Active Manifolds: Dimension Reduction via Nonlinear Spaces

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The Curse of Dimensionality

Understanding $f : \mathbb{R}^m \to \mathbb{R}$

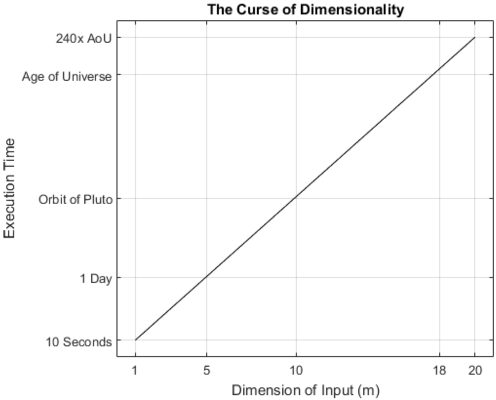

Figure : Execution time scales exponentially with dimension.

Solution: Reduce m – *Dimension Reduction*

Active Subspaces

Consider $f(\mathbf{x}) = \frac{1}{2}(.7x_1 + .3x_2)^2$

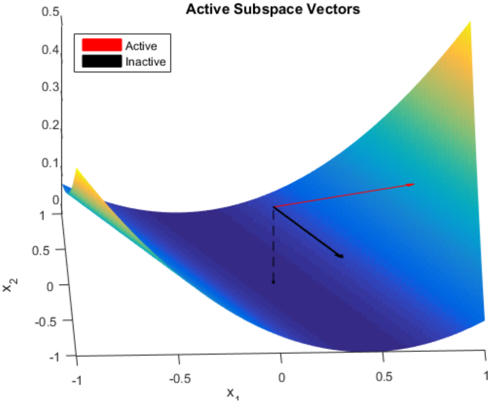

Figure : Active Subspace vectors over surface plot of f.

More details available in Reference [Constantine, 2015].

Early Results

$$f(\mathbf{x}) = x + y - 2z^2$$

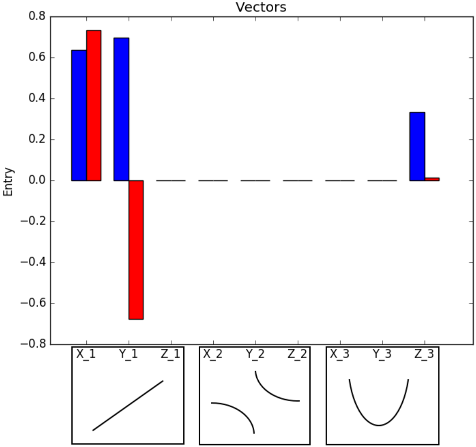

Figure : Vector components

Paul Constantine (2015)
Active Subspaces: Emerging Ideas for Dimension Reduction in Parameter Studies
Publisher SIAM Philadelphia

Active Manifolds

New idea: Allow $\mathbf{W} = \mathbf{W}(\mathbf{x})$ to vary in parameter space.

Approximate Equation **??** above at a set of sample points $\mathbf{x}_i$.

$$\begin{aligned} \text{min.} \quad & \|\mathbf{M}^T \boldsymbol{\alpha}\|_2 + \beta \|\boldsymbol{\alpha}\|_1 \\ \text{s.t.} \quad & \|\boldsymbol{\alpha}\|_2 \geqslant 1. \end{aligned} \tag{1}$$

Each $\boldsymbol{\alpha}_k^*$ defines a manifold; we successively reparameterize to find orthogonal $\boldsymbol{\alpha}_k^*$ until we fill the space.