# Dataflow Matrix Machines and V-values: a Bridge between Programs and Neural Nets

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**HERE Technologies** 

Joint work with Jon Anthony (Boston College)

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# Introduction Neural model of computation V-values and variadic neurons

V-values and variadic neurons Programming patterns and self-referential facilities

- A strong generalization of neural networks
- Powerful enough to write programs in this formalism
- A program is determined by a matrix of numbers
- Any program P can be transformed to any program Q in this formalism by continuously transforming the matrices defining those programs.
- Synthesize a matrix of numbers to synthesize a program.

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- Neural networks process streams of numbers
- DMMs process linear streams
- Streams are sequences
- Linear streams: the notion of
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- To distinguish between different spaces of linear streams we talk about kinds of linear streams.
- Every vector space V gives rise to the corresponding kind of linear streams (streams of vectors from that space)
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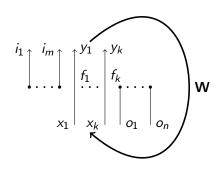
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#### Recurrent neural networks



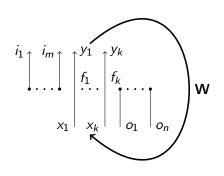
"Two-stroke engine" for an RNN:

 $\text{``Down movement'':} \\ (x_1^{t+1}, \dots, x_k^{t+1}, o_1^{t+1}, \dots, o_n^{t+1})^\top = \\ \mathbf{W} \cdot (y_1^t, \dots, y_k^t, i_1^t, \dots, i_m^t)^\top.$ 

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$$y_1^{t+1} = f_1(x_1^{t+1}), \dots, y_k^{t+1} = f_k(x_k^{t+1}).$$

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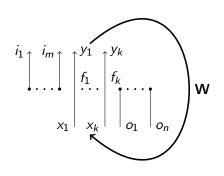


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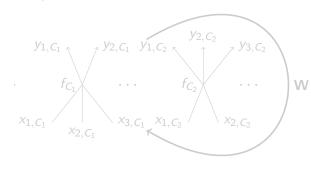
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#### Dataflow matrix machines

#### Countable network with finite active part at any moment of time.

Countable matrix **W** with finite number of non-zero elements at any moment of time.



For all inputs  $x_i, C_k$  where there is a non-zero weight  $w_{(i, C_k), (j, C_l)}^t$ :

$$\begin{aligned} x_{i,C_{k}}^{t+1} &= \\ &\sum \{(j,C_{l})|w_{(i,C_{k}),(j,C_{l})}^{t} \neq 0\} \\ w_{(i,C_{k}),(j,C_{l})}^{t} &* y_{j,C_{l}}^{t}. \end{aligned}$$

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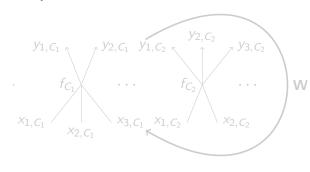
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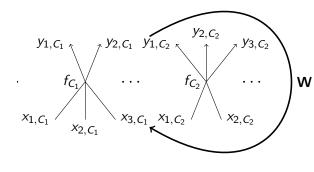
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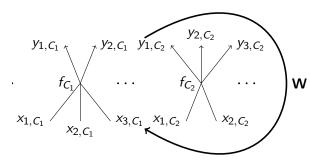
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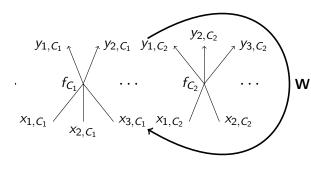
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## Type correctness condition

We need to keep input and output arity of each neuron in mind, and we also need to impose a type correctness condition in order for the formulas in the previous slide to be well-defined:

 $w_{(i,C_k),(j,C_l)}^t$  is allowed to be non-zero only if  $x_{i,C_k}$  and  $y_{j,C_l}$  belong to the same *kind* of linear streams.

We are now going to introduce linear streams of *V-values* based on *nested dictionaries*, which are sufficiently universal and expressive to save us from the need to impose type correctness conditions.

Moreover, they will allow us to define *variadic neurons*, so that we don't need to keep track on input and output arity either.

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It might be the case when computational resources are sufficient, that if one has to base a formalism on a single kind of nested data structure, this structure should be nested dictionaries.

The simplest way to build a vector space of nested dictionaries is to require all atoms (leaves) to be numbers.

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Consider ordinary words ("labels") to be letters of a countable "super-alphabet" L.

V-values can be understood as

- Finite linear combinations of finite strings of letters from *L*;
- Finite prefix trees with numerical leaves;
- Sparse "tensors of mixed rank" with finite number of non-zero elements;
- Recurrent maps (that is, nested dictionaries) from  $V \cong \mathbb{R} \oplus (L \to V)$  admitting finite descriptions.

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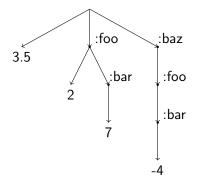
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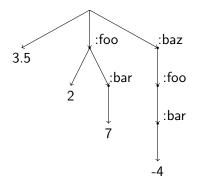
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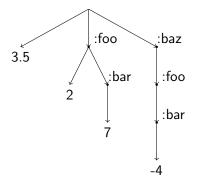
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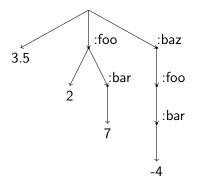
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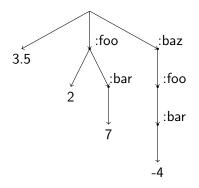
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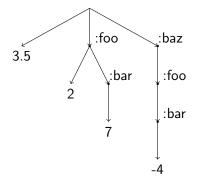
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# Dataflow matrix machines based on streams of V-values and variadic neurons

$$x_{f,n_f,i}^{t+1} = \sum_{g \in F} \sum_{n_g \in L} \sum_{o \in L} w_{f,n_f,i;g,n_g,o}^t * y_{g,n_g,o}^t \text{ (down movement)}$$

$$y_{f,n_f}^{t+1} = f(x_{f,n_f}^{t+1}) \text{ (up movement)}$$

$$f \qquad g \qquad W$$

#### Powerful neurons

With powerful variadic neurons and streams of V-values we have a formalism which is much more expressive than neural nets based on streams of numbers.

Many tasks can be accomplished by compact networks, with single neurons functioning as layers or modules.

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The dictionary mapping words to their respective counts is an infinite-dimensional vector with a finite number of non-zero elements.

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The computational architecture promoted by the functional programming community is to work with immutable data sharing common substructures and garbage collecting the obsolete parts.

The experience of the field shows that this approach is pretty efficient in general and allows easy parallelization.

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#### Self-referential facilities

It is easy to represent the network matrix  $\mathbf{W}$  as a V-value.

Therefore, it is easy to designate a particular neuron Self and a particular output of that neuron (say, :active) as emitting the streams of network matrices.

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This is something which is theoretically possible, but practically very difficult to do in neural nets based on streams of numbers.

Our current implementation uses Self which works as an accumulator. It accumulates the value of the network matrix while accepting additive updates from other neurons in the network.

In turn, the other neurons can use Self outputs to become aware of the current network structure and weights.

Our experiments show that this mechanism together with simple constant update neurons is capable of producing waves of connectivity patterns propagating within the network matrix **W**.

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# Applications...

We hope that this formalism will be useful in various learning to learn setups (the systems which learn how to learn better), in program synthesis, and more...

#### Electronic resources

The companion paper will be linked from my page on partial inconsistency and vector semantics of programming languages:

 $http://www.cs.brandeis.edu/{\sim}bukatin/partial\_inconsistency.html$ 

It will also be linked from our open source project which implements core DMM primitives in Clojure:

https://github.com/jsa-aerial/DMM