# Dimensional analysis



## Dimensional consistency

Important dimensions: time [T], amount [N], temperature [Θ], length [L]. Rules for dimensional consistency:

1. Only quantities with the same dimensions can be added, subtracted, compared or equated.
2. Quantities with different dimensions can be multiplied or divided.

**Example: leaky bucket model**

Let $N$ be the state variable with dimensions of [N]. Let $h$ be the size of a time step with dimensions of [T]. Let $T$ be the residence time with dimensions of [T]. Let $a$ be the supply rate with dimensions $[\mathrm{NT}^{-1}]$. $\underbrace{\frac{N(t+h)}{h}}_{[\mathrm{NT}^{-1}]} - \underbrace{\frac{N(t)}{h}}_{[\mathrm{NT}^{-1}]} = \underbrace{a}_{[\mathrm{NT}^{-1}]} - \underbrace{\frac{1}{T}}_{[\mathrm{T}^{-1}]} \underbrace{N(t)}_{[\mathrm{N}]}$. This equation is dimensionally consistent. Why?

## Non-dimensionalization

Can help with interpretation. Three steps: (1) identify state and time variables, (2) divide these by quantities with the same units, and (3) substitute back into the original equation.

**Example: leaky bucket model**

Let $n(t) = \frac{N(t)}{aT} = \frac{N(t)}{N_\star}$ (ratio of state to the state at equilibrium) and $\tau = \frac{h}{T}$ (ratio of time step size to the residence time at equilibrium). Note that these are dimensionless quantities. Why? Show it. Substituting yields $\frac{n(t+1)-n(t)}{\tau} = 1 - n(t)$. Now there is only one parameter in the model, $\tau$, with a simple interpretation as the average number of time steps a molecule stays in the bucket. With large $\tau$, few time steps required to clear, small $\tau$, many time steps.