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1. Syntax and Semantics.

   (a) Yes

   (b) Yes

   (c) No

   (d) No,because an expression can have multiple expression rules applied to it at one time. For example:

      i. $(3+7)+(1+2) \Rightarrow 10+(1+2)$

      ii. $(3+7)+(1+2) \Rightarrow (3+7)+3$

   (e) $e := \cdots \mid$ if0 e_1 then e_2 else e_3

   (f) if0 0 then e_2 else $e_3 \rightarrow e_2$

   (g) if0 i then e_2 else $e_3 \rightarrow e_3 (i \neq 0)$

   (h) $K := \cdots \mid$ if0 K then e_1 else e_2

   (i) $\frac{e_1 \Downarrow 0 \;\; e_2 \Downarrow i_2}{\text{if0 } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow i_2}$ $\frac{e_1 \Downarrow i_1 (i_1 \neq 0) \;\; e_3 \Downarrow i_3}{\text{if0 } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow i_3}$

   (j) if0 $y + x + x$ then 42 else 0

(k)

D

let $x := -3$ in let $y := 6$ in
if0 $y + x + x$ then 42 else 0

$\Rightarrow$ (let $y := 6$ in if0 $y + x + x$ then 42
else 0) $[-3/x]$

$\Rightarrow$ (let $y := 6$ if0 $y + (-3) + (-3)$
then 42 else 0

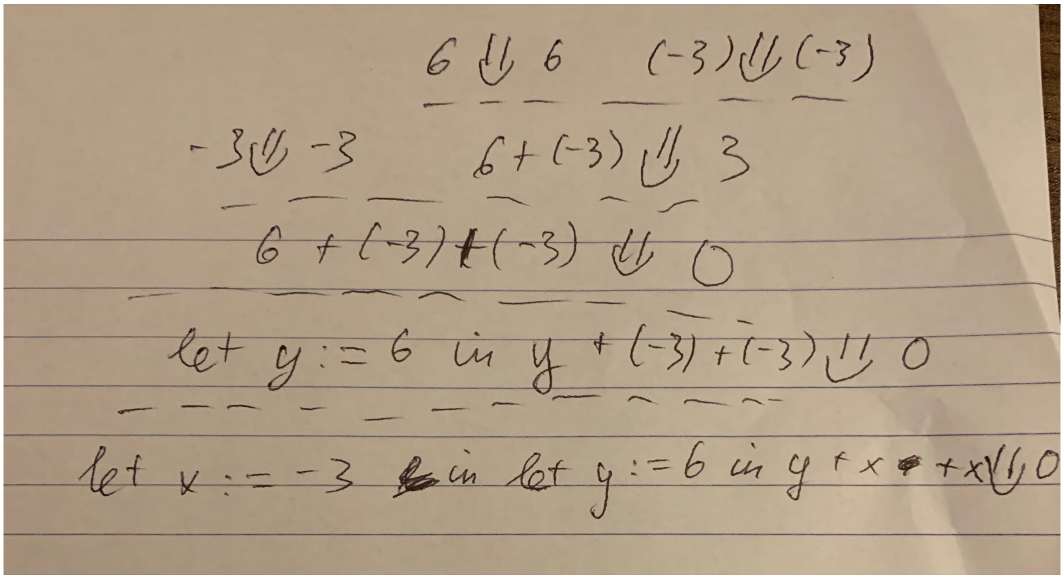
$\Rightarrow$ (if0 $y + (-3) + (-3)$
then 42 else 0) $[6/y]$

$\Rightarrow$ if0 $6 + (-3) + (-3)$
then 42 else 0

$\Rightarrow$ if0 $3 + (-3)$
then 42 else 0

$\Rightarrow$ if0 0 then 42 else 0

$\Rightarrow$ 42

(l) 

2. Untyped λ-calculus

   (a) hd := λ x . fst (snd x)

   (b) tl := λ x . snd (snd x)

   (c) isnil := λ x . fst x

   (d) hd (cons x l) $\equiv$ fst (snd (pair false (pair x l)))
   $\equiv$ fst (pair x l)
   $\equiv$ x

   (e) tl (cons x l) $\equiv$ snd (snd (pair false (pair x l)))
   $\equiv$ snd (pair x l)
   $\equiv$ l

   (f) isnil nil $\equiv$ fst (λ x . x)
   $\equiv$ (λ x . x) (true)
   $\equiv$ true

   (g) isnil (cons x l) $\equiv$ fst (pair false (pair x l))
   $\equiv$ false

   (h) hd nil $\equiv$ fst (snd (λ x . x))
   $\equiv$ fst ((λ p . p false)(λ x . x))
   $\equiv$ (λ p . p true) (false)
   $\equiv$ false true