

Active Manifolds: Dimension Reduction via Nonlinear Spaces

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The Curse of Dimensionality

Understanding $f : \mathbb{R}^m \rightarrow \mathbb{R}$

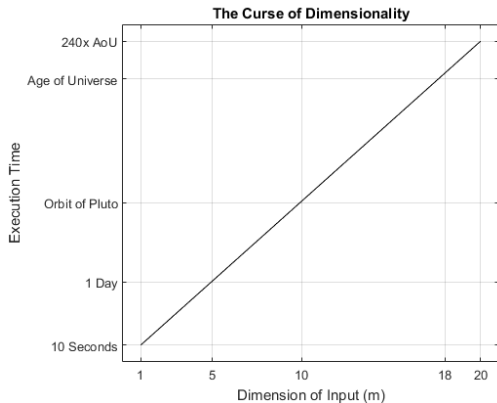


Figure : Execution time scales exponentially with dimension.

Solution: Reduce m

Find linear subspaces in the domain $\mathcal{D} \subseteq \mathbb{R}^m$ along which $f(\mathbf{x})$ does not change appreciably, and ignore those directions, roughly

$$\mathbf{W}^T \nabla f(\mathbf{x}) \approx 0, \text{ for all } \mathbf{x} \in \mathcal{D}. \quad (1)$$

More details available in Reference [Constantine, 2015]. New idea: Allow $\mathbf{W} = \mathbf{W}(\mathbf{x})$ to vary in parameter space.

Active Manifolds

Approximate Equation 1 above at a set of sample points \mathbf{x}_i .
Parameterizing $\mathbf{W}(\mathbf{x})$ on $\boldsymbol{\alpha}$ in a linear fashion yields $\mathbf{M}^T \boldsymbol{\alpha}$. Add an L1 term for sparsity, and constrain to have a minimum L2 norm.

$$\begin{aligned} \min. \quad & \|\mathbf{M}^T \boldsymbol{\alpha}\|_2 + \beta \|\boldsymbol{\alpha}\|_1 \\ \text{s.t.} \quad & \|\boldsymbol{\alpha}\|_2 \geq 1. \end{aligned} \tag{2}$$

Each $\boldsymbol{\alpha}$ defines a manifold; we successively reparameterize to find orthogonal $\boldsymbol{\alpha}$ until we fill the space.

Early Results

- Can recover Active Subspaces
- Can recover exact Active Manifold cases
- Solver sometimes gets 'stuck'



Paul Constantine (2015)

Active Subspaces: Emerging Ideas for Dimension Reduction in
Parameter Studies

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