# SISMID Spatial Statistics in Epidemiology and Public Health 2016 R Notes: Clustering and Cluster Detection

2016 R Notes: Clustering and Cluster Detection for Count Data

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#### North Carolina SIDS Data

The nc.sids data frame has 100 rows and 21 columns and can be found in the spdep library.

It contains data given in Cressie (1991, pp. 386-9), Cressie and Read (1985) and Cressie and Chan (1989) on sudden infant deaths in North Carolina for 1974–78 and 1979–84.

The data set also contains the neighbour list given by Cressie and Chan (1989) omitting self-neighbours (ncCC89.nb), and the neighbour list given by Cressie and Read (1985) for contiguities (ncCR85.nb).

Data are available on the numbers of cases and on the number of births, both dichotomized by a binary indicator of race.

The data are ordered by county ID number, not alphabetically as in the source tables.

#### North Carolina SIDS Data

The code below plots the county boundaries along with the observed SMRs for 1974.

The expected numbers are based on internal standardization with a single stratum. So the single reference probability is the incidence of SIDS in 1974.

```
library(maptools)
library(spdep)
nc.sids <- readShapePoly(system.file("etc/shapes/sids.shp",</pre>
    package = "spdep")[1], ID = "FIPSNO",
    proj4string = CRS("+proj=longlat +ellps=clrk66"))
nc.sids2 <- nc.sids # Create a copy, to add to
Y <- nc.sids$STD74
E <- nc.sids$BIR74 * sum(Y)/sum(nc.sids$BIR74)
nc.sids2$SMR74 <- Y/E
nc.sids2$EXP74 <- E
brks <- seq(0, 5, 1)
rm(nc.sids) # We load another version of this later, so tidy up
```

#### SMR Plot

The map of the SMRs shows a number of counties with high relative risks (the risk relative to the state wide risk).

```
spplot(nc.sids2, "SMR74", at = brks,
    col.regions = grey.colors(5, start = 0.9,
    end = 0.1))
```

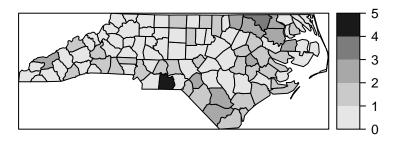


Figure 1: Map of SMRs for SIDS in 1974 in North Carolina

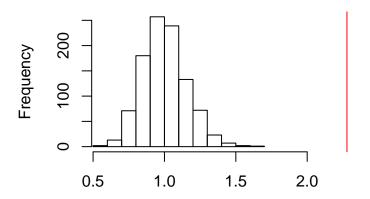
#### Overdispersion

Examine  $\kappa$ , the overdispersion statistic, and use a Monte Carlo test to examine significance.

```
library(spdep)
kappaval <- function(Y, fitted, df) {</pre>
    sum((Y - fitted)^2/fitted)/df
mod <- glm(Y ~ 1, offset = log(E), family = "quasipoisson")</pre>
kappaest <- kappaval(Y, mod$fitted, mod$df.resid)</pre>
nMC < -1000
ncts <- length(E)</pre>
yMC <- matrix(rpois(n = nMC * ncts, lambda = E),</pre>
    nrow = ncts, ncol = nMC)
kappaMC <- NULL
for (i in 1:nMC) {
    modMC <- glm(yMC[, i] ~ 1, offset = log(E),</pre>
        family = "quasipoisson")
    kappaMC[i] <- kappaval(yMC[, i], modMC$fitted,</pre>
        modMC$df.resid)
```

# Overdispersion: $\widehat{\kappa}$ is significantly different from 1

```
hist(kappaMC, xlim = c(min(kappaMC),
    max(kappaMC, kappaest)), main = "",
    xlab = expression(kappa))
abline(v = kappaest, col = "red")
```



κ

We first fit a non-spatial random effects model:

$$Y_i|\beta_0, \epsilon_i \sim_{iid} \text{Poisson}(E_i e^{\beta_0 + \epsilon_i}),$$
  
 $\epsilon_i|\sigma_{\epsilon}^2 \sim_{iid} N(0, \sigma_{\epsilon}^2)$ 

with the default priors on  $\beta_0$  and  $\sigma_{\epsilon}^2$ .

```
library(INLA)
nc.sids2$ID <- 1:100
m0 <- inla(SID74 ~ f(ID, model = "iid"),
    family = "poisson", E = EXP74, data = as.data.frame(nc.sids2 control.predictor = list(compute = TRUE))</pre>
```

The control.predictor argument indicates we want fitted values.

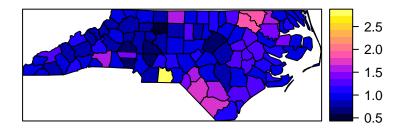
Examine the first few "fitted values", summaries of the posterior distribution of  $\exp(\beta_0 + \epsilon_i)$ , i = 1, ..., n.

```
head(m0$summary.fitted.values)
##
                                         sd 0.025quant 0.5quant 0.975quant
                             mean
## fitted.predictor.001 1.2515021 0.2930181
                                             0.7548490 1.2250844
                                                                   1.899824
## fitted.predictor.002 0.7665958 0.2700582
                                             0.3481650 0.7299039
                                                                   1.397177
## fitted.predictor.003 0.9149708 0.3494437
                                                                   1.751025
                                             0.3989681 0.8598644
## fitted.predictor.004 2.7309425 0.7626511 1.5074088 2.6400575
                                                                   4.470065
## fitted.predictor.005 0.9027425 0.3177245
                                             0.4165809 0.8575336
                                                                   1,650257
## fitted.predictor.006 0.8544442 0.3152039
                                             0.3789292 0.8076757
                                                                   1.601193
##
                             mode
## fitted.predictor.001 1.1747221
## fitted.predictor.002 0.6631748
## fitted.predictor.003 0.7637463
## fitted.predictor.004 2.4583333
## fitted.predictor.005 0.7763712
## fitted.predictor.006 0.7245109
```

#### Create two interesting inferential summaries:

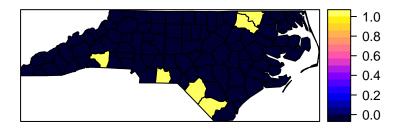
- the posterior mean of the relative risk
- ▶ a binary indicator of whether the posterior median is greater than 1.5 (which we assume is an epidemiologically significant value). This value can be changed, based on the context.

```
# Display posterior means of
# relative risks
spplot(nc.sids2, "RRpmean0")
```



A number of counties have high mean values.

```
# Display indicators of whether 0.5
# points above 1.5
spplot(nc.sids2, "RRind0")
```



Six counties have posterior median relative risk greater than 1.

We now fit a model with non-spatial and ICAR spatial random effects.

```
nc.sids2$ID2 <- 1:100
m1 <- inla(SID74 ~ 1 + f(ID, model = "iid") + f(ID2,
    model = "besag", graph = "examples/NC.graph"),
    family = "poisson", E = EXP74, data = as.data.frame(nc.sids2 control.predictor = list(compute = TRUE))
# Define summary quantities of interest as with iid
# model
nc.sids2$RRpmean1 <- m1$summary.fitted.values[, 1]
nc.sids2$RRind1 <- m1$summary.fitted.values[, 4] >
    1.5
```

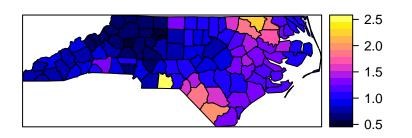
#### Summarize the IID+ICAR model

```
summary(m1)
##
## Call:
## c("inla(formula = SID74 ~ 1 + f(ID, model = \"iid\") + f(ID2, model = \"besag\", ", " graph = \"ex
##
## Time used:
## Pre-processing Running inla Post-processing
                                                      Total
##
           0.9511
                          0.4588
                                        0.0746
                                                     1 4845
##
## Fixed effects:
##
                 mean
                        sd 0.025quant 0.5quant 0.975quant mode kld
## (Intercept) -0.0554 0.054 -0.1642 -0.0545 0.0479 -0.0524 0
##
## Random effects:
## Name Model
## ID IID model
## ID2 Besags ICAR model
##
## Model hyperparameters:
##
                                    sd 0.025quant 0.5quant 0.975quant
                        mean
## Precision for ID 17946 355 1 751e+04 1204 782 12805 631
                                                            64499 90
## Precision for ID2
                       2.299 8.844e-01 1.092 2.125
                                                                4.50
##
                       mode
## Precision for ID 3275.258
## Precision for ID2 1.823
##
## Expected number of effective parameters(std dev): 34.19(5.792)
## Number of equivalent replicates : 2.925
##
## Marginal log-Likelihood: -314.28
## Posterior marginals for linear predictor and fitted values computed
```

As an aside, if we wanted to create a neighbour list based on regions with contiguous boundaries we can use the poly2nb function in the spdep library.

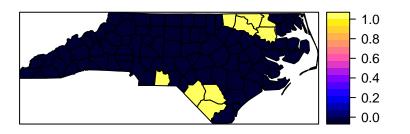
Display posterior means of relative risks.

```
spplot(nc.sids2, "RRpmean1")
```



Display areas with medians above 1.5, ie those areas with greater than 50% chance of exceedence of 1.5.

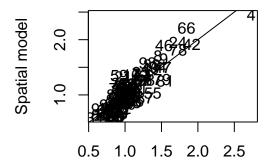
spplot(nc.sids2, "RRind1")



Both summaries show differences with the iid only model, with changes in the obvious direction. The spatial smoothing model gives a larger collection in the north-east, for example, and a single area in the west is not highlighted. As we will see later, the spatial model is supported by the data in this example.

# Disease Mapping: Comparison of posterior mean RRs

```
plot(nc.sids2$RRpmean1 ~ nc.sids2$RRpmean0,
    type = "n", xlab = "Non-spatial model",
    ylab = "Spatial model")
text(nc.sids2$RRpmean1 ~ nc.sids2$RRpmean0)
abline(0, 1)
```



Non-spatial model

We now examine the variances of the spatial and non-spatial random effects.

Recall that the ICAR model variance has a conditional interpretation.

To obtain a rough estimate of the marginal variance we obtain the posterior median of the  $S_i$ 's and evaluate their variance.

From the output below, we conclude that the spatial random effects dominate for the SIDS data so that we conclude there is clustering of cases in neighboring areas.

# Proportion of variation that is spatial

```
nareas <- 100
mat.marg \leftarrow matrix(NA, nrow = nareas, ncol = 1000)
m <- m1$marginals.random$ID2</pre>
for (i in 1:nareas) {
    Sre <- m[[i]]
    mat.marg[i, ] <- inla.rmarginal(1000, Sre)</pre>
}
var.Sre <- apply(mat.marg, 2, var)</pre>
var.eps <- inla.rmarginal(1000, inla.tmarginal(function(x))</pre>
    m1$marginals.hyper$"Precision for ID"))
mean(var.Sre)
## [1] 0.1989025
mean(var.eps)
## [1] 0.0001426175
perc.var.Sre <- mean(var.Sre/(var.Sre + var.eps))</pre>
perc.var.Sre
## [1] 0.9992725
```

We evaluate Moran's test for spatial autocorrelation using the "W" style weight function: this standardizes the weights so that for each area the weights sum to 1. Also define the "B" style for later.

To obtain a variable with approximately constant variance we form residuals from an intercept only model.

```
library(spdep)
# Note the nc.sids loaded from the data() command
# is in a different order to that obtained from the
# shapefile
data(nc.sids)
col.W <- nb2listw(ncCR85.nb, style = "W", zero.policy = TRUE)
col.B <- nb2listw(ncCR85.nb, style = "B", zero.policy = TRUE)
rm(nc.sids)
quasipmod <- glm(SID74 ~ 1, offset = log(EXP74), data = nc.sids2,
    family = quasipoisson())
sidsres <- residuals(quasipmod, type = "pearson")</pre>
```

```
moran.test(sidsres, col.W)
##
##
   Moran I test under randomisation
##
## data: sidsres
## weights: col.W
##
## Moran I statistic standard deviate = 2.4351, p-value = 0.0074
## alternative hypothesis: greater
## sample estimates:
## Moran I statistic
                                             Variance
                       Expectation
## 0.147531140
                        -0.010101010
                                         0.004190361
```

This analysis suggests significant clustering.

Moran's test may suggest spatial autocorrelation if there exists a non-constant mean function.

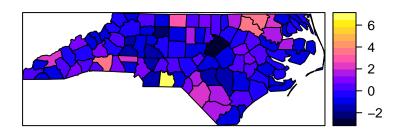
Hence, we should endeavor to remove the large-scale trends.

Below we fit a model with Eastings and Northings (of the County seat) as covariates – both show some at least some association.

```
quasipmod2 <- glm(SID74 ~ east + north, offset = log(EXP74),
   data = nc.sids2, family = quasipoisson())
summary(quasipmod2)
##
## Call:
## glm(formula = SID74 ~ east + north, family = quasipoisson(),
##
      data = nc.sids2, offset = log(EXP74))
##
## Deviance Residuals:
##
      Min
              10 Median 30
                                        Max
## -2.7961 -1.0249 -0.3475 0.6043 4.7261
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -0.2465437 0.2680159 -0.920 0.35992
## east 0.0020105 0.0006469 3.108 0.00247 **
## north -0.0028032 0.0014545 -1.927 0.05687.
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
  (Dispersion parameter for quasipoisson family taken to be 2.039456)
##
##
      Null deviance: 203.34 on 99 degrees of freedom
## Residual deviance: 171.80 on 97 degrees of freedom
## ATC: NA
##
```

We map the residuals to get a visual on the clustering.

```
sidsres2 <- residuals(quasipmod2, type = "pearson")
nc.sids2$res <- sidsres2
par(mar = c(0.1, 0.1, 0.1, 0.1))
spplot(nc.sids2, "res")</pre>
```



The significance of the Moran statistic is reduced, though still significant if judged by conventional levels.

```
moran.test(sidsres2, col.W)
##
##
   Moran I test under randomisation
##
## data: sidsres2
## weights: col.W
##
## Moran I statistic standard deviate = 2.1328, p-value = 0.0164
## alternative hypothesis: greater
## sample estimates:
## Moran I statistic
                         Expectation
                                              Variance
       0.127428361
                         -0.010101010
##
                                          0.004157993
```

## Neighborhood options

There are various coding schemes for the weights.

B has 0/1 corresponding to non-neighbor/neighbor – this means areas with many neighbors are more influential.

W has rows standardized by the number of neighbors so that the sum for each row (area) is unity.

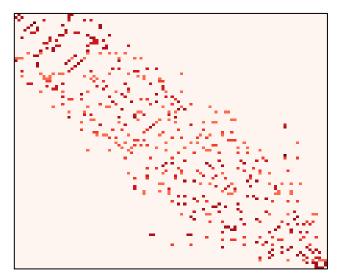
Weights can be more complex, depending on inverse distance, for example. See Bivand et al. (2013, Section 9.2).

#### Neighborhood options

```
library(RColorBrewer)
pal <- brewer.pal(9, "Reds")</pre>
z <- t(listw2mat(col.W))</pre>
brks \leftarrow c(0, 0.1, 0.143, 0.167, 0.2, 0.5, 1)
nbr3 <- length(brks) - 3
image(1:100, 1:100, z[, ncol(z):1], breaks = brks,
    col = pal[c(1, (9 - nbr3):9)], main = "W style",
    axes = FALSE)
box()
z <- t(listw2mat(col.B))</pre>
brks \leftarrow c(0, 0.1, 0.143, 0.167, 0.2, 0.5, 1)
nbr3 <- length(brks) - 3
image(1:100, 1:100, z[, ncol(z):1], breaks = brks,
    col = pal[c(1, (9 - nbr3):9)], main = "B style",
    axes = FALSE)
box()
```

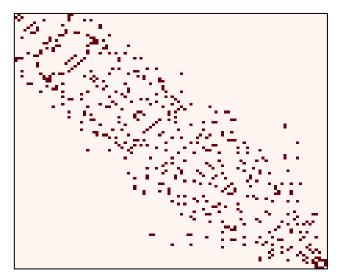
# W style

1:100



# B style





## Moran's I with a different neighborhood structure

We now use Moran's statistic on the detrended residuals, but with the binary "B" weight option. This option has unstandardized weights.

The conclusion, evidence of spatial autocorrelation, is the same as with the standardized weights option.

```
moran.test(sidsres2, col.B)
##
   Moran I test under randomisation
##
##
## data: sidsres2
## weights: col.B
##
## Moran I statistic standard deviate = 2.2357, p-value = 0.0126
## alternative hypothesis: greater
## sample estimates:
## Moran I statistic
                         Expectation
                                              Variance
                         -0.010101010
        0.125344196
                                          0.003670354
##
```

#### Clustering via Geary's c

We now use Geary's statistic on the detrended residuals, and come to the same conclusion

```
geary.test(sidsres2, col.W)
##
   Geary C test under randomisation
##
##
## data: sidsres2
## weights: col.W
##
## Geary C statistic standard deviate = 2.3479, p-value = 0.0094
## alternative hypothesis: Expectation greater than statistic
## sample estimates:
## Geary C statistic
                        Expectation
                                               Variance
##
          0.8195420
                            1.0000000
                                              0.0059072
```

## North Carolina SIDS Data: Clustering Conclusions

The disease mapping model shows that almost all of the residual variation is spatial.

Both of the Moran's *I* and Geary's *c* methods suggest that there is evidence of clustering in these data.

So all methods in agreement!

#### Cluster detection

We now turn to cluster detection: detecting areas or contiguous collections of areas that appear to be at elevated risk.

We concentrate on the SatScan method but for illustration show the methods of Openshaw and Besag and Newell in action.

The results from these two methods are difficult to interpret formally (Openshaw's in particular) because of the multiple testing problem.

# Cluster detection: Openshaw

We implement Openshaw's method using the centroids of the areas in data.

Circles of radius 30 are used and the centers are placed on a grid of size 10.

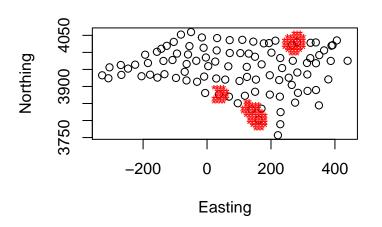
For multiple radii, multiple calls are required.

The significance level for calling a cluster is 0.002.

```
library(spdep)
data(nc.sids)
sids <- data.frame(Observed = nc.sids$SID74)
sids <- cbind(sids, Expected = nc.sids$BIR74 * sum(nc.sids$SID74)/sum(nc.sids$B
sids <- cbind(sids, x = nc.sids$x, y = nc.sids$y)
# GAM
library(DCluster)
sidsgam <- opgam(data = sids, radius = 30, step = 10,
    alpha = 0.002)
names(sidsgam)
## [1] "x"
                   "v"
                               "statistic" "cluster"
                                                        "pvalue"
                                                                    "size"
dim(sidsgam)
## [1] 106 6
```

#### Cluster detection: Openshaw

```
plot(sids$x, sids$y, xlab = "Easting", ylab = "Northing")
# Plot centroids of locations flagged as clusters
points(sidsgam$x, sidsgam$y, col = "red", pch = "*")
```



#### Clustering via Openshaw

A subset of the 106 signinificant Openshaw results.

```
head(sidsgam, n = 15)
##
                y statistic cluster pvalue size
## 1
     151.96 3776.92
                       15
                               1 1.743356e-03
                       15
## 2
     161.96 3776.92
                               1 1.743356e-03
## 3 171.96 3776.92 15
                               1 1.743356e-03
    141.96 3786.92 15
                               1 1.743356e-03
##
                  15
                               1 1.743356e-03
## 5
    151.96 3786.92
##
    161.96 3786.92 15
                               1 1.743356e-03
    171.96 3786.92
                       15
                               1 1.743356e-03
## 7
## 8 181.96 3786.92
                       15
                               1 1.743356e-03
     131.96 3796.92
                       15
                               1 1.743356e-03
##
## 10 141.96 3796.92
                       15
                               1 1.743356e-03
  11 151.96 3796.92
                       15
                               1 1.743356e-03
  12 161.96 3796.92
                       15
                               1 1.743356e-03
## 13 171.96 3796.92
                       15
                               1 1.743356e-03
  14 181.96 3796.92
                       15
                               1 1.743356e-03
## 15 131.96 3806.92
                       46
                               1 5.531787e-06
```

We illustrate the Besag and Newell method using k=20 as the cluster size.

```
library(maptools)
library(maps)
library(ggplot2)
library(sp)
nc.sids <- readShapePoly(system.file("etc/shapes/sids.shp",</pre>
    package = "spdep")[1], ID = "FIPSNO",
    proj4string = CRS("+proj=longlat +ellps=clrk66"))
referencep <- sum(nc.sids$SID74)/sum(nc.sids$BIR74)</pre>
population <- nc.sids$BIR74
cases <- nc.sids$SID74
E <- nc.sids$BIR74 * referencep
SMR <- cases/E
n <- length(cases)</pre>
```

We need to form a matrix containing the centroids.

```
getLabelPoint <- function(county) {</pre>
    Polygon(county[c("long", "lat")])@labpt
df <- map_data("county", "north carolina") # NC region county d
centNC <- by(df, df$subregion, getLabelPoint) # Returns list</pre>
centNC <- do.call("rbind.data.frame", centNC) # Convert to Data</pre>
names(centNC) <- c("long", "lat") # Appropriate Header</pre>
centroids <- matrix(0, nrow = n, ncol = 2)</pre>
for (i in 1:n) {
    centroids[i, ] <- c(centNC$lat[i], centNC$long[i])</pre>
}
colnames(centroids) <- c("x", "y")</pre>
rownames(centroids) <- 1:n</pre>
```

# SpatialEpi package

The most recent version of SpatialEpi can be installed from github (see below for the relevant command).

Otherwise the usual install.packages("SpatialEpi") should be used.

```
# devtools::install_github('rudeboybert/SpatialEpi')
library(SpatialEpi)
k < -20
alpha.level <- 0.01
geo <- centroids
BNresults <- besag_newell(geo, population, cases, expected.cases = NULL,
    k, alpha.level)
BNsig <- length(BNresults$p.values[BNresults$p.values <
    alpha.level])
cat("No of sig results = ", BNsig, "\n")
## No of sig results = 11
resmat <- matrix(NA, nrow = BNsig, ncol = 100)
reslen <- NULL
for (i in 1:length(BNresults$clusters)) {
    reslen[i] <- length(BNresults$clusters[[i]]$location.IDs.included)</pre>
    resmat[i, 1:reslen[i]] <- BNresults$clusters[[i]]$location.IDs.included
}
```

Now we set up the polygons for plotting the results.

```
NCTemp <- map("county", "north carolina",
    fill = TRUE, plot = FALSE)
NCIDs <- substr(NCTemp$names, 1 + nchar("north carolina,"),
    nchar(NCTemp$names))
NC <- map2SpatialPolygons(NCTemp, IDs = NCIDs,
    proj4string = CRS("+proj=longlat"))
# Fix currituck county which is 3 islands
index <- match(c("currituck:knotts", "currituck:main",
    "currituck:spit"), NCIDs)
currituck <- list()
for (i in c(27:29)) currituck <- c(currituck,
    list(Polygon(NC@polygons[[i]]@Polygons[[i]]@coords)))
currituck <- Polygons(currituck, ID = "currituck")</pre>
```

```
# SANITY CHECK: Reorder Spatial Polygons of list to
# match order of county
names <- rep("", 100)
for (i in 1:length(NC@polygons)) names[i] <- NC@polygons[[i]]@ID</pre>
identical(names, NCIDs)
## [1] FALSE
index <- match(NCIDs, names)</pre>
NC@polygons <- NC@polygons[index]</pre>
rm(index)
names <- rep("", 100)
for (i in 1:length(NC@polygons)) names[i] <- NC@polygons[[i]]@ID</pre>
identical(names, NCIDs)
## [1] TRUE
```

















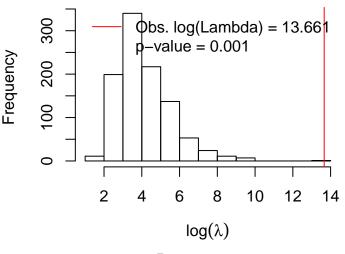






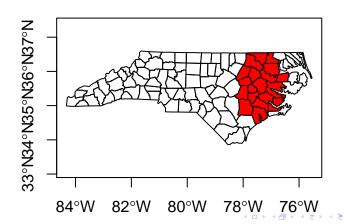
We now turn to SatScan and set 20% as the upper bound on the proportion of the population to be contained in any one potential cluster.

### Monte Carlo Distribution of Lambda



```
plot(NC.new, axes = TRUE)
plot(NC.new[Kcluster], add = TRUE, col = "red")
title("Most Likely Cluster")
```

# **Most Likely Cluster**

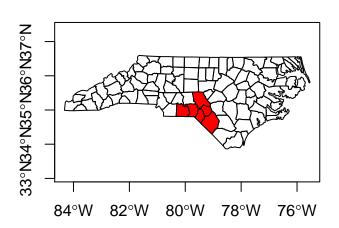


Now look at secondary clusters.

Two are significant, and indicated in the figures below,

```
K2cluster <- Kpoisson$secondary.clusters[[1]]$location.IDs.inclu
plot(NC.new, axes = TRUE)
plot(NC.new[K2cluster], add = TRUE, col = "red")
title("Second Most Likely Cluster")</pre>
```

# **Second Most Likely Cluster**

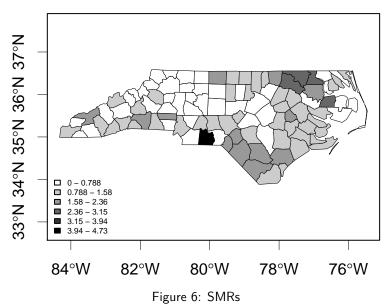


```
# Load NC map and obtain geographic
# centroids
library(maptools)
sp.obj <- readShapePoly(system.file("etc/shapes/sids.shp",
    package = "spdep")[1], ID = "FIPSNO",
    proj4string = CRS("+proj=longlat +ellps=clrk66"))
centroids <- latlong2grid(coordinates(sp.obj))</pre>
```

## Bayes cluster model: running the MCMC algorithm

```
y <- sp.obj$SID74
population <- sp.obj$BIR74
E <- expected(population, y, 1)
max.prop <- 0.2
k < -5e - 05
shape <- c(2976.3, 2.31)
rate \leftarrow c(2977.3, 1.31)
J <- 7
pi0 < -0.95
n.sim.lambda <- 0.5 * 10^4
n.sim.prior <- 0.5 * 10^4
n.sim.post <- 0.5 * 10^5
output <- bayes_cluster(y, E, population, sp.obj, centroids,
    max.prop, shape, rate, J, pi0, n.sim.lambda, n.sim.prior,
    n.sim.post)
## [1] "Algorithm started on: Sun Jul 3 10:42:37 2016"
## [1] "Importance sampling of lambda complete on: Sun Jul 3 10:42:41 2016"
## [1] "Prior map MCMC complete on: Sun Jul 3 10:42:43 2016"
## [1] "Posterior estimation complete on: Sun Jul 3 10:44:25 2016"
```

```
SMR <- y/E
plotmap(SMR, sp.obj, nclr = 6, location = "bottomleft",
    leg.cex = 0.5)
plotmap(output$prior.map$high.area, sp.obj, nclr = 6,
    location = "bottomleft", leg.cex = 0.5)
plotmap(output$post.map$high.area, sp.obj, nclr = 6,
    location = "bottomleft", leg.cex = 0.5)
barplot(output$pk.y, names.arg = 0:J, xlab = "k", ylab = "P(k|y)
plotmap(output$post.map$RR.est.area, sp.obj, log = TRUE,
    nclr = 6, location = "bottomleft", leg.cex = 0.5)</pre>
```



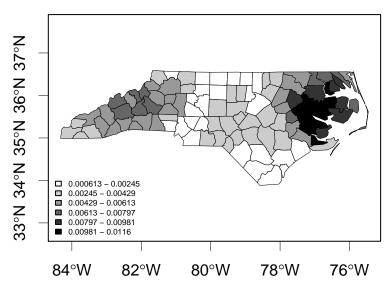


Figure 7: Prior probabilities of lying in a cluster

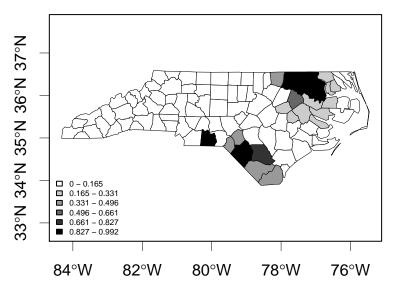


Figure 8: Posterior probability of a cluster

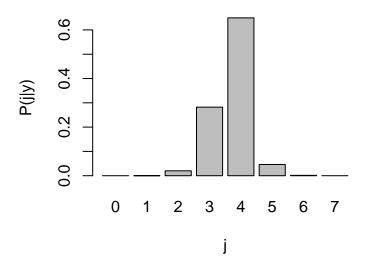


Figure 9: Posterior on the number of clusters

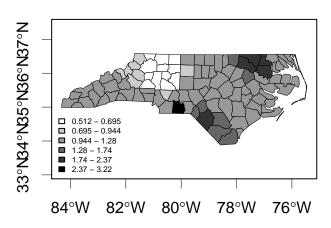


Figure 10: Posterior relative risk estimates