Instrumental Variable Estimation 1: Framework

Instructor: Yuta Toyama

Last updated: 2020-03-30

Section 1

Introduction

Introduction: Endogeneity Problem and its Solution

- When $Cov(x_k, \epsilon) = 0$ does not hold, we have **endogeneity problem**
  - We call such x_k an **endogenous variable**.
- In this chapter, I introduce an **instrumental variable** estimation method, a solution to this issue.
- The lecture plan
  1. More on endogeneity issues
  2. Framework
  3. Implementation in R
  4. Examples

Section 2

Endogeneity

Examples of Endogeneity Problem

- Here, I explain a bit more about endogeneity problems.
  1. Omitted variable bias
  2. Measurement error
  3. Simultaneity

More on Omitted Variable Bias

- Remember the wage regression equation (true model)

$$\begin{aligned} \log W_i = & \ \beta_0 + \beta_1 S_i + \beta_2 A_i + u_i \\ E[u_i|S_i, A_i] = & \ 0 \end{aligned}$$

  where W_i is wage, S_i is the years of schooling, and A_i is the ability.
- Suppose that you omit A_i and run the following regression instead.

$$\log W_i = \alpha_0 + \alpha_1 S_i + v_i$$

  Notice that $v_i = \beta_2 A_i + u_i$, so that S_i and v_i is likely to be correlated.
- You might want to add more and more additional variables to capture the effect of ability.
  - Test scores, GPA, SAT scores, etc...
- However, can you make sure that S_i is indeed exogenous after adding many control variables?
- Multivariate regression cannot deal with the presence of **unobserved heterogeneity** that matters both in wage and years of schooling.

Measurement error

- Measurement error in variables
  - Reporting error, respondent does not understand the question, etc...
- Consider the regression

$$y_i = \beta_0 + \beta_1 x_i^* + \epsilon_i$$

- Here, we only observe x_i with error:

$$x_i = x_i^* + e_i$$

where e_i is measurement error.
  - e_i is independent from ϵ_i and x_i^* (called classical measurement error)
  - You can think e_i as a noise added to the data.
- The regression equation is

$$\begin{aligned} y_i &= \beta_0 + \beta_1(x_i - e_i) + \epsilon_i \\ &= \beta_0 + \beta_1 x_i + (\epsilon_i - \beta_1 e_i) \end{aligned}$$

- Then we have correlation between x_i and the error $\epsilon_i - \beta_1 e_i$, violating the mean independence assumption

Simultaneity (or reverse causality)

- Dependent variable and explanatory variable (endogenous variable) are determined simultaneously.
- Consider the demand and supply curve

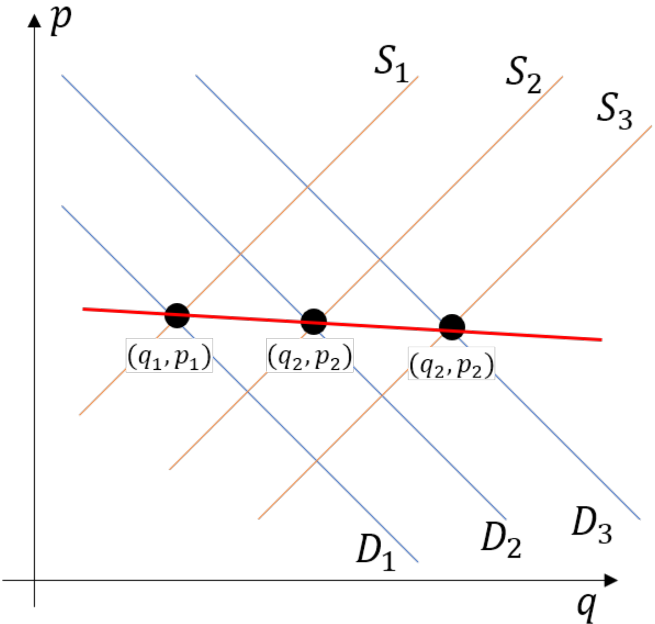
$$q^d = \beta_0^d + \beta_1^d p + \beta_2^d x + u^d$$
$$q^s = \beta_0^s + \beta_1^s p + \beta_2^s z + u^s$$

- The equilibrium price and quantity are determined by $q^d = q^s$.
- In this case,

$$p = \frac{(\beta_2^s z - \beta_2^d z) + (\beta_0^s - \beta_0^d) + (u^s - u^d)}{\beta_1^d - \beta_1^s}$$

  implying the correlation between the price and the error term.
- Putting this differently, the data points we observed is the intersection of these supply and demand curves.

- How can we distinguish demand and supply?



Section 3

IV Idea

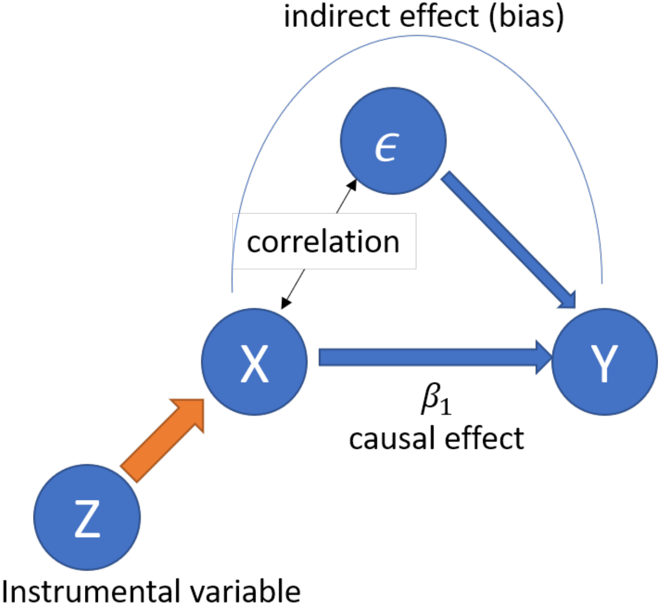
Idea of IV Regression

- Let's start with a simple case.

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,$$

  and $Cov(x_i, \epsilon_i) \neq 0$.
- Now, we consider another variable z_i, which we call **instrumental variable (IV)**.
- Instrumental variable z_i should satisfies the following two conditions:
  1. **Independence**: $Cov(z_i, \epsilon_i) = 0$. No correlation between IV and error.
  2. **Relevance**: $Cov(z_i, x_i) \neq 0$. There should be correlation between IV and endogenous variable x_i.
- Idea: Use the variation of x_i **induced by instrument** z_i to estimate the direct (causal) effect of x_i on y_i, that is β_1!.

- More on this:
  1. Intuitively, the OLS estimator captures the correlation between x and y.
  2. If there is no correlation between x and ϵ, it captures the causal effect β_1.
  3. If not, the OLS estimator captures both direct and indirect effect, the latter of which is bias.
  4. Now, let's capture the variation of x due to instrument z,
     - Such a variation should exist under **relevance** assumption.
     - Such a variation should not be correlated with the error under **independence assumption**
  5. By looking at the correlation between such variation and y, you can get the causal effect β_1.


indirect effect (bias)
ϵ
correlation
X
Y
β_1
causal effect
Z
Instrumental variable

Section 4

IV framework

Model

- We now introduce a general framework with multiple endogenous variables and multiple instruments.
- Consider the model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_K X_{Ki} + \beta_{K+1} W_{1i} + \cdots + \beta_{K+R} W_{Ri} + u_i,$$

  with $i = 1, \ldots, n$ is the general instrumental variables regression model where
  - Y_i is the dependent variable
  - $\beta_0, \ldots, \beta_{K+R}$ are $1 + K + R$ unknown regression coefficients
  - $X_{1i}, \ldots, X_{Ki}$ are K endogenous regressors: $Cov(X_{ki}, u_i) \neq 0$ for all k.
  - $W_{1i}, \ldots, W_{Ri}$ are R exogenous regressors which are uncorrelated with u_i. $Cov(W_{ri}, u_i) = 0$ for all r.
  - u_i is the error term
  - $Z_{1i}, \ldots, Z_{Mi}$ are M instrumental variables
- I will discuss conditions for valid instruments later.

Estimation by Two Stage Least Squares (2SLS)

- We can estimate the above model by **Two Stage Least Squares (2SLS)**
- Step 1: **First-stage regression(s)**
  - Run an OLS regression for each of the endogenous variables $(X_{1i}, \ldots, X_{ki})$ on all instrumental variables $(Z_{1i}, \ldots, Z_{mi})$, all exogenous variables $(W_{1i}, \ldots, W_{ri})$ and an intercept.
  - Compute the fitted values $(\widehat{X}_{1i}, \ldots, \widehat{X}_{ki})$.
- Step 2: **Second-stage regression**
  - Regress the dependent variable Y_i on **the predicted values** of all endogenous regressors $(\widehat{X}_{1i}, \ldots, \widehat{X}_{ki})$, all exogenous variables $(W_{1i}, \ldots, W_{ri})$ and an intercept using OLS.
  - This gives $\widehat{\beta}_0^{TSLS}, \ldots, \widehat{\beta}_{k+r}^{TSLS}$, the 2SLS estimates of the model coefficients.

Intuition

- Why does this work? Let's go back to the simple example with 1 endogenous variable and 1 IV.
- In the first stage, we estimate

$$x_i = \pi_0 + \pi_1 z_i + v_i$$

  by OLS and obtain the fitted value $\widehat{x}_i = \widehat{\pi}_0 + \widehat{\pi}_1 z_i$.
- In the second stage, we estimate

$$y_i = \beta_0 + \beta_1 \widehat{x}_i + u_i$$

- Since $\widehat{x}_i$ depends only on z_i, which is uncorrelated with u_i, the second stage can estimate β_1 without bias.
- Can you see the importance of both independence and relevance asssumption here? (More formal discussion later)

Section 5

Conditions for IV

Conditions for Valid IVs: Necessary condition

- Depending on the number of IVs, we have three cases
  1. Over-identification: $M > K$
  2. Just identification] $M = K$
  3. Under-identification $M < K$
- The necessary condition is $M \geq K$.
  - We should have more IVs than endogenous variables!!

Sufficient condition

- How about sufficiency?
- In a general framework, the sufficient condition for valid instruments is given as follows.
  1. **Independence**: $Cov(Z_{mi}, \epsilon_i) = 0$ for all m.
  2. **Relevance**: In the second stage regression, the variables

     $$\left(\widehat{X}_{1i}, \ldots, \widehat{X}_{ki}, W_{1i}, \ldots, W_{ri}, 1\right)$$

     are not perfectly multicollinear.
- What does the relevance condition mean?

Relevance condition

- In the simple example above, The first stage is

$$x_i = \pi_0 + \pi_1 z_i + v_i$$

and the second stage is

$$y_i = \beta_0 + \beta_1 \widehat{x}_i + u_i$$

- The second stage would have perfect multicollinarity if $\pi_1 = 0$ (i.e., $\widehat{x}_i = \pi_0$).
- Back to the general case, the first stage for X_k can be written as

$$X_{ki} = \pi_0 + \pi_1 Z_{1i} + \cdots + \pi_M Z_{Mi} + \pi_{M+1} W_{1i} + \cdots + \pi_{M+R} W_{Ri}$$

and one of $\pi_1, \cdots, \pi_M$ should be non-zero.
- Intuitively speaking, **the instruments should be correlated with endogenous variables after controlling for exogenous variables**

Check Instrument Validity: Relevance

- Instruments are **weak** if those instruments explain little variation in the endogenous variables.
- Weak instruments lead to
  1. imprecise estimates (higher standard errors)
  2. The asymptotic distribution would deviate from a normal distribution even if we have a large sample.
- Here is a rule of thumb to check the relevance conditions.

- Consider the case with one endogenous variable X_{1i}.
- The first stage regression

$$X_k = \pi_0 + \pi_1 Z_{1i} + \cdots + \pi_M Z_{Mi} + \pi_{M+1} W_{1i} + \cdots + \pi_{M+R} W_{Ri}$$

- And test the null hypothesis

$$H_0 : \pi_1 = \cdots = \pi_M = 0$$
$$H_1 : \textit{otherwise}$$

  - This is F test (test of joint hypothesis)
- If we can reject this, we can say no concern for weak instruments.
- A rule of thumbs is that the F statistic should be larger than 10.

Independence (Instrument exogeneity)

- Arguing for independence is hard and a key in empirical analysis.
- Justification of this assumption depends on a context, institutional features, etc. . .
- We will see this through examples in the next chapter.