Active Manifolds: Dimension Reduction via Nonlinear Spaces

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The Curse of Dimensionality

Understanding $f : \mathbb{R}^m \to \mathbb{R}$

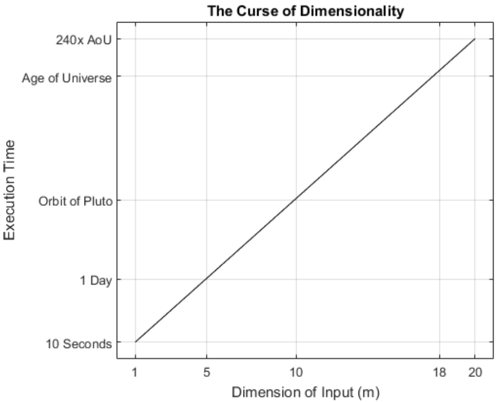

Figure: Execution time scales exponentially with dimension.

Solution: Reduce m

Active Subspaces

Find linear subspaces in the domain $\mathcal{D} \subseteq \mathbb{R}^m$ along which $f(\boldsymbol{x})$ does not change appreciably, and ignore those directions, roughly

$$\boldsymbol{W}^T \nabla f(\boldsymbol{x}) \approx 0, \text{ for all } \boldsymbol{x} \in \mathcal{D}. \tag{1}$$

More details available in Reference [Constantine, 2015]. New idea: Allow $\boldsymbol{W} = \boldsymbol{W}(\boldsymbol{x})$ to vary in parameter space.

Active Manifolds

Approximate Equation 1 above at a set of sample points $\boldsymbol{x}_i$. Parameterizing $\boldsymbol{W}(\boldsymbol{x})$ on $\boldsymbol{\alpha}$ in a linear fashion yields $\boldsymbol{M}^T\boldsymbol{\alpha}$. Add an L1 term for sparsity, and constrain to have a minimum L2 norm.

$$\begin{aligned} \text{min.} \quad & \|\boldsymbol{M}^T\boldsymbol{\alpha}\|_2 + \beta\|\boldsymbol{\alpha}\|_1 \\ \text{s.t.} \quad & \|\boldsymbol{\alpha}\|_2 \geqslant 1. \end{aligned} \tag{2}$$

Each $\boldsymbol{\alpha}$ defines a manifold; we successively reparameterize to find orthogonal $\boldsymbol{\alpha}$ until we fill the space.

Paul Constantine (2015)
Active Subspaces: Emerging Ideas for Dimension Reduction in Parameter Studies
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