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CS161, Spring 2021

Homework 5

1.

a. *$P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$*
$P \Rightarrow \neg Q \equiv \neg P \vee \neg Q$
$Q \Rightarrow \neg P \equiv \neg Q \vee \neg P$

P	Q	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

$M(P \Rightarrow \neg Q) = \{w_2, w_3, w_4\}$
$M(Q \Rightarrow \neg P) = \{w_2, w_3, w_4\}$
$M(P \Rightarrow \neg Q) = M(Q \Rightarrow \neg P)$, so the sentences are equivalent.

b. *$P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$*
$P \Leftrightarrow \neg Q \equiv (P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)$

P	Q	$P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

$M(P \Leftrightarrow \neg Q) = \{w_2, w_3\}$
$M((P \wedge \neg Q) \vee (\neg P \wedge Q)) = \{w_2, w_3\}$
$M(P \Leftrightarrow \neg Q) = M((P \wedge \neg Q) \vee (\neg P \wedge Q))$, so the sentences are equivalent.

2.

a. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

Smoke	Fire	(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
T	T	T
T	F	T
F	T	F
F	F	T

The sentence is satisfiable since it holds in worlds w_1, w_2, and w_4.
It is not valid since it does not satisfy all worlds.

b. (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

Smoke	Fire	Heat	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

The sentence is satisfiable since it holds in worlds w_1, w_2, w_3, w_4, w_5, w_6, and w_8.
It is not valid since it does not satisfy world w_7.

c. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Smoke	Fire	Heat	((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

The sentence is satisfiable in all worlds.
The sentence is valid since it satisfied all worlds.

3. a. Knowledge Base

   Let the Propositional Symbols be:

   A = Mythical
   B = Immortal
   C = Mammal
   D = Horned
   E = Magical

   1. $A \Rightarrow B$
   2. $\neg A \Rightarrow (\neg B \vee C)$
   3. $(B \vee C) \Rightarrow D$
   4. $D \Rightarrow E$

   b. Converting to CNF

   1. $A \Rightarrow B$
      $\equiv \neg A \vee B$
   2. $\neg A \Rightarrow (\neg B \wedge C)$
      $\equiv A \vee (\neg B \wedge C)$
      $\equiv (A \vee \neg B) \wedge (A \vee C)$
   3. $(B \vee C) \Rightarrow D$
      $\equiv \neg(B \vee C) \vee D$
      $\equiv (\neg B \wedge \neg C) \vee D$
      $\equiv (\neg B \vee D) \wedge (\neg C \vee D)$
   4. $D \Rightarrow E$
      $\equiv \neg D \vee E$

   CNF: $(\neg A \vee B) \wedge (A \vee \neg B) \wedge (A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D) \wedge (\neg D \vee E)$

   c.

   i. Prove that the unicorn is mythical (A)

      1. $\neg A \vee B$
      2. $A \vee \neg B$
      3. $A \vee C$
      4. $\neg B \vee D$
      5. $\neg C \vee D$
      6. $\neg D \vee E$
      7. $\neg A$ ($-\alpha$)
      8. $\neg B$ (2,7)
      9. C (3,7)
      10. D (5,9)
      11. E (6,10)

We cannot apply any more rules, and since we did not find any contradictions, $\Delta \wedge -\alpha$ is satisfiable, and we cannot use the knowledge base to prove that the unicorn is mythical.

ii. Prove that the unicorn is magical
   1. $\neg A \vee B$
   2. $A \vee \neg B$
   3. $A \vee C$
   4. $\neg B \vee D$
   5. $\neg C \vee D$
   6. $\neg D \vee E$
   7. $\neg E$ ($-\alpha$)
   8. $\neg D$ (6,7)
   9. $\neg C$ (5,8)
   10. $\neg B$ (4,8)
   11. $\neg A$ (1,10)
   12. A (3,9)
   13. Contradiction (11,12)

Since we found a contradiction, Δ ^ $-\alpha$ is unsatisfiable, and we proved that the unicorn is magical.

iii. Prove that the unicorn is horned
   1. $\neg A \vee B$
   2. $A \vee \neg B$
   3. $A \vee C$
   4. $\neg B \vee D$
   5. $\neg C \vee D$
   6. $\neg D \vee E$
   7. $\neg D$ ($-\alpha$)
   8. $\neg B$ (4,7)
   9. $\neg C$ (5,7)
   10. $\neg A$ (1,8)
   11. A (3,9)
   12. Contradiction (10,11)

Since we found a contradiction, Δ ^ $-\alpha$ is unsatisfiable, and we proved that the unicorn is horned.

4.

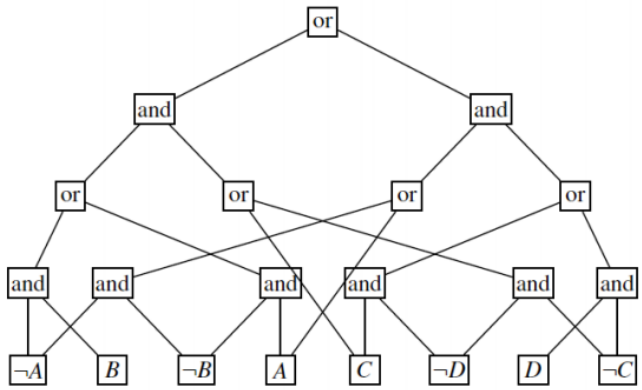

Figure 1

The NNF is decomposable because all of the sub-circuits feeding into the ‘and’ gates do not share variables.
The NNF is not deterministic because the topmost ‘or’ gate does not have at most 1 true input. For example, the assignment A=True, B=False, C=True, D=False will result in 2 true inputs being fed into the ‘or’ gate.
The NNF is not smooth because the subcircuits feeding into the second ‘or’ gate in the 3rd level is composed of {C} vs. {C,D} and the third ‘or’ gate in the 3rd level is composed of {A,B} vs. {A}, and these sub-circuits do not share variables.

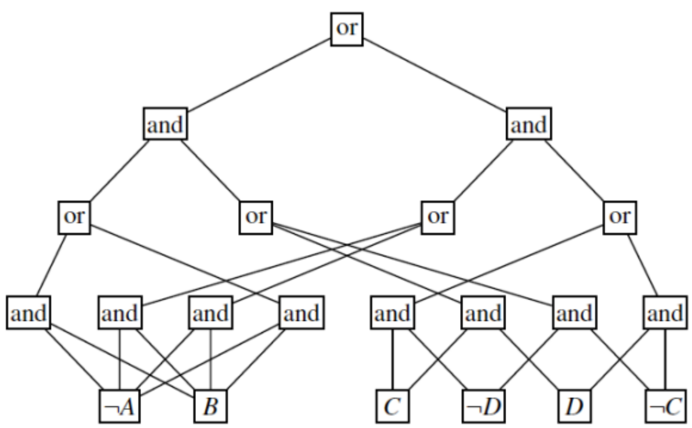

Figure 2

The NNF is decomposable because all of the sub-circuits feeding into the ‘and’ gates do not share variables.
The NNF is not deterministic because the first and third ‘or’ gate on the third level receive 2 true inputs with the assignment A=False, B=True.
The NNF is smooth because all of the sub-circuits feeding into the ‘or’ gates share variables.

5.

a) $\omega(A) = 0.1$
$\omega(\neg A) = 0.9$
$\omega(B) = 0.3$
$\omega(\neg B) = 0.7$
$\omega(C) = 0.5$
$\omega(\neg C) = 0.5$
$\omega(D) = 0.7$
$\omega(\neg D) = 0.3$

A	B	$(\neg A \land B) \lor (\neg B \land A)$
T	T	F
T	F	T
F	T	T
F	F	F

$(\neg A \land B) \lor (\neg B \land A)$
WMC = $\omega(\neg A)\omega(B) + \omega(\neg B)\omega(A) = (0.9)(0.3) + (0.7)(0.1) = 0.34$

b)

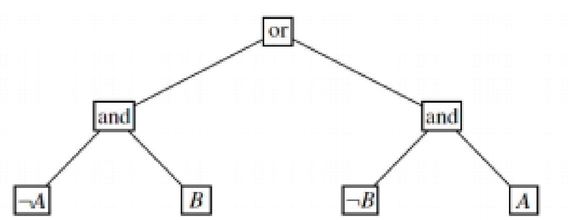

Figure 3

1st 'and' gate: $\omega(\neg A)\omega(B) = (0.9)(0.3) = 0.27$
2nd 'and' gate: $\omega(\neg B)\omega(A) = (0.7)(0.1) = 0.07$
(root node) top 'or' gate: $0.27+0.07 = 0.34$

The formula for the root node is $(\neg A \land B) \lor (\neg B \land A)$.
The count for the root node is equal to the WMC of its formula = $\omega(\neg A)\omega(B) + \omega(\neg B)\omega(A)$ = 0.34

c) Formula of the NNF:
$(((\neg A \wedge B) \vee (\neg B \wedge A)) \wedge ((C \wedge D) \vee (\neg C \wedge \neg D))) \vee (((\neg A \wedge \neg B) \vee (A \wedge B)) \wedge ((C \wedge \neg D) \vee (\neg C \wedge D)))$
WMC = $(\omega(\neg A)\omega(B) + \omega(\neg B)\omega(A)) * (\omega(C)\omega(D) + \omega(\neg C)\omega(\neg D))$ +
$(\omega(\neg A)\omega(\neg B) + \omega(A)\omega(B)) * (\omega(C)\omega(\neg D) + \omega(\neg C)\omega(D))$

$$= ((0.9)(0.3)+(0.7)(0.1))*((0.5)(0.7) + (0.5)(0.3)) + ((0.9)(0.7)+(0.1)(0.3))*((0.5)(0.3)+(0.5)(0.7)) = 0.5$$

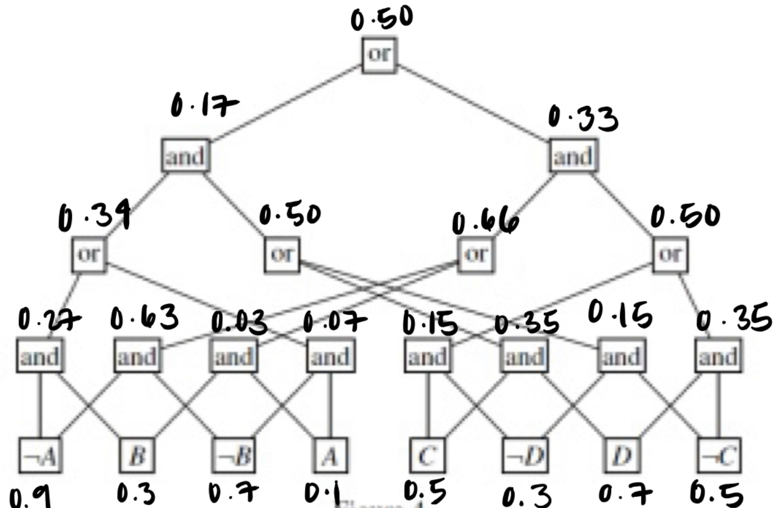

Figure 4