Univariate nonlinear discrete-time models

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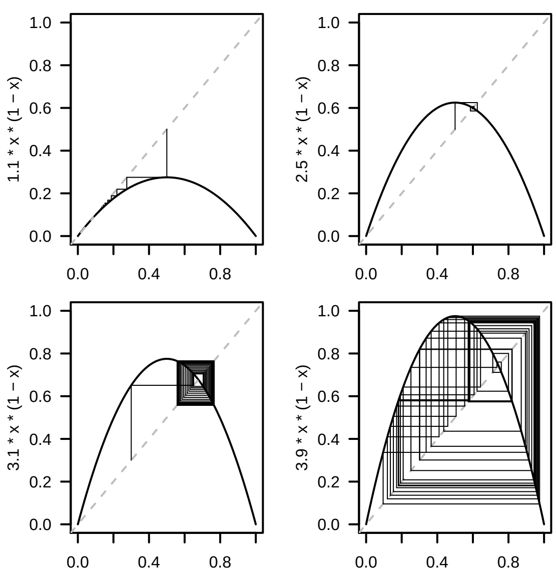
Logistic model

Impose bounds on an otherwise ridiculous growth process. Begin with the geometric difference equation, $N(t+1) - N(t) = RN(t)$. Set R equal to a decreasing linear function of $N(t)$ with x-intercept, $N_{\max}$, and y-intercept $R_{\max}$. This yields the logistic difference equation, $N(t+1) - N(t) = R_{\max} N(1 - N(t)/N_{\max})$; can set $N_{\max} = 1$ (*non-dimensionalization*). Fixed points: $N(t+1) - N(t) = 0 = RN*(1 - N*/N_{\max})$ has two solutions, $N* = 0$ and $N* = K$.

Stability

The geometric recurrsion, $N(t+1) = f(N(t)) = RN(t)$, is stable at the fixed point $N* = 0$, whenever $|R| < 1$. For general scalar function, f, and fixed point $N*$, this criterion becomes $|f'(N*)| < 1$, where $f'(N)$ is the first derivative of f with respect to N. Note that this is a true generalization because $f'(N) = R$ for the geometric model.

The derivative of the function defining the logistic recurrsion, $f(N) = N + R_{\max} N(1 - N/N_{\max})$, is $f'(N) =$



Alternative parameterizations

An ecologist or other normal person might choose to parameterize the discrete logistic model as above. A mathematician would choose $x(t+1) = Rx(1-x)$. The mathematician has chosen $R = r/K \rightarrow K = 1 - 1/R$. Mathematically equivalent parameterizations often have quite different meanings (or statistical properties), as well as cultural connotations. Get used to it.

More nonlinear models

Other 1-D discrete nonlinear models: *Ricker* model ($N = rNe^{-bN}$); population genetics; approximations of continuous models. Epidemic models (SI) (equivalent to discrete logistic).

$$\begin{aligned} S(t+1) =& m(N-S) - bSI + gI \\ =& m(N-S) - bS(N-S) + g(N-S) \\ =& m(1-S) - bS(1-S) + g(1-S) \\ =& mI - bI(1-I) + gI \\ =& (m+g-b)I + bI^2 \end{aligned} \tag{1}$$

$$\begin{aligned} N(t+1) =& N + rN(1 - N/K) \\ =& (1+r)N - (r/K)N^2 \\ =& (1+r)N - rN^2 \end{aligned} \tag{2}$$

Graphical approaches, continued: *Allee effects*. Bistability, multiple stable states.