THE UNIVERSITY OF MELBOURNE

MECHANICS AND MATERIAL (MCEN30017)

# Finite Element Analysis Assignment

*Anh Kiet Ha, 1213835*

09/05/2023

# Contents

# 1 FEA Analytical Approach

## 1.1 Question 1

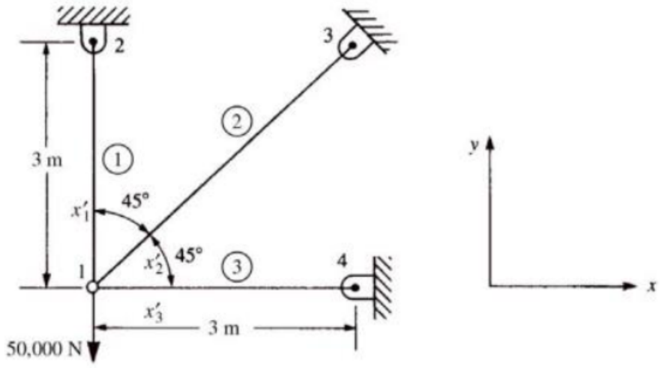



Figure 1: Question 1

Summary:

$l_1 = l_3 = l = 3\ m$ $\qquad$ $l_2 = 3\sqrt{2}\ m$

$\theta_1 = 90°$ $\qquad$ $\theta_2 = 45°$

$\theta_3 = 0°$ $\qquad$ $k_1 = k_3 = \frac{AE}{l}$

$k_2 = \frac{AE}{l\sqrt{2}}$ $\qquad$ $A = 6 \times 10^{-4}\ m^2$

$E = 200\ GPa$

We got the general expression for two-dimensional truss elements:

$$K^e = k^e \begin{bmatrix} \cos^2\theta^e & \cos\theta^e\sin\theta^e & -\cos^2\theta^e & -\cos\theta^e\sin\theta^e \\ \cos\theta^e\sin\theta^e & \sin^2\theta^e & -\cos\theta^e\sin\theta^e & -\sin^2\theta^e \\ -\cos^2\theta^e & -\cos\theta^e\sin\theta^e & \cos^2\theta^e & \cos\theta^e\sin\theta^e \\ -\cos\theta^e\sin\theta^e & -\sin^2\theta^e & \cos\theta^e\sin\theta^e & \sin^2\theta^e \end{bmatrix}$$

Element 1:

$$K^{(1)} = \frac{AE}{l}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Element 2:

$$K^{(2)} = \frac{AE}{l}\begin{bmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix}$$

Element 3:

$$K^{(3)} = \frac{AE}{l}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Global Stiffness:

$$K = \frac{AE}{l}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -1 & 0 \\ \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & 0 & -1 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Global System of equations:

$$\frac{AE}{l}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -1 & 0 \\ \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & 0 & -1 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ R_{2x} \\ R_{2y} \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{bmatrix}$$

Applying the constraint:

$$\frac{AE}{l}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -1 & 0 \\ \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & 0 & -1 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -50000 \\ R_{2x} \\ R_{2y} \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{bmatrix}$$

Solving for the the displacement on element 1:

$$\frac{AE}{l}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} = \begin{bmatrix} 0 \\ -50000 \end{bmatrix}$$

$$\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} = \frac{l}{AE}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ -50000 \end{bmatrix}$$

$$\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} = 2.5\times10^{-8}\times\frac{1}{4}\begin{bmatrix} -\dfrac{2\sqrt{2}}{2+\sqrt{2}} & \dfrac{8+2\sqrt{2}}{2+\sqrt{2}} \\ \dfrac{2\sqrt{2}}{2+\sqrt{2}} & \dfrac{8+2\sqrt{2}}{2+\sqrt{2}} \end{bmatrix}\begin{bmatrix} 0 \\ -50000 \end{bmatrix} \text{(Appendix. Question 1)}$$

$$\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} = 2.5\times10^{-8}\times\frac{1}{4}\begin{bmatrix} \dfrac{8+2\sqrt{2}}{2+\sqrt{2}} & -\dfrac{2\sqrt{2}}{2+\sqrt{2}} \\ -\dfrac{2\sqrt{2}}{2+\sqrt{2}} & \dfrac{8+2\sqrt{2}}{2+\sqrt{2}} \end{bmatrix}\begin{bmatrix} 0 \\ -50000 \end{bmatrix}$$

$$\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} = \begin{bmatrix} 2.5888 \\ -9.9112 \end{bmatrix}\times10^{-4}\ m$$

Stress on each element:

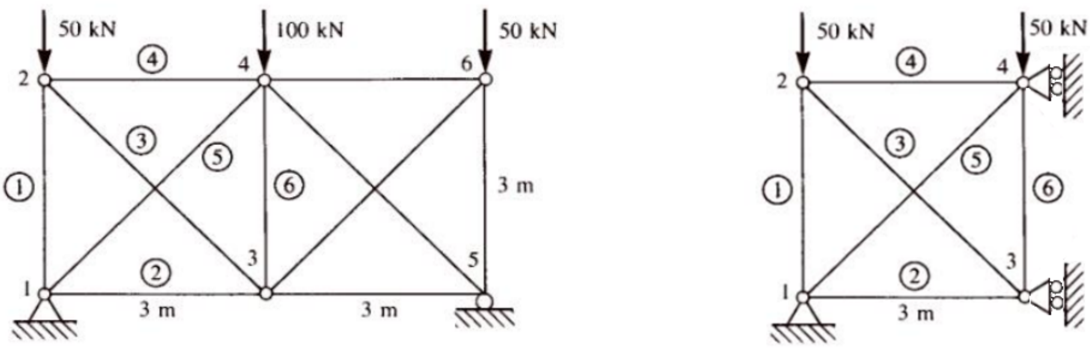
$$\sigma^{(1)} = \frac{E}{l_1}\begin{bmatrix} -\cos\theta_1 & -\sin\theta_1 & \cos\theta_1 & \sin\theta_1 \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix} \Longrightarrow \sigma^{(1)} = 66.07\ MPa$$

$$\sigma^{(2)} = \frac{E}{l_2}\begin{bmatrix} -\cos\theta_2 & -\sin\theta_2 & \cos\theta_2 & \sin\theta_2 \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{3x} \\ u_{3y} \end{bmatrix} \Longrightarrow \sigma^{(2)} = 24.408\ MPa$$

$$\sigma^{(3)} = \frac{E}{l_3}\begin{bmatrix} -\cos\theta_3 & -\sin\theta_3 & \cos\theta_3 & \sin\theta_3 \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{4x} \\ u_{4y} \end{bmatrix} \Longrightarrow \sigma^{(3)} = -17.3\ MPa$$

## 1.2 Question 2

Applying the symmetry to split the model in the middle, half the force applied on the node 4, and constraint node 3 and 4 with the vertical roller. By using the symmetry, we have to half the area of the element 6.




(a) Original problem (b) Problem redraw

Figure 2: Problem 2

Summary:

$$\begin{aligned} &l_1 = l_2 = l_4 = l_6 = l = 3\ m && l_3 = l_5 = 3\sqrt{2}\ m \\ &\theta_1 = \theta_6 = 90^\circ && \theta_5 = 45^\circ \\ &\theta_2 = \theta_4 = 0^\circ && \theta_3 = -45^\circ \\ &k_1 = k_2 = k_4 = \frac{AE}{l} && k_3 = k_5 = \frac{AE}{l\sqrt{2}} \\ &k_6 = \frac{AE}{2l} && A = 3 \times 10^{-4}\ m^2 \\ &E = 700\ GPa \end{aligned}$$

We got the general expression for two-dimensional truss elements:

$$K^e = k^e \begin{bmatrix} \cos^2\theta^e & \cos\theta^e\sin\theta^e & -\cos^2\theta^e & -\cos\theta^e\sin\theta^e \\ \cos\theta^e\sin\theta^e & \sin^2\theta^e & -\cos\theta^e\sin\theta^e & -\sin^2\theta^e \\ -\cos^2\theta^e & -\cos\theta^e\sin\theta^e & \cos^2\theta^e & \cos\theta^e\sin\theta^e \\ -\cos\theta^e\sin\theta^e & -\sin^2\theta^e & \cos\theta^e\sin\theta^e & \sin^2\theta^e \end{bmatrix}$$

Element 1:

$$K^{(1)} = \frac{AE}{l}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Element 2 and 4:

$$K^{(2)} = K^{(4)} = \frac{AE}{l}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 3:

$$K^{(3)} = \frac{AE}{l}\begin{bmatrix} \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix}$$

Element 5:

$$K^{(5)} = \frac{AE}{l}\begin{bmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix}$$

Element 6:

$$K^{(6)} = \frac{AE}{l}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

Global Stiffness:

$$K = \frac{AE}{l}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 & -1 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & 0 & -1 & 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ 0 & 0 & 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -1 & 0 \\ 0 & -1 & -\frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ -1 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{2\sqrt{2}} & 0 & -\frac{1}{2} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -1 & 0 & 0 & 0 & 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{2\sqrt{2}} \end{bmatrix}$$

Global System of equations:

$$\frac{AE}{l}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 & -1 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & 0 & -1 & 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ 0 & 0 & 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -1 & 0 \\ 0 & -1 & -\frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ -1 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{2\sqrt{2}} & 0 & -\frac{1}{2} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -1 & 0 & 0 & 0 & 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{2\sqrt{2}} \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix} = \begin{bmatrix} R_{1x} \\ R_{1y} \\ F_{2x} \\ F_{2y} \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ F_{4y} \end{bmatrix}$$

Applying constraint:

$$\frac{AE}{l}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 & -1 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & 0 & -1 & 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ 0 & 0 & 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -1 & 0 \\ 0 & -1 & -\frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ -1 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{2\sqrt{2}} & 0 & -\frac{1}{2} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -1 & 0 & 0 & 0 & 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{2\sqrt{2}} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ u_{2x} \\ u_{2y} \\ 0 \\ u_{3y} \\ 0 \\ u_{4y} \end{bmatrix} = \begin{bmatrix} R_{1x} \\ R_{1y} \\ 0 \\ -50000 \\ R_{3x} \\ 0 \\ R_{4x} \\ -50000 \end{bmatrix}$$

Solving for the displacement which are free to be calculated:

$$\frac{AE}{l}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 \\ -\frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\ \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{2\sqrt{2}} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2}+\frac{1}{2\sqrt{2}} \end{bmatrix}\begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{3y} \\ u_{4y} \end{bmatrix} = \begin{bmatrix} 0 \\ -50000 \\ 0 \\ -50000 \end{bmatrix}$$

$$\begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{3y} \\ u_{4y} \end{bmatrix} = \frac{l}{AE}\begin{bmatrix} 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 \\ -\frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\ \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{2\sqrt{2}} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2}+\frac{1}{2\sqrt{2}} \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ -50000 \\ 0 \\ -50000 \end{bmatrix} \text{(Appendix. Question 2)}$$

$$\begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{3y} \\ u_{4y} \end{bmatrix} = 1.4286\times 10^{-8}\times\frac{1}{16}\begin{bmatrix} \frac{12+16\sqrt{2}}{1+\sqrt{2}} & \frac{4}{1+\sqrt{2}} & -8 & -\frac{8\sqrt{2}}{1+\sqrt{2}} \\ \frac{4}{1+\sqrt{2}} & \frac{12+16\sqrt{2}}{1+\sqrt{2}} & 8 & \frac{8\sqrt{2}}{1+\sqrt{2}} \\ -8 & 8 & \frac{4(12+8\sqrt{2})}{1+\sqrt{2}} & \frac{4(8+4\sqrt{2})}{1+\sqrt{2}} \\ -\frac{8\sqrt{2}}{1+\sqrt{2}} & \frac{8\sqrt{2}}{1+\sqrt{2}} & \frac{4(8+4\sqrt{2})}{1+\sqrt{2}} & 32 \end{bmatrix}\begin{bmatrix} 0 \\ -50000 \\ 0 \\ -50000 \end{bmatrix}$$

$$\begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{3y} \\ u_{4y} \end{bmatrix} = \begin{bmatrix} 1.35 \\ -8.5 \\ -13.67 \\ -16.38 \end{bmatrix}\times 10^{-4}\ m$$

Reaction force:

$$7 \times 10^7 \times \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} \\ 0 & -1 & 0 & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\ -1 & 0 & 0 & \frac{1}{2\sqrt{2}} \end{bmatrix} \begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{3y} \\ u_{4y} \end{bmatrix} = \begin{bmatrix} R_{1x} \\ R_{1y} \\ R_{3x} \\ R_{4x} \end{bmatrix}$$

$$\begin{bmatrix} 40538.43 \\ 100038.43 \\ 9454.02 \\ -49988.43 \end{bmatrix} N = \begin{bmatrix} R_{1x} \\ R_{1y} \\ R_{3x} \\ R_{4x} \end{bmatrix}$$

Stress on each element:

$$\sigma^{(1)} = \frac{E}{l_1} \begin{bmatrix} -\cos\theta_1 & -\sin\theta_1 & \cos\theta_1 & \sin\theta_1 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix} \Longrightarrow \sigma^{(1)} = -198.33\ MPa \Longrightarrow F_1 = \sigma^{(1)} A = -59499\ N$$

$$\sigma^{(2)} = \frac{E}{l_2} \begin{bmatrix} -\cos\theta_2 & -\sin\theta_2 & \cos\theta_2 & \sin\theta_2 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{3x} \\ u_{3y} \end{bmatrix} \Longrightarrow \sigma^{(2)} = 0\ MPa \Longrightarrow F_2 = \sigma^{(2)} A = 0\ N$$

$$\sigma^{(3)} = \frac{E}{l_3} \begin{bmatrix} -\cos\theta_3 & -\sin\theta_3 & \cos\theta_3 & \sin\theta_3 \end{bmatrix} \begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \end{bmatrix} \Longrightarrow \sigma^{(3)} = 44.57\ MPa \Longrightarrow F_3 = \sigma^{(3)} A = 13371\ N$$

$$\sigma^{(4)} = \frac{E}{l_4} \begin{bmatrix} -\cos\theta_4 & -\sin\theta_4 & \cos\theta_4 & \sin\theta_4 \end{bmatrix} \begin{bmatrix} u_{2x} \\ u_{2y} \\ u_{4x} \\ u_{4y} \end{bmatrix} \Longrightarrow \sigma^{(4)} = -31.5\ MPa \Longrightarrow F_4 = \sigma^{(4)} A = -9450\ N$$

$$\sigma^{(5)} = \frac{E}{l_5} \begin{bmatrix} -\cos\theta_5 & -\sin\theta_5 & \cos\theta_5 & \sin\theta_5 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{4x} \\ u_{4y} \end{bmatrix} \Longrightarrow \sigma^{(5)} = -191.1\ MPa \Longrightarrow F_5 = \sigma^{(5)} A = -57330\ N$$

$$\sigma^{(6)} = \frac{E}{l_6} \begin{bmatrix} -\cos\theta_6 & -\sin\theta_6 & \cos\theta_6 & \sin\theta_6 \end{bmatrix} \begin{bmatrix} u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix} \Longrightarrow \sigma^{(6)} = -63.23\ MPa \Longrightarrow F_6 = \sigma^{(6)} A = -18969\ N$$

Verifying the equilibrium at Node 2:

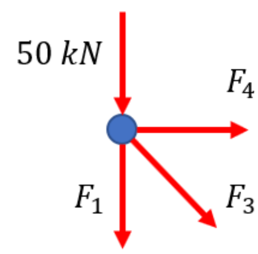



Figure 3: Node 2

$$\sum F_x = F_4 + F_3 \cos 45° = 4.725 \ N$$

$$\sum F_y = -50000 - F_1 - F_3 \sin 45° = 44.27 \ N$$

Compared to the overall force on each element, 4.725N and 44.27N can be negligible due to the numeric error. Therefore, it is in equilibrium in node 2.

Verifying the equilibrium at Node 4:

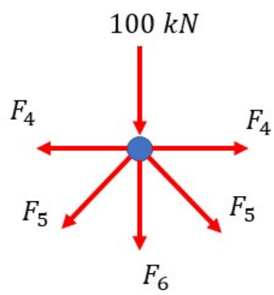



Figure 4: Node 4

$$\sum F_x = F_4 - F_4 + F_5 \cos 45° - F_5 \cos 45° = 0 \ N$$

$$\sum F_y = -100000 - F_6 - 2F_5 \sin 45° = 45.86 \ N$$

As compared to the overall force on each element, 45.86N can be negligible due to the numeric error. Therefore, it is in equilibrium in node 4.

# 2 Computer-based stress and strain analysis

## 2.1 Part 1

### 2.1.1 Question 1

In this model, the 15mm global mesh size is applied to have the initial figure.

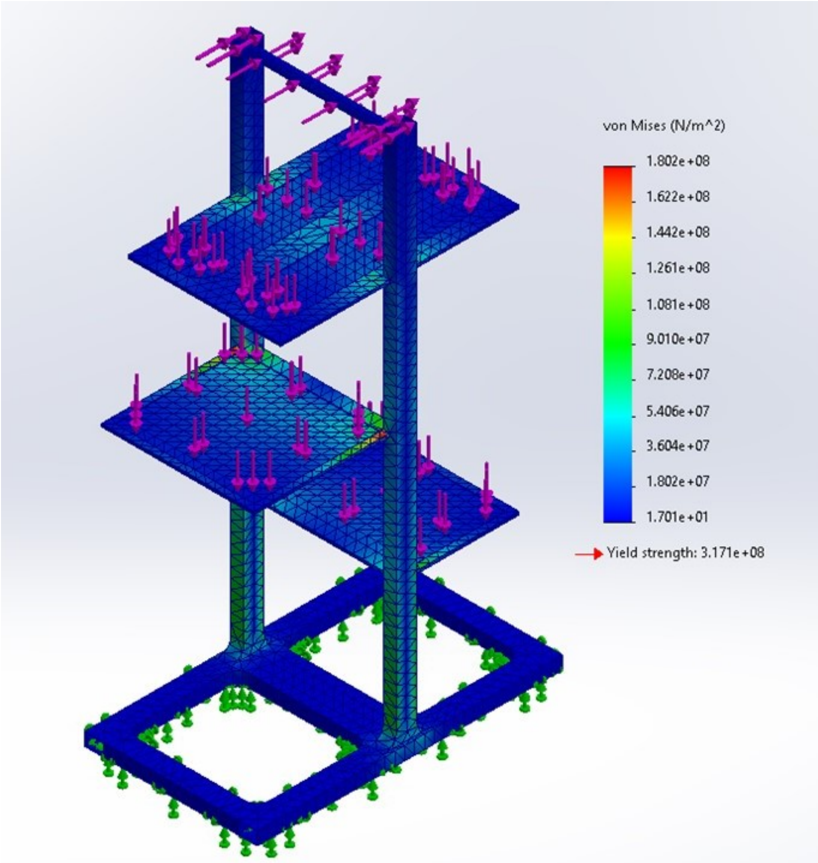
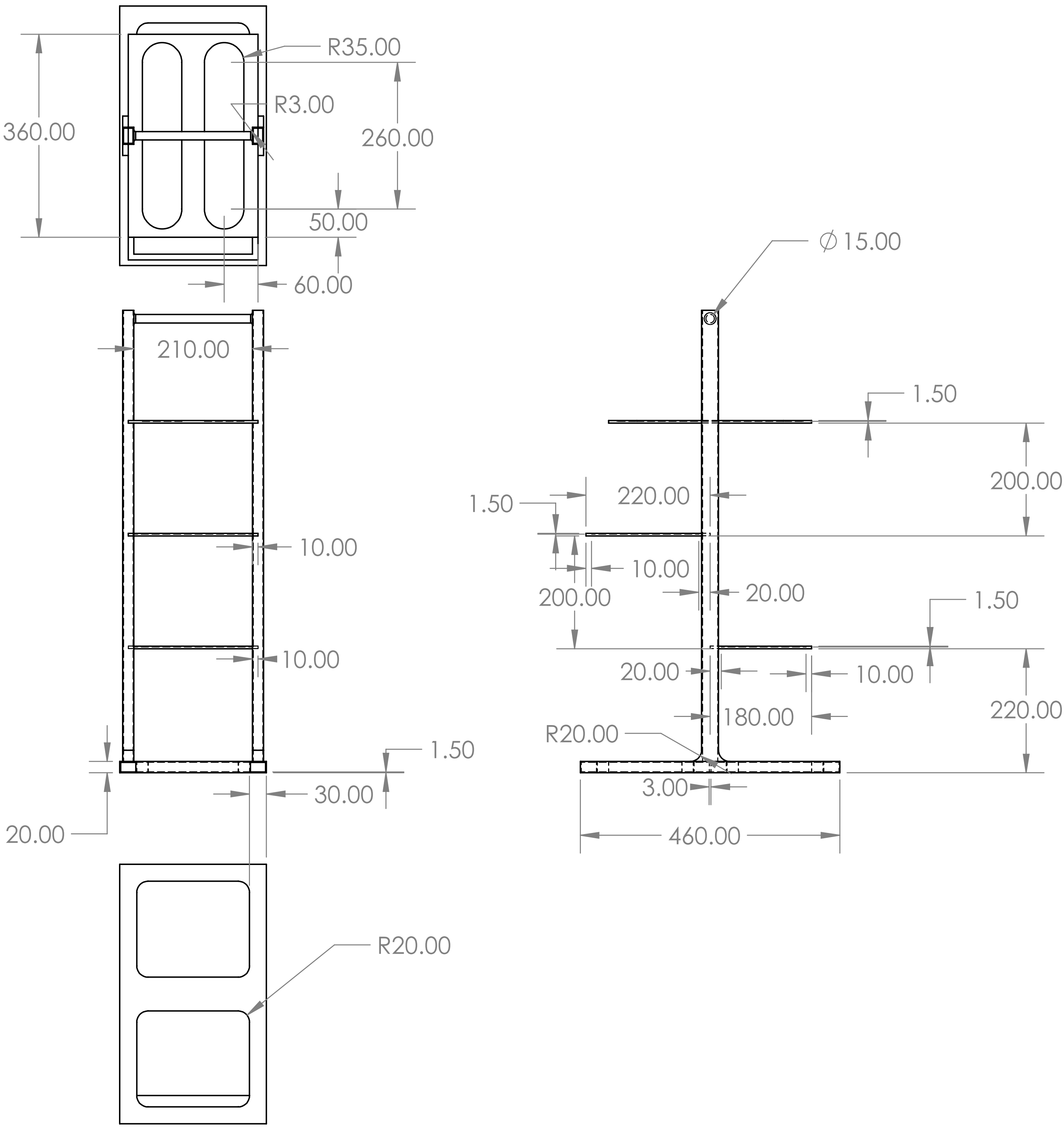



Figure 5: Model with final mesh, loads, and constraints




| | | | | | | |
|---|---|---|---|---|---|---|
| UNLESS OTHERWISE SPECIFIED:<br>DIMENSIONS ARE IN MILLIMETERS<br>SURFACE FINISH:<br>TOLERANCES:<br>LINEAR:<br>ANGULAR: | FINISH: | DEBURR AND BREAK SHARP EDGES | DO NOT SCALE DRAWING | REVISION | | |
| | NAME | SIGNATURE | DATE | | | TITLE: |
| DRAWN | | | | | | |
| CHK'D | | | | | | |
| APPV'D | | | | | | |
| MFG | | | | | | |
| Q.A | | | | MATERIAL: | | DWG NO. model |
| | | | | | | A4 |
| | | | | WEIGHT: | | SCALE:1:10 |
| | | | | | | SHEET 1 OF 1 |

### 2.1.2 Question 2

The load pointing downward on each shelve is the modelling of the weight of the pots put on it. For this modelling, the load on each shelve is equal to the weight of the pot multiply by g which is supposed to be $10m/s^2$. The load on the above shaft can be the model of pulling from the rope or from our hand whenever it is moved. Moreover, there is a constraint on the basement of the plant stand which has a fix to the ground or on other objects with the weld along the line on the basement. In addition, for each shelve, it might be fixed to the vertical beams by welding, which makes them merge into the vertical beams as a single object. Therefore, for this boundary condition, it helps to investigate the situation in which this plant stand sticks on other objects, such as small-size movable storage, with the shaft as a holder to move both of the objects together.

### 2.1.3 Question 3

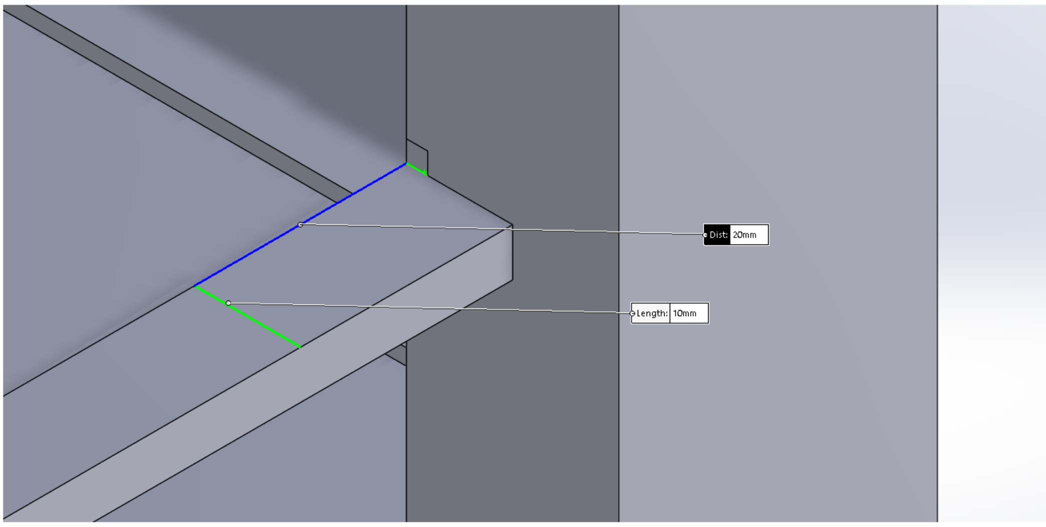
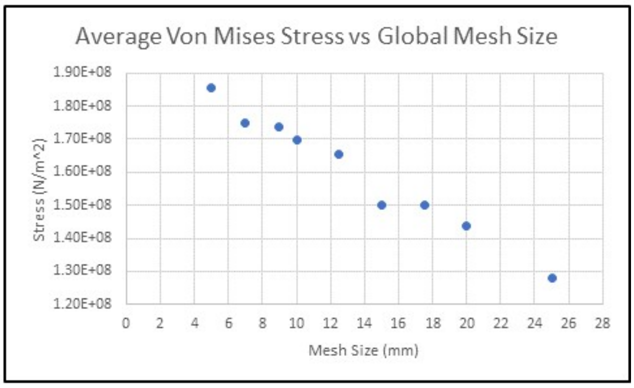
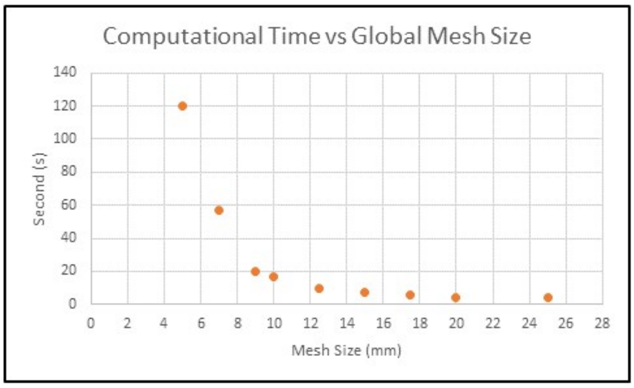



Figure 6: The region for extracting of average Von Mises Stress

For refining the global mesh size, I choose the average Von Mises to stress on the face of the shelve where the maximum stress occurs. Therefore, according to the initial image in question, the region of 20mm away from the edge of the second shelve is chosen to refine the global mesh size. In addition, during the meshing process, I observed that it was a small stress concentration on the edge between the vertical beam and the second shelf. Therefore, I applied the split line on a small piece around that point to cover it. Moreover, because the global mesh size cannot catch all of the stress so that I choose the global mesh size which the decrease in global mesh size causes a low change in the average stress on that region.




(a) Average Von Mises Stress vs Global Mesh Size




(b) Computational Time vs Global Mesh Size

Figure 7: Global Mesh Refinement

According to the graph, it can be observed that the decrease of the global mesh from 9mm to 7mm or 5mm does not have a significant change in the average stress on the maximum face. Moreover, there is an exponential increase in the computational time rising from 20 seconds for 9mm to 57 and 120 seconds for 7mm and 5mm respectively. Therefore, 9mm will be chosen as the global mesh size for this model.

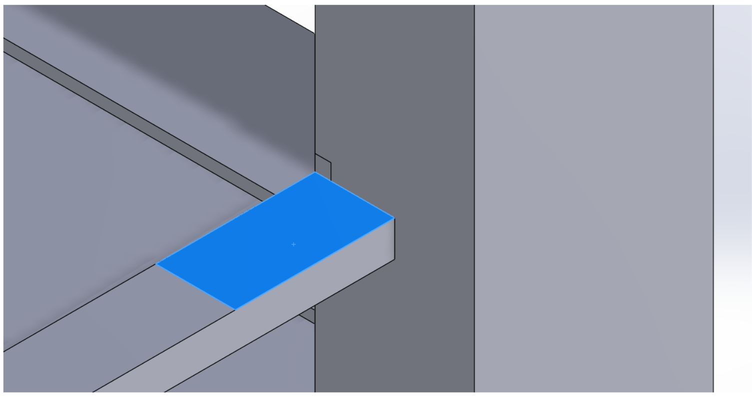


Figure 8: Region for refining local mesh size

For refining the local mesh, we will focus on the decrease in the mesh in the region above. Moreover, I also choose a small region on the vertical beam which is showing the failure. However, the point on the edge is where the stress concentration on the structure, which leads to the Von Mises stress in the study keeps rising as the decrease in local mesh. Therefore, to resolve this problem, in practice, they will use the small radius fillet to resolve this problem of the point. As a result, we can ignore that problem on that point and focus on the failure of the overall shelve due to the high load.

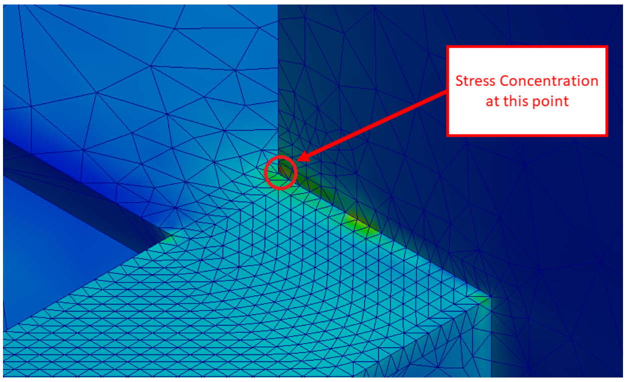
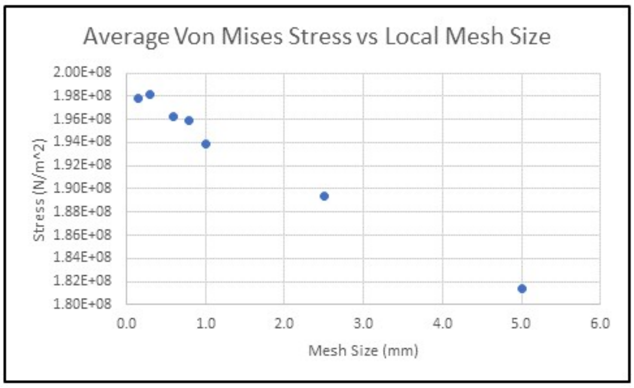
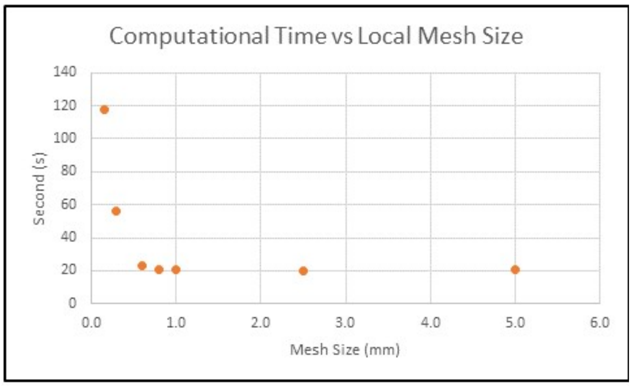



Figure 9: The point that we should ignore




(a) Average Von Mises Stress vs Local Mesh Size




(b) Computational Time vs Local Mesh Size

Figure 10: Local Mesh Refinement

According to the graph in figure 10, as the decrease in local mesh size, the average on the selected region converged and stopped starting to increase dramatically when the mesh size reduced from 0.6mm to 0.15mm. Moreover, there was a significant change in computational time while reducing the mesh size from 0.6mm to 0.3mm and 0.15 which cost 56 seconds and 120 seconds respectively. Therefore, the 0.6mm local mesh was chosen for the final mesh which would be filled in the optimization.

## 2.2 Question 4

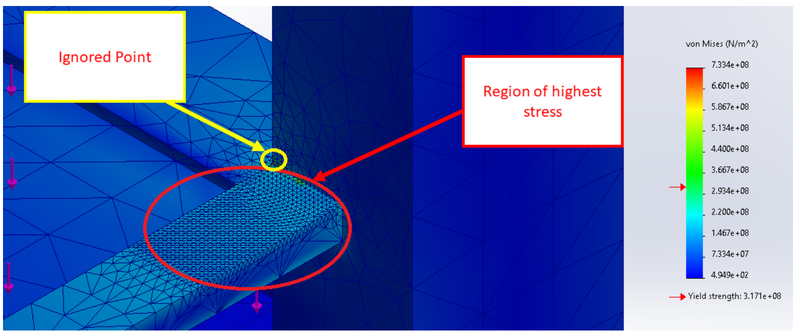



Figure 11: Von Mises stress contour plot the final mesh

In the contour plot in figure 11, because of the reason in the above question, the maximum region is described as in the red circle. And, the average Von Mises stress on that region is around 196.2 MPa.

### 2.3 Question 5

Because the Finite Elements Analysis is the numerical method in which we split all the models into smaller elements to analyze the stress and displacement for complicated models. Because it is a numerical method, this method is not infallible such as:

- Meshing problems:

  For each value of the mesh size, there is a difference in the number of elements in the model which causes a significant difference in the result compared to another mesh size. Therefore, if the result has an error if it is not a convergence result. Therefore, choosing a bad meshing size will lead to unreliable which can cause an enormous failure in the actual model.

- Boundaries and Constraints problem:

  Different boundary conditions on the model such as fixed geometry or slider will lead to different FEA results. Therefore, the FEA result will be unreliable if the model is modelled differently from its application. As a result, the model will be fallible.

- Assumption:

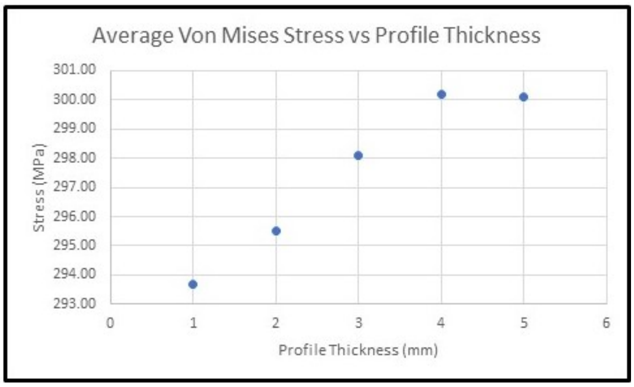
  The assumption of the load applied to the model also affects the result. Inaccurate in magnitude, distribution, and location can introduce the error in the analysis.

The method that should be used to verify the FEA result is testing with the smaller-scale model. By creating a smaller-scale model of the plant stand, we are able to apply a smaller load on the beam and shelves to verify our FEA result. Simultaneously, we also run the FEA on the smaller scale model so that we can match the result between the actual model with our FEA model. As a result, if the result matches each other so that we can conclude our work with the bigger scale model is accurate.

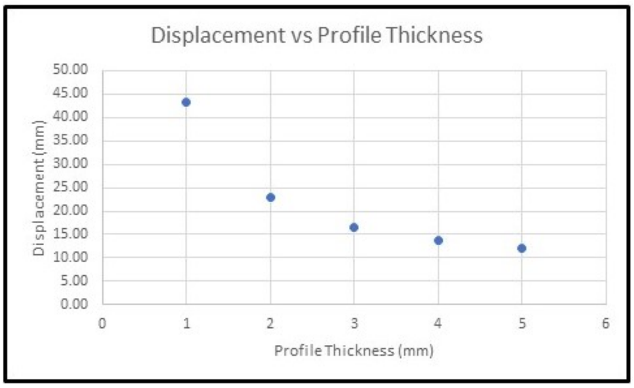
## 2.4 Part 2

### 2.4.1 Question 6

To examine the independent increase in the profile thickness change in the maximum stress and the displacement in the body, I kept the fillet radius at 20mm and increased the profile thickness from 1mm to 5mm with a 1mm step. According to figure 12(a) below, the average stress increases linearly as an increase in the profile thickness except that the stress slightly drops when the profile thickness increase from 4mm to 5mm. Additionally, when increasing the profile thickness from 1mm to 4mm, the stress has a 0.73% average increase. On the other hand, according to figure 12(b) below, the displacement has an exponential downward trend as the increase in profile thickness. However, the rate of change in the displacement always decreases as the increase in profile thickness. The rate of change drops from 47%, when profile thickness increases from 1mm to 2mm, to 11.4%, when profile thickness increases from 4mm to 5mm.



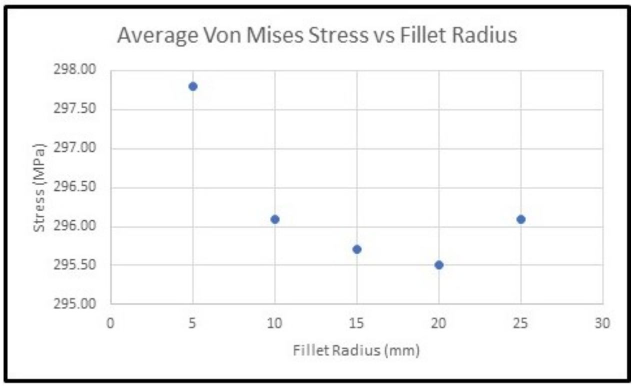

(a) Change of Average Von Mises Stress



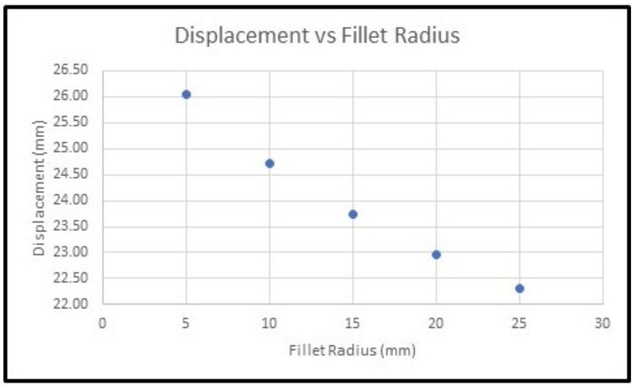

(b) Change of Displacement

Figure 12: Independent increase in Profile Thickness

To examine the independent increase in the fillet radius change the maximum stress and the displacement in the body, I kept the profile thickness at 2mm and increased the fillet radius from 5mm to 25mm with a 5mm step. According to figure 13(a) below, there is an exponential decrease in the average Von Mises stress as an increase in the fillet radius. However, there is a bounce back of the stress when the fillet radius increases from 20mm to 25mm. In addition, the stress drops 0.57% as the radius increase from 5mm to 10mm while there is only a 0.06% change in the stress when the radius increase from 15mm to 20mm. On the other hand, according to the figure 13(b) below, the displacement decreases linearly with the increase in the fillet radius. The average decrease in stress is 3.8%.




(a) Change of Average Von Mises Stress




(b) Change of Displacement

Figure 13: Independent increase in Fillet Radius

As we discussed above, the change in profile thickness caused a significant change in the Von Mises stress 0.73% compared to 0.57% from the change in fillet radius. Moreover, the increase in the profile thickness also caused the displacement to drop dramatically from 45mm to nearly 10mm. Therefore, the profile thick has a greater effect on the plant stand as comparing the fillet radius.

#### 2.4.2 Question 7

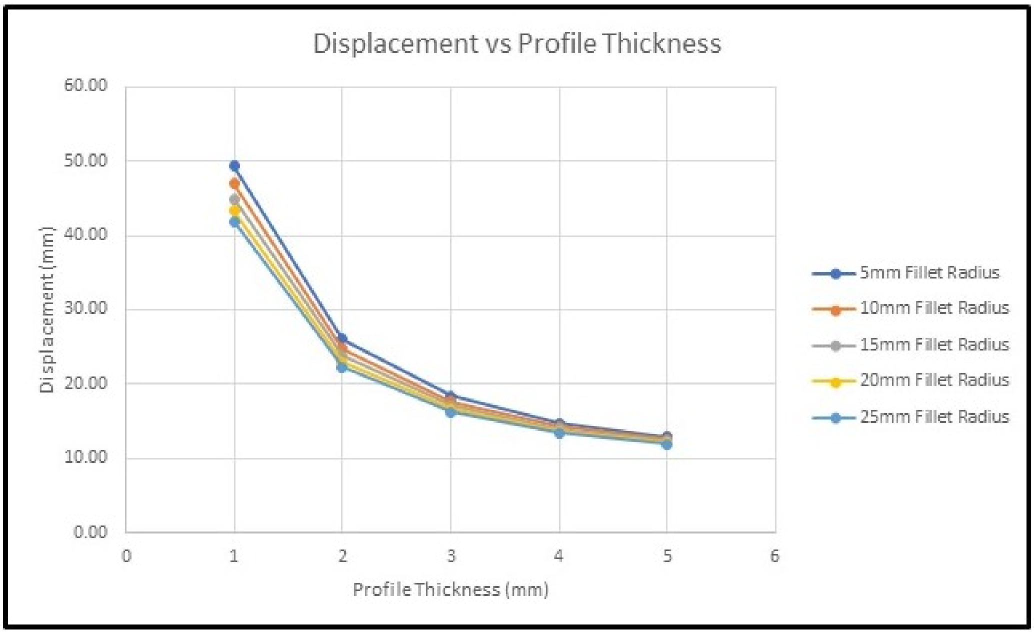



Figure 14: URES displacement vs beam profile thickness

| Design Study 1 | | | |
|---|---|---|---|
| Scenarios/Iterations: | 25 | | |
| Parameter Constraint or Goal | Format | Unit | Optimal Value |
| | | | |
| Fillet Radius | | mm | 5 |
| Profile Thickness | | N/A | 4 |
| Displacement2 | < 16mm | mm | 14.71 |
| Stress5 | < 317.1 N/mm^2 | N/mm^2 (MPa) | 300.70 |
| Mass1 | Minimize | g | 1764.45 |

Figure 15: The optimal geometry for the plant stand

### 2.4.3 Question 8

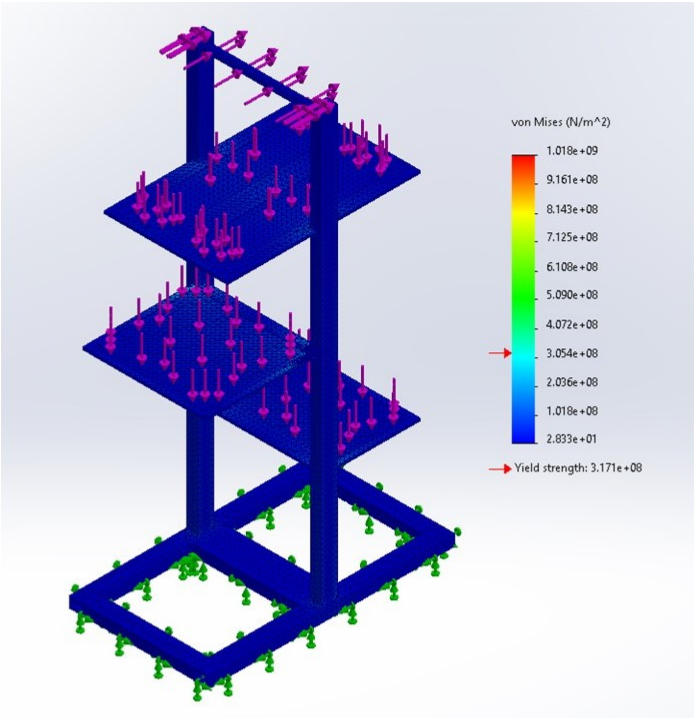



Figure 16: the optimal model with its stress contour

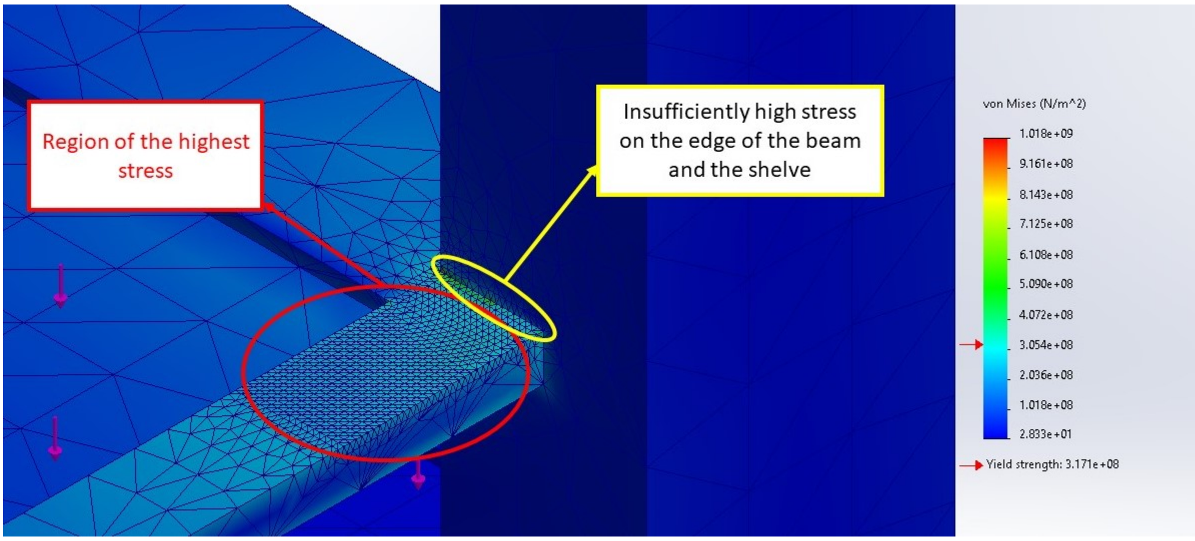



Figure 17: Location of the maximum stress of the optimal geometry model

The edge has insufficiently high stress because it is a shaft edge in the design which should not be taken into account for the highest stress in the model. Therefore, the region near that edge should be the region that will cause the failure of the plant stand. Moreover, the average stress in that region is approximately 300.4 MPa.

# 3 Appendix

## 3.1 Section 1

### 3.1.1 Question 1

The solution of the inverse of the matrix from Wolfram:

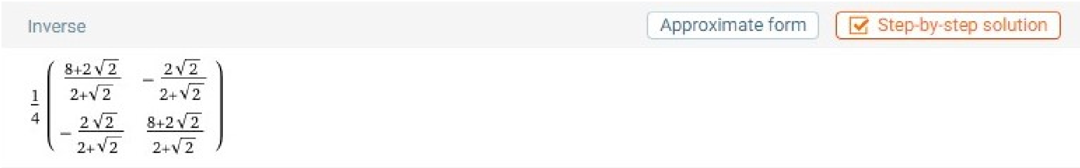



Figure 18: Inverse of the matrix from problem 1

### 3.1.2 Question 2

The solution of the inverse of the matrix from Wolfram:

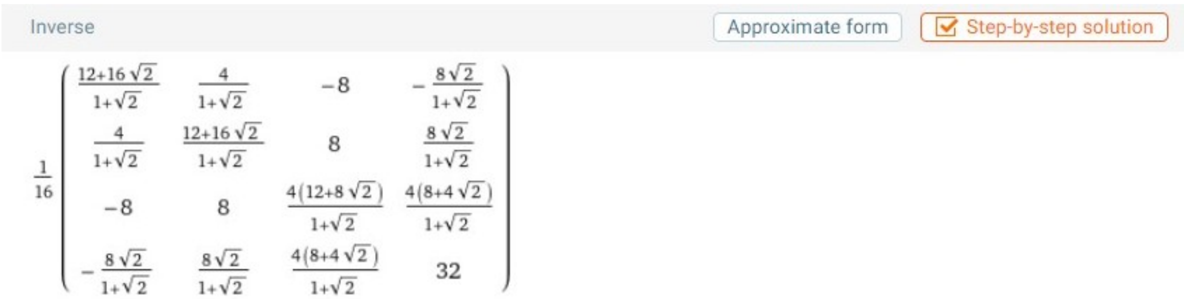



Figure 19: Inverse of the matrix from problem 2