

## 812 Section # 9

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Happy Halloween!

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### 1 Mean and Variance of a Random Variable

**Exercise 1** [Walpole et al. Ex 4.2] In a gambling game, a player tosses three fair coins. The player wins \$5 if all of the coins match (i.e. he gets three heads or three tails). If the three coins do not match, he has to pay \$3. What is his expected gain or loss? If  $X$  is the winnings, what is the variance of  $X$ ?

**Exercise 2** [Walpole et al. Ex 4.4] Assume that the number of cars  $X$  entering a carwash between 4 and 5 pm has the following probability distribution:

x:	4	5	6	7	8	9
$P(X = x)$ :	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$

- What is the expected number of cars that will enter the car wash?
- What is the variance of  $X$ ?
- Let's say the attendant is paid a commission based on the number of cars entering the car wash each hour. Specifically, he receives 2 dollars for every car that enters, but he has to pay 1 dollar to the manager (so he earns  $2X - 1$ ). What are his expected earnings over this time period?
- What is the variance of his payment?

**Exercise 3** Two random variables  $X$  and  $Y$  are distributed with the following probability distribution:

	Y=1	Y=2	Y=3
X=2	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{6}$
X=4	$\frac{1}{12}$	0	$\frac{1}{12}$
X=8	$\frac{1}{6}$	$\frac{1}{12}$	0

- What is  $E(X)$ ?
- What is  $E(Y)$ ?
- What is  $E(XY)$ ?
- What is  $Var(X)$ ?
- What is  $E\left(\frac{Y}{X}\right)$ ?
- What is  $E(X^2 + 3Y)$ ?
- Assume that  $X = 4Z - 2$ . What is  $E(Z)$ ?
- What is  $Cov[X, Y]$ ?

**Exercise 4** *BONUS!* [Casella and Berger 2.20] A couple decides to continue to have children until a daughter is born. Assume that the probability that a child is a girl is  $\frac{1}{2}$ . What is the expected number of children that the couple will have? (Hint: model this using the geometric distribution)

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