1 The Problem

One of the most challenging difficulties facing high-fidelity modeling is the treatment of high-dimensional parameter spaces: the Curse of Dimensionality. Consider a parameter study on some quantity of interest (QoI) f in a space of dimension d; a simple heuristic is to use 10 points per dimension, in order to well represent the parameter space. Then the total number of sample points is 10^d. If a computer code implementing our model executes in a fixed time of 1 second, then our parameter study execution time scales exponentially. Figure 1 depicts the aforementioned scenario.

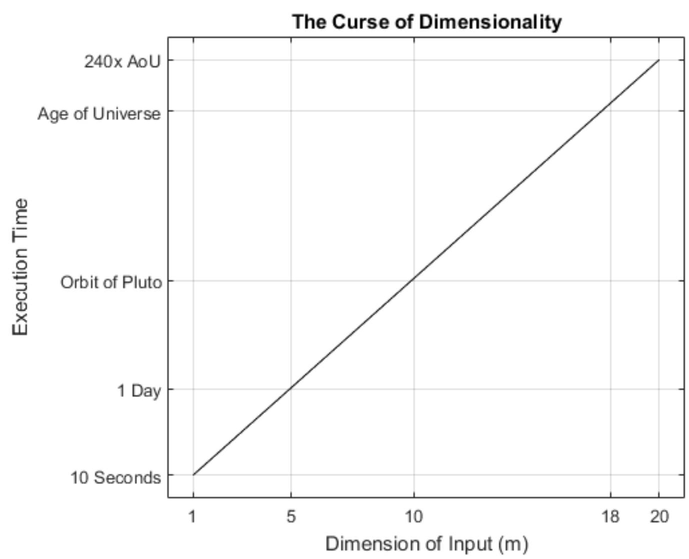

Figure 1: The Curse of Dimensionality; as dimensionality increases, the cost of many dimension-dependent studies (parameter studies, integration, etc.) increases exponentially. In this example, for modestly high-dimensional systems (say 10) the study is already completely intractable.

The only reasonable strategy to mitigate this challenge is to perform *dimension reduction*, that is, to reduce d. One scheme for dimension reduction of this sort is to seek *Active Subspaces* − linear subspaces in parameter space along which the majority of variation in our QoI is captured. [1] Active Subspaces gives a 'perfect' dimension reduction in the case that our QoI is a Ridge Function; that is, for $\boldsymbol{x} \in \mathbb{R}^d$ and $A \in \mathbb{R}^{d\times k}$ with $k < d$, we have

$$f(\boldsymbol{x}) = f(A^T\boldsymbol{x}) \tag{1}$$

Active Subspaces have already proven to be a useful strategy in numerous engineering and scientific computing applications, though the technique has some limitations. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(\boldsymbol{x}) = x_1^2 - 2x_2^2. \tag{2}$$

Note that such a function does not

References

[1] P. Constantine. *Active Subspaces: Emerging Ideas for Dimension Reduction in Parameter Studies.* SIAM Philadelphia, 2015.