Active Manifolds: Dimension Reduction via Nonlinear Spaces

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The Curse of Dimensionality

Understanding $f : \mathbb{R}^m \to \mathbb{R}$

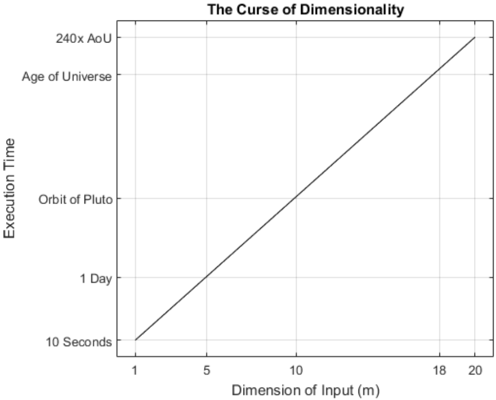

Figure : Execution time scales exponentially with dimension.

Solution: Reduce m – *Dimension Reduction*

Active Subspaces

Find linear subspaces in the domain $\mathcal{D} \subseteq \mathbb{R}^m$ along which

$$\mathbf{W}^T \nabla f(\mathbf{x}) \approx 0, \text{ for all } \mathbf{x} \in \mathcal{D}. \tag{1}$$

More details available in Reference [Constantine, 2015].

Active Manifolds

New idea: Allow $\mathbf{W} = \mathbf{W}(\mathbf{x})$ to vary in parameter space.

Approximate Equation 1 above at a set of sample points $\mathbf{x}_i$.

$$\begin{aligned} \text{min.} \quad & \|\mathbf{M}^T \boldsymbol{\alpha}\|_2 + \beta \|\boldsymbol{\alpha}\|_1 \\ \text{s.t.} \quad & \|\boldsymbol{\alpha}\|_2 \geqslant 1. \end{aligned} \tag{2}$$

Each $\boldsymbol{\alpha}_k^*$ defines a manifold; we successively reparameterize to find orthogonal $\boldsymbol{\alpha}_k^*$ until we fill the space.

Early Results

$$f(\mathbf{x}) = \frac{1}{2}(0.3x_1 + 0.3x_2 + 0.7x_3)^2$$

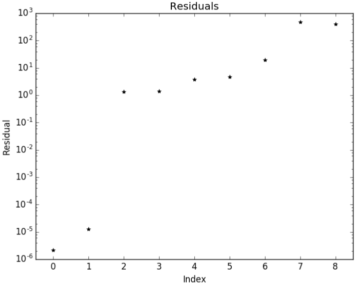

Figure : Residuals

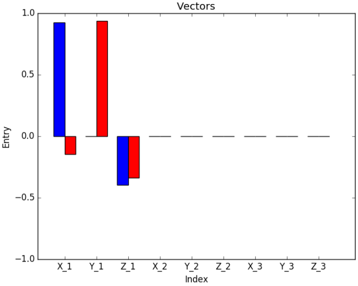

Figure : Vectors

Paul Constantine (2015)
Active Subspaces: Emerging Ideas for Dimension Reduction in Parameter Studies
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