Quiz 1, STATS 401 W18

In lab on 10/5

This document produces different random quizzes each time the source code generating it is run. The actual quiz will be a realization generated by this random process, or something similar.

Instructions. You have a time allowance of 40 minutes, though the quiz may take you less time and you can leave lab once you are done. The quiz is closed book, and you are not allowed access to any notes. Any electronic devices in your possession must be turned off and remain in a bag on the floor.

Formulas

The following formulas will be provided. To use these formulas properly, you need to make appropriate definitions of the necessary quantities.

(1) $\mathbf{b} = \left(\mathbb{X}^{\mathsf{T}}\mathbb{X}\right)^{-1}\mathbb{X}^{\mathsf{T}}\mathbf{y}$

(2) $\mathrm{Var}(X) = \mathrm{E}\big[(X - \mathrm{E}[X])^2\big] = \mathrm{E}[X^2] - \big(\mathrm{E}[X]\big)^2$

(3) The probability density function of the standard normal distribution is $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$

(4) Syntax from `?pnorm`:

```
pnorm(q, mean = 0, sd = 1)
qnorm(p, mean = 0, sd = 1)
q: vector of quantiles.
p: vector of probabilities.
```

Q1. Matrix exercises

Q1-1.

(a). Evaluate $\mathbb{A}\mathbb{B}$ when

$$\mathbb{A} = \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ -1 & -2 \end{bmatrix}, \quad \mathbb{B} = \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}$$

Solution:

$$\mathbb{A}\mathbb{B} = \begin{bmatrix} 6 & 5 \\ 3 & 4 \\ -3 & -3 \end{bmatrix}$$

(b). For $\mathbb{A}$ as above, write down $\mathbb{A}^{\mathsf{T}}$.

Solution:

$$\mathbb{A}^\intercal = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 3 & -2 \end{bmatrix}$$

(c). For $\mathbb{B}$ as above, find $\mathbb{B}^{-1}$ if it exists. If $\mathbb{B}^{-1}$ doesn't exist, explain how you know this.

Solution:

$$\mathbb{B}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$$

Q1-2.

(a). Evaluate $\mathbb{A}\mathbb{B}$ when

$$\mathbb{A} = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 0 & 3 \end{bmatrix}, \quad \mathbb{B} = \begin{bmatrix} -1 & 1 & -2 \\ 0 & 0 & 0 \\ -2 & 3 & 0 \end{bmatrix}$$

Solution:

$$\mathbb{A}\mathbb{B} = \begin{bmatrix} -5 & 8 & 2 \\ -8 & 11 & -4 \end{bmatrix}$$

(b). For $\mathbb{A}$ as above, write down $\mathbb{A}^\intercal$.

Solution:

$$\mathbb{A}^\intercal = \begin{bmatrix} -1 & 2 \\ -1 & 0 \\ 3 & 3 \end{bmatrix}$$

(c). For $\mathbb{A}$ as above, find $\mathbb{A}^{-1}$ if it exists. If $\mathbb{B}^{-1}$ doesn't exist, explain how you know this.

Solution:

Only square matrices can be invertible. $\mathbb{A}$ is 2×3 and so cannot have an inverse.

Q2. Summation exercises

Q2-1.

Calculate $\sum_{i=k}^{k+3}(i+3)$, where k is a whole number. Your answer should depend on k.

Solution:

TBD

Q2-2.

Evaluate $\sum_{i=1}^{30} 10 - \sum_{i=10}^{20} 20$.

Solution:

TBD

Q2-3.

Calculate $\sum_{k=m}^{n} a$, where m and n are whole numbers and a is a real number.

Solution:

TBD

Q3. R exercises

Q3-1.

(a) Which of the following is the output of `matrix(c(rep(0,times=4),rep(1,times=4)),ncol=2)`

$$(a)\begin{bmatrix}0 & 1\\0 & 1\\0 & 1\\0 & 1\end{bmatrix};\quad (b)\begin{bmatrix}0 & 0\\0 & 0\\1 & 1\\1 & 1\end{bmatrix};\quad (c)\begin{bmatrix}0 & 0\\1 & 1\\0 & 0\\1 & 1\end{bmatrix};\quad (d)\begin{bmatrix}0 & 0 & 1 & 1\\0 & 0 & 1 & 1\end{bmatrix}$$

Solution:

TBD

(b) Suppose we define an R vector by `y <- c(3,NA,-1,4,NA,-2)`. What will `y[y>0]` give you?

(i). A vector of the positive elements and NA values of `y`.

(ii). A vector of the negative elements of `y`.

(iii). A vector of all `NA`s.

(iv). A vector of `TRUE`s and `FALSE`s.

(v). A vector of `TRUE`s and `FALSE`s and `NA`s.

Solution:

TBD

Q3-2.

(a) Which of the following code successfully construct the matrix $\mathbb{A} = \begin{bmatrix}1 & 1\\2 & 2\\3 & 3\end{bmatrix}$

(i). `A <- matrix(c(1,1,2,2,3,3) ,nrow=3)`

(ii). `A <- cbind(c(1,1),c(2,2),c(3,3))`

(iii). `A <- t(matrix(c(1,1,2,2,3,3) ,nrow=2))`

(iv). `A <- c(c(1:3),c(1:3))`

Solution:

TBD

(b) Suppose `X` is a matrix in R. Which of the following is NOT equivalent to `X`?

(a). `t(t(X))`

(b). `X %*% matrix(1,ncol(X)`

(c). `X*1`

(d). `X%*%diag(ncol(X))`

Solution:

TBD

Q3-3.

(a) Which of the following is the matrix $\mathbb{A}$ generated by

```
A <- t(matrix(c(rep(1,times=2),rep(3,times=2), 6, 4),ncol=3))
```

(i) $\mathbb{A} = \begin{bmatrix} 1 & 1 \\ 3 & 3 \\ 6 & 4 \end{bmatrix}$

(ii) $\mathbb{A} = \begin{bmatrix} 1 & 3 & 6 \\ 1 & 3 & 4 \end{bmatrix}$

(iii) $\mathbb{A} = \begin{bmatrix} 1 & 3 \\ 1 & 6 \\ 1 & 3 \end{bmatrix}$

(iv) $\mathbb{A} = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 6 & 4 \end{bmatrix}$

Solution:

TBD

(b) Which of the following successfully select the first five odd elements of the vector $x = c(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)$? (List all that apply. Do not list commands that will give an error)

(c) `x[rep(c(TRUE,FALSE),each=5)]`

(ii) `x[rep(c(TRUE,FALSE),times=5)]`

(iii) `x[rep(c(TRUE,FALSE),length=9)]`

(iv) `x[rep(c(TRUE,FALSE)][1:5]`

(v) `x[rep(c("TRUE","FALSE"),5)]`

(vi) None of the above

(vii) All of the above

Solution:

TBD

Q4. Fitting a linear model by least squares

Q4-1.

TBD

Solution:

TBD

Q5. Probability exercises

Q5-1.

TBD

Solution:

TBD
