812 Section # 3
"Pie or Death"

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Conditional Probability: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Remember:

$$P(A \cap B) = P(A \mid B)P(B)$$

Bayes Rule: $P(A \mid B) = \frac{P(B|A)P(A)}{P(B)}$

Law of Total Probability: $P(A) = P((A \cap B) \cup (A \cap B)^C) = P(A \cap B) + P(A \cap B^C)$

Independence: Events A and B are independent iff ($\Leftrightarrow$) $P(A \cap B) = P(A)P(B)$

Equivalently, $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = P(A)$ or $P(B \mid A) = P(B)$.

Exercise 1

Refresher from last week: For the department picnic every year, it's very important that *someone* bring Oreos. It's also really important that we have chips and guacamole. These are the key ingredients to any successful picnic, particularly to keep our vegan friends happy. We have 38 faculty and 93 graduate students.

a. Suppose I assign 2 graduate students to bring Oreos, just in case one forgets. What's the probability that any given graduate student is chosen to bring Oreos?

b. Suppose that we conduct a lottery to see who has to bring guacamole, but we've already chosen our two Oreo-bringers. What's the probability that a given member of the department draws the guacamole ticket in the lottery?

Exercise 2

Let's assume we don't coordinate who brings what snacks to the picnic, but that there's some general probability of bringing Oreos (O) and some general probability of bringing chips and guacamole (G). Calculate the following given that $P(O) = \frac{2}{5}$, $P(G) = \frac{3}{8}$, and $P(O \cap G) = \frac{3}{20}$.

a. $P(O \mid G)$
b. $P(G \mid O)$
c. $P(O \mid O \cup G)$
d. $P(O \mid O \cap G)$
e. $P(O \cap G \mid O \cup G)$

Exercise 3

Easy as pie: We have surveyed a group of undergraduates and obtained their preferences over pie as well as where they're from:

	Northern US	Southern US
Apple	14	11
Sweet Potato	6	17
Cherry	16	8

If a person is picked at random from this group, find the probability that:

a. they are from the South given that they like sweet potato pie
b. they are from the North given that they like cherry pie
c. they like apple pie

* Fun fact courtesy of Crisco and the American Pie Council (yes that's a real thing): Nearly one out of five (19%) of Americans prefer apple pie, followed by pumpkin (13%), pecan (12%), banana cream (10%) and cherry (9%).

Exercise 4

Three prisoners A, B, and C are on death row. The governor decides to pardon one at random. She tells the prison warden her choice, but asks him not to inform the prisoners. Prisoner A asks the warden who will be pardoned, but the warden refuses to answer. Prisoner A tries again, but this time asks who will be executed. The warden thinks for a while, and then tells Prisoner A that B will be executed. The warden believes he has given A no additional information about the probability that A himself will be pardoned. Even so, A is happy because he believes that he now has a 50% chance of being pardoned. Who is right? Why? [from Casella and Berger 1.3.4]

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