**Numerical Capacitance Extraction using a Discretized Method**

ECE 107 Fall 2021 Project

Hao Le

The objective of this project is to discretize the problem of finding the surface charge density of the plates of a parallel-plate capacitor, thus enabling the extraction of capacitance. We are given the voltages of the two plates: Vo /2 of the top plate and - Vo /2 of the bottom. Additionally, we also know the geometry of the plates: with negligible thickness, the width is *w*, and the distance between the plates is *d*.

A picture containing text, clock

Description automatically generatedThe electric potential due to a point charge Q is given as:

Equation taken from: https://en.wikipedia.org/wiki/Electric\_potential

Where r is the distance between the charge and the observation point. We can extend this to a surface charge distribution, where Q is now a function that involves multiplying infinitesimal area dA by the surface charge density evaluated at the area. If we discretized the plates’ surfaces into squares:

Diagram

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**Notice that the coordinate system is placed on the top left corner of the bottom plate, thus dictating that the patches are indexed as such in the diagram**. For every patch, we can find the potential at its center rm:

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Description automatically generatedTo find the potential for a patch, we first fix the observation point to be rm. Then, using the formula above with the dA being discretized to delta S, we find the individual N potentials, from the other N charged patches including the observation patch, that contribute to the final potential of V(rm):

From the plate voltages given, we can form equalities for the potentials at each patch – those on the bottom plate are - Vo /2 and conversely those on the top plate are Vo /2. From these equalities, we can solve for the discrete charge density at each patch. Turning the problem into a matrix problem:

Diagram

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Q is the charge vector that describes the discrete charges of all the patches. By dividing the vector by delta S, we get the discrete charge densities. Notice that the voltage contribution of a charged patch an observation point that is located on/near itself can be approximated as follows:

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To solve for Q, multiply the inverse of Z with V:

Q = Z-1V

To find the capacitance, we simply sum up the N /2 charges of the positively charged plate (indexed from N/2 to N-1), and then divide by Vo.

**Computer code**

The problem will be solved using an implementation in Python.

Functions used are defined:

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To find and plot the surface charge distribution of the positive plate, as well as extract the numerical capacitance, given w, d, and V0:

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For width of 1cm and gap of 3mm, and with a patch subdivision of 15 (225 patches per plate), the code yields the following distribution plot:

Chart, radar chart

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The plot appears probable, considering that the charges get denser near the edge of the plate, which is explained by the fringing effects. Experimenting with lower/higher subdivisions that yield coarser/finer distribution plots respectively as well as varying calculated capacitances:

Chart, radar chart, surface chart

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The next experiment involves fixing the width of the plates to 1cm while varying the gap distance to compare the numerical capacitance extracted from our discrete method against using the approximation of:

C = (ε0 (A^2 )/ d

Where A is the area of a plate, which in our case is w\*w.

Modification to the code involves adding the array of distances that are to be used for the capacitance calculation; we vary from 0.5mm up to 40mm, with 0.1mm increments. We fixed the subdivision number to 5 for faster computation time.

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The calculated capacitances are plotted against each other as a function of the gap distance

Chart

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From observation, the discrete method’s results converge towards the approximation as the gap distance gets larger. However, as the distance gets smaller, the approximation diverges from the discrete method’s results.

It is worth noting that the discrete method fails at smaller gap distances and produces a negative, invalid capacitance likely due to floating point errors incurred by the nature of discretization. Furthermore, as the gap distance increases, the calculated capacitance increases steadily until it converges to a fixed capacitance. This is converse to the approximation that exhibits an exponential decrease until convergence.