

# 812 Section # 11

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## 1 Finishing off our example from class...

## 2 Method of Moments and Maximum Likelihood Estimation

**Exercise 1** For random variables  $X_1, X_2, \dots, X_n \stackrel{\text{iid}}{\sim} \text{Pois}(\lambda)$ , where  $\lambda > 0$ :

- What is the method-of-moments estimator of  $\lambda$  ( $\hat{\lambda}_{\text{MM}}$ )?
- Is the estimator  $\hat{\lambda}_{\text{MM}}$  unbiased? If not, find the bias.
- Is  $\hat{\lambda}_{\text{MM}}$  consistent?
- What is the likelihood function? The log likelihood?
- What is the maximum likelihood estimator of  $\lambda$  ( $\hat{\lambda}_{\text{MLE}}$ )?
- Is  $\hat{\lambda}_{\text{MLE}}$  unbiased? If not, find the bias.
- Is  $\hat{\lambda}_{\text{MLE}}$  consistent?

**Exercise 2** For random variables  $Y_1, Y_2, Y_3 \stackrel{\text{iid}}{\sim} \text{Pois}(\lambda)$ , consider the following estimators of  $\lambda$ :

$$\hat{\theta}_1 = Y_1$$

$$\hat{\theta}_2 = \frac{Y_1 + Y_2}{2}$$

$$\hat{\theta}_3 = \frac{Y_1 + 2Y_2}{3}$$

$$\hat{\theta}_4 = \min(Y_1, Y_2, Y_3)$$

$$\hat{\theta}_5 = \frac{Y_1 + Y_2 + Y_3}{3}$$

- Which of these estimators are unbiased?
- Which unbiased estimator has the smallest variance?

**Exercise 3** For random variables  $X_1, X_2, \dots, X_n \stackrel{\text{iid}}{\sim} \text{Bernoulli}(p)$ :

- What is the method-of-moments estimator of  $p$  ( $\hat{p}_{\text{MM}}$ )?
- Is the estimator  $\hat{p}_{\text{MM}}$  unbiased? Is it consistent?
- Tossing a coin 10 times and allowing Heads = 1 and Tails = 0, we observe the following outcomes:

0 1 1 0 1 0 1 1 1 0

What is the moment estimate for  $p$  (the probability of a “success”—in this case, Heads)?

**Exercise 4** For random variables  $X_1, X_2, \dots, X_n \stackrel{\text{iid}}{\sim} \text{Gamma}(\alpha, \beta)$ :

- What are the method of moments estimators for  $\alpha$  and  $\beta$ ?

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