Linear Algebra - Worksheet

Read this article before beginning the exercises: Linear Algebra Explained in 4 Pages

This assignment consists of 3 parts:

- Matrix Dimensions
- Vector Operations
- Matrix Operations

After completing the exercises by hand, use Python to check your work.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix}$$
 $v = \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 5 \end{bmatrix}$

1. Matrix Dimensions

Write the dimensions of each matrix.

- $1.1) A 2 \times 3$
- 1.2) B a a
- 1.3)~C~ 3x2
- 1.4) D 2x3
- 1.5) u / 4
- 1.6) w 4x/

2. Vector Operations

Perform the following operations. Assume $\alpha = 6$.

$$2.1) \vec{u} + \vec{v} = [9 + 49]$$

2.2)
$$\vec{u}$$
 - \vec{v} = [3 3 -2]

$$2.3)$$
 $\alpha \vec{u} =$ [36 12 -18 39]

$$2.4) \vec{u} \cdot \vec{v} = 18 + 10 + 3 + 20 = 5/$$

2.5)
$$\|\vec{u}\| = \sqrt{36 + 4 + 9 + 25} = \sqrt{74} = 8.6$$

3. Matrix Operations

Evaluate each of the following expressions, if it is defined; else fill in with "not defined." Do your work by hand on scratch paper.

$$3.1) A + C = not defined$$

$$3.2) A - C^{T} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 6 \end{bmatrix} = \begin{bmatrix} -4 & -7 & -3 \\ 3 & 6 & 4 \end{bmatrix}$$

$$3.3) C^{T} + 3D = \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 9 & -6 & -3 \\ 3 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 14 & 3 & 3 \\ 2 & 7 & 9 \end{bmatrix}$$

3.4)
$$BA = \begin{bmatrix} -1 & -5 & -1 \\ 2 & 7 & 4 \end{bmatrix}$$

$$3.5)$$
 $BA^T = not defined$

Optional

$$3.6)$$
 $BC = Not defined$

$$3.7) CB = \begin{bmatrix} 5 & -6 \\ 9 & -8 \\ 6 & -6 \end{bmatrix}$$

$$3.8) B^4 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

3.9)
$$AA^{T} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 7 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 28 \\ 28 & 69 \end{bmatrix}$$

$$3.10) \ D^T D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 10 & -4 & 0 \\ -4 & 8 & 8 \\ 0 & 8 & 10 \end{bmatrix}$$