MA1101R Revision Lecture

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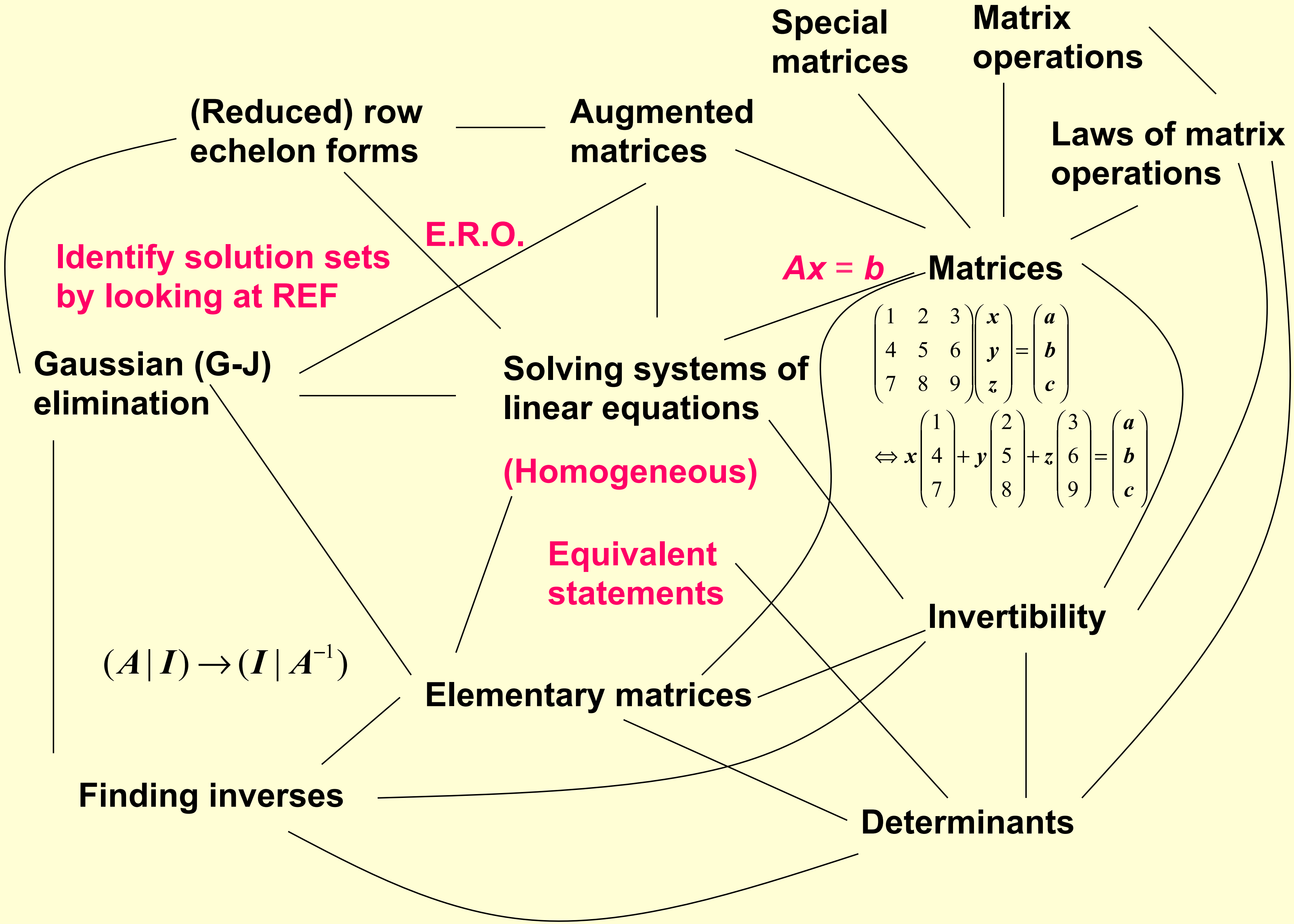
Chapters 1 and 2

Linear systems and matrices

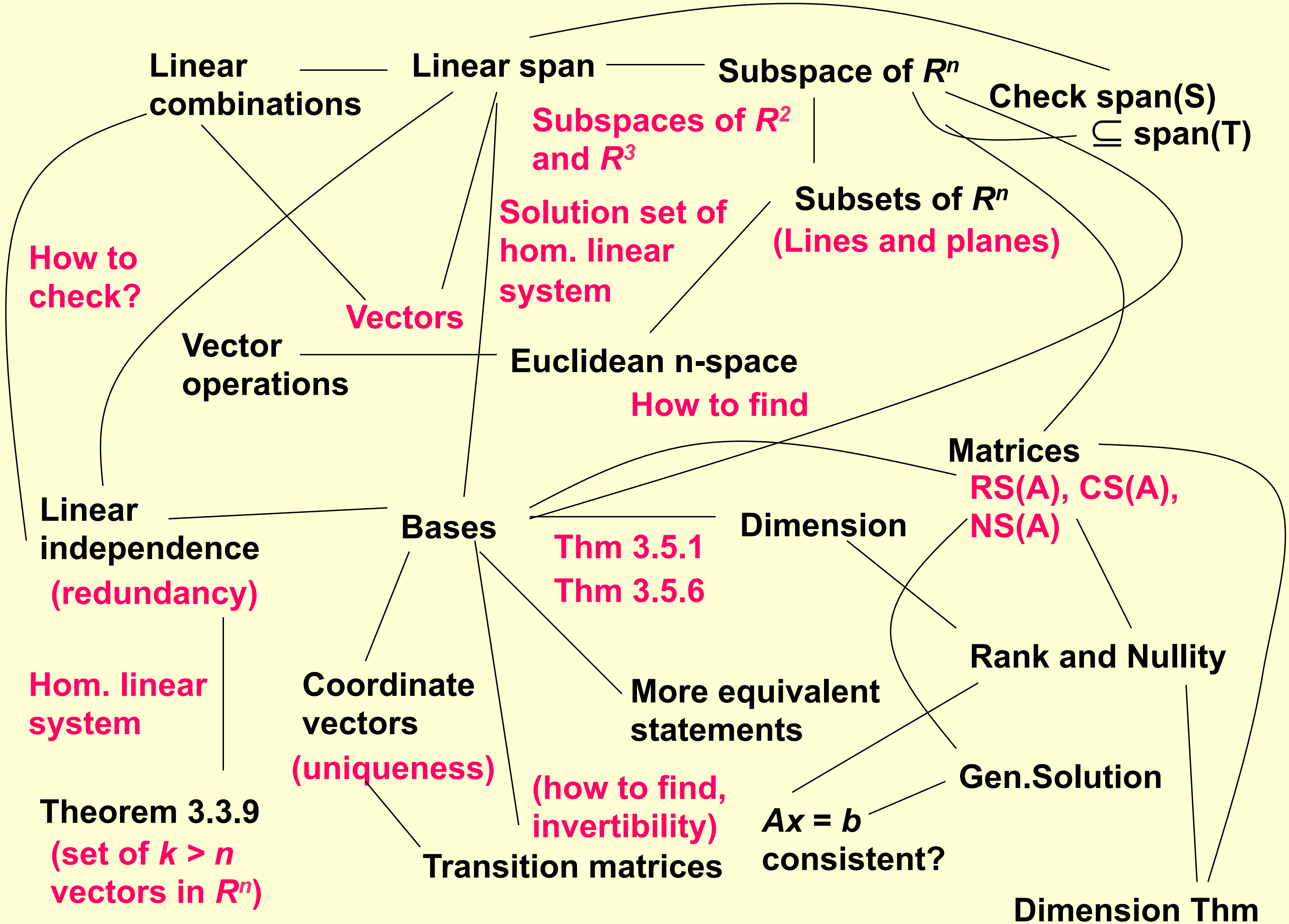

Special matrices
Matrix operations
(Reduced) row echelon forms
Augmented matrices
Laws of matrix operations
E.R.O.
Identify solution sets by looking at REF
Ax = b
Matrices
$\begin{pmatrix}1&2&3\\4&5&6\\7&8&9\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}a\\b\\c\end{pmatrix}$
$\Leftrightarrow x\begin{pmatrix}1\\4\\7\end{pmatrix}+y\begin{pmatrix}2\\5\\8\end{pmatrix}+z\begin{pmatrix}3\\6\\9\end{pmatrix}=\begin{pmatrix}a\\b\\c\end{pmatrix}$
Gaussian (G-J) elimination
Solving systems of linear equations
(Homogeneous)
Equivalent statements
Invertibility
$(A \mid I) \to (I \mid A^{-1})$
Elementary matrices
Finding inverses
Determinants

Chapters 1 and 2: Linear systems, Gaussian Elimination and Matrices

- What is a linear system? What is the solution set of a linear system?
- How to solve a linear system (systematically)? (Reduced) row-echelon forms.
- Some geometric interpretations of linear systems. Homogeneous linear systems.
- Matrices and its operations. (i,j) representation of matrices. How to prove a matrix equation.
- Different representations of $\boldsymbol{AB}$. Matrix representations of a linear system.
- Invertibility. Existence of $\boldsymbol{A}^{-1}$. How to determine if a matrix is invertible?
- Elementary matrices (how to obtain) and their relationship with E.R.O.
- Determinants – definitions and cofactor expansions. Properties of determinants and their relationship with E.R.O. Adjoint of a matrix.

Chapters 3 and 4

Vector spaces and subspaces


Linear combinations
Linear span
Subspace of R^n
Check span(S) $\subseteq$ span(T)
Subspaces of R^2 and R^3
Solution set of hom. linear system
Subsets of R^n
(Lines and planes)
How to check?
Vectors
Vector operations
Euclidean n-space
How to find
Matrices
RS(A), CS(A), NS(A)
Linear independence
(redundancy)
Bases
Dimension
Thm 3.5.1
Thm 3.5.6
Rank and Nullity
Hom. linear system
Coordinate vectors
(uniqueness)
More equivalent statements
Theorem 3.3.9
(set of $k > n$ vectors in R^n)
(how to find, invertibility)
Gen.Solution
$Ax = b$ consistent?
Transition matrices
Dimension Thm

Chapters 3 and 4: Vector spaces and subspaces

• Euclidean n-spaces. Vectors in $\mathbf{R}^n$. Vector operations and its relationship with matrices.

• Subsets of $\mathbf{R}^n$. Lines and planes in $\mathbf{R}^2$ and $\mathbf{R}^3$.

• Linear combinations and linear spans. What is ***a*** linear combination? Remember that linear span is "the set of ***all*** linear combinations".

• Definition of a subspace. The "span" definition and the "closure" definition.

• What properties must a subspace have? The notion of self sufficiency. How to prove or disprove that a subset is a subspace. Subspaces in $\mathbf{R}^2$ and $\mathbf{R}^3$.

• Solution set of a homogeneous linear system.

• How to check if span(S) is a subset of span(T)?

• The notion of redundancy in linear span, leading to the definition of linear independence. Two equivalent notions of linear independence.

• Bases – definition. Uniqueness representation and coordinate vectors. Dimensions – "not too few, not too many". Importance of knowing the dimension of a subspace.

• Transition matrices – how to find? Inverse of a transition matrix.

Chapters 3 and 4: Vector spaces and subspaces (continued)

• Row space and column spaces – definition.

• Finding a basis for the row space or a column space of a matrix. The notion of Row equivalence. Finding a basis for span{$\boldsymbol{u}_1$, $\boldsymbol{u}_2$,…, $\boldsymbol{u}_n$}.

• $\boldsymbol{Ax} = \boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ belongs to the column space of $\boldsymbol{A}$.

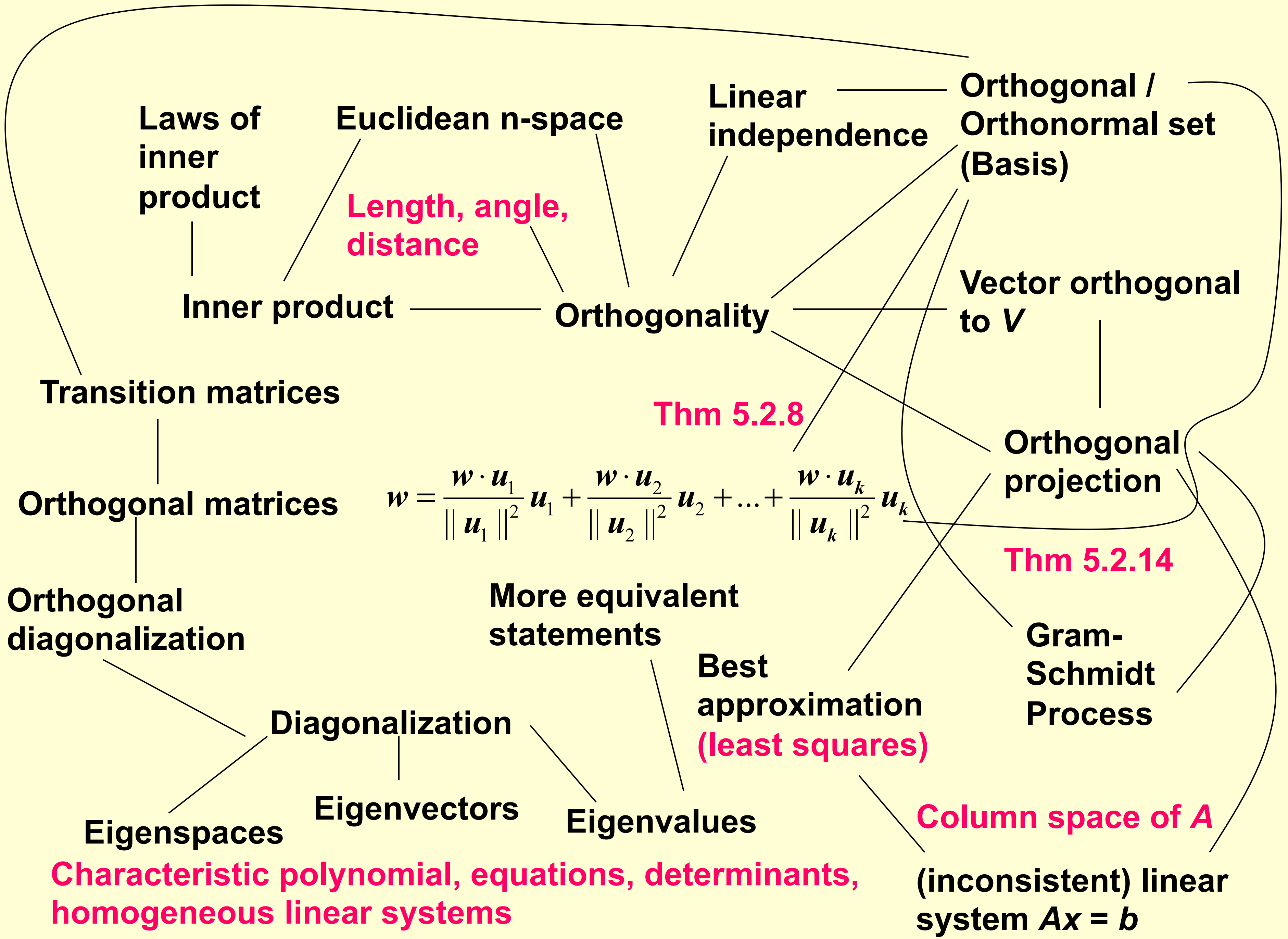
• Dimensions of the row space and column space of a matrix. Definition of rank.

• Nullspace of a matrix and nullity. Finding a basis for the nullspace.

• Dimension theorem for matrices.

Chapters 5 and 6

Orthogonality and Diagonalization



Laws of inner product
Euclidean n-space
Linear independence
Orthogonal / Orthonormal set (Basis)
Length, angle, distance
Inner product
Orthogonality
Vector orthogonal to V
Transition matrices
Thm 5.2.8
Orthogonal projection
Orthogonal matrices
$w = \frac{w \cdot u_1}{\| u_1 \|^2} u_1 + \frac{w \cdot u_2}{\| u_2 \|^2} u_2 + \ldots + \frac{w \cdot u_k}{\| u_k \|^2} u_k$
Thm 5.2.14
Orthogonal diagonalization
More equivalent statements
Gram-Schmidt Process
Best approximation
(least squares)
Diagonalization
Column space of A
Eigenspaces
Eigenvectors
Eigenvalues
Characteristic polynomial, equations, determinants, homogeneous linear systems
(inconsistent) linear system Ax = b

Chapters 5 and 6: Orthogonality and Diagonalization

• Inner products, length, distance and angles.

• Matrix representation for dot products. Properties of dot products.

• Orthogonal implies independence (but not conversely). Orthogonal and orthonormal bases.

• $S = \{\boldsymbol{u}_1,\ldots,\boldsymbol{u}_k\}$ is a orthogonal (orthonormal) basis for V (a subspace of $\mathbf{R}^n$). How to write a vector in V as a linear combination of $\boldsymbol{u}_1,\ldots,\boldsymbol{u}_k$?

• When is a vector orthogonal to a subspace? Orthogonal projection.

• $S = \{\boldsymbol{u}_1,\ldots,\boldsymbol{u}_k\}$ is a orthogonal (orthonormal) basis for V (a subspace of $\mathbf{R}^n$). How to project a vector in $\mathbf{R}^n$ onto V? (and write it as a linear combination of $\boldsymbol{u}_1,\ldots,\boldsymbol{u}_k$?)

• Using the idea of orthogonal projection, leading up to G-S process.

• Best approximations, least squares solutions and their relationship with $\boldsymbol{Ax} = \boldsymbol{b}$ and the column space of $\boldsymbol{A}$.

• Orthogonal matrices – transition matrices between orthonormal bases.

Chapters 5 and 6: Orthogonality and Diagonalization (continued)

• Diagonalization – why do we want to diagonalize a matrix.

• Eigenvalues, eigenvectors and eigenspaces. Characteristic polynomials and characteristic equations.

• How to determine whether a matrix can be diagonalized?

• Two facts (Remark 6.2.5.2 and 6.2.5.3) whose proofs are non-trivial. A related and "not so difficult" statement to prove.

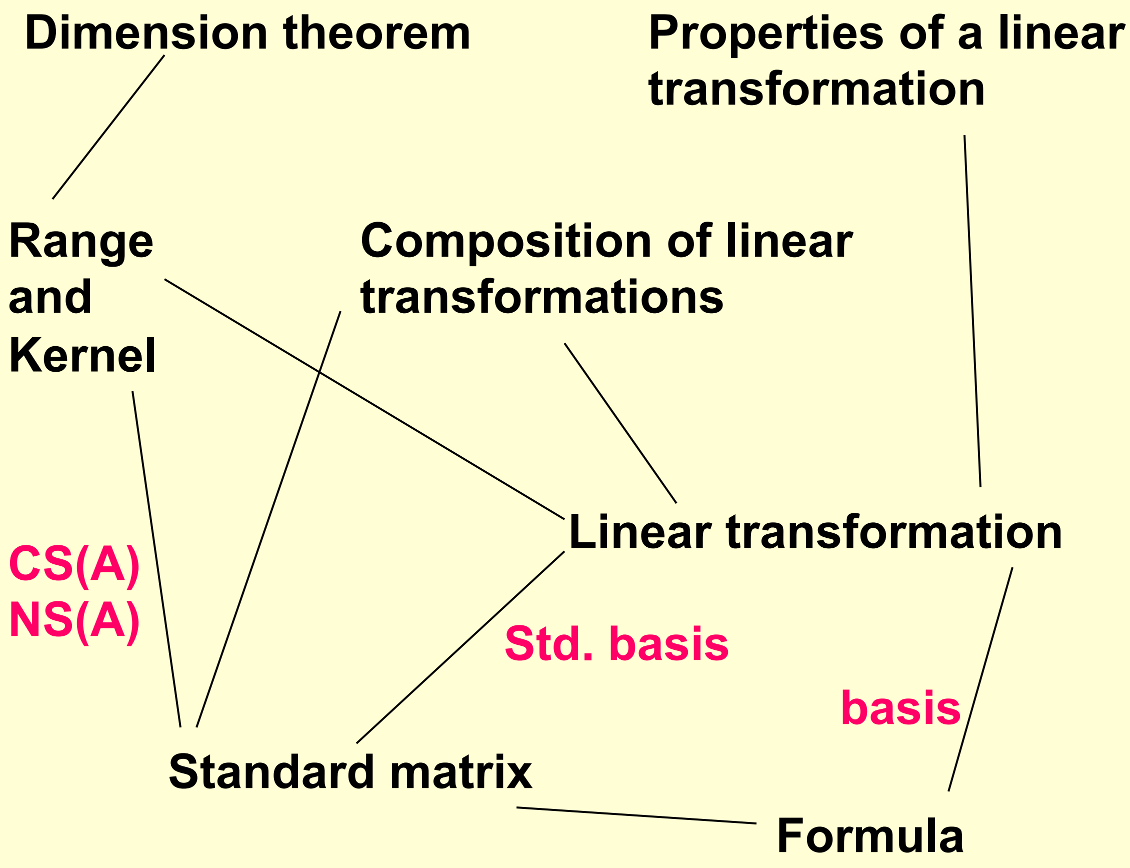
"Let $\lambda_1, \ldots, \lambda_k$ be distinct eigenvalues of $\boldsymbol{A}$ and $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_k$ be the corresponding eigenvectors. Prove that $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_k$ are linearly independent."

• Applications of diagonalization.

• Orthogonal diagonalization and characterization of orthogonally diagonalizable matrices.

Chapter 7

Linear transformations


Dimension theorem
Properties of a linear transformation
Range and Kernel
Composition of linear transformations
Linear transformation
CS(A)
NS(A)
Std. basis
basis
Standard matrix
Formula

Chapter 7: Linear and geometric transformations

- Definition of a linear transformation T. How to find formula for T and standard matrix for T?
- Properties of linear transformations.
- $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$. Determining T completely once we know what T does to a basis of $\mathbf{R}^n$.
- Composition of linear transformations and their respective standard matrices.
- Ranges and Kernels and their relationships with column spaces and nullspaces.