Univariate linear (and affine) continuous-time deterministic models

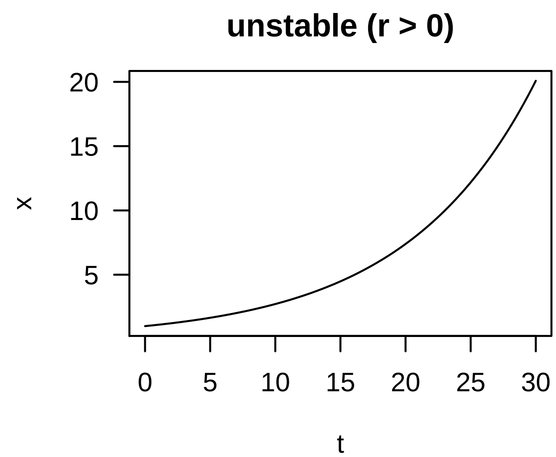
Basic model: $\frac{dx(t)}{dt} = rx(t)$, where r is constant. Can be thought of as the limit of the discrete-time linear model as the time step gets smaller, $\frac{dx}{dt} = \lim_{h\to 0} \frac{x(t+h)-x(t)}{h} = \lim_{h\to 0} \frac{R(h)x(t)}{h} = \left[\lim_{h\to 0} \frac{R(h)}{h}\right] x(t) = rx(t)$. When does it make sense to let the time-step go to zero?

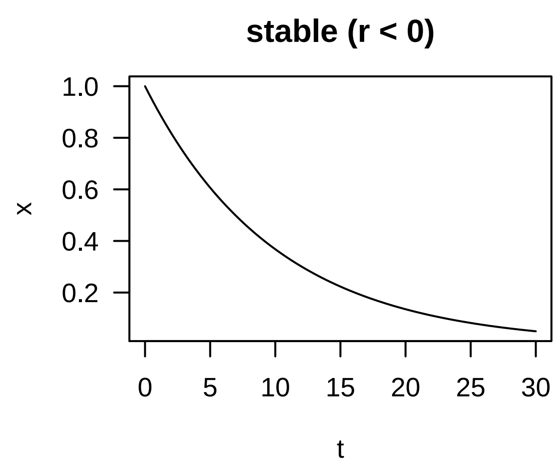
Fixed points and stability

Set $\frac{dx}{dt} = 0 = rx\star$. If $r \neq 0$, $x\star = 0$, else any $x\star$ is a fixed point. Note that this is just like the discrete case. However, the stability properties of $x\star = 0$ differ from the discrete case. In particular, $x\star = 0$ is stable if $r < 0$. The logic is that when $x(t) < 0$, x will go towards zero because $\frac{dx}{dt} > 0$; and when $x(t) > 0$, x will also go towards zero because $\frac{dx}{dt} < 0$. How do these stability properties differ from the discrete case? Are oscillations possible? Overshooting?

Time-dependent solution

Solve by seperable variables. $\frac{dx}{dt} = rx$, $\frac{dx}{x} = rdt$, $\int_{x(0)}^{x(t)} \frac{dx}{x} = r\int_0^t dt$, $\log(x(t)) - \log(x(0)) = rt$, $x(t) = x(0)e^{rt}$.





Affine models

Leaky bucket model. $\frac{dx}{dt} = a - bx$. Fixed points $0 = a - bx\star$, $x\star = a/b$ assuming $b \neq 0$. Stability whenever $b > 0$. Can you find time-dependent solution? Hint: seperation of variables again.