SISMID Spatial Statistics in Epidemiology and Public Health 2016 R Notes: Infectious Disease Data

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As a first example of the Held et al. (2005) approach we examine data on measles considered in this paper (actually an updated version).

These data are in the surveillance package.

The data consist of weekly measles counts over 2001 and 2002, for each of 17 administrative district Weser-Ems region of Lower Saxony, Germany.

Included in the dataset are a 17×17 matrix of 0/1 entries indicating which areas share a common boundary.

There is also the population that is contained in each area, and various other data including vaccination information.

Lower Saxony Measles: Reading in the Data

data("measlesWeserEms") is of sts class.

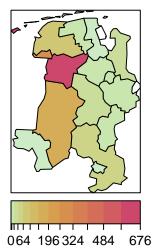
The data object also contains a map of the region, as a SpatialPolygonsDataFrame

```
library(surveillance)
data("measlesWeserEms")
```

Map of total counts

plot(measlesWeserEms, type = observed ~ unit)

2001/1 - 2002/52



Map of total counts with labels

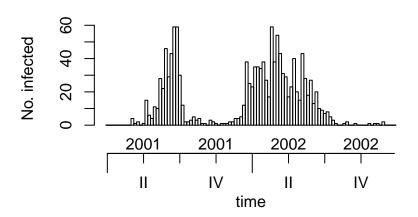
```
plot(measlesWeserEms, type = observed ~ unit, population = measl
    labels = list(labels = "GEN", cex = 0.3, font = 3))
```

2001/1 - 2002/52



Time series of all data

```
plot(measlesWeserEms, type = observed ~ time,
    legend.opts = NULL)
```

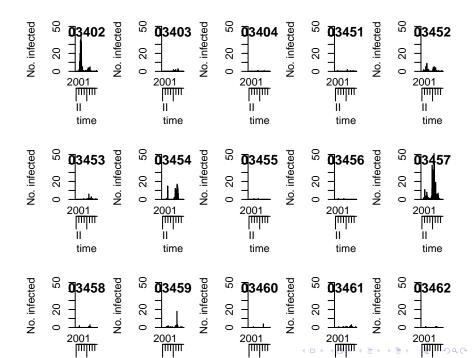


Time series with areas of all zeroes excluded

Two areas contain all zeroes

```
measlesWeserEms15 <- measlesWeserEms[, colSums(observed(measlesWeserEms))

plot(measlesWeserEms15)</pre>
```



Animation

The commands
library(gridExtra)
animate(measlesWeserEms)
produce an animation.
Try them yourself!

Model framework

We have a negative binomial model with

$$E[Y_{it}|\mu_{it}] = \mu_{it}$$

and

$$var(Y_{it}|\mu_{it}] = \mu_{it}(1 + \psi\mu_{it})$$

so that $\psi={\rm 0}$ corresponds to a Poisson model.

First model

We first fit the model

$$\mu_{it} = \underbrace{\lambda^{\text{AR}}_{\text{exp}(\alpha_0^{\text{AR}})} y_{i,t-1} + N_i \lambda_t^{\text{EN}}}_{\text{exp}(\alpha_0^{\text{AR}})},$$

with endemic term:

$$\log(\lambda_t^{\rm EN}) = \alpha_0^{\rm EN} + \alpha_1 t + \gamma \sin(\omega t) + \delta \cos(\omega t)$$

where

- \triangleright λ^{AR} is the epidemic force,
- \triangleright N_t are population counts in area i,
- $\triangleright \lambda_t^{\text{EN}}$ is the endemic term,
- $ightharpoonup lpha_1$ is a slope parameter describing the large scale endemic temporal trend,
- γ and δ are seasonal parameters and do not vary across areas, $\omega = (2\pi)/52$.



First model

NegBin1 gives a single overdisperion parameter, i.e., common to all areas.

```
f.end1 <- addSeason2formula(f = ~1 + t, S = 1, period = 52)
measles.mod1 <- list(ar = list(f = ~1), end = list(f = f.end1,
    offset = population(measlesWeserEms)), family = "NegBin1")
results.mod1 <- hhh4(measlesWeserEms, control = measles.mod1)
confint(results.mod1, parm = "overdisp")
## 2.5 % 97.5 %
## overdisp 1.608914 2.840724</pre>
```

This interval suggests a Poisson model, with $\psi=0$ would

Results for model 1

```
summary(results.mod1)
##
## Call:
## hhh4(stsObj = measlesWeserEms, control = measles.mod1)
##
## Coefficients:
##
                         Estimate Std. Error
## ar.1
                         -0.384089 0.122443
## end.1
                         0.397918 0.197906
## end.t
                        0.002941 0.003321
## end.sin(2 * pi * t/52) 1.027359 0.146790
## end.cos(2 * pi * t/52) -0.526122 0.130008
## overdisp
                          2.224819 0.314243
##
## Log-likelihood: -991.34
## AIC:
                   1994.68
## BIC:
                2027.49
##
## Number of units:
                       17
## Number of time points: 103
```

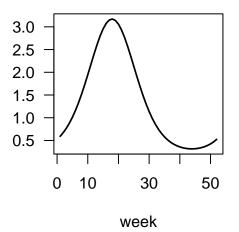
Results for model 1

idx2Exp = 1:4 gives exponentiated parameters and amplitudeShift tramsforms to the amplitude and phase shift.

```
summary(results.mod1, idx2Exp = 1:4, amplitudeShift = TRUE)
##
## Call:
## hhh4(stsObj = measlesWeserEms, control = measles.mod1)
##
## Coefficients:
##
                      Estimate Std. Error
## exp(ar.1)
                     0.681071 0.083392
## exp(end.1)
            1.488722 0.294627
## exp(end.t)
                1.002945 0.003331
## end.A(2 * pi * t/52) 1.154241 0.147789
## end.s(2 * pi * t/52) -0.473290 0.103303
## overdisp
              2.224819 0.314243
##
## Log-likelihood: -991.34
## AIC:
                  1994.68
## BTC:
               2027.49
##
## Number of units: 17
## Number of time points: 103
```

Plot of endemic component

```
# Plot for the 7th area:
plot(results.mod1, type = "season", components = "end",
    main = "")
```



A second model

We now add a neigborhood term, with simple binary weights given by

$$\mu_{it} = \underbrace{\lambda^{\text{AR}}_{\exp(\alpha_0^{\text{AR}})} y_{i,t-1}}_{\exp(\alpha_0^{\text{NE}})} \underbrace{\sum_{j \in \text{ne}(i)} y_{j,t-1}}_{j \in \text{ne}(i)} + N_{it} \lambda_{it}^{\text{EN}},$$

with endemic term:

$$\log(\lambda_{it}^{\rm EN}) = \alpha_0^{\rm EN} + \alpha_1 t + \gamma \sin(\omega t) + \delta \cos(\omega t)$$

where

- \triangleright λ^{AR} is the epidemic force,
- \triangleright λ^{NE} is the neighborhood effect,
- N_{it} are (possibly standardized) population counts in area i at time t,
- λ_{it}^{EN} is the endemic term,
- $ightharpoonup \gamma$ and δ are seasonal parameters and do not vary across areas, $\omega = (2\pi)/52$.



The neighbourhood matrix is a distance matrix (neighbourhood orders)

wji is a standardized weight

```
wji <- neighbourhood(measlesWeserEms)/rowSums(neighbourhood(measles.mod2 <- list(ar = list(f = ~1), ne = list(f = ~1,
    weights = wji), end = list(f = f.end1, offset = population(mfamily = "NegBin1")
results.mod2 <- hhh4(measlesWeserEms, control = measles.mod2)</pre>
```

```
summary(results.mod2)
##
## Call:
## hhh4(stsObj = measlesWeserEms, control = measles.mod2)
##
## Coefficients:
##
                          Estimate Std. Error
                         -3.841e-01 1.224e-01
## ar.1
## ne.1
                         -2.094e+01 3.864e+03
## end.1
                         3.979e-01 1.979e-01
## end.t.
                         2.941e-03 3.321e-03
## end.sin(2 * pi * t/52) 1.027e+00 1.468e-01
## end.cos(2 * pi * t/52) -5.261e-01 1.300e-01
                           2.225e+00 3.142e-01
## overdisp
##
## Log-likelihood: -991.34
## ATC:
                    1996.68
## BIC:
                    2034.96
##
## Number of units:
                         17
## Number of time points: 103
```

Hence, the mean model is

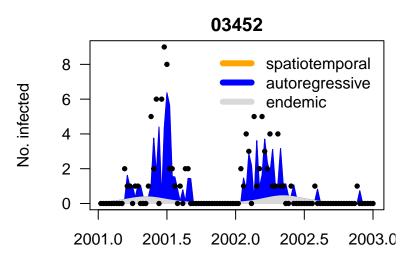
$$\begin{split} E[Y_{i,t+1}|y_{it},y_{jt}] &= \exp(-0.38)y_{it} + \exp(-25.05)\sum_{j\in\mathsf{ne}(i)}y_{jt} \\ &+ \exp\left[0.40 + 0.0029 \times t + 1.0\sin(\omega t) - 0.53\cos(\omega t)\right] \end{split}$$

The variance is

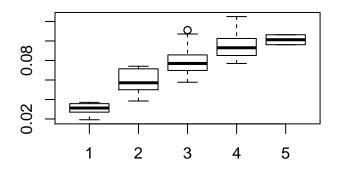
$$var(Y_{i,t+1}|y_{it}) = E[Y_{it}|y_{it}](1 + 2.23 \times E[Y_{it}|y_{it}]).$$

The fits are different in each of the areas due to the two epidemic components (AR and NE).

```
# Plot for the 7th area:
plot(results.mod2, unit = 7, legend = TRUE)
```



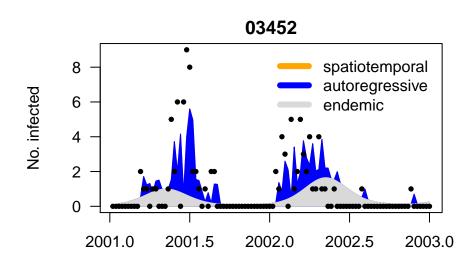
Neighbors summary



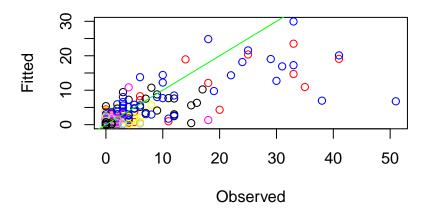
We now extend the model to add area-specific intercepts (fixed effects) to the endemic component.

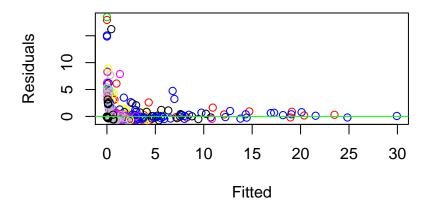
```
cbind(results.mod3$coeff, results.mod3$se)
##
                                   [,1]
                                                 [,2]
                           -0.580739100 1.289739e-01
## ar.1
## ne.1
                          -23.379640570 7.734442e+03
## end.t
                            0.009601913 3.476919e-03
## end.sin(2 * pi * t/52) 1.282329563 1.571930e-01
## end.cos(2 * pi * t/52) -0.638093991 1.333572e-01
## end.1.03401
                          -21.481281768 1.628901e+04
## end.1.03402
                            2.261499246 3.210989e-01
## end.1.03403
                          -0.278109315 3.987178e-01
## end.1.03404
                          -1.717665726 6.406479e-01
## end.1.03405
                          -21.543083205 1.592327e+04
## end.1.03451
                          -0.829303555 5.242275e-01
## end.1.03452
                            0.968107877 3.098019e-01
## end.1.03453
                            0.124479801 3.683747e-01
## end.1.03454
                           0.255424890 3.091657e-01
## end.1.03455
                          -1.646575261 7.590931e-01
## end.1.03456
                          -1.918957639 7.590523e-01
## end.1.03457
                           1.778743964 2.957993e-01
## end.1.03458
                           -0.577112364 4.694557e-01
## end.1.03459
                          -0.406827611 3.602242e-01
## end.1.03460
                           -1.056395570 5.294573e-01
## end.1.03461
                           0.514041619 3.795217e-01
## end.1.03462
                          -0.406028743 5.731908e-01
## -log(overdisp)
                           -0.388969337 1.408852e-01
```

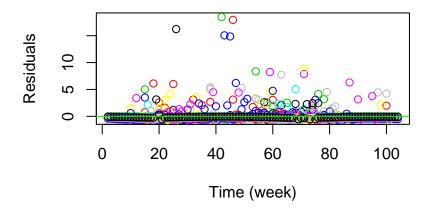
plot(results.mod3, unit = 7, legend = TRUE)



```
y <- measlesWeserEms@observed[2:104, ] # Notice no first
time <- matrix(rep(measlesWeserEms@epoch, 17), nrow = 104,
   ncol = 17
time \leftarrow time [-1,]
mu3 <- fitted(results.mod3)</pre>
res3 <- (y - mu3)/sqrt(mu3 * (1 + 1.74 * mu3))
par(mfrow = c(1, 3))
plot(mu3 ~ y, xlab = "Observed", ylab = "Fitted", type = "n")
for (i in 1:17) points(mu3[, i] ~ y[, i], col = i)
abline(0, 1, col = "green")
plot(res3 ~ mu3, xlab = "Fitted", ylab = "Residuals",
   tvpe = "n")
for (i in 1:17) points(res3[, i] ~ mu3[, i], col = i)
abline(0, 0, col = "green")
plot(res3 ~ time, xlab = "Time (week)", ylab = "Residuals",
   type = "n")
for (i in 1:17) points(res3[, i] ~ time[, i], col = i)
abline(0, 0, col = "green")
```



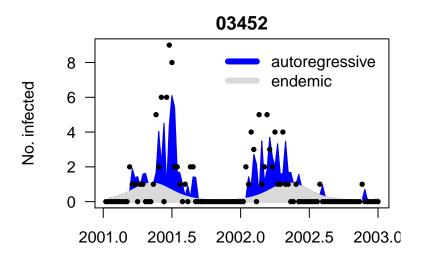




Random effects model, no neighborhood

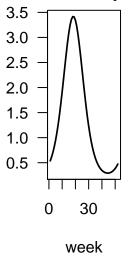
```
measles.mod4 <- list(ar = list(f = ~1), end = list(f = addSeason
    ri(type = "iid"), S = 1, period = 52), offset = population(m
    family = "NegBin1")
measles.res4 <- hhh4(measlesWeserEms, measles.mod4)</pre>
```

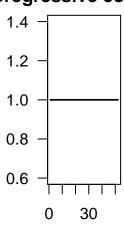
fitted values for a single unit



plot the multiplicative effect of seasonality

endemic compor autoregressive com

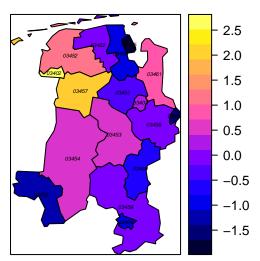




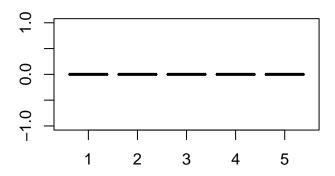
week

random intercepts of the endemic component

```
plot(measles.res4, type = "ri", component = "end",
    labels = list(cex = 0.3, font = 3))
```



Neighbors summary



Random effects model

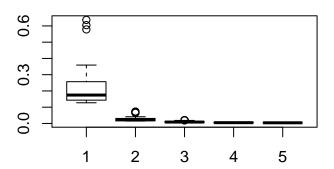
Without normalization, power-law weights are $w_{ji} = o_{ji}^{-d}$, where o_{ji} is the order of neighbourhood between regions i and j.

```
measles.mod5 <- list(end = list(addSeason2formula(~0 +
    ri(type = "iid"), S = 1, period = 52)), ar = list(f = ~1),
    ne = list(f = ~-1 + ri(type = "iid", corr = "all"),
        weights = W_powerlaw(maxlag = 5)), family = "NegBin1")
measles.res5 <- hhh4(measlesWeserEms, measles.mod5)</pre>
```

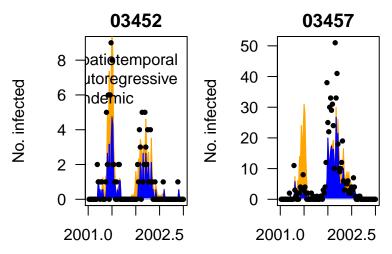
Neighborhood weights

Plot the (estimated) neighbourhood weights (neweights) as a function of neighbourhood order (shortest-path distance between regions), i.e., $w_{ji} \sim o_{ji}$.

```
plot(measles.res5, type = "neweights")
```



random intercepts of the endemic component



Random effects in all

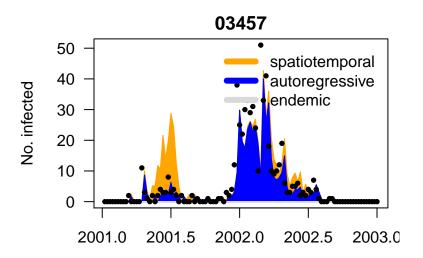
```
f.end <- addSeason2formula(~0 + ri(type = "iid", corr = "all"),
    S = 1, period = 52)
measles.mod6 <- list(end = list(f.end, offset = population(measl
    ar = list(f = ~-1 + ri(type = "iid", corr = "all")),
    ne = list(f = ~-1 + ri(type = "iid", corr = "all"),
        weights = W_powerlaw(maxlag = 5)), family = "NegBin1")
measles.res6 <- hhh4(measlesWeserEms, measles.mod6)</pre>
```

Results

```
summary(measles.res6)
##
## Call:
## hhh4(stsObj = measlesWeserEms, control = measles.mod6)
##
## Random effects:
##
             Var
                   Corr
## ar.ri(iid) 1.5565
## ne.ri(iid) 2.5279 0.8981
##
## Fixed effects:
##
            Estimate Std. Error
## ar.ri(iid) -1.9596 0.3950
## ne.ri(iid) -2.0552 0.4349
## end.1 -0.7821 0.2150
## neweights.d 3.0418 0.6195
## overdisp 1.3802 0.1987
##
## Penalized log-likelihood: -907.11
## Marginal log-likelihood:
                            -37.62
##
## Number of units:
                      17
## Number of time points: 103
```

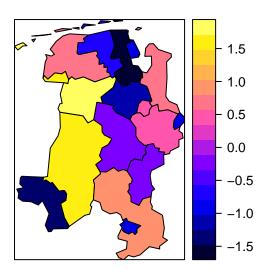
random intercepts of the endemic component

plot(measles.res6, type = "fitted", units = c(12))



Random intercepts of the endemic component

```
plot(measles.res6, type = "ri", component = "ar")
```



Random intercepts of the endemic component

```
plot(measles.res6, type = "ri", component = "ne")
```

