

Summary of Hypothesis Testing

The idea of testing is that if a given model is “right” then the real world data should look consistent with the models predictions. Hypothesis is a claim about an unknown parameter in a model. If something we observe would be really unlikely if the hypothesis were true, then we reject the hypothesis. We use sampling distribution of an estimator to decide if something is unlikely.

To test the null hypothesis $H_0 : p = p_0$ against the alternative $H_a : p \neq p_0$, we use the following test statistic

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

We reject at the α if $|Z| > z_{1-\alpha/2}$, otherwise, we fail to reject. Recall that $P[N(0,1) < z_\alpha] = \alpha$. We base our decision on the observation that if the null is true, then the test statistic is approximately normal, that is, $Z \sim N(0,1)$. A test at the α level has the interpretation that it will reject in α fraction of the times when the null is true. In a 5% level test, we use 2 as the critical value.

To test the null hypothesis $H_0 : \mu = \mu_0$ against the alternative $H_a : \mu \neq \mu_0$, we use the following test statistic

$$Z = \frac{\bar{x} - \mu_0}{s_x/\sqrt{n}}.$$

We reject at the α if $|Z| > z_{1-\alpha/2}$, otherwise, we fail to reject.

In general, we may have an estimator $\hat{\theta}$ of a parameter θ . Often (but not always) the sampling distribution of $\hat{\theta}$ will be approximately normal based on the central limit theorem and we will use

$$Z = \frac{\hat{\theta} - \theta}{S.E.(\hat{\theta})}$$

to construct confidence intervals and perform hypothesis tests.

A **p-value** allows us to communicate more information to other people than just reporting reject or fail to reject. A p-value asks: “If we assume the null hypothesis is true, what is the probability of observing a sample with this much or more evidence against the null hypothesis?” The p-value is the probability of getting something as far or farther out in the tails than the observed value for test statistic. We can use p-value to reject a null hypothesis. To test the null hypothesis at level α , we reject if the p-value is less than α .