Operation risk: Economic capital calculation from loss distribution simulation in AMA approach

Phan Anh Khoi - 11212902
DSEB 63

May 2024

1 Introduction

Basel Committee defines Operational Risk as: *The risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events.*

Operational risk is much more difficult to quantify than market or credit risk because of the lack of data available. Many serious operational risks facing institutions involve rare and novel events.

The Basel II Accord was published in June 2004. It was a new framework for international banking standards, superseding the Basel I framework, to determine the minimum capital that banks should hold to guard against the financial and operational risks. The Basel II rules for operational risk had three approaches:

- The basic indicator approach (BIA)
- The standardized approach (SA)
- The advanced measurement approach (AMA)

The AMA is much more complicated than the other two approaches. According to Basel: "*This approach allows a bank to calculate its regulatory capital charge using internal models, based on internal risk variables and profiles, and not on exposure proxies such as gross income (i.e. BIA and SA). This is the only risk-sensitive approach for operational risk allowed and described in Basel II.*"

2 Methodology

Economic capital calculations require an operational risk loss distribution which key determinants are:

- Average loss frequency: the average number of times in a year that large losses occur.
- Loss severity: the probability distribution of the size of each loss.

A Poisson distribution is often assumed for loss frequency. If the expected number of losses in a period of time is λ, the probability of n loss events during this period given by Poisson distribution is:

$$\frac{e^{-\lambda}\lambda^n}{n!} \tag{1}$$

For loss severity, the mean and standard deviation (SD) of the loss are often fitted to the lognormal distribution, a distribution where the natural logarithm of the variable is normal. Under the lognormal distribution, the mean of the logarithm of the loss size is:

$$m = \ln\left(\frac{\mu}{\sqrt{1+w}}\right) \tag{2}$$

and the variance of the logarithm of the loss size is:

$$s^2 = \ln(1+w) \tag{3}$$

where $w = (\frac{\sigma}{\mu})^2$.
From loss frequency and loss distribution, we carry out many Monte Carlo simulation trials. By the Central Limit Theorem, we get the probability distribution for the total loss, from which the required percentile can be calculated.

3 Experiment and results

Given lambda = 2, we derive the probability mass function of n losses during the period.
With mean = 70+a, SD = 20+a, we could calculate the mean and variance of the natural exponential of loss size from (2) and (3) as 4.232 and 0.089 respectively. The probability density function of loss size can be plotted by fitting natural logarithm of loss size to the normal distribution with mu = 4.232 and sigma = 0.089.

Then we carry out many Monte Carlo simulation trials. To sample the frequency of loss events, we get a random number between 0 and 1 by RAND() function in Excel. The sampled number will lie between two cumulative probabilities, for example, 0.245 lies between 0.135 and 0.436, from table in Figure 2, this corresponds to one loss event.

Then we sample the loss size of each loss event. We get a random number from the lognormal distribution by using function

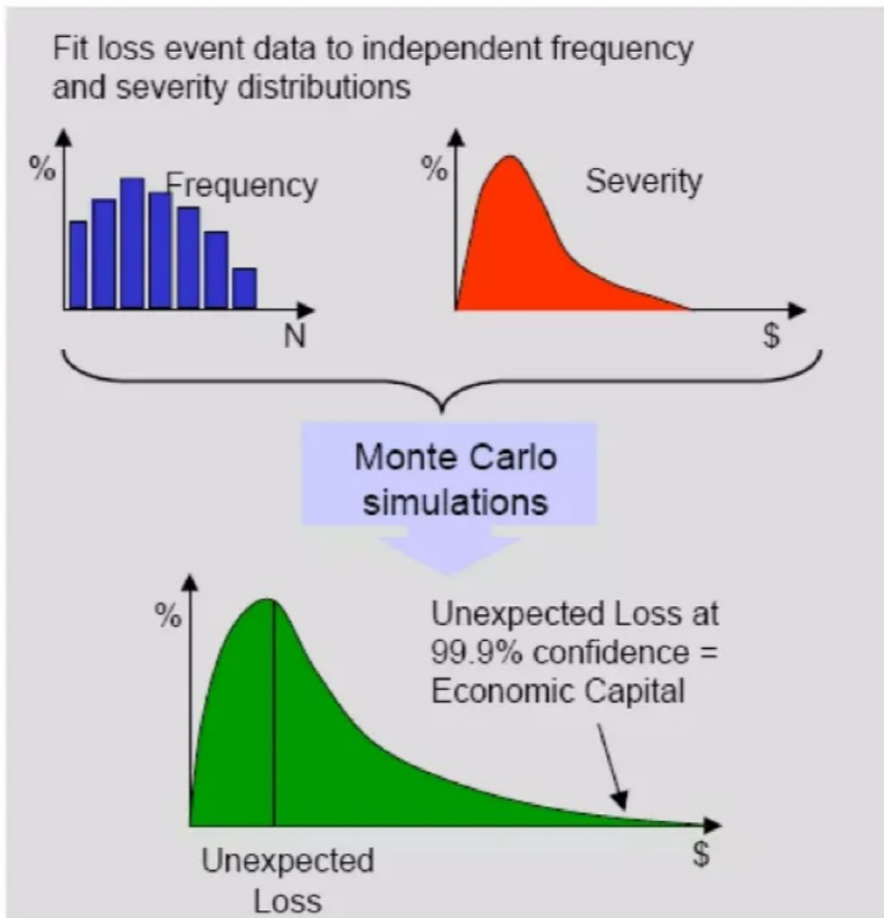

Figure 1: Determination of loss distribution from loss frequency and loss severity

no. of losses	Probability	Cumulative
0	0.135	0.135335283
1	0.271	0.40600585
2	0.271	0.676676416
3	0.180	0.85712346
4	0.090	0.947346983
5	0.036	0.983436392
6	0.012	0.995466194
7	0.003	0.998903281
8	0.001	0.999762553
9	0.000	0.999953502
10	0.000	0.999991692
11	0.000	0.999998635
12	0.000	0.999999793
13	0.000	0.999999971
14	0.000	0.999999996
15	0.000	1

Figure 2: Probability Mass Function for number of losses in a year for $\lambda = 2$

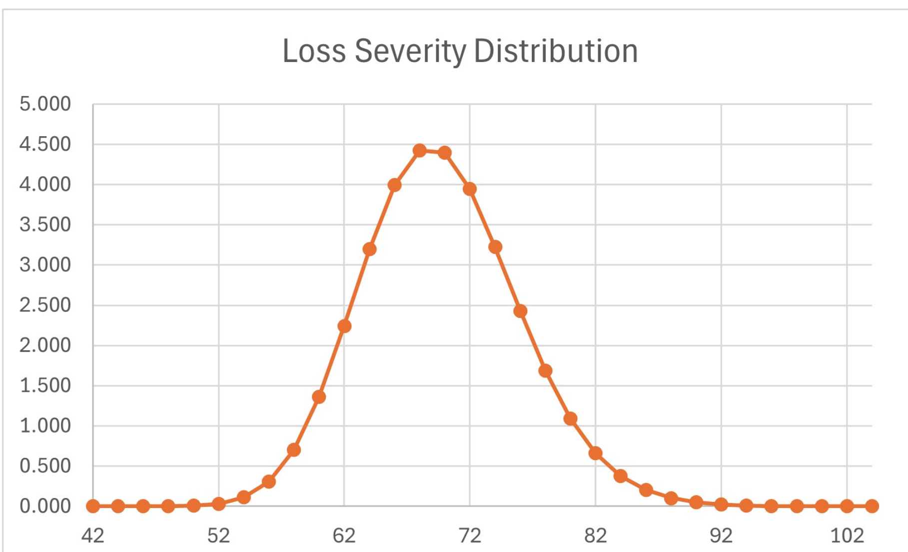

Figure 3: Distribution of loss size

			Loss Severity															
Simulation trial	Random between 0 and 1	Frequency	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
1	0.100944763	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.623235034	2	74.63	76.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	151.27
3	0.646054588	2	81.17	64.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	145.22
4	0.625798992	2	59.51	60.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	120.11
5	0.319450883	1	77.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	77.60
6	0.431043289	2	56.49	79.77	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	136.27
7	0.076654004	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.610449923	2	62.18	75.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	137.37
9	0.279681598	1	77.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	77.95
10	0.439543435	2	78.65	82.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	160.91
11	0.981858765	5	61.71	93.15	65.18	56.26	95.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	372.01
12	0.653426797	2	68.22	99.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	168.01
13	0.435956828	2	61.20	75.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	136.89
14	0.317250522	1	82.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	82.80
15	0.26240858	1	99.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	99.12
16	0.809298389	3	82.83	43.22	75.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	201.83
17	0.85536469	3	52.73	53.21	102.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	208.91
18	0.640166429	2	88.01	92.77	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	180.78

Figure 4: Monte Carlo simulation trials

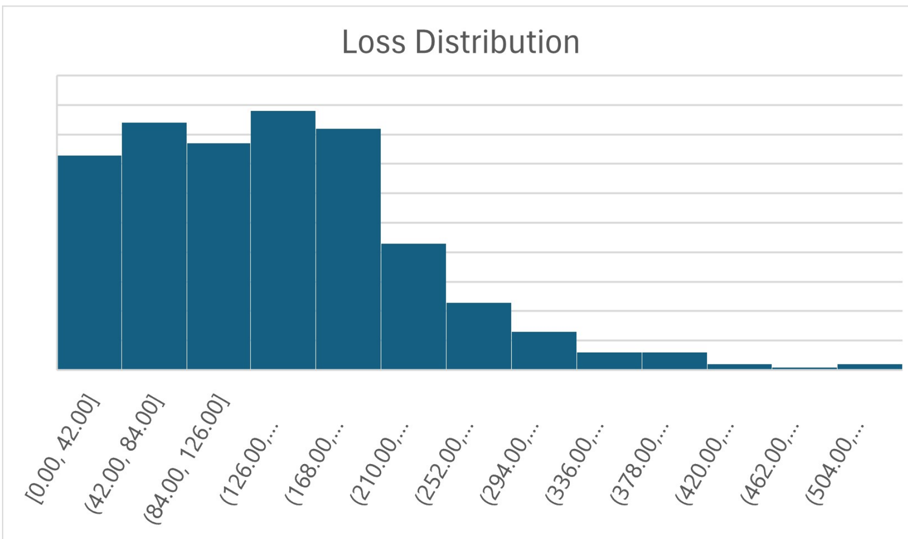

Figure 5: Distribution of total loss

NORM.INV(RAND(),mean,SQRT(sd)) in Excel, with mean and sd are mu and sigma calculated. Then we get the loss sizes as the natural exponential of these random numbers.

The total loss is the sum of the loss size of loss events for each simulation trial. We could plot the histogram of the total loss to get the shape of the distribution loss, following the Central Limit Theorem. The Value at Risk is calculated as the 99th percentile of the loss size distribution.

Loss distribution		
Mean	138.53	
SD	95.643	
Confidence interval		99.00%
Value at Risk (VaR)		418.5088

Figure 6: Value at Risk calculated from loss distibution