(ARTIFICIAL) NEURAL NETWORKS

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ASSESSMENT:

oral exam. (60%) + lab. (40%)

Lecture notes: http://www.mini.pw.edu.pl/~mandziuk → Teaching

Outline

Associative memories (Lectures 1-3)

- capacity, attraction radius, error-correction,
- online vs. offline learning,
- Hebb rule and its modifications,
- non-hebbian learning,
- perceptron-type rules,
- bi-directional memories (BAM).

Outline

<u>*Neural Networks in classification* (Lecture 4)</u>

- Review of f-f networks
    - Perceptron and Multilayer-perceptron
    - Backpropagation algorithm

<u>*Partly Recurrent Networks* (Lecture 5)</u>

Outline

Application of Hopfield models to solving optimization problems
(Lectures 6-7)

- Hopfield model as electrical circuit,
- reprezentation of the combinatorial problem,
- energy function,
- choice of coefficients,
- deterministic, chaotic and stochastic extensions of HMs,
- practical applications.

Assessment method

EXAM:

- without lecture notes
- four questions
- 0-60 pts
- passed if > 30 pts

LAB:

- 0-40 pts
- passed if > 20 pts

FINAL SCORE:

exam+lab

- passed if > 50 pts

To pass the course **BOTH** exam and project must be passed

What NNs are used for?

When it is worth to use them?

?

Recent years → A resurgence of NNs

Some Thought-Provoking Examples

- Deep Mind → AlphaGo beats Lee Sedol (4:1, March 2016)
- Deep Mind → AlphaGo Zero (learning from scratch, TensorFlow)

- Facebook → Deep Face → Face recognition rate of 97,35% → comparable (or excelling) that of humans

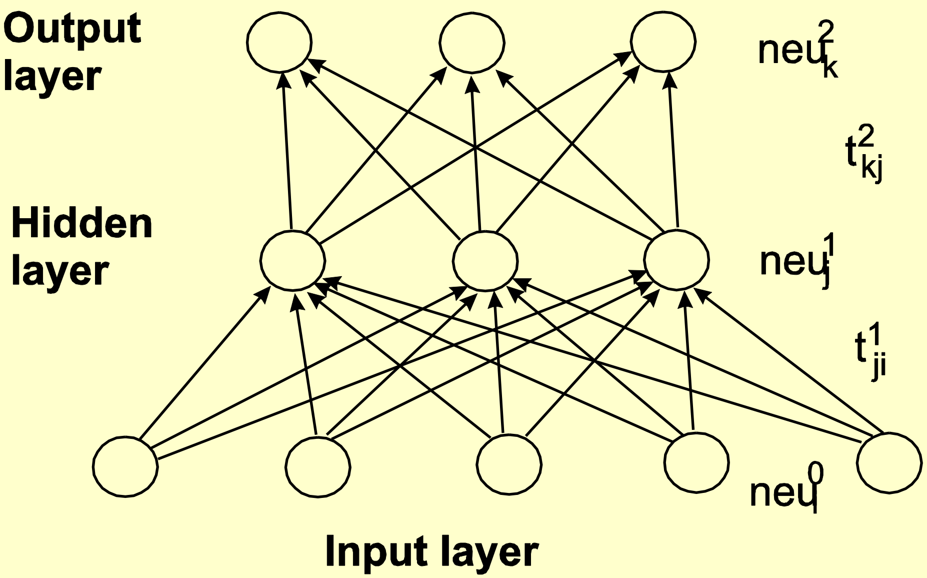
- IBM → Watson → *Jeopardy* , but also … nontrivial recipies (cooking) → it composed a cooking book basing on a combination of several world cuisines by means of analysis of the properties of the components of dishes representative for these cuisines

- ➔ Human-level intelligence (or beyond …)

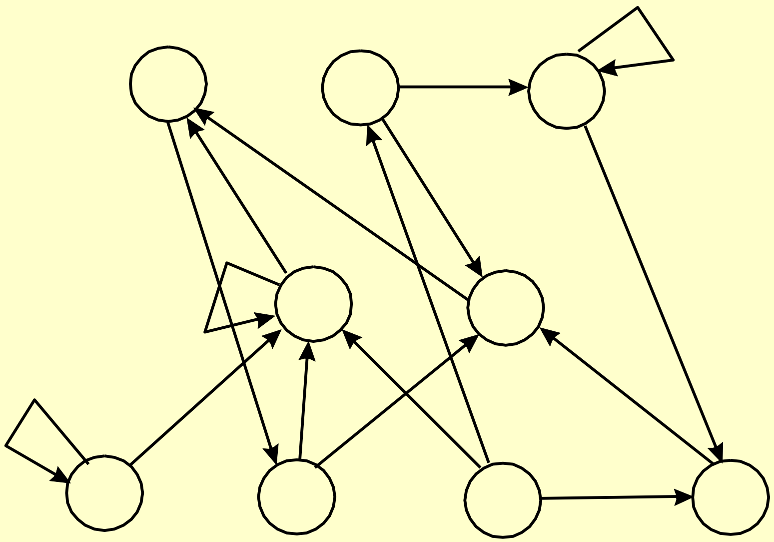
- ➔ Quo vadis AI (NNs)?

Feed-forward vs. Recurrent neural networks

Feed-forward network



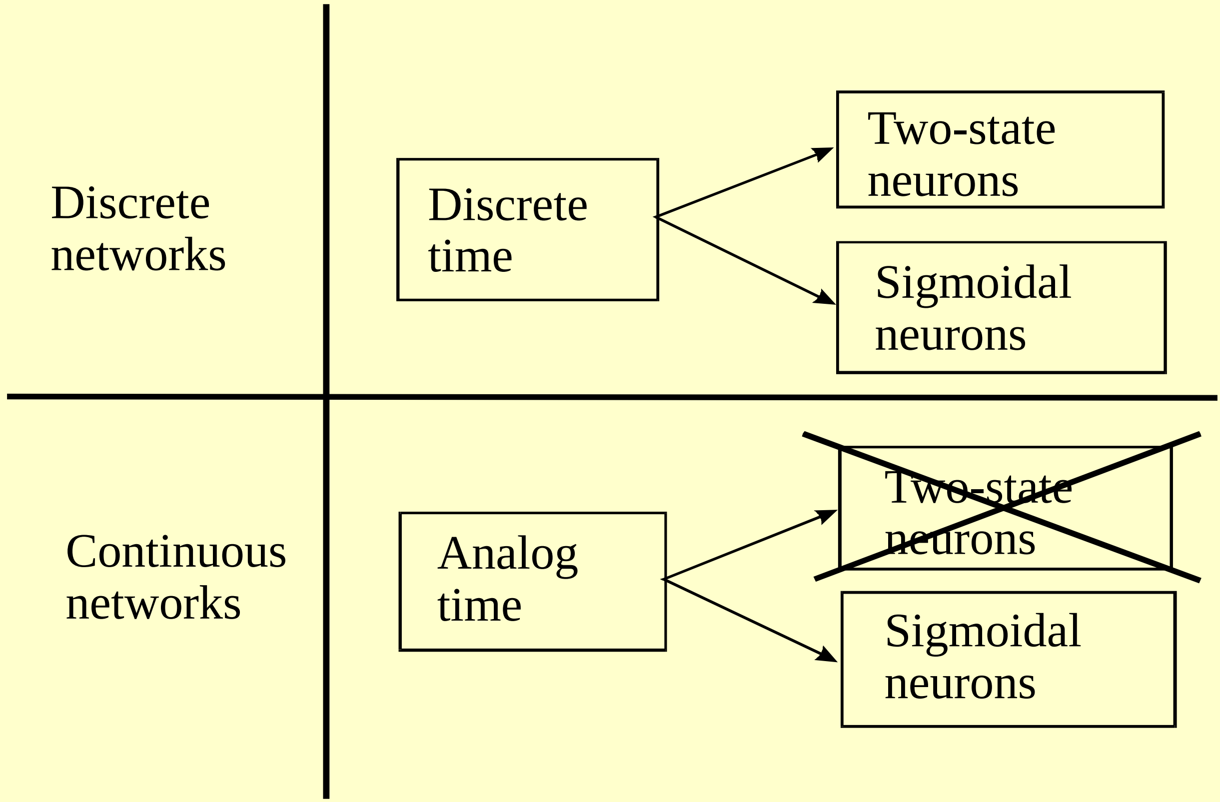
Recurrent network



Recurrent Hopfield net

discrete:

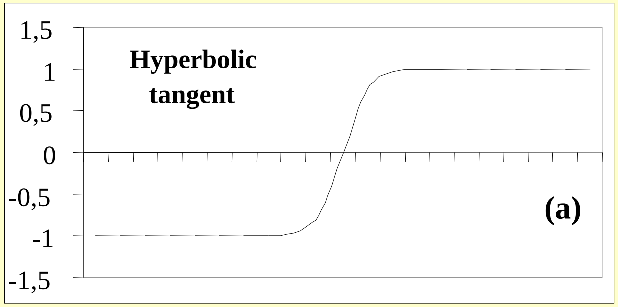
- combinatorial optimization
- associative memories

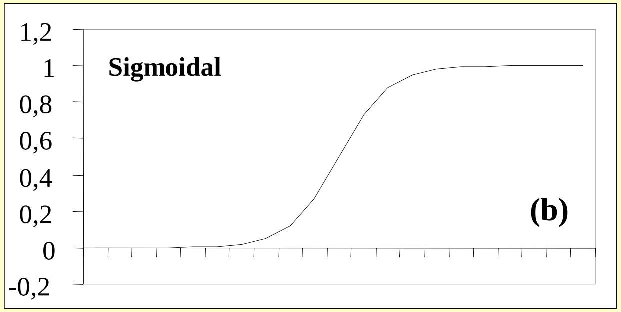


continuous:

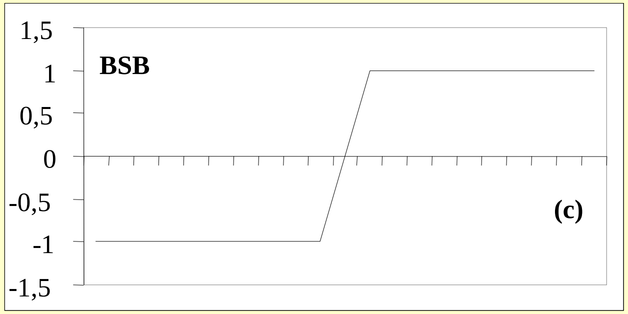
- solving combinatorial optimization problems

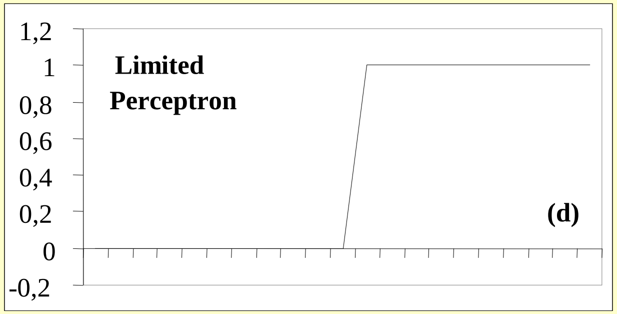
$$g(x)=\frac{1}{2}\left(1+\tanh(\alpha x)\right)$$



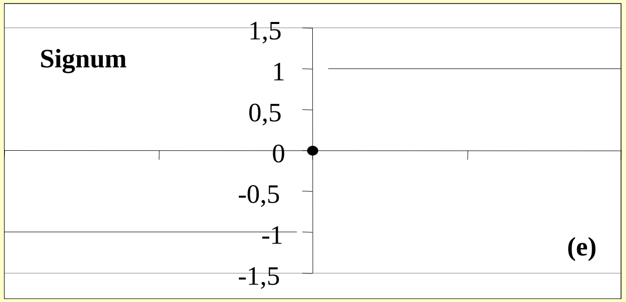


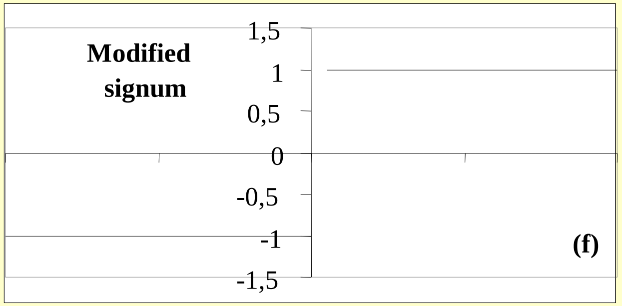
$$g(x)=\frac{1}{1+e^{-\alpha x}}$$



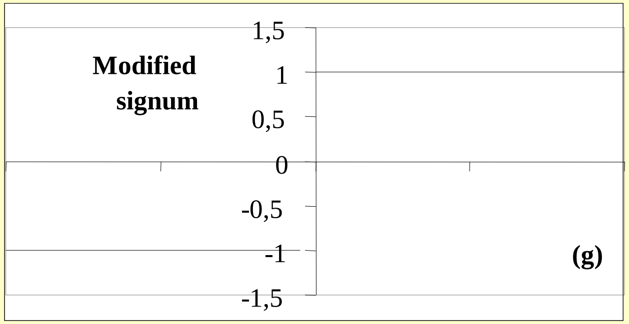


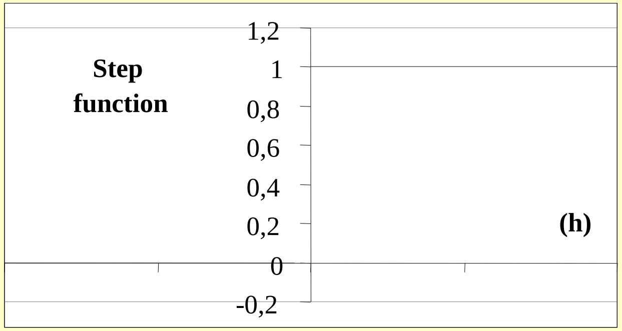
$$g(x)=\begin{cases}1 & \text{for} \quad x\geq 0\\ 0 & \quad\quad\; x<0\end{cases}$$





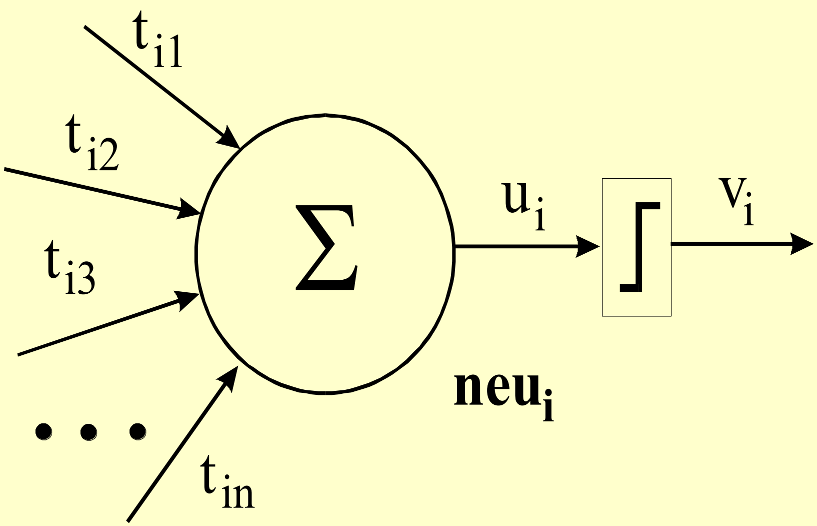
$$g(x)=\begin{cases}1 & \text{for} \quad x> 0\\ 0 & \quad\quad\; x\leq 0\end{cases}$$





Discrete Hopfield net

McCulloch-Pitts neurons (1943)

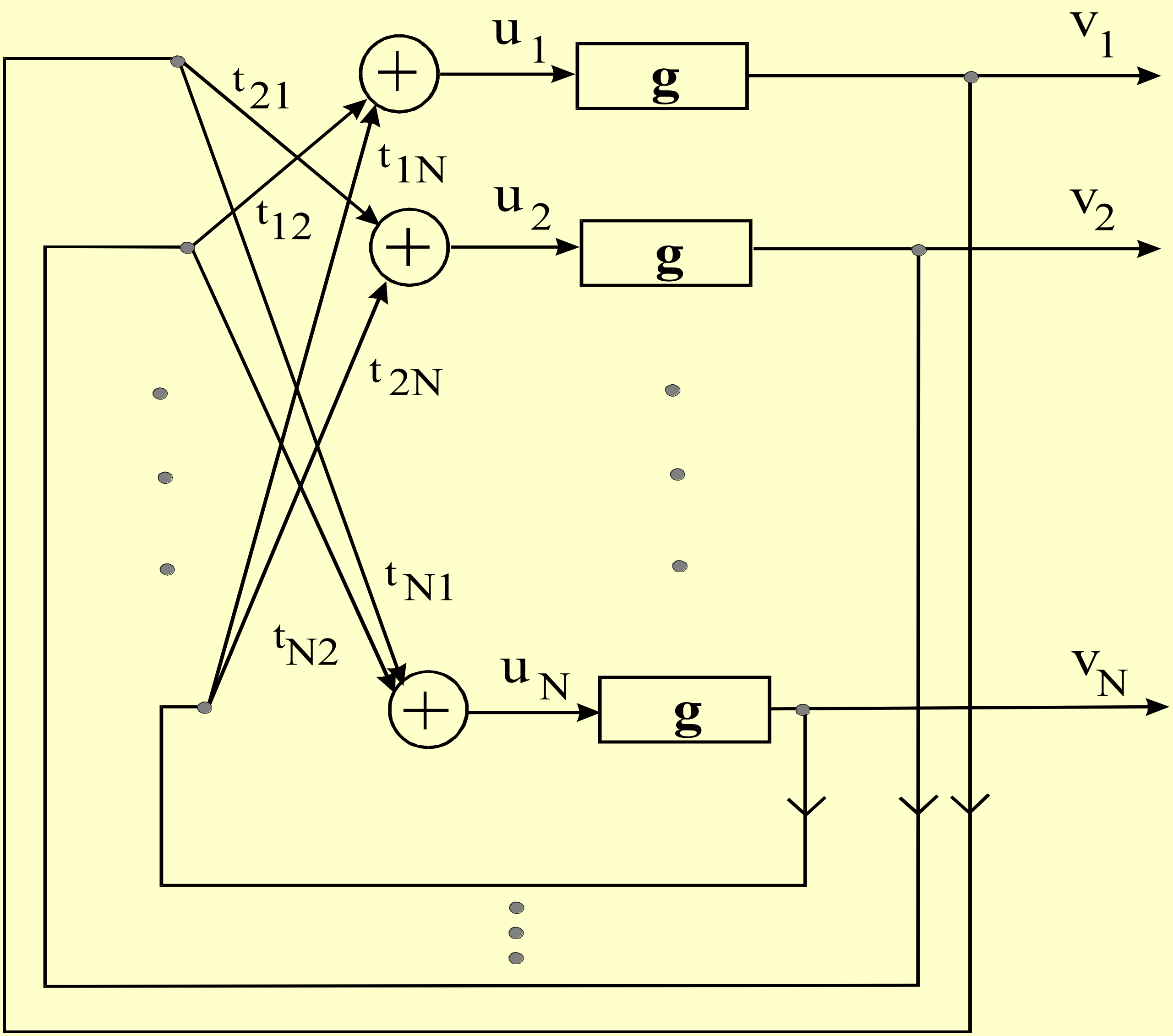


Ising model – spin glass theory – statistical quantum mechanics

Little model (1974, 78) – ***synchronous*** long-term memory (LTM)

Hopfield – associative memories, ***asynchronous*** *updating*

Hopfield net (1982, 84, 85)



Dynamics (discrete net)

Two modes:

SYNCHRONOUS mode: all neurons are updated simultaneously

$$u_i(t+1) = \sum_{j=1}^{N} t_{ij}\, v_j(t) + I_i, \qquad v_i(t+1) = g\big(u_i(t)\big), \qquad i, j = 1, \ldots, N$$

ASYNCHRONOUS mode: at time *t* one neuron neu_i is selected at random; the neuron updates its input and output potentials

- index is selected based on uniform distribution
- a neuron is selected according to predefined permutaion (cycle)

Dynamics (discrete net) – cont.

BIPOLAR net

$$g(x)=\begin{cases}1 & \text{for } x\geq 0\\ -1 & \text{for } x<0\end{cases}$$

$$g(x)=\begin{cases}1 & \text{for } x>0\\ -1 & \text{for } x\leq 0\end{cases}$$

$$g(x)=\begin{cases}1 & \\ v(t) & \text{for}\\ -1 & \end{cases}\begin{array}{l}u(t+1)>0\\ u(t+1)=0\\ u(t+1)<0\end{array}$$

The only difference in case of: $u(t+1)=0$

UNIPOLAR net – analogously for {0,1}

Mathematical equivalence, but **functionally significantly distinct, because …**

Associative memories

Associative memory – elements are addressed by the content of the input rather than a physical address

POSTULATES

• the memory should respond with the library (training) vector in case this vector is shown in the input

• the memory should respond with the correct version of a library vector in case the „noisy version" (within some limits of the amount of noise added) of this vector is presented in the input

Associative memories – Hebb rule

HEBB'S POSTULATES:

[1] In human brains some psychological concepts may be represented by simultaneous activation of some (a group of) neurons.

[2] Functional groups of neurons are being formed in the learning process by strengthening connections (weights) of all simultaneously activated neurons.

[3] Memorized (learned) concept can be recalled if sufficient number (not neccessarily all) of neurons representing this conception is simultaneously activated – the so-called *context addressing*.

Associative memories – Hebb rule

Set X composed of M bipolar vectors $X^i=[x^1_i,\ldots,x^N_i]$, i=1,...,M. A memory is composed of N neurons $neu_1,\ldots neu_N$.

WEIGHT MATRIX → HEBB RULE ('49)

$$t_{ij} = \begin{cases} 0 & for \quad i = j \\ \dfrac{1}{N}\sum_{s=1}^{M} x_i^s x_j^s & for \quad i \neq j \end{cases} \qquad i, j = 1,\ldots, N$$

$$T = \frac{1}{N}(XX^T - MI)$$

I, X – matricies

Associative memories – Hebb rule

UNIPOLAR VECTORS

$$t_{ij} = \begin{cases} 0 & for \quad i = j \\ \dfrac{1}{N}\sum_{s=1}^{M}(2x_i^s - 1)(2x_j^s - 1) & for \quad i \neq j \end{cases} \qquad i, j = 1, \ldots, N$$

EXAMPLES

X={[1,1,1,1], [-1,-1,-1,-1],[1,-1,1,-1]}^{T} →

X={[1,0,1], [1,1,1],[0,0,1], [1,0,0]}^{T} →

Associative memories

Rules local/non-local

Rules online/offline

Hebb rule is ...

ITERATIVE VERSION

$$t_{ij}^{s} = \begin{cases} 0 & for \quad s = 0 \\ 0 & for \quad i = j \\ t_{ij}^{s-1} + \frac{1}{N} x_i^s x_j^s, & for \quad i \neq j,\, s = 1,\ldots,M \end{cases} \qquad i, j = 1,\ldots, N$$

Associative memories

RECOGNITION (TEST) PHASE

$$u_i = \sum_{j=1}^{N} t_{ij} z_j, \qquad v_i = g(u_i), \qquad i = 1, \ldots, N$$

- iterative
- synchronously / asynchronously

EXAMPLES

Two possible scenarios:

- either stabilization
- or a cycle of length 2: A→B→A→B→... (only in synchronous mode)

Associative memories – synchronous mode

EXAMPLE (generation of cycle)

$$X^1 = [0,0] \quad X^2 = [1,1] \quad \rightarrow \quad T =$$

$$Z = [1,0]$$

THEOREM (estimation of the number of iterations)

For any input vector the maximal number of synchronous iterations of Hopfield net before the stable state is achieved or the net enters a cycle of length 2 does not exceed

$$4^{2^{M-1}}$$

where M is the number of library vectors stored in the network.

Associative memories – synchronous mode

EXAMPLE (adding noise)

$$\begin{matrix} 0 & 1 & 3 & 1 \\ 1 & 0 & 1 & 3 \\ 3 & 1 & 0 & 1 \\ 1 & 3 & 1 & 0 \end{matrix} \rightarrow \begin{matrix} 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \\ 3 & 1 & 0 & 1 \\ 1 & 3 & 1 & 0 \end{matrix} \qquad \begin{matrix} 0 & 1 & 3 & 1 \\ 1 & 0 & -1 & 3 \\ 3 & -1 & 0 & 1 \\ 1 & 3 & 1 & 0 \end{matrix}$$

Memory loss　　　　Memory error

EXAMPLE (sensitivity – flipping one bit in one of the vectors)

$X_1=[1, 1, 1, -1]^T$, $X_2=[-1, -1, -1, -1]^T$, $X_3=[1, -1, 1, -1]^T$

Associative memories – asynchronous mode

In asynchronous mode – when the weights are updated according to a predefined permutation – the problem of oscillation DOES NOT OCCUR

THEOREM (sufficient convergence conditions)

In asynchronous mode the sufficient condition for net's convergence to a stable state is that matrix T is symmetrical and has zeros on the main diagonal.

Proof: unipolar net with 3-value activation function; define auxiliary Energy function E; calculate one-step change of the energy; E is non-increasing in time, bounded and finite.

Associative memories – asynchronous mode

EXAMPLE – Z = [1,0], order {2,1} or {1,2}

THEOREM (estimation of the number of iterations)

If matrixT of bipolar Hopfield net is symmetric, with non-negative diagonal elements, then the number of asynchronous steps required for the networks to converge to a stable state does not exceed

$$\frac{\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,\, j\neq i}^{N}\left|t_{ij}\right| + \sum_{i=1}^{N} e_i}{1+\min_i t_{ii}}$$

where

$$e_i = \begin{cases} 1 & for \quad \sum_j t_{ij} \ even \\ 0 & in\ the\ opposite\ case \end{cases}$$

Associative memories – cont.

Construction of matrix T as optimization problem

<u>THEOREM</u>

A sufficient condition for an N-element set of bipolar vectors $X=\{X^{\mathbf{1}},...,X^{\mathbf{M}}\}$ to be properly memorized and retrieved in the Hopfield net is to fulfill the following set of inequalities:

$$(\sum_{j=1}^{N} t_{ij} x_j^k) x_i^k > 0, \qquad k = 1,...,M; i = 1,...,N$$

Proof:

Associative memories – quality

- capacity
- attraction radius

THEOREM (asymptotic network capacity)

Capacity of bipolar (synchronous or asynchronous) Hopfield net with Hebb rule converges asymptotically to

$$M \to \frac{N}{4 \ln N}, \qquad N \to \infty$$

Practical estimation: (Hopfield, 1982): $M \approx 0{,}15N$

Associative memories – capacity

In case the probability of perfect recall of a vector equals 0.994 the net's capacity equals

$$M \approx \frac{N + 2[\ln N + 3] - 1}{2[\ln N + 3]}$$

For $N \to \infty$ $M \approx \dfrac{N}{2\ln N}$

Advantage: the above ARE NOT asymptotic results.

E.g. N=10 → M=1.85, N=20 →M=2.58, N=100 → M=7.85

Associative memories – weaker restrictions

Weaker restrictions on the main diagonal

Bipolar net; experimental results; positive values on the main diagonal of T and modification of the input-output function leads to increase of the capacity of the memory:

$$v_i(t+1) = \begin{cases} 1 & dla \quad u_i(t+1) > t_{ii} \\ v_i(t) & dla \quad -t_{ii} \le u_i(t+1) \le t_{ii} \\ -1 & dla \quad u_i(t+1) < -t_{ii} \end{cases}$$

Increase of capacity and more stable memory →

Only sufficiently big changes are accepted.

Properties of associative memories

Classical Hopfield model with Hebb rule:

- the efficiency of the memory (drastically) degrades along with the increase of the number of stored patterns

- the efficiency degrades also in case of storing rare/dense patterns

➔Alternative learning rules have been proposed with the architecture of the network and its operational principles (in the restoring mode) remaining unchanged

Thank you!