Multidisciplinary Optimization of a Two-Stage Hybrid Rocket

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The primary goal of this project is to minimize the vehicle mass of a two-stage hybrid rocket Mars ascent vehicle designed to launch a given payload to a specific circular orbit. A barrier method was chosen to optimize this constrained problem. A penalty function was used to modify the objective function to account for the constraints.

Nomenclature

v	Velocity, km/s
h	Altitude, km
γ	Flight Path Angle, degrees
ρ	Density, kg/m^3
T	Temperature
P	Pressure
m	Mass, kg
$\dot{m}$	Mass flow rate, kg/s
L	Length, kg
D	Diameter, m
A_e	Nozzle exit area, m^2
A_t	Nozzle throat area, m^2
C_d	Coefficient of drag

Subscript

f	Final
s	Structure
l	Payload
p	Propellant
ox	Oxidizer

I. Introduction

When designing an orbital launch vehicle, mass is one of the most critical components to consider. The payload is only a small fraction of the total mass of any rocket. Reducing the mass the the launch vehicle itself can often translate into higher payload capabilities and is therefore something that is highly desirable. The purpose of this project is to minimize the vehicle mass of a rocket designed for a specific mission profile that calls for a given payload mass and target circular orbit. This work is based on a previous class project. In the original design, several parameter values were chosen simply for convenience and therefore the design is sub-optimal. The goal with this project is to use optimization techniques to find optimal design parameters and compare them with the original design.

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I.A. Background and Prior Work

The design of a hybrid rocket is traditionally broken down into several disciplines, each encompassing a major aspect of the rocket. These disciplines can be implemented into a model as separate modules. Based on the desired complexity and detail for the design, a fewer or greater number of modules can be used. For optimization, these modules can be optimized individually or combined into one optimization problem.

The majority of prior work on this subject uses genetic algorithms (GAs) for the optimization, such as in the work presented by Schoonover, Crossley and Heister in 2000.[1] Their highly detailed model contains dozens of different modules that allow for a very comprehensive design of the rocket. One benefit of using this GA technique is that it allows for optimization of both the continuous and discrete design parameters. In this implementation of GA, all of the design parameters were represented using a binary coding. Continuous variables were also represented this way, meaning they are represented as discrete values with a resolution determined by the number of bits used to represent them. Combining the strings that represent different design parameters creates a ”chromosome” that represents the overall rocket. Changing the bits in the chromosome changes some of the components of the rocket.

The optimization was performed by starting with random chromosomes for the initial gene pool. Based on the design variables decoded from each chromosome, the individuals are assigned fitness values and a tournament selection method is then used to determine which individuals will move on to produce the next generation and which will die out. The tournament is repeated so that there are two rounds for each generation (each individual competes twice). The selected parents then produce offspring using crossover and mutation. For the purposes of their paper, the GA was run for 100 generations. Using a population size of 80 individuals, this method is able to present several different values for design parameters that can create an optimal hybrid rocket design.

Optimization techniques can also be used to trade-off the benefits of different parameters in rocket design. For example, minimizing vehicle mass may be one objective while another is to maximize the altitude, as in the case of the work presented by Oyama et al. in 2011.[2] This group used a divided range multi-objective genetic algorithm (DRMOGA) to optimize a single stage hybrid rocket. This algorithm started with randomly produced individuals that were then ranked based on their values of the objective function. The individuals were then subdivided into m equally sized subpopulations. Within each subpopulation, a multi-objective genetic algorithm (MOGA) was used to find the optimum. The optimization was performed to both maximize altitude and to minimize weight. A Pareto ranking method was used to show the trade-off between the two parameters. It is interesting to note that by using this optimization method, the group was able to isolate a single design parameters as having the greatest effect on the final numbers: the oxidizer mass flow rate. This type of information is useful when designing systems since it tells you which parameters are more lenient and which ones must be tightly controlled.

II. Objective and Constraints

The primary objective of this project is to minimize the vehicle mass of the rocket, subject to some constraints. There are both equality and inequality constraints to consider. First, the rocket must obtain its target circular orbit. This can be broken down into three separate equality constraints: a target altitude, a target velocity, and a target flight path angle (FPA, γ). The final altitude, velocity and FPA must be equal to these target values.

$$h_f = 500km \tag{1}$$

$$v_f = 3.318km/s \tag{2}$$

$$\gamma = 0^\circ \tag{3}$$

In addition to the orbital constraints there are also two inequality constraints regarding the size of the rocket. The overall length to diameter ratio (L/D) of the rocket must be reasonable. Also, the ratio between the area of the nozzle throat (A_t) and the area of the nozzle exit (A_e) for each stage must be small enough to be feasible.

$$L/D \leq 25 \tag{4}$$

$$A_e/A_t \leq 75 \tag{5}$$

III. Approach

As is common in rocket design problems, the rocket components were divided into smaller modules. There were two main modules to cover different aspects of the rocket: propulsion and trajectory. The propulsion module handles the physical sizing of the rocket. The mass of the rocket can be broken down into payload mass, propellant mass, and structural mass. The payload mass is fixed for this project, leaving the propellant and structural masses as parameters to optimize. The trajectory module handles the flight of the rocket to achieve the final target orbit. This includes things like launch angle, thrust vector control angle, and coast time between stages. Each of these modules contains several different variables that can be adjusted Two different approaches were used for this optimization problem. First an overarching optimization in which all of the various modules of the rocket were simultaneously optimized. The second method used a distributed architecture to optimize the different modules individually at each iteration.

For the single large optimization, a barrier method was used to incorporate all of the different constraints. The key design parameters were treated as inputs to an objective function (*f(x)*), which was then modified to include the constraints using a penalty function, illustrated in Eq. 6, where m is the total number of inequality constraints (in this case $m = 3$) and l is the total number of equality constraints (in this case $l =$ 5).

$$P(x) = \sum_{i=1}^{m} max[0, g_i(x)] + \sum_{j=1}^{l} |h_j(x)| \tag{6}$$

This penalty was added to the original objective function to create the modified objective function shown in Eq. 7

$$T(x) = f(x) + rP(x) \tag{7}$$

where r is an optimization parameter that is adjusted at each iteration of the minimization of $T(x)$.

A lot of work was put into choosing the relative importance of the constraints. First, they were all normalized to one. Second, the circular orbit trajectory constraints were given a multiplier of 10 so that they would be prioritized by the optimization. The constraint equations are as follows:

$$g_1 = \frac{A_e/A_{t,S1} - 75}{75}$$

$$g_2 = \frac{A_e/A_{t,S2} - 75}{75}$$

$$g_3 = \frac{L/D - 25}{25}$$

$$h_1 = 10\frac{|h_{final} - h_{target}|}{h_{target}}$$

$$h_2 = 10\frac{|V_{final} - V_{target}|}{V_{target}}$$

$$h_3 = 10\frac{|\gamma_{FPA,final} - \gamma_{FPA,target}|}{\gamma_{FPA,target}}$$

IV. Results

The single large scale optimization approach described above was carried out over the 12-dimensional problem. The initial value of the penalty function multiplier, r, was chosen to be 1000 with an increasing schedule of $\times 10$ with each convergence iteration. Due to the computational cost of each function evaluation with the full trajectory simulation, a maximum number of iterations was chosen to be 200 for each convergence iteration and value of r.

The resulting trajectory of the converged solution can be seen in figure 2. This trajectory shows excellent convergence on the circular orbit altitude of 500 km and flight path angle of 0° however it does not meet the target velocity of 3.3 km/s. Additional results from the propulsion model are also shown in figure 1. These

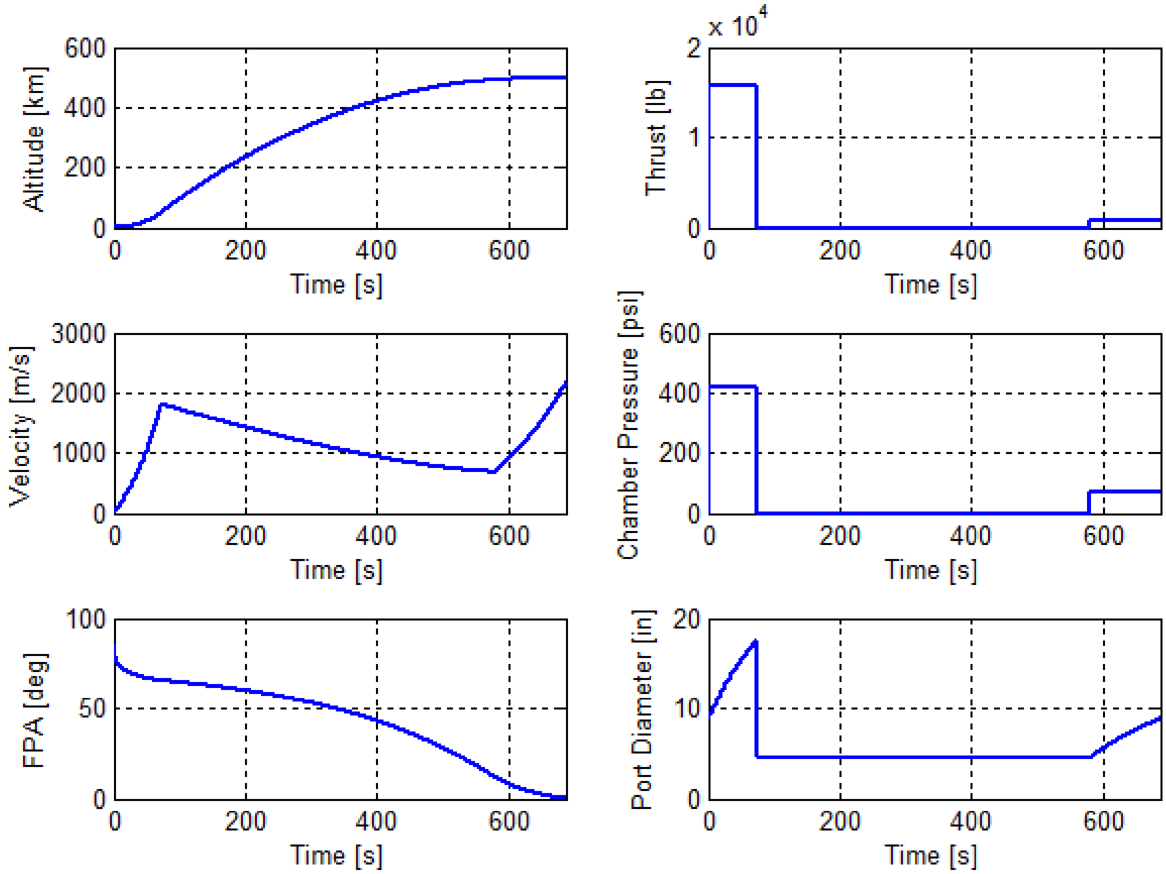

Figure 1. Trajectory of converged solution from bulk optimization.

show the thrust, chamber pressures, and fuel grain port diameters of each stage of the vehicle over time. These plots reveal the optimized burn time for each stage as well as the optimized coast time (in between stages) of the trajectory.

Finally, the converged solution of the design variables is provided in figure 2.

	Propellant Mass (kg)	O/F Ratio	Burn Time (s)	TVC Angle (°)	Nozzle Area Ratio
Stage 1	2118	5.17	72.2	-0.29	20.1
Stage 2	163	0.86	108.2	0.02	54.4
	Initial FPA (°)	Coast Time (s)			
Trajectory	85.1	506.2			

Figure 2. Final values of design variables from converged solution.

It can be noted that the optimized solution seems to converge on a vehicle with a very small second stage compared to the first stage. The second stage O/F ratio appears quite low and some further work could be performed to investigate why the optimization moved to that point. It is a good check to see that the nozzle area ratio of the first stage is much lower than that of the second stage leading to less expansion in the lower, more dense part of the atmosphere.

V. Conclusion

While this optimization scheme was able to meet some of the constraints and objectives of the problem, there is a lot more work that can be performed to improve the optimization process and solution. An important step is determining the optimal scheduling for the penalty function multiplier r and optimal relative weights of constraint equations in the penalty function. It was interesting to note how prioritizing the circular orbit trajectory constraints with a multiplier of 10 allowed the optimizer to do a better job of meeting the target orbit.

References

[1]Schoonover, P. L., Crossley, W. A., & Heister, S. D. (2000). *Application of a genetic algorithm to the optimization of hybrid rockets.* Journal of Spacecraft and Rockets, 37(5), 622-629.

[2]Oyama, A., Fujii, K., & Kanazaki, M. (2011). *Multidisciplinary and multi-objective design exploration methodology for conceptual design of a hybrid rocket.*