1}{2}\operatorname{sgn}\left[\sum_{j=1}^{n} v_j x_j(t) - \Theta\right] + \frac{1}{2}$$

or

$$y(t+1) = \begin{cases} 1 \ \textbf{when} \ \sum_{j=1}^{n} v_j x_j(t) \geq \Theta \\ 0 \ \textbf{when} \ \sum_{j=1}^{n} v_j x_j(t) < \Theta \end{cases}$$

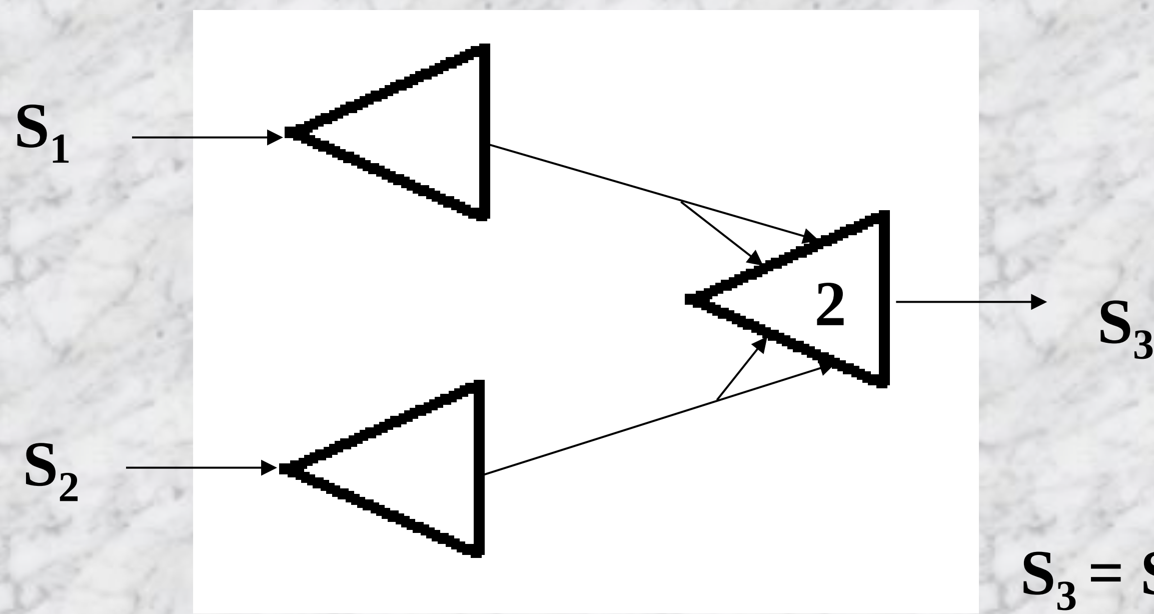
Neural cell models

McCulloch and Pitts models



Simple nets build from McCulloch &Pitts elements

From these simple elements - formal neurons - the nets simulating complex operations or some forms of the behavior of living organisms can be modeled



$S_3 = S_1 \vee S_2$

Simple nets build from McCulloch &Pitts elements

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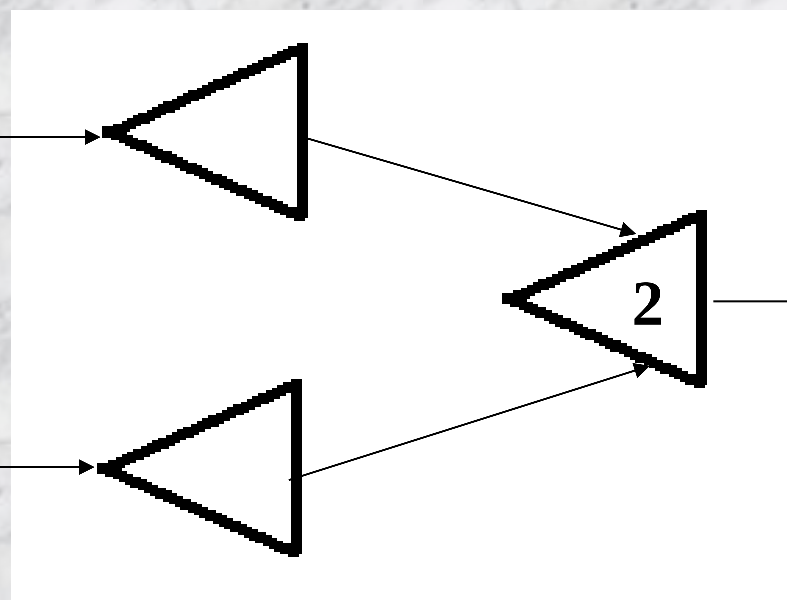
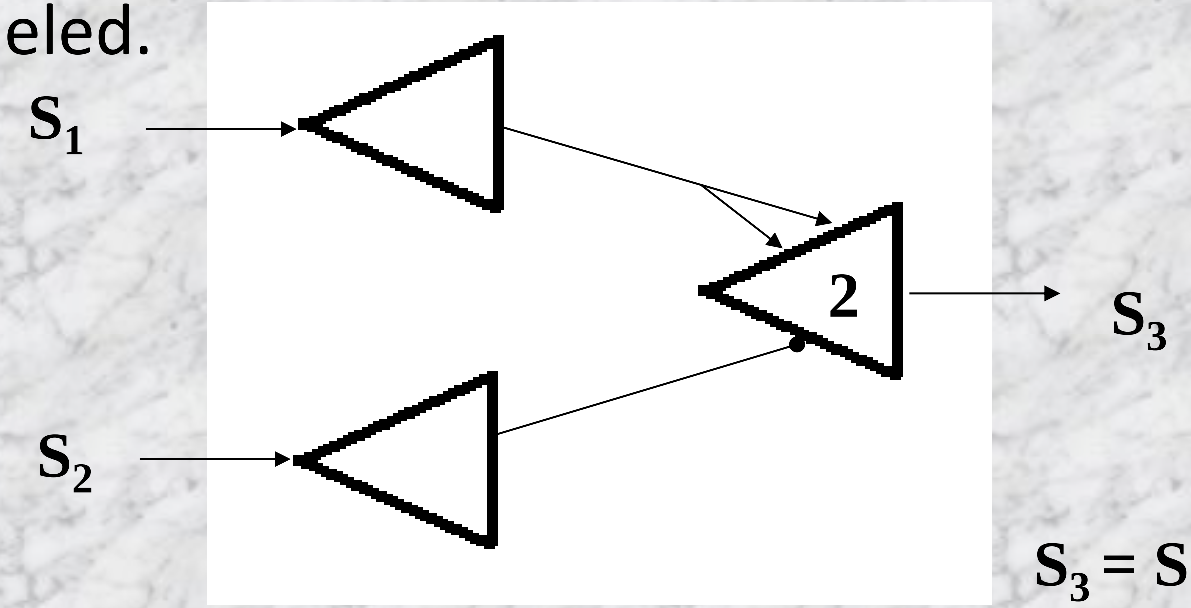


$S_3 = S_1 \wedge S_2$

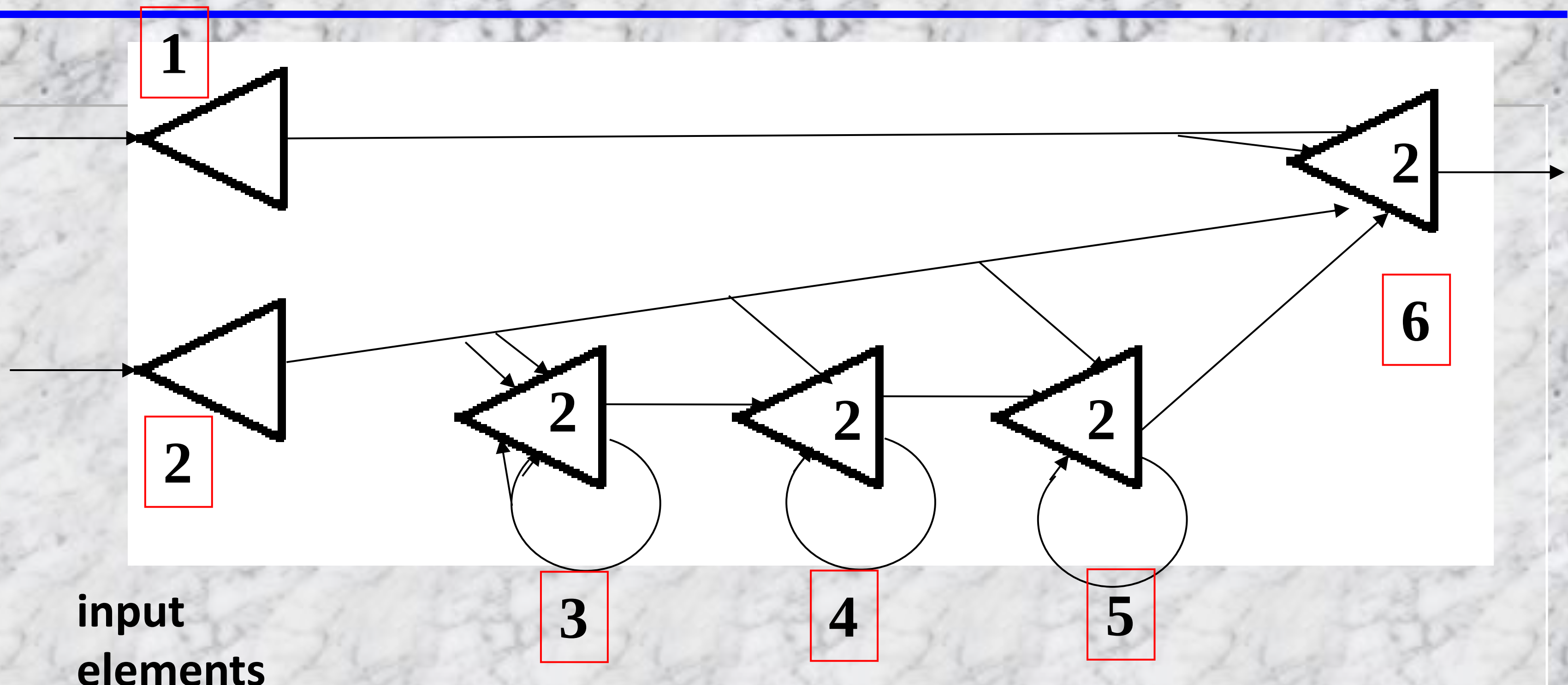
Simple nets build from McCulloch &Pitts elements

From these simple elements - formal neurons - the nets simulating complex operations or some forms of the behavior of living organisms can be modeled.



$S_3 = S_1 \wedge \sim S_2$

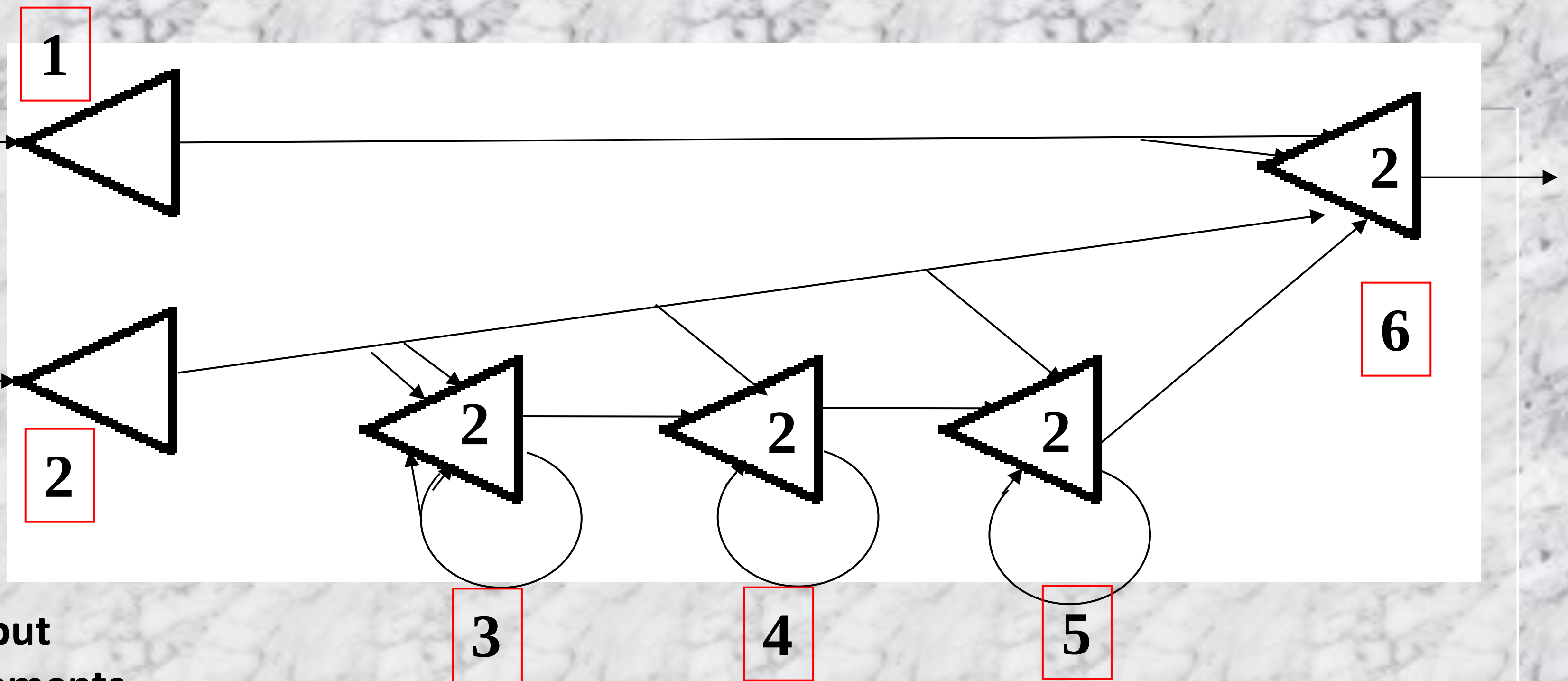
Simple nets build from McCulloch &Pitts elements, facilitation phenomenon



Signals incoming to input: 1 directly excite the element 6

Signal incoming to input: 2 excites 6 after 3 times repetition

Simple nets build from McCulloch &Pitts elements, facilitation phenomenon



input elements

First excitation of 2 excite 3 but is not enough to excite the others

This excitation yields to self excitation (positive feedback) of the 3

Output from 3 approaches 4 (but below threshold). Totally 2 and 3 excite 4