Recitation 3

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Question 1: oil company - decision making

An oil company wants to drill in a new location. There is a 20% chance of finding a small amount of oil, a 50% chance of a moderate amount and a 30% chance of a large amount of oil. The company has a choice of either a standard drill that simply burrows deep into the earth or a more sophisticated drill that is capable of horizontal drilling but is far more expensive.

	small	medium	large
standard	\$20M	\$30M	\$50M
horizontal	-\$20M	\$40M	\$90M

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1. Find the mean payoffs of the two different drilling strategies.
2. Find the variance in payoffs of each strategy.
3. Which strategy would you advocate for and why?
4. How much are you willing to pay for a geological evaluation that would determine with certainty the quantity of oil at the site prior to drilling?

Question 2: real estate - joint and marginal distributions

Cooper Realty is a small real estate company located in Albany, New York, specializing primarily in residential listings. They have recently become interested in determining the likelihood of one of their listings being sold within a certain number of days. Based on historical data, they produced the following figures based on the past 800 homes sold.

Days Listed until Sold	Under 20	31-90	Over 90	Total
Under \$50K	50	40	10	100
\$50-\$100K	20	150	80	250
\$100-\$150K	20	280	100	400
Over \$150K	10	30	10	50

Question2. continued

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1. What is the probability that a randomly selected home is listed over 90 days before being sold?
2. What is the probability that a randomly selected initial asking price is under \$50K?
3. What is the probability of both of the previous two events happening? Are these two events independent?
4. Assuming that a contract has just been signed to list a home that has an initial asking price less than \$100K, what is the probability that the home will take Cooper Realty more than 90 days to sell

Question 3: back to oil - updating probability

An oil company has purchased an option on land in Alaska. Preliminary geologic studies have assigned the following probabilities of finding oil:

event	high quality (H)	medium quality (M)	no oil (N)
Probability	0.50	0.20	0.30

After 200 feet of drilling on the first well, a soil test is taken which shows a certain soil type. The probabilities of finding this particular soil type (event S), given the type of oil present (if any) are:

$$\mathrm{P}(S \mid H) = 0.20 \qquad \mathrm{P}(S \mid M) = 0.80 \qquad \mathrm{P}(S \mid N) = 0.20$$

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1. What are the revised (post-soil-test) probabilities of finding the three types of oil?
2. How should the firm interpret the soil test?

Question 4: retirement income investment

The following joint probability distribution is based on survey data collected by a major financial publication in 2002. The columns represent the percentage of retirement income invested in the stock market (X) by age division (Y).

		X		
		10%	30%	60%
Y	$<$ 30 yrs old	0.04	0.05	0.01
	30-50 yrs old	0.05	0.23	0.19
	$>$ 50 yrs old	0.10	0.26	0.07

Question 4. continued

		X		
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1. Among persons investing 60% of retirement income in the stock market, what proportion are younger than 30?
2. Among persons younger than 30, what percentage invest more than 10% of their retirement income in the stock market?
3. Re-express the information in the table as the conditional probability of X *given* Y.
4. What is the conditional expectation of X given that $Y = \{\text{older than 50}\}$.
5. Compute the correlation between X and Y.

Question 5: how to filter spam emails - Bayesian approach

In a sample of 100,000 emails you find that 550 are spam. Your next email contains the word "bigger." From historical experience, you know half of all spam email contains the word "bigger" and only 2% of non-spam emails contain it. Find the probability that this new email is spam.