# Section 2: Estimation, Confidence Intervals and Testing Hypothesis

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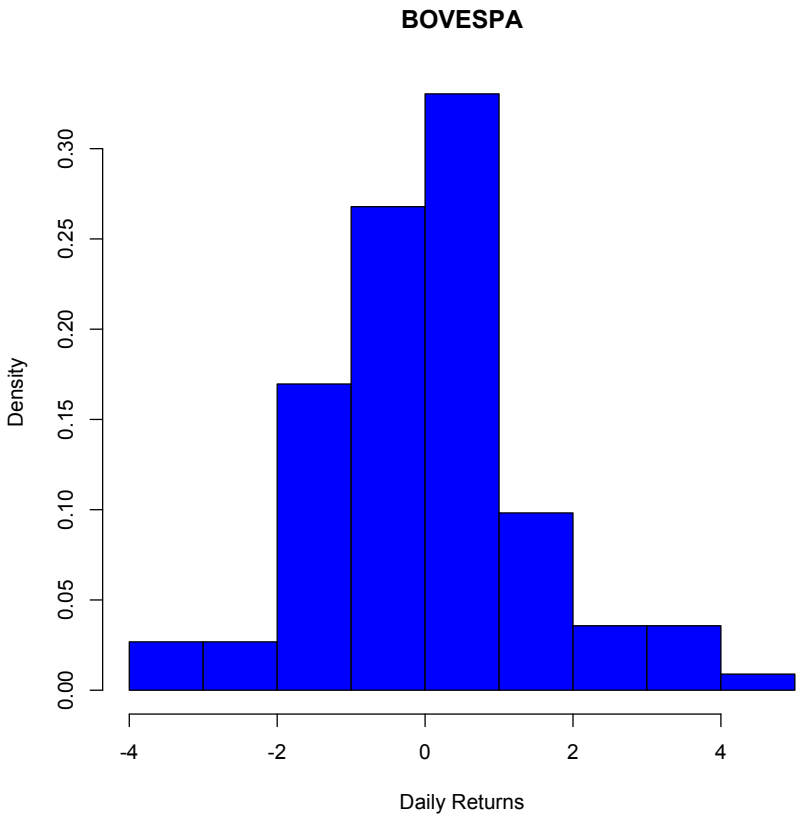
`https://tyliang.github.io/BUS41000/`

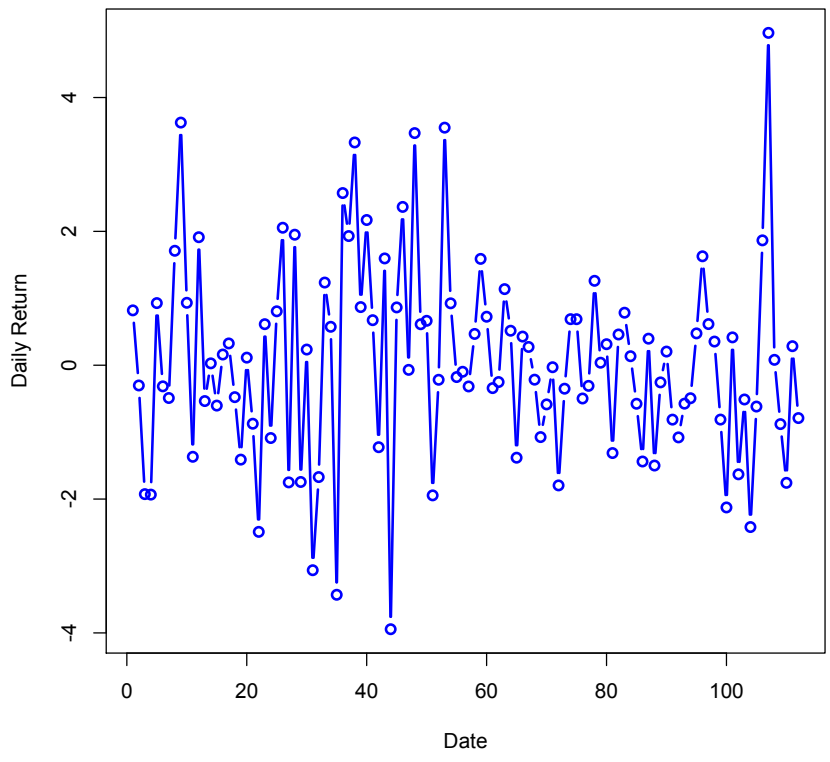
Suggested Reading:
Naked Statistics, Chapters 7, 8, 9 and 10
OpenIntro Statistics, Chapters 4, 5 and 6

# A First Modeling Exercise

- I have US$ 1,000 invested in the Brazilian stock index, the IBOVESPA. I need to predict tomorrow's value of my portfolio.
- I also want to know how risky my portfolio is, in particular, I want to know how likely am I to lose more than 3% of my money by the end of tomorrow's trading session.
- What should I do?

# IBOVESPA - Data



As a first modeling decision, let's call the random variable associated with daily returns on the IBOVESPA $X$ and assume that returns are independent and identically distributed as

$$X \sim N(\mu, \sigma^2)$$

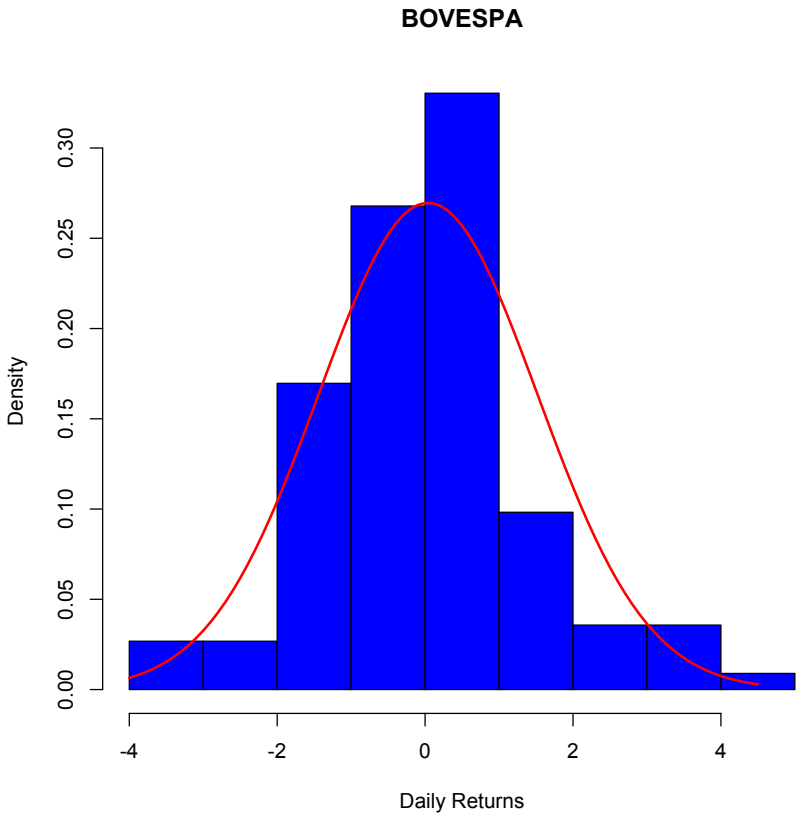
- Question: What are the values of $\mu$ and $\sigma^2$ ?
- We need to estimate these values from the sample in hands ($n$=113 observations)...

- Let's assume that each observation in the random sample $\{x_1, x_2, x_3, \ldots, x_n\}$ is independent and distributed according to the model above, i.e., $x_i \sim N(\mu, \sigma^2)$
- An usual strategy is to estimate $\mu$ and $\sigma^2$, the mean and the variance of the distribution, via the sample mean ($\bar{X}$) and the sample variance ($s^2$)... (their sample counterparts)

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left( x_i - \bar{x} \right)^2$$

For the IBOVESPA data in hands,




$\bar{X} = 0.04$ and $s^2 = 2.19$

- The red line represents our "model", i.e., the normal distribution with mean and variance given by the estimated quantities $\bar{X}$ and $s^2$.
- What is $Pr(X < -3)$?

# Annual Returns on the US market...

Assume I invest some money in the U.S. stock market. Your job is to tell me the following:

- what is my expected one year return?
- what is the standard deviation (volatility)?
- what is the probability my investment grow by 10%?

- What happens in 20 years if I invest \$1 today on the market?

# Building Portfolios

- Let's assume we are considering 3 investment opportunities
  1. IBM stocks
  2. ALCOA stocks
  3. Treasury Bonds (T-bill)
- How should we start thinking about this problem?

# Building Portfolios

Let's first learn about the characteristics of each option by assuming the following models:

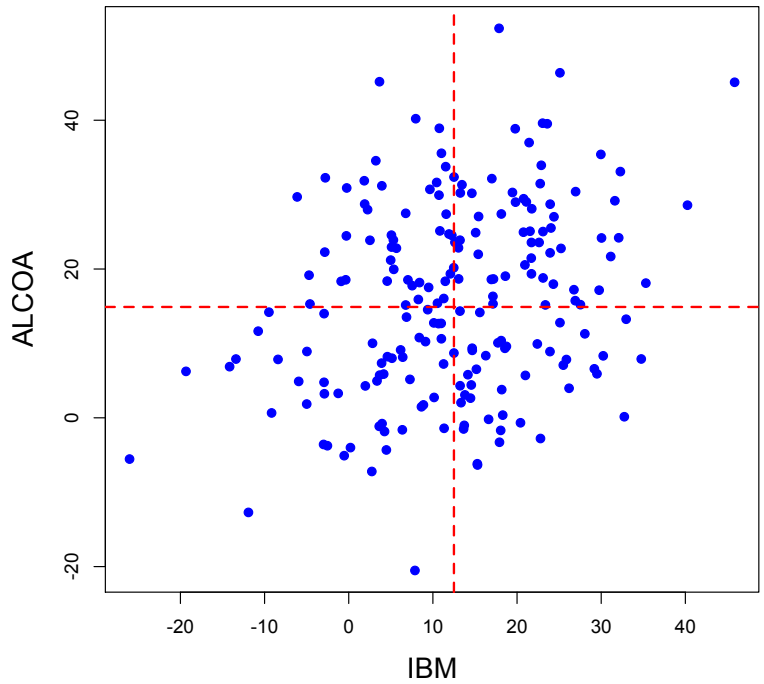
- $IBM \sim N(\mu_I, \sigma_I^2)$
- $ALCOA \sim N(\mu_A, \sigma_A^2)$

and

- The return on the T-bill is 3%

After observing some return data we can came up with estimates for the means and variances describing the behavior of these stocks

# Building Portfolios




| IBM | ALCOA | T-bill |
|---|---|---|
| $\hat{\mu}_I = 12.5$ | $\hat{\mu}_A = 14.9$ | $\mu_{Tbill} = 3$ |
| $\hat{\sigma}_I = 10.5$ | $\hat{\sigma}_A = 14.0$ | $\sigma_{Tbill} = 0$ |

$$corr(IBM, ALCOA) = 0.33$$

# Building Portfolios

- How about combining these options? Is that a good idea? Is it good to have all your eggs in the same basket? Why?
- What if I place half of my money in ALCOA and the other half on T-bills...

- Remember that:

$$\begin{aligned} E(aX + bY) &= aE(X) + bE(Y) \\ Var(aX + bY) &= a^2 Var(X) + b^2 Var(Y) + 2ab * Cov(X, Y) \end{aligned}$$

# Building Portfolios

- So, by using what we know about the means and variances we get to:

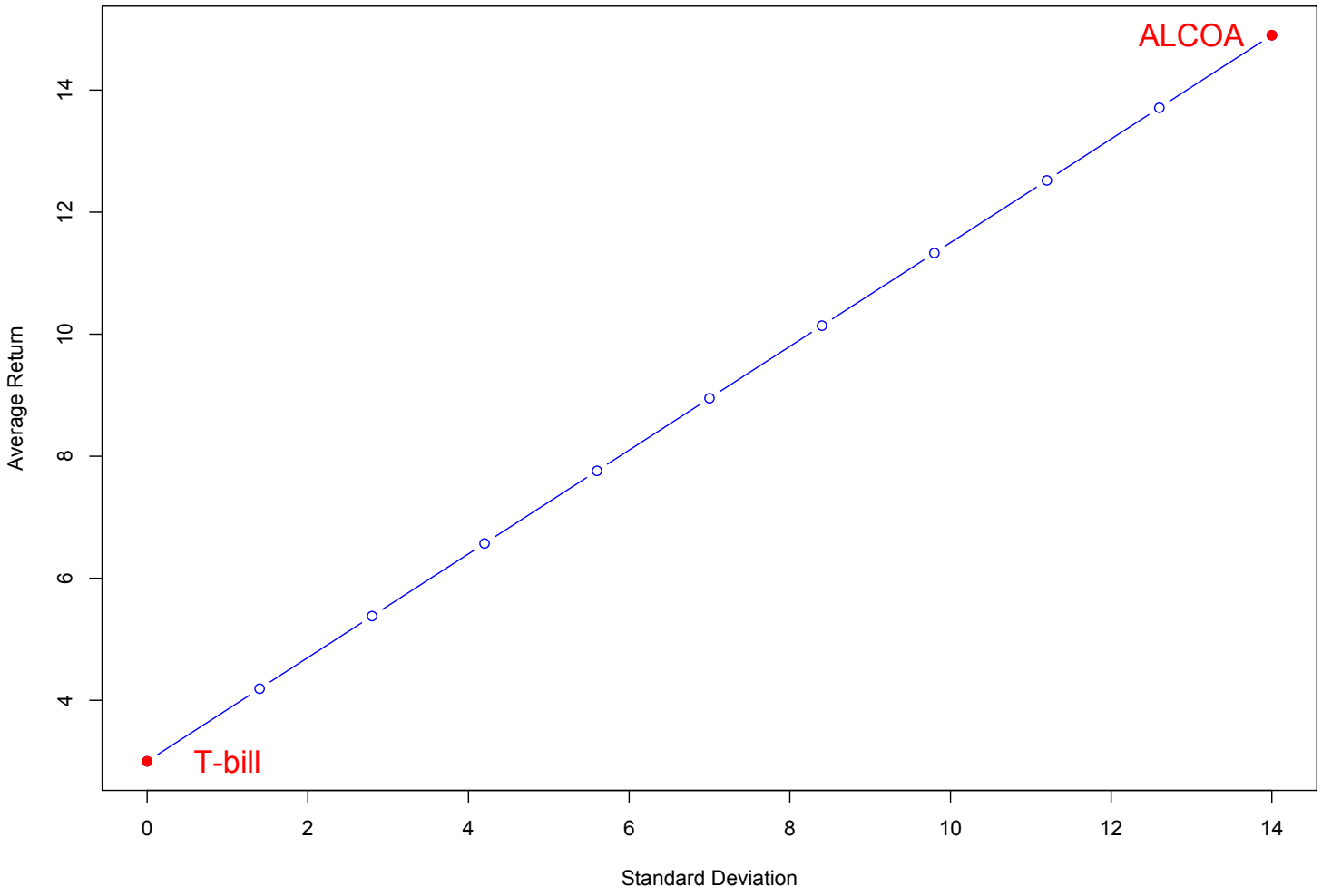
$$\begin{aligned} \hat{\mu}_P &= 0.5\hat{\mu}_A + 0.5\mu_{Tbill} \\ \hat{\sigma}_P^2 &= 0.5^2\hat{\sigma}_A^2 + 0.5^2 * 0 + 2 * 0.5 * 0.5 * 0 \end{aligned}$$

- $\hat{\mu}_P$ and $\hat{\sigma}_P^2$ refer to the estimated mean and variance of our portfolio

- What are we assuming here?

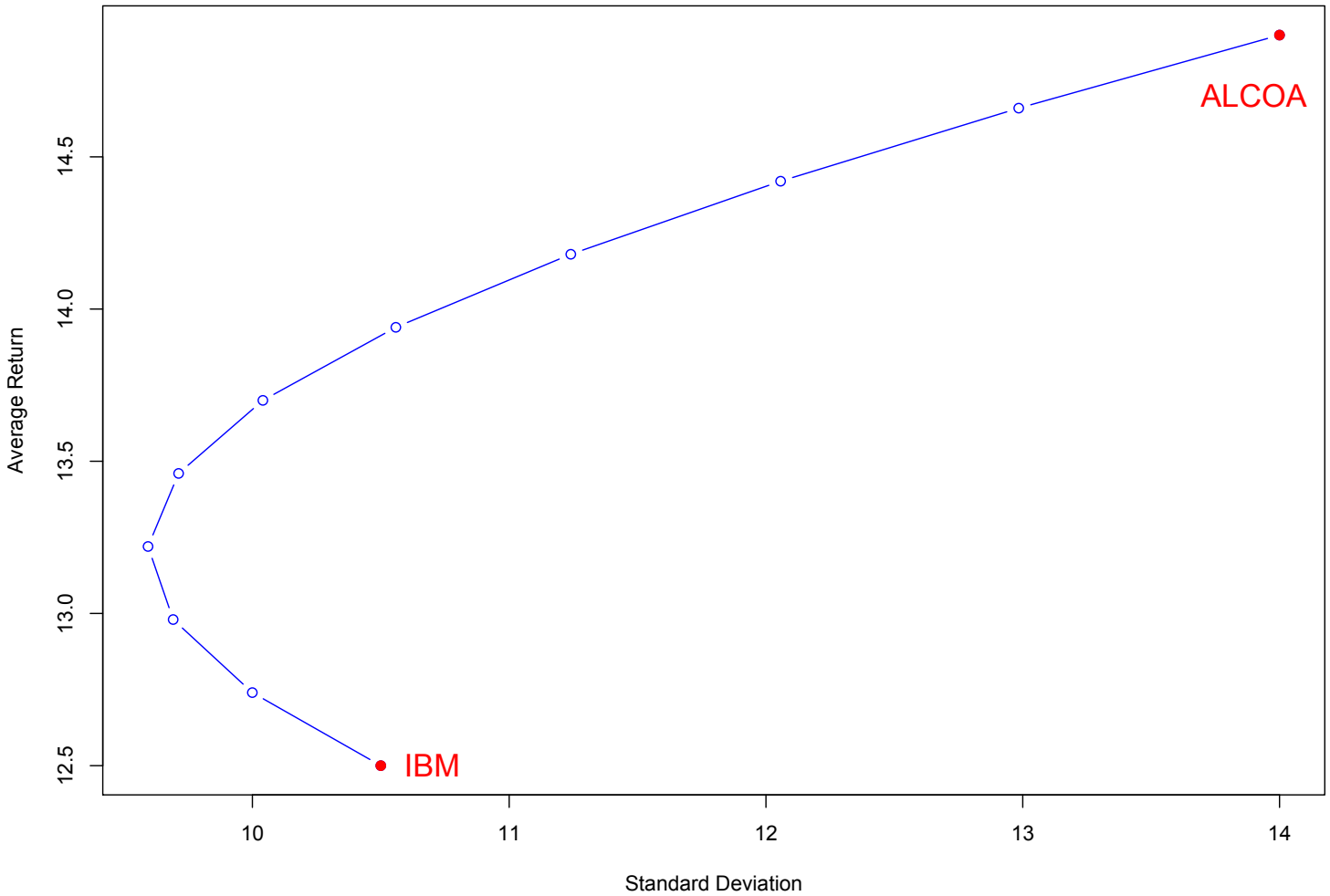
# Building Portfolios

- What happens if we change the proportions. . .



# Building Portfolios

- What about investing in IBM and ALCOA?




How much more complicated this gets if I am choosing between 100 stocks?

# R Markdown

This is an R Markdown presentation. Markdown is a simple formatting syntax for authoring HTML, PDF, and MS Word documents. For more details on using R Markdown see `http://rmarkdown.rstudio.com`.

When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document.

# Slide with Bullets

- Bullet 1
- Bullet 2
- Bullet 3

# Slide with R Output

```
summary(cars)
```

```
##      speed           dist       
##  Min.   : 4.0   Min.   :  2.00  
##  1st Qu.:12.0   1st Qu.: 26.00  
##  Median :15.0   Median : 36.00  
##  Mean   :15.4   Mean   : 42.98  
##  3rd Qu.:19.0   3rd Qu.: 56.00  
##  Max.   :25.0   Max.   :120.00
```

# Slide with Plot