HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY
FACULTY OF APPLIED SCIENCE

Report

GENERAL PHYSIC 1

Topic 12 | **Study of oscillations** | Date: 5th December of 2023

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TABLE OF CONTENTS

Content

The oscillation of any body due to elastic force can be described by the differential equation:

$$\frac{d^2y}{dt^2} + b\frac{dy}{dt} + \omega_0^2 y = F\cos(\omega t)$$

In which, y is oscillation displacement, b is a damped coefficient, ω_0 is the angular frequency of free oscillation, □ is the angular frequency of stimulating force.

This project requires students to use Matlab to solve the above equation to study harmonic oscillation (no damped, no stimulated force: b = F = 0), damped oscillation (b ≠ 0, F = 0), stimulated oscillation (b ≠ 0, F ≠ 0).

Task

Examine the command dsolve to solve differential equation in MATLAB symbolic calculation.
Write Matlab program to solve and plot the graph depending on time (with initial conditions y(0) = 5; y'(0) = 0):
a) Harmonic oscillation (α_0 = 3; b = F = 0; t = 20s)
b) Damped oscillation (α_0 = 10; b = 0.01, 0.1, 1.0, 10.0 ; F = 0; t = 20s) % many values of b
c) Stimulated oscillation (α_0 = 10; b = 0.1 ; F = 10; □ = 10.0, 5.0, 3.0, 0.0; t = 150s) % many values of □
Discuss about the obtained results. Note: Students can use other non-symbolic approaches. Submitting report has to contain text explaining the content of the program and the entire code verified to run properly in Matlab.

Chapter 1: Theory

The equation $\frac{d^2y}{dt^2} + b\frac{dy}{dt} + \omega_0^2 y = F\cos(\omega t)$ is a second-order linear differential equation that describes the motion of an oscillating system. Let's break down its components:

1. y represents the oscillation displacement. It's a function of time, t, and it describes how the position of the system changes over time.
2. $\frac{d^2y}{dt^2}$ is the second derivative of y with respect to time. It represents the acceleration of the system. In the context of oscillation, this term captures how the system's acceleration depends on its position.
3. b is the damping coefficient. It accounts for the damping or resistance in the system. Damping reduces the amplitude of oscillation over time. $\frac{dy}{dt}$ is the first derivative of y with respect to time, which represents the system's velocity. It shows how the system's velocity changes over time.
4. ω_0 is the angular frequency of free oscillation, representing the system's natural or intrinsic frequency of oscillation when there are no external forces or damping. It's related to the stiffness and mass of the system.
5. F is the amplitude of the stimulating force. This force can be applied externally to the system and may vary with time.
6. $\cos(\square t)$ represents a cosine function of angular frequency ω and time t. It models a periodic external force acting on the system.

The equation combines these components to describe the behavior of a system undergoing oscillatory motion. The left-hand side $\frac{d^2y}{dt} + b\frac{dy}{dt} + \omega_0^2 y$ represents the system's response to its position, velocity, and intrinsic frequency. The right-hand side $F\cos(\square t)$ represents the effect of the external force applied to the system.

Solving this equation allows you to study how the system responds to different parameters (such as b, F, and ω) and initial conditions, and it can help you understand the behavior of oscillatory systems in various physical contexts, including mechanical vibrations, electrical circuits, and more.

The equation is derived from Hooke's law, which states that the force required to extend or compress a spring by some distance is proportional to that distance 12. The elastic force is the force that restores the spring to its original length when it is stretched or compressed 2. The differential equation describes the motion of any body that oscillates due to an elastic force, such as a mass attached to a spring 1. The equation is used to model the behavior of many physical systems, including mechanical oscillators, electrical circuits, and chemical reactions 2.

1.1 Harmonic oscillation:

Periodic motion: In harmonic oscillation, the motion repeats itself in equal intervals of time. The system returns to its initial state after a fixed period. This repeating pattern is known as one complete cycle or period.

Restoring force: The motion occurs as a result of a restoring force that brings the system back toward its equilibrium or central position when it is displaced from that position. The restoring is directly proportional to the displacement from equilibrium, and it acts in the opposite direction to the displacement.

Constant Frequency: The oscillation occurs at a constant and specific frequency (ω). The system's characteristics determine this frequency and is independent of the amplitude of the oscillation.

Sinusoidal (Sine or Cosine) Motion: The displacement of the system as a function of time is described by a sinusoidal function, typically a sine or cosine function. The displacement varies sinusoidally, resulting in a smooth, back-and-forth motion. In the differential equation of the oscillation of any body, the term $\Box_0^2 y$, represents the restoring force that is characteristic of harmonic oscillation. The angular frequency ω_0 is associated with the natural or intrinsic frequency of the oscillation when there is no damping or external forcing. That is why b=F=0. The term $\omega_0^2 y$ captures the behavior of the system when it oscillates around its equilibrium position.

1.2 Damped oscillation:

Damped oscillation is a type of oscillatory motion exhibited by physical systems where the amplitude of the oscillations gradually decreases over time due to the presence of a damping force or resistance. In damped oscillation, the system's motion eventually comes to rest, reaching an equilibrium position.
Key characteristics of damped oscillation include:

1. Restoring Force: Similar to undamped harmonic oscillation, damped oscillation is characterized by a restoring force that acts on the system and pulls it back toward its equilibrium position when it is displaced. This restoring force is proportional to the displacement from equilibrium and is responsible for the oscillatory behavior.
2. Damping Force: In damped oscillation, there is an additional force called the damping force, represented by the term $b\,\frac{dy}{dt}$ in the differential equation. This force opposes the motion of the system and is proportional

to the velocity of the system $\frac{dy}{dt}$. It is directed opposite to the velocity of the system and results in energy dissipation. The damping force is responsible for reducing the amplitude of the oscillations.

3. Decay in Amplitude: Due to the damping force, the amplitude of the oscillations gradually decreases over time. The system loses energy to the surroundings through the damping process. As a result, the oscillations become smaller and eventually come to a stop, reaching equilibrium.
4. Exponential Decay: The displacement of the system as a function of time in damped oscillation often follows an exponential decay pattern. The exponential decay is influenced by the damping coefficient b, and the larger the damping coefficient, the faster the oscillations decay.

In summary, the equation $\frac{d^2y}{dt^2} + b\frac{dy}{dt} + \omega_0^2 y = F\cos(\omega t)$ is a representation of damped oscillation, where the oscillatory behavior (harmonic oscillation) is affected by both damping (damping term) and external forcing (external force term). It allows for the analysis of systems that exhibit oscillations that decrease in amplitude due to damping while being influenced by an external driving force. Damped oscillation is a common phenomenon in various physical systems and is important in engineering, physics, and other fields for understanding the behavior of systems with resistance or energy dissipation.

1.3 Forced Oscillation:

1. Definition: Forced oscillation is a phenomenon where an external periodic force is applied to a system that is already undergoing oscillation. The system oscillates with a frequency that is different from its natural frequency. The amplitude of this oscillation depends on the frequency of the external force and the damping factor of the system.
2. Driving force: $F(t) = F_0\cos(\square t)$
3. Equation:

$$\sum F_x = ma_x \rightarrow F_0 \sin \square\, t - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

4. Characteristic: After a sufficiently long time, when the energy input per cycle from the driving force equals the amount of mechanical energy transformed to internal energy for each cycle, a steady-state condition is reached in which the oscillations proceed with constant amplitude. In this situation, the solution for the equation is:

$$x = A\cos(\square t + \phi)$$

Where:

$$A = \frac{\frac{F_0}{m}}{\sqrt{(\square^2 - \square_0^2)^2 + \left(\frac{b\square}{m}\right)^2}}$$

5. Resonance: the dramatic increase in amplitude near the natural frequency is called resonance. That means, when the frequency of the driving force is near the natural frequency of oscillation, or $\omega = \omega_0$, the amplitude is large. At resonance, the applied force is in phase with the velocity and the power transferred to the oscillator is a maximum.

The syntax used in code:

variable_name = value;: Assigns a value to a variable.
‘dydt=@(t, y)’ : Defines an anonymous function taking ‘t’ and ‘y’ as input arguments.
‘[y(2); -w0^2 * y(1)]’: Represents the system of differential equations.
‘initial_conditions’ : Creates a vector with initial conditions.
‘[t , y]= ode45(.....)’: Calls the ODE45 to solve the system of differential equations.
‘y(:, 1)’: Extracts the first column of the ‘y’ matrix.
‘plot(t , y_values)’ : Plots the graph of the displacement against time.
‘title’ , ‘xlabel’, and ‘ylabel’: Set the title and axis labels for the plot.
‘for’ loop to iterate over different damping stored in the array ‘b_values’.
‘legend’:Adds a legend to the plot, indicating the different damping coefficients.
‘grid on’ adds a grid to the plot.
‘hold off’ releases the hold state, allowing subsequent plots to replace the current plot.
dsolve(ode, initial_conditions) solves the differential equation with the specified initial conditions and stores the solution in the cell array.
cell(1, length(w_values)) initializes a cell array to store solutions for different values of w.
linspace(0, t_end, 1000) generates 1000 evenly spaced time values from 0 to t_end.
syms t y(t) declares symbolic variables t and y.

Chapter 2: Method

2.1 Method of Harmonic Oscillation

Step 1: Define symbolic variables and input the parameters

t is time

y(t) is displacement function

ω_0 = 3 is the angular frequency of free oscillation

Step 2: Define the differential equation

ode = diff (y, t, t) + $\omega_0^2 . y$ == 0 is

the syntax of second-order linear homogeneous ODE:

$\frac{d^2y}{dt^2} + \omega_0^2 y = 0$. This ODE represent harmonic oscillation.

Step 3: Set initial conditions

y (0) = 5 is the initial displayment

y’ (0) = 0 is the initial velocity

Step 5: Solve the differential equation

Use the ‘dsolve’ function to find the symbolic solution to the differential equation with the specified initial conditions

solution = dsolve (ode, initial_conditions)

Step 6: Generate time value

t_values = 0:0.01:20 to create an array of time values from 0 to 20 with a step of 0.01 seconds

Step 7: Evaluate the solution

y_values = subs (solution, t, t_values) is evaluate the symbolic equation at each time point specified in t_values

Step 8: Plot the graph

plot (t_values, y_values) is visualize the solution by plotting the displacement y against time (t_values)

2.2 Method of damping oscillation

Step 1: Input the parameters:

w_0 = 10: angular frequency of free oscillation

b = [0.01 0.1 1 10.0] : different values damping coefficient

F = 0 : magnitude of the stimulating force (since there is no stimulating force, we set F = 0)

t_end = 20: Total time recorded since the beginning

Step 2: Generate time values

The linspace function in MATLAB

y = linspace(x1,x2,n) generates n points. The spacing between the points is (x2-x1)/(n-1)..

Application: we use linspace to generate time value of the oscillation from the beginning (0s) to end (20s) with the step of around 0.02s

t_values = linspace(0, t_end, 1000)

Step 3: Initialize a cell array to store a solution for each value of b

Cell array in MATLAB

A cell array is a data type with indexed data containers called cells, where each cell can contain any type of data. Cell arrays commonly contain either lists of text, combinations of text and numbers, or numeric arrays of different sizes. Refer to sets of cells by enclosing indices in smooth parentheses, (). Access the contents of cells by indexing with curly braces, {}.

The cell function in MATLAB

C = cell(sz1,...,szN) returns a sz1-by-...-by-szN cell array of empty matrices where sz1,...,szN indicate the size of each dimension. For example, cell(2,3) returns a 2-by-3 cell array.

The length function in MATLAB

L = length(X) returns the length of the largest array dimension in X. For vectors, the length is simply the number of elements. For arrays with more dimensions, the length is max(size(X)). The length of an empty array is zero.

Application: By using both the cell function and the length function as in the code below, we can store the solution for each value of b during the simulation

```
solutions = cell(1, length(b_values))
```

Step 4: Create a loop over different values of damping coefficient b

We use the function for to initiate a loop that will allow us to solve the differential equation with each value of b

```
for i = 1:length(b_values)
  b = b_values(i)
```

Step 5: Define the differential equation:

We defines the second-order linear homogenous ordinary differential equation (ODE) with damping term

```
syms t y(t)
  ode = diff(y, t, t) + b*diff(y, t) + w_0^2*y - F*cos(w_0*t) == 0
```

Since F=0, this ODE represents a damped oscillation

Step 6: Set up the initial condition

y(0) = 5 : initial displacement

y’(0) = 0 : initial velocity

Step 7: Use the dsolve function to solve the differential equation

```
solutions{i} = dsolve(ode, initial_conditions)
```

Step 8: Evaluate the solution for each time point

We uses the sub

```
y_values = subs(solutions{i}, t, t_values)
```

Step 9: Plot the graph

```
plot(t_values, y_values, 'DisplayName', ['b = ' num2str(b)])
```

2.3 Method of Stimulated Oscillation

Step 1: Input the parameters

w_0 = 10: Angular frequency of free oscillation

b= 0.1: Damping coefficient

F= 10: Stimulating Force

w_values = [10.0, 5.0, 3.0, 0.0]: Different values of stimulating force angular frequency

t_end = 150s: Total time for stimulation

Step 2: Generate time values

t_values = linspace (0, t_end, 1000) : Time ranges from 0 to 150s with the step of 0.01s

Step 3: Initialize a cell array to store a solution for each value of ω

solutions = cell(1, length(w_values)): This step's purpose is to store the solutions for each value of angular frequency (w) during the simulation

Step 4: Loop over different values of angular frequency

```
for i = 1:length(w_values)
   w = w_values(i)
```

Initiating a loop to go through each value of angular frequency in the array w_values and solve the differential equation for each case

Step 5: Define the differential equation

```
syms t y(t)
   ode = diff(y, t, t) + 2*b*w_0*diff(y, t) + w_0^2*y - F*cos(w*t) == 0;
```

Defining the second-order linear homogeneous ordinary differential equation (ODE) with damping and external force terms

$\frac{d^2y}{dt^2} + b\frac{dy}{dt} + \omega_0^2 y - Fcos(\omega t) = 0$. This ODE represents forced oscillation.

Step 6: Set initial conditions

y (0) = 5 is the initial displayment

y' (0) = 0 is the initial velocity

Step 7: Solve the differential equation

Use the 'dsolve' function to find the symbolic solution to the differential equation with the specified initial conditions solutions{i} = dsolve(ode, initial_conditions)

Step 8: Evaluate the solution for each time point

y_values = subs(solutions{i}, t, t_values) is evaluate the symbolic equation at each time point specified in t_values

Step 9: Plot the graph

plot(t_values, y_values, 'DisplayName', ['\omega = ' num2str(w)])is plotting the displacement over time for the current angular frequency on the same figure, with a legend indicating the value of the angular frequency

Chapter 3: Result

3.1 Harmonic Oscillation:

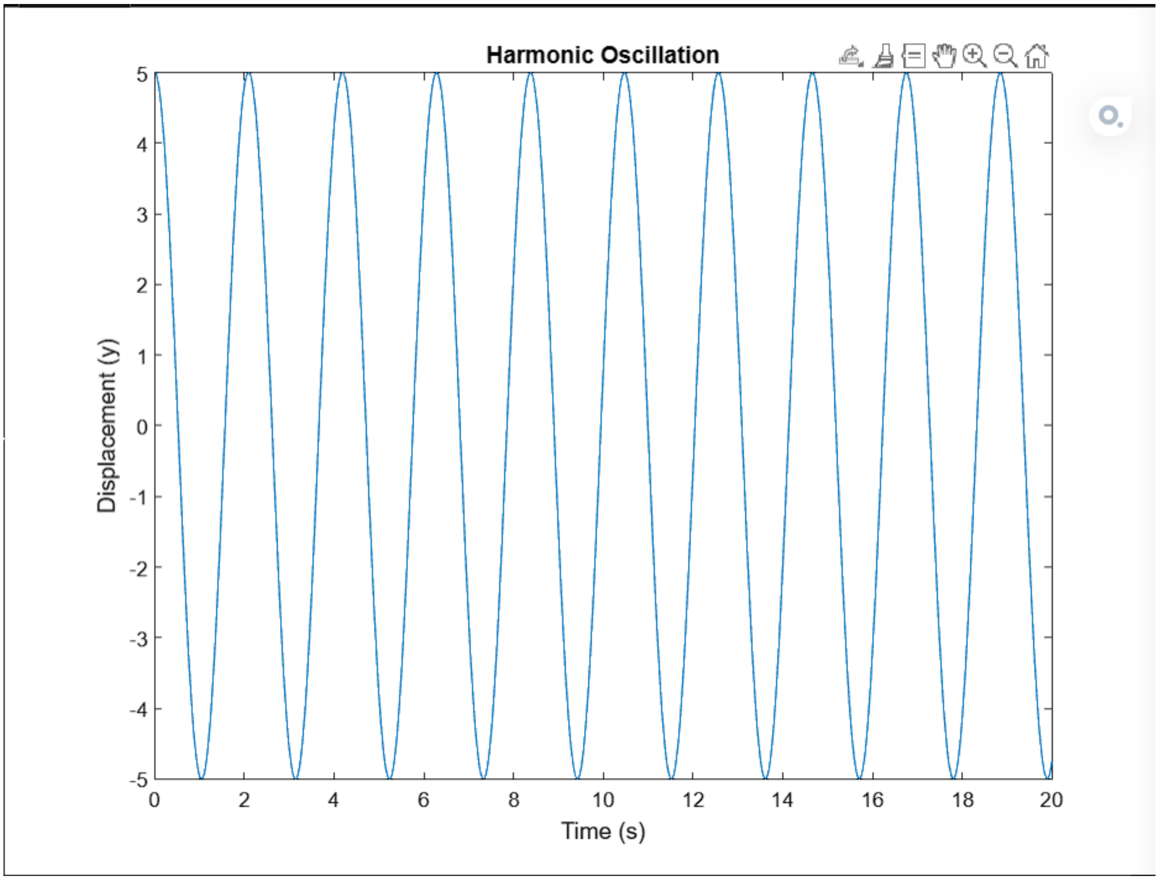

Figure 1: The graph of oscillation over time of harmonic oscillation

Comment:

The graph depicts harmonic oscillation, where y changes over time t in a regular manner and according to the laws of harmonic oscillation.

Because there is no damping coefficient (b=0) and no stimulated forces, this graph illustrates a simple harmonic oscillation that has a stable period, frequency, and amplitude.

In general, this graph is the result of a simple harmonic oscillation, no damping coefficient as well as stimulated forces. It clearly shows the fluctuation and stability of the system under this condition.

3.2 Damped Oscillation:

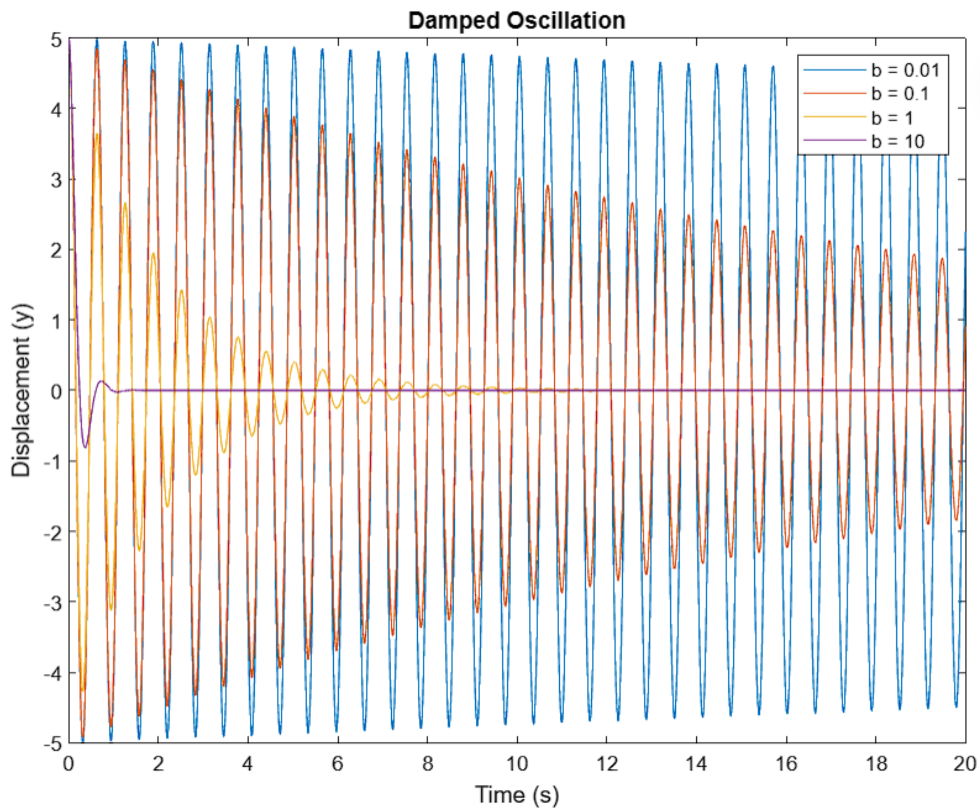

Figure 2: The graph of oscillation over time of damped oscillation

Comment:

The graph shows a significant decrease over time when the damping coefficient b increases. The gradual decrease leads to the decline of the amplitude of oscillation over time.

The graph lines correspond to different b values, and they describe how the damping (b) affects the damping oscillation. When b is large, the decay is stronger and the oscillation easily decays to zero.

For smaller b-values, oscillations can maintain the amplifier and frequency for a longer period of time before gradually decreasing to zero. On the contrary, for larger values of b, oscillations will rapidly decrease and converge to zero.

In general, the graph illustrates an overview of the decreasing oscillation and the way it depends on the damping coefficient.

3.3 Stimulated Oscillation:

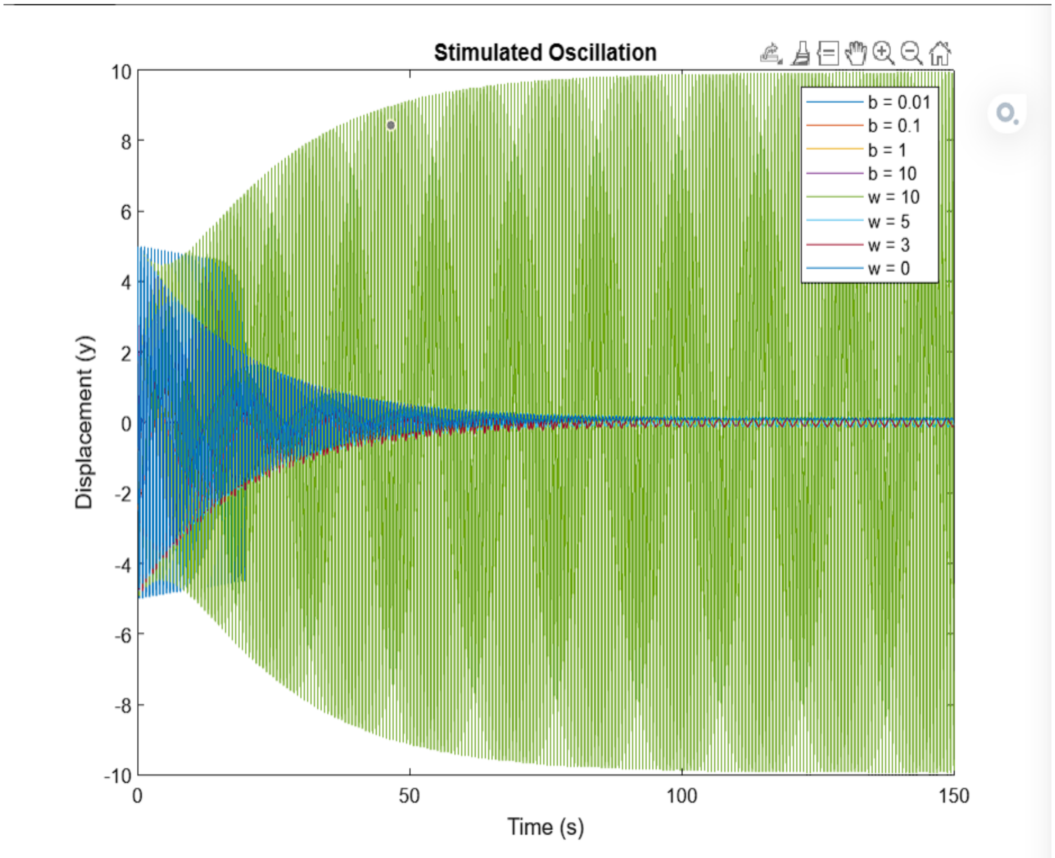

Figure 3 The graph of oscillation over time of stimulated oscillation

Comment:

The graphs show how the amplitude and shape of the oscillation change based on different values of □. When □ has a non-zero value, the oscillation is excited by a force of frequency □ and reacts in a complex way.

The damping coefficient (b) is still a major influencing factor for oscillation. Damping affects both the amplitude and phase shift of the oscillation.

The plots provide information about the phase difference between the excitation force and the transformation. This phase shift can be varied based on the excitation frequency and the damping factor.

Overall, the plots provide information about the phase difference between the excitation force and the transformation. This phase shift can be varied based on the excitation frequency and the damping factor.

Chapter 4: Conclusion

In conclusion, the oscillation of objects affected by elastic forces can be divided into three forms. The first type of oscillation is harmonic oscillation, this kind of oscillation has the zero-damped coefficient and stimulated force. This results in the stable periodic and frequency. It also has an unchanged amplitude and a sinusoidal oscillation graph. Furthermore, if the damped coefficient is different from 0 and the stimulated force is still zero, the object's oscillation is damping oscillation. The damped coefficient significantly affects the amplitude of the object. The bigger the damped coefficient, the smaller the amplitude. As a result, its oscillation does not have a sinusoidal shape and stable periodic as well as frequency. Finally, when the damping coefficient and stimulated force are both different from zero, the oscillation will be stimulating oscillation. The amplitude of this motion is increasing over time when w is bigger than the damping coefficient. It leads to the complication of the graph.

Chapter 5: Reference

A. L. Garcia and C. Penland, *MATLAB Projects for Scientists and Engineers*, Prentice Hall, Upper Saddle River, NJ, 1996. http://www.algarcia.org/fishbane/fishbane.html. Or https://www.mathworks.com/matlabcentral/fileexchange/2268-projects-for-scientists-and-engineers

Appendix:

Full code:

```
**a) Harmonic Oscillation (No Damping, No Stimulating Force: \(b = F = 0\)):**
syms t y(t)
w0 = 3;
ode = diff(y, t, t) + w0^2 * y == 0;
initial_conditions = [y(0) == 5, subs(diff(y, t), t, 0) == 0];
solution = dsolve(ode, initial_conditions);
t_values = 0:0.01:20;
y_values = subs(solution, t, t_values);
figure;
plot(t_values, y_values);
title('Harmonic Oscillation (Symbolic Solution)');
xlabel('Time (s)');
ylabel('Displacement (y)');
grid on;
```

```
**b) Damped Oscillation (with varying damping coefficients \(b\)):**
w_0 = 10;
F = 0;
t_end = 20;
b_values = [0.01, 0.1, 1.0, 10.0];
t_values = linspace(0, t_end, 1000);
solutions = cell(1, length(b_values));
for i = 1:length(b_values)
  b = b_values(i);
    syms t y(t)
  ode = diff(y, t, t) + b*diff(y, t) + w_0^2*y - F*cos(w_0*t) == 0;
  initial_conditions = [y(0) == 5, subs(diff(y, t), t, 0) == 0];
    solutions{i} = dsolve(ode, initial_conditions);
  y_values = subs(solutions{i}, t, t_values);
  hold on;
  plot(t_values, y_values, 'DisplayName', ['b = ' num2str(b)]);
end
legend('show');
xlabel('Time (s)');
ylabel('Displacement (y)');
title('Damped Oscillation for Different b Values');
grid on;
hold off;
```

c) Stimulated Oscillation (with varying stimulating frequencies \(w\)):

```
w_0 = 10;
b = 0.1;
F = 10;
w_values = [10.0, 5.0, 3.0, 0.0];
t_end = 150;

t_values = linspace(0, t_end, 1000);
solutions = cell(1, length(w_values));
for i = 1:length(w_values)
   w = w_values(i);
   syms t y(t)
   ode = diff(y, t, t) + b*diff(y, t) + w_0^2*y - F*cos(w*t) == 0;
   initial_conditions = [y(0) == 5, subs(diff(y, t), t, 0) == 0];
   solutions{i} = dsolve(ode, initial_conditions);
   y_values = subs(solutions{i}, t, t_values);
   hold on;
   plot(t_values, y_values, 'DisplayName', ['\omega = ' num2str(w)]);
end
legend('show');
xlabel('Time (s)');
ylabel('Displacement (y)');
title('Stimulated Oscillation for Different \omega Values');
grid on;
hold off;
```