Bayesian Statistics and Bayesian Cognitive Modeling

Lei Zhang

Institute of Systems Neuroscience, University Medical Center Hamburg-Eppendorf

Special thanks to Antonius Wiehler!

10-12. Oct. 2016, Hamburg

lei.zhang@uke.de

Recap

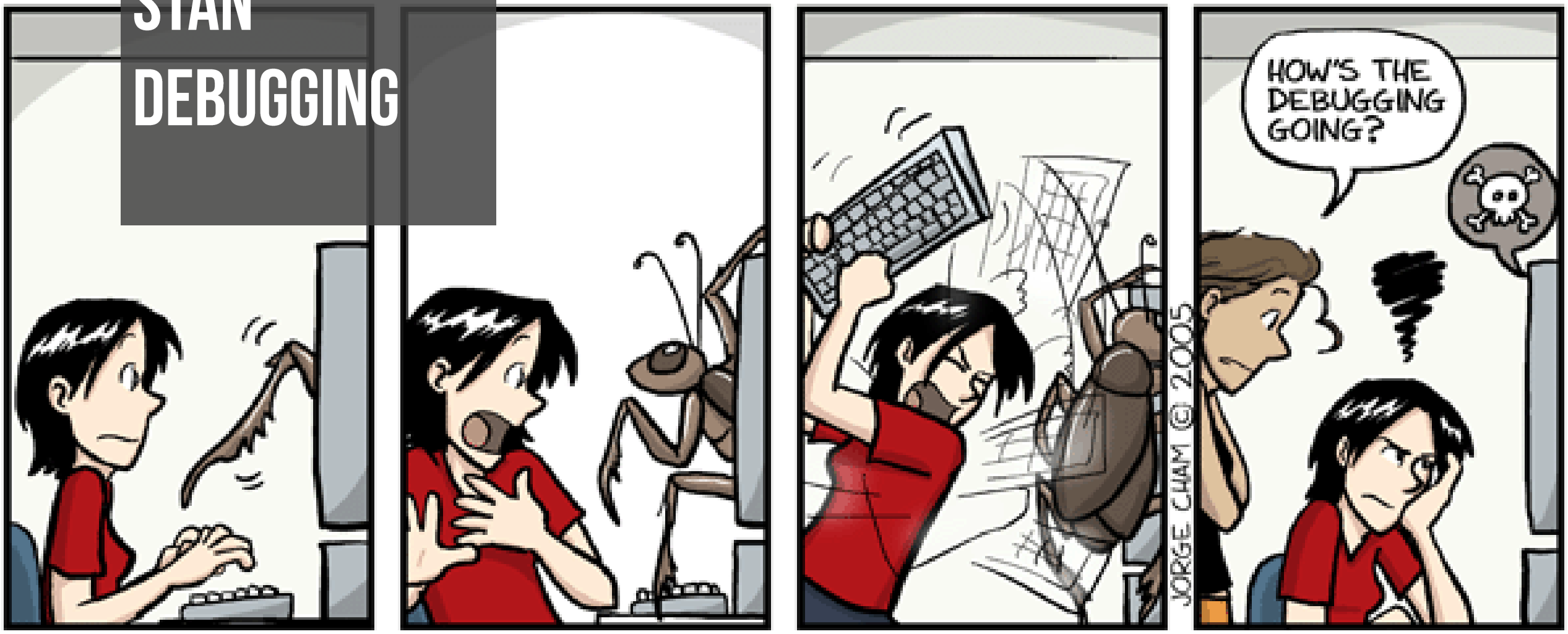
What we've learned…

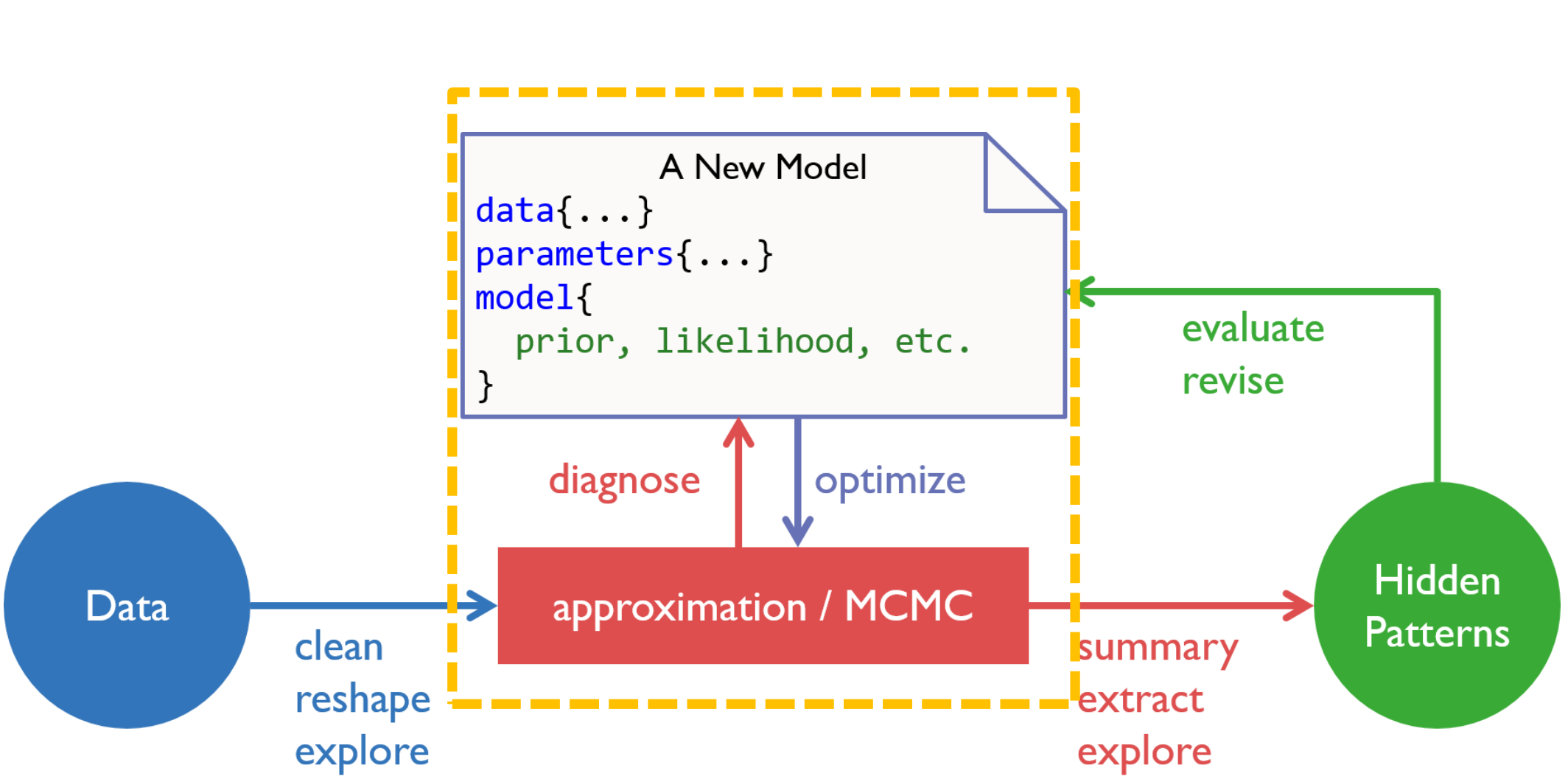
- Linear Regression Model
- Predictive Check
- Cognitive Modeling
- Reinforcement Learning Model
- Hierarchical Modeling
- Optimizing Stan Codes
- Model Comparison

DAY3

09:00 – 09:30	Implementing Model Comparison
09:30 – 10:15	Stan Style Tips and Debugging
10:15 – 11:00	Introduction to Model-based fMRI
11:00 – 11:15	Coffee break
11:15 – 12:15	Capstone Project: Delay Discounting
12:15 – 13:00	Summary, Misc., Q&A

STAN DEBUGGING




cognitive model
statistics
computing
A New Model
data{...}
parameters{...}
model{
prior, likelihood, etc.
}
diagnose
optimize
evaluate
revise
Data
clean
reshape
explore
approximation / MCMC
summary
extract
explore
Hidden
Patterns

Stan Style Tips

cognitive model
statistics
computing

Make it Reproducible

- Scripts are good documentation!
- Save your seed

Make it Readable

- Choose a consistent style
- Give meaningful variable names

Start with Simulated Data

Design Top-Down, Code Bottom-Up

Write Comments

- Code never lies!

The Editor of your Choice

```
data {
  int<lower=0> w;
  int<lower=0> N;
}

parameters {
  real<lower=0,upper=1> p;
}

model {
  p ~ uniform(0,1);
  w ~ binomial(N, p);
}
```


```
data {
  int<lower=0> w;
  int<lower=0> N;
}

parameters {
  real<lower=0,upper=1> p;
}

model {
  p ~ uniform(0,1);
  w ~ binomial(N, p);
}
```



```
data {
  int<lower=0> w;
  int<lower=0> N;
}

parameters {
  real<lower=0,upper=1> p;
}

model {
  p ~ uniform(0,1);
  w ~ binomial(N, p);
}
```

Common Error / Warning Types

cognitive model
statistics
computing

ERRORS

forget " ; "
mis-indexing: mismatch or constant
support mismatch
improper constrain
improper dimension declaration
vectorizing when not supported
wrong data type
wrong distribution names
forget priors
miss spelling

WARNINGS

forget last blank line
use earlier version of Stan
numerical problems
divergent transitions
hit max_treedepth
improper prior

Debugging in Stan

cognitive model
statistics
computing

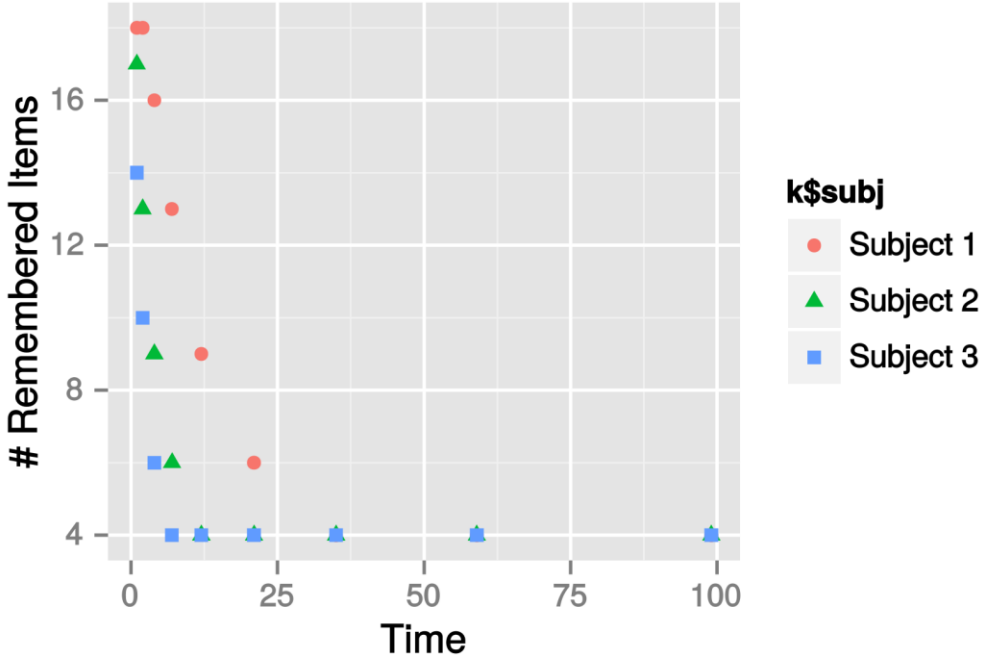
- always use a *.stan file
- press Check in RStudio
- use `lookup()`
- start with simulated data
- be careful with copy/paste
- run 1 chain, 1 sample
- debugging by printing

```
for (s in 1:1) {
  vector[2] v;
  real pe;
  v <- initV;

  for (t in 1:nTrials) {
    choice[s,t] ~ categorical_logit( tau[s] * v );

    print("s = ", s, ", t = ", t, ", v = ", v);

    pe <- reward[s,t] - v[choice[s,t]];
    v[choice[s,t]] <- v[choice[s,t]] + lr[s] * pe;
  }
}
```

```
> lookup(dnorm)
    StanFunction                              Arguments ReturnType Page SamplingStatement
344       normal           (reals mu, reals sigma)       real  369              TRUE
348   normal_log (reals y, reals mu, reals sigma)       real  369             FALSE
```

Example: Memory Retention

cognitive model
statistics
computing

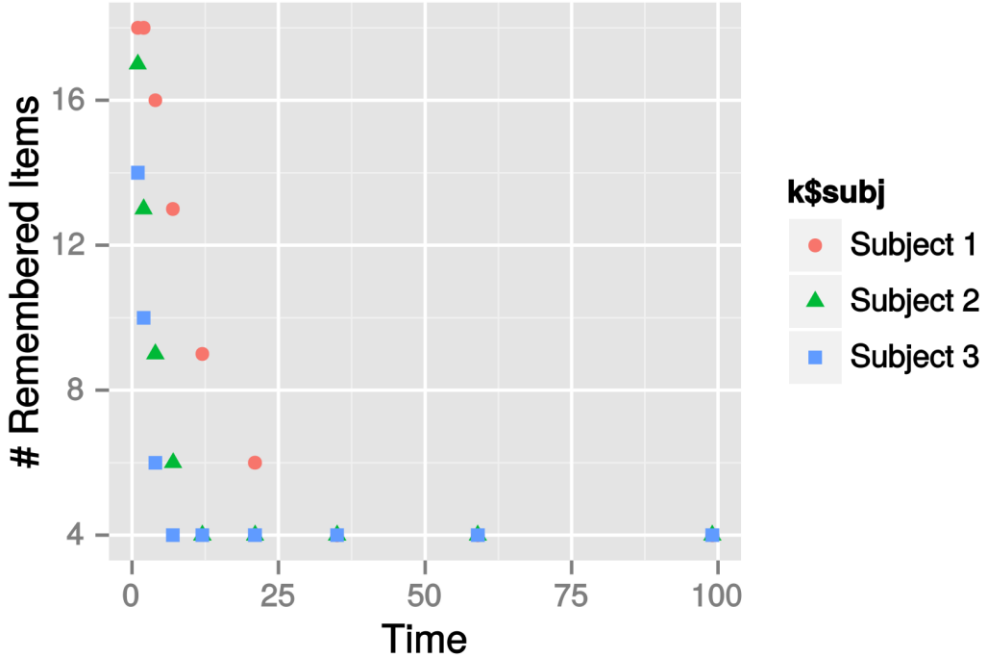


	Time Interval								
Subject	1	2	4	7	12	21	35	59	99
1	18	18	16	13	9	6	4	4	4
2	17	13	9	6	4	4	4	4	4
3	14	10	6	4	4	4	4	4	4

Lee & Wagenmakers (2013)

Simple Exponential Decay Model

cognitive model
statistics
computing



$$\theta_t = \min(1.0, \exp(-\alpha t) + \beta)$$

p(remember) decay rate baseline

Lee & Wagenmakers (2013)

Exercise I

cognitive model
statistics
computing

```
…/BayesCog/09.debugging/_scripts/exp_decay_main.R
```

TASK: Debugging the Memory retention model

>= 10 errors!

Viel Spaß!

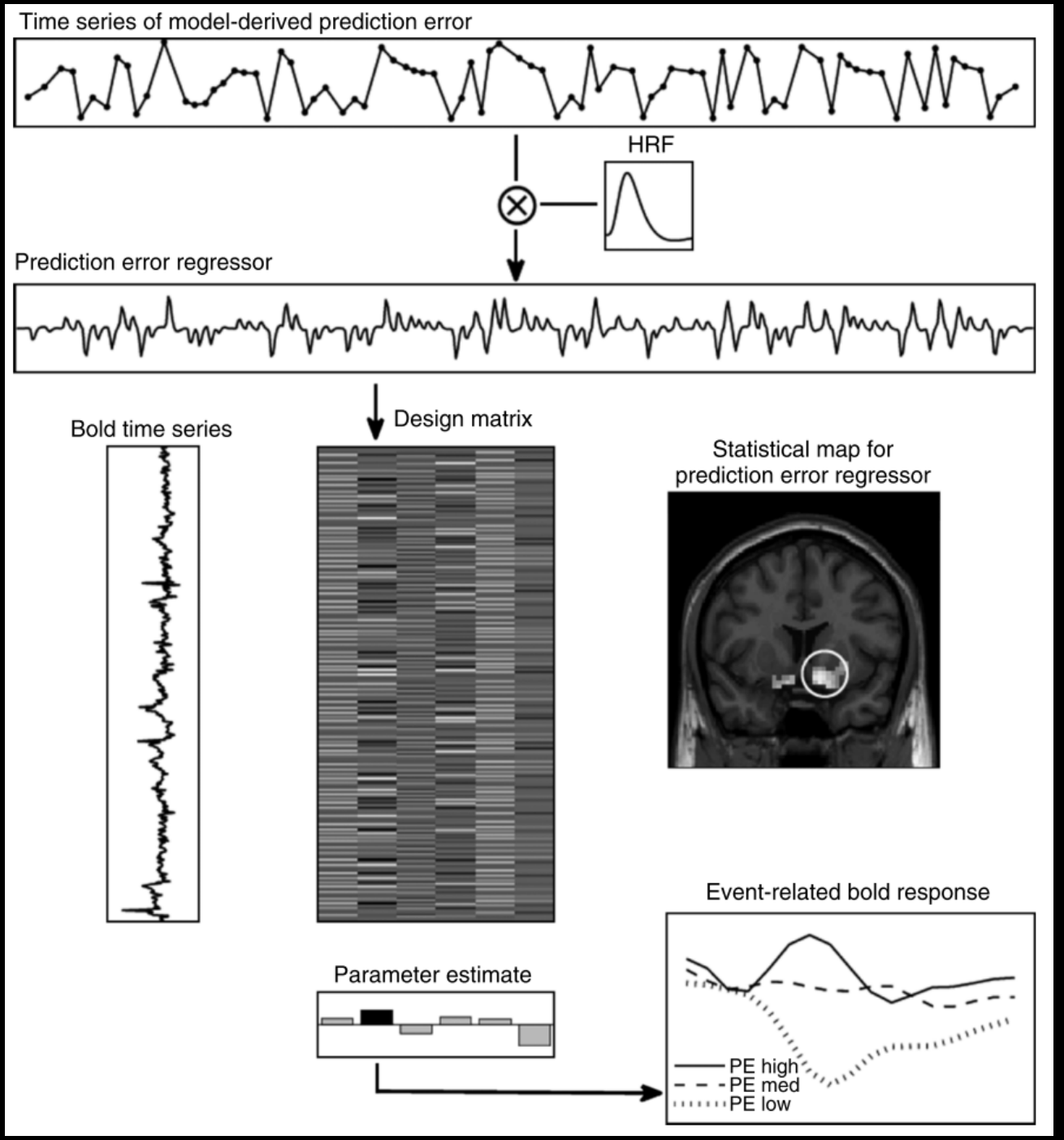
Why Stan Fails?

cognitive model
statistics
computing

```
for (s in 1:ns) {
  for (t in 1:nt) {
    theta[s,t] = fmin(1.0, exp(-alpha[s] * intervals[t]) + beta[s]);
    k[s,t] ~ binomial(nItem, theta[s,t]);
  }
}
```

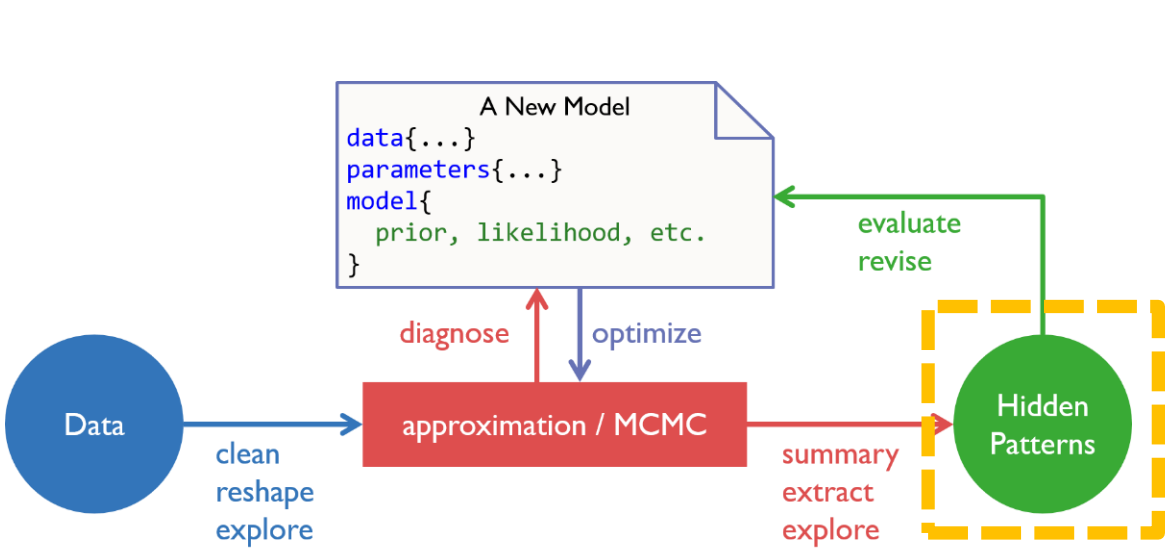
Non-differentiable link (likelihood)functions are bad news, particularly in Stan, which relies on derivatives.

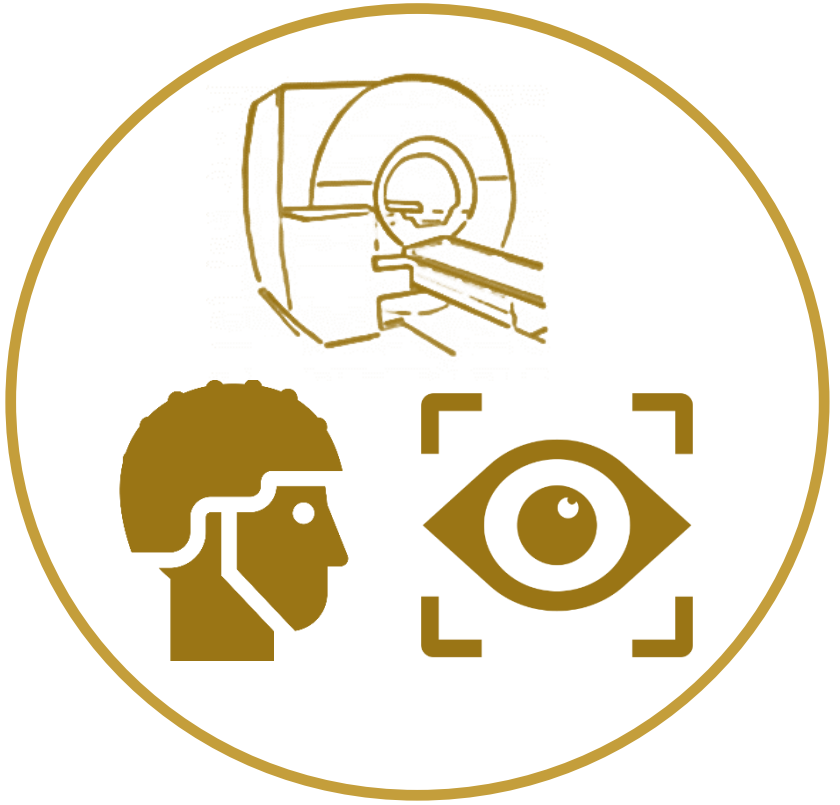
INTRODUCTION TO MODEL-BASED FMRI



Model-based Analysis

cognitive model
statistics
computing





Explore Hidden Patterns in Stan

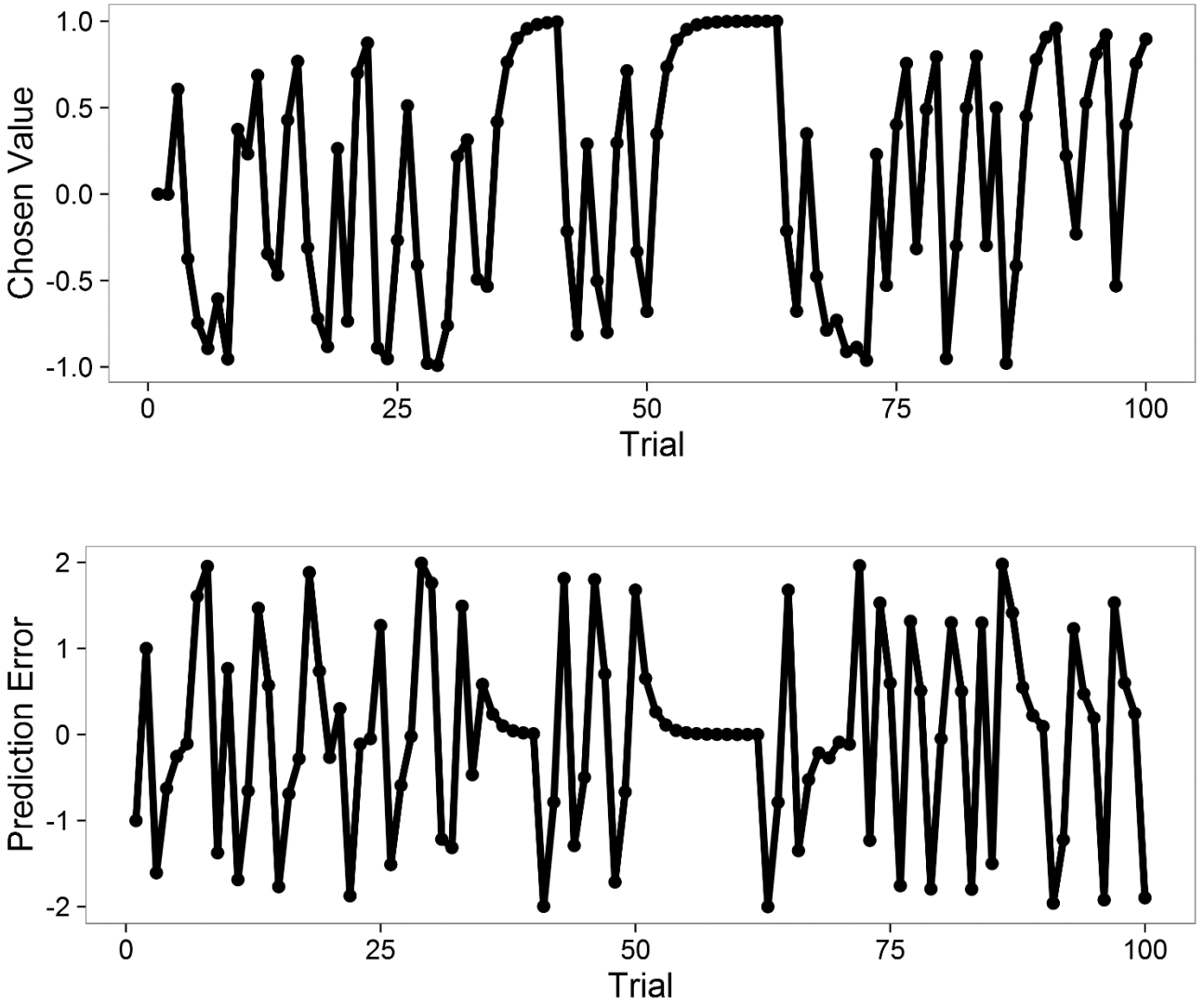
cognitive model
statistics
computing

```
generated quantities {
  ...

  vector[2] v[nSubjects, nTrials+1];
  real vc[nSubjects,nTrials]; //chosen value
  real pe[nSubjects,nTrials];

  {
    for (s in 1:nSubjects) {
      log_lik[s] <- 0;
      v[s,1] <- initV;

      for (t in 1:nTrials) {
        log_lik[s] = log_lik[s] + categorical_logit_log(choice[s,t], tau[s] * v[s,t] );

        vc[s,t] = v[s,t,choice[s,t]];

        pe[s,t] = reward[s,t] - v[s,t,choice[s,t]];

        v[s,t+1] = v[s,t];
        v[s,t+1,choice[s,t]] = v[s,t,choice[s,t]] + lr[s] * pe[s,t];
```

Obtain Decision Variables

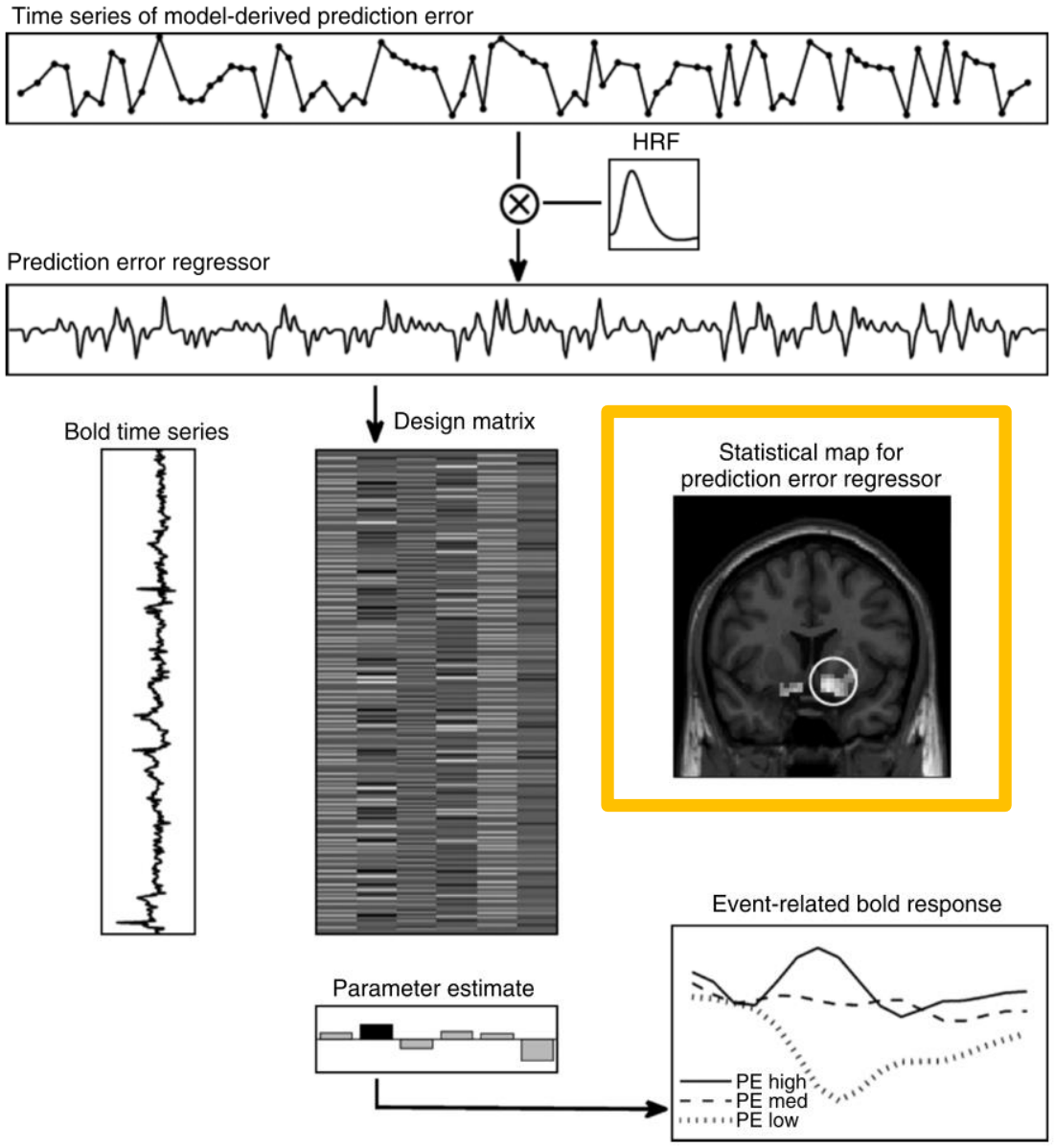
cognitive model
statistics
computing

subject01



Perform Model-based fMRI

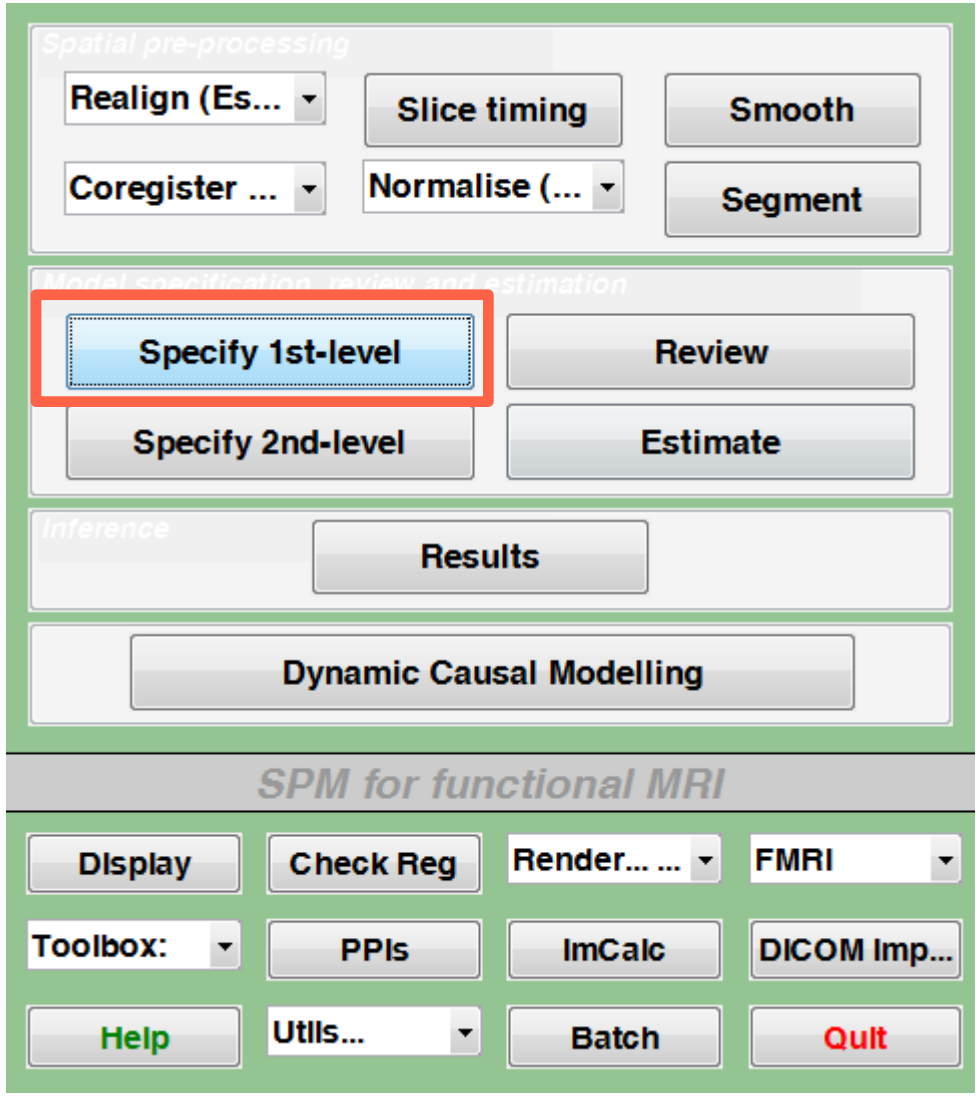
cognitive model
statistics
computing

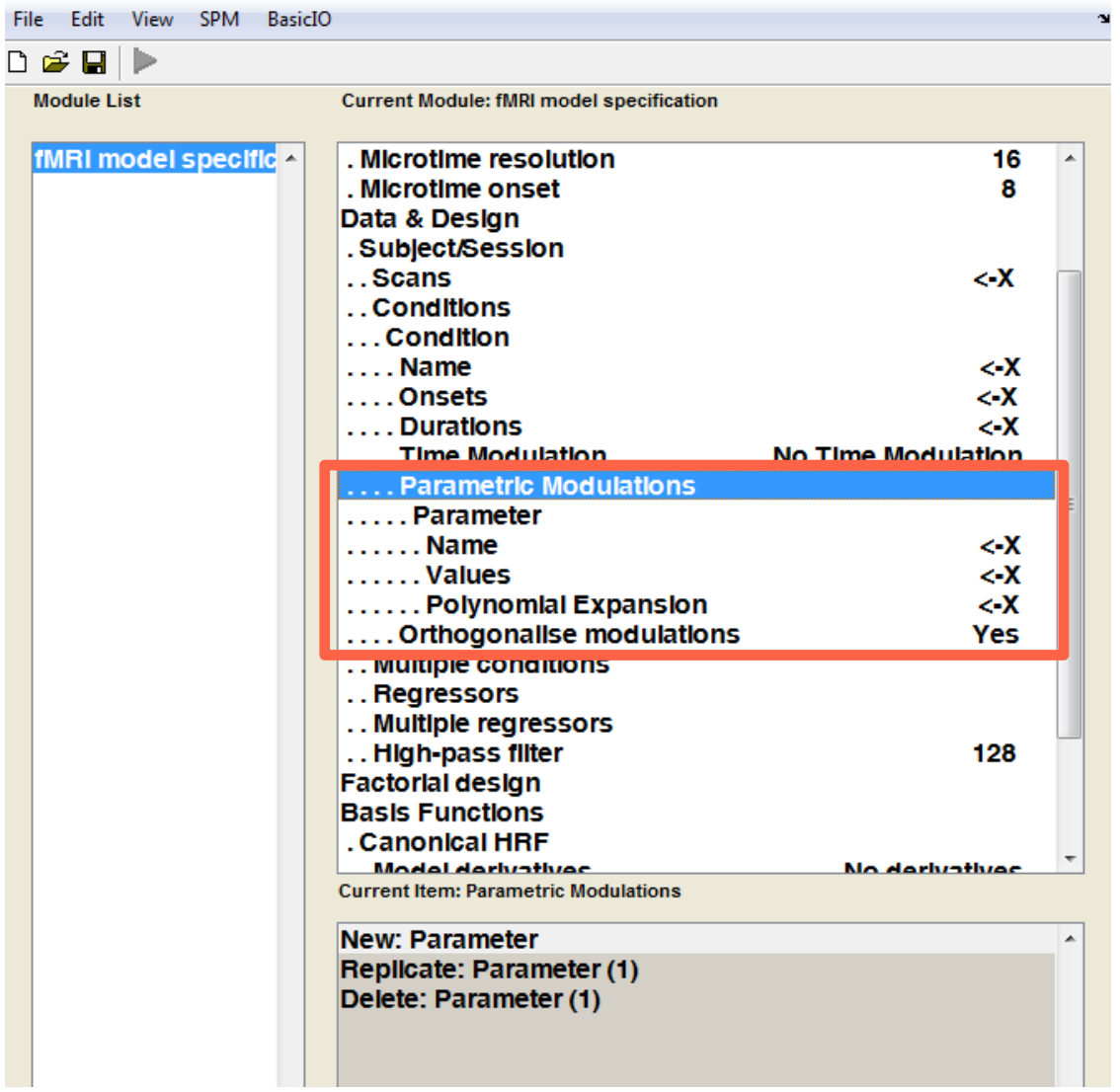


Gläscher & O'Doherty (2010)

Implementing in SPM12

cognitive model
statistics
computing





SPM12 – batch scripting

cognitive model
statistics
computing

```
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).name = 'onsetPE';
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).onset = onscat.sub(i_sub).cueoutcome;
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).duration = 0;
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).tmod = 0;
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).pmod.name = 'PE';
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).pmod.param = pe(i_sub);
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).pmod.poly = 1;
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).orth = 0;
```

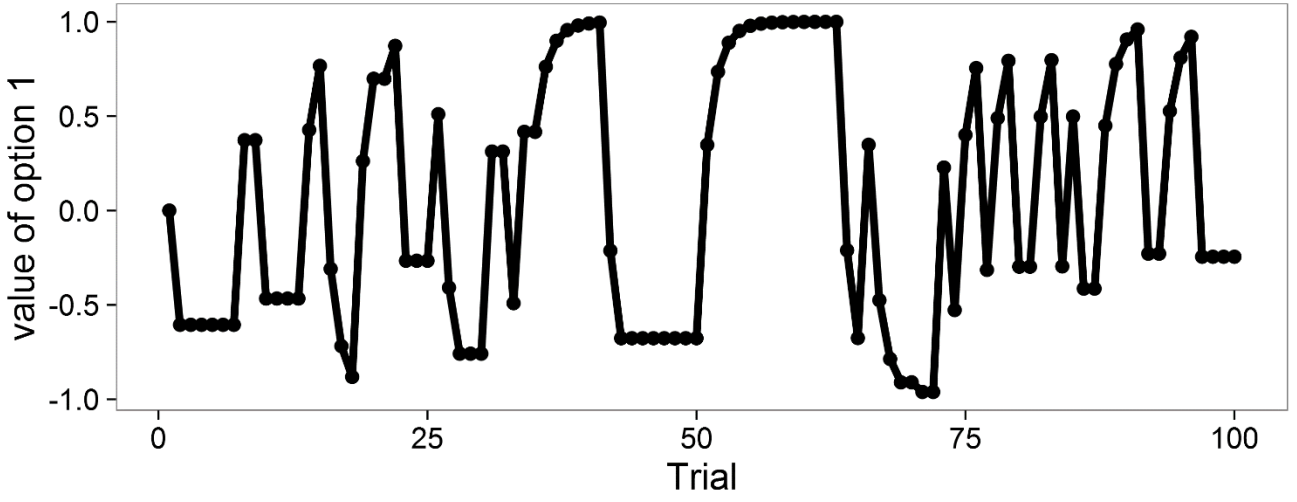
Exercise II

cognitive model
statistics
computing

`…/BayesCog/10.model_based/_scripts/reinforcement_learning_model_based_main.R`

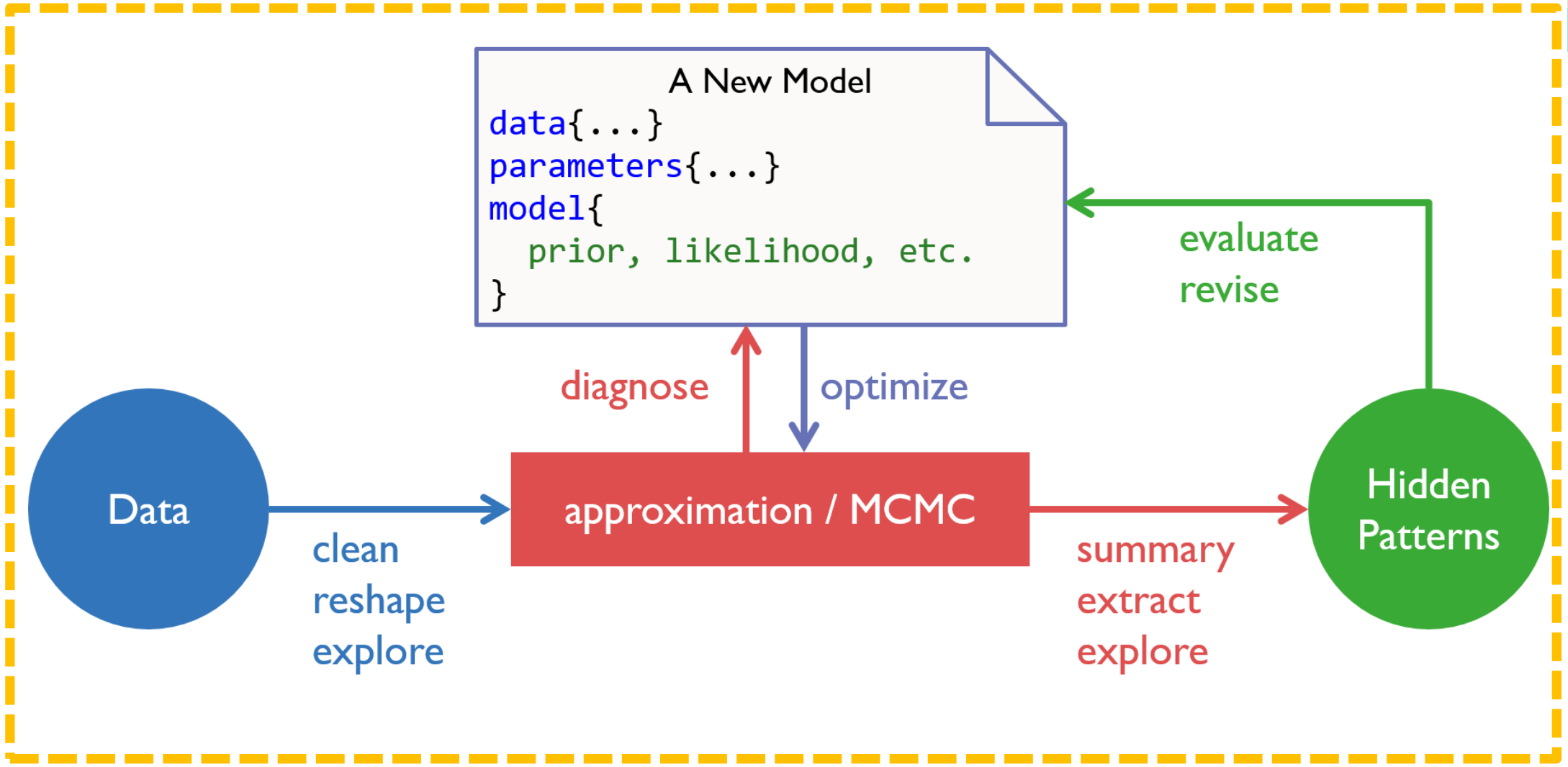
TASK: extract and plot V(option=1), for subject01 (from L64/L105)

TIP: `fit_rl <- readRDS('_outputs/fit_rl.RData')`



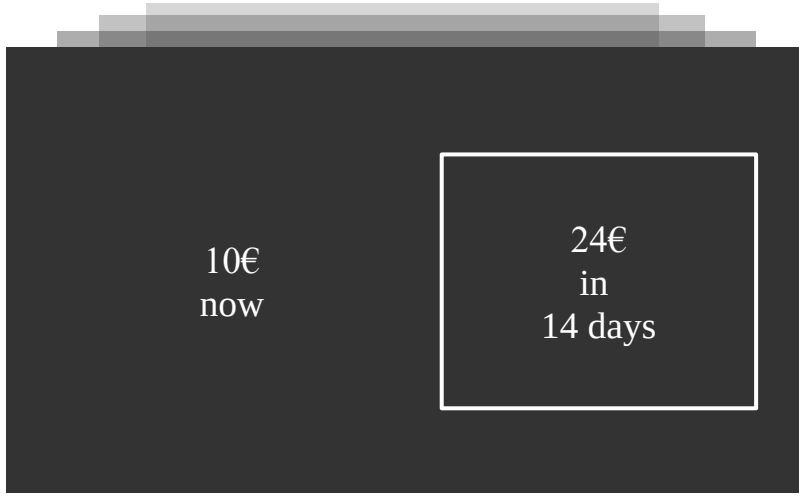
DELAY DISCOUNTING

cognitive model
statistics
computing



Delay Discounting Task and Models

cognitive model
statistics
computing



Hyperbolic Discounting Model

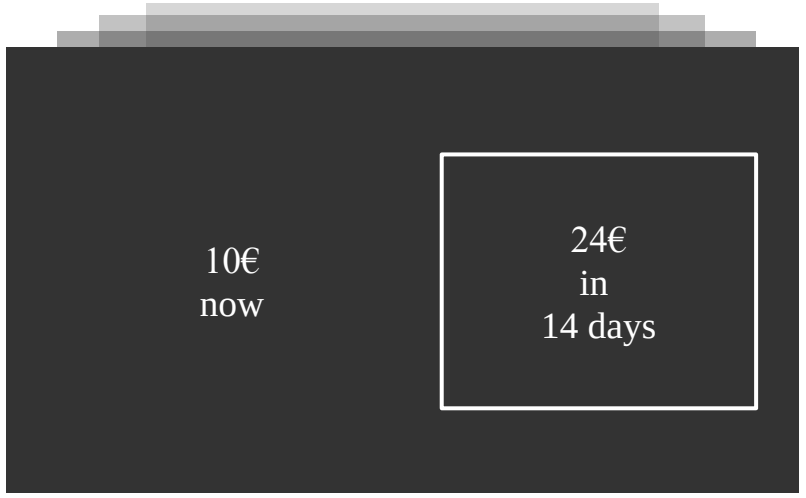
$$SV = \frac{A}{1 + k * delay}$$

$$p(LL) = \frac{1}{1 + \exp^{temp(v(SS) - v(LL))}}$$

LL – late large option
SS – soon small option

Delay Discounting Task and Models

cognitive model
statistics
computing



Exponential Discounting Model

$$SV = A * \exp(-r * delay)$$

$$p(LL) = \frac{1}{1 + \exp^{temp(v(SS) - v(LL))}}$$

Exercise III

cognitive model
statistics
computing

`…/BayesCog/11.delay_discounting/_scripts/delay_discounting_main.R`

TASK:

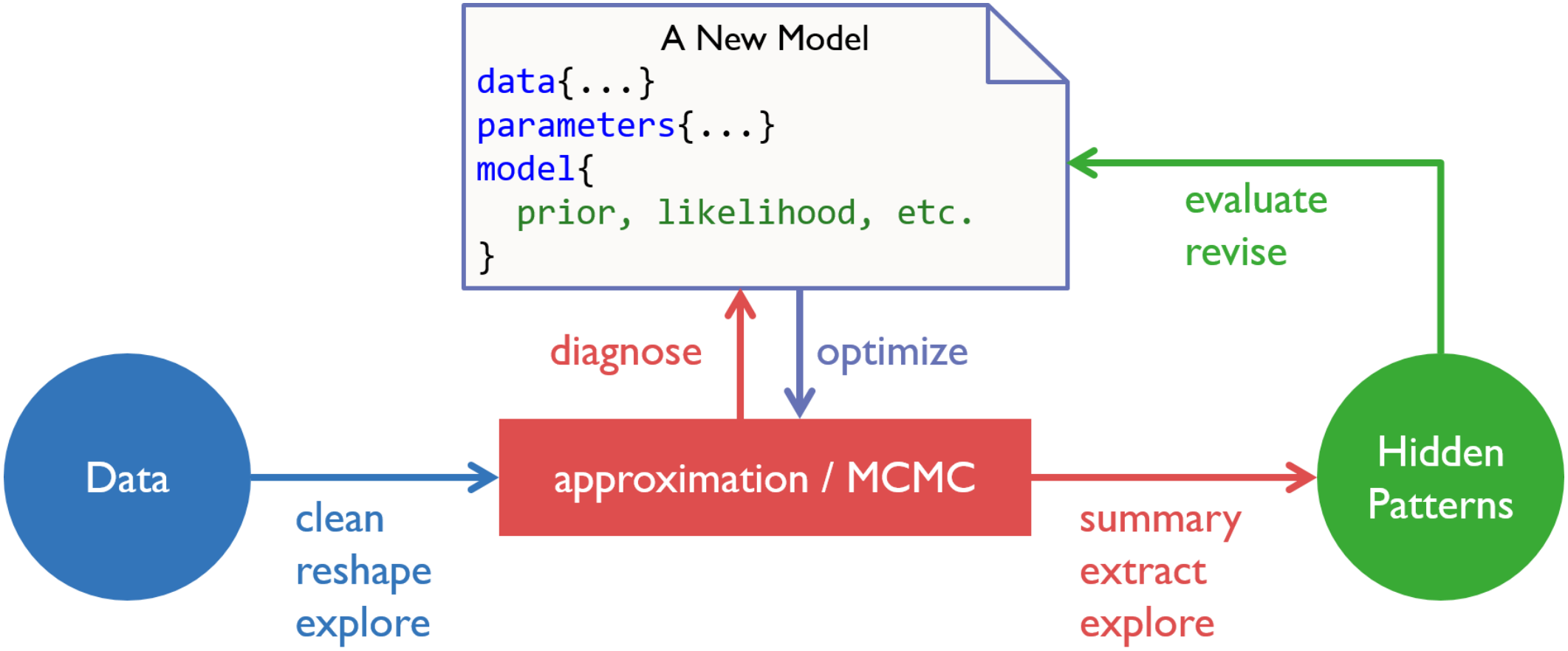
(1) understand how to deal with missing trials

(2) complete and fit both models

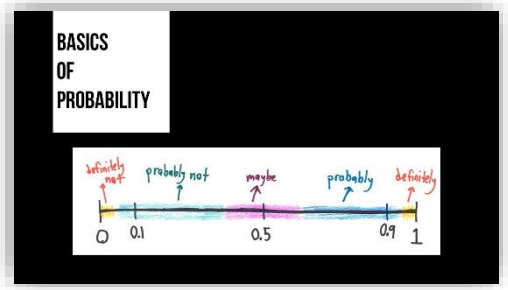
(3) complete the main script for comparing the two models

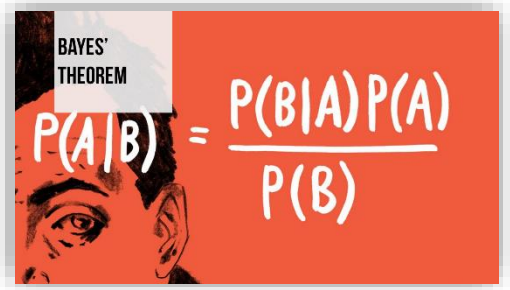
Summary

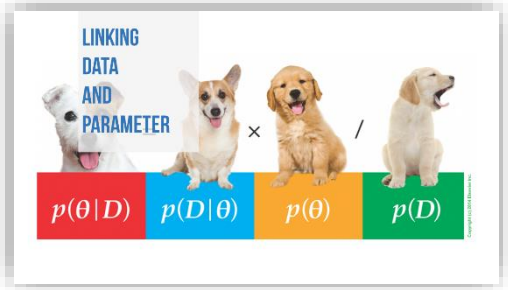
cognitive model
statistics
computing

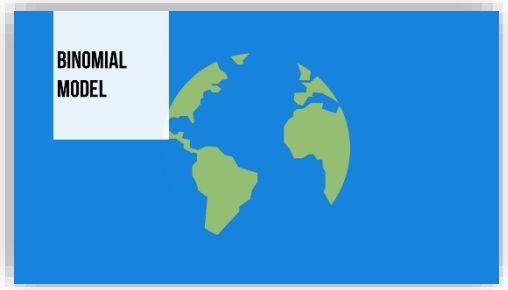


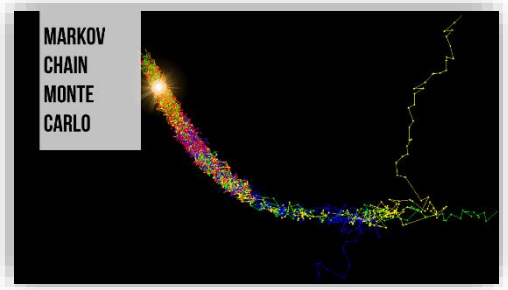
Summary of Topics

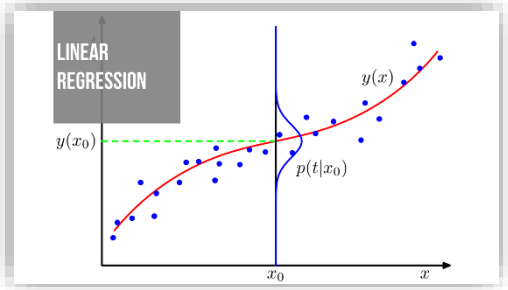


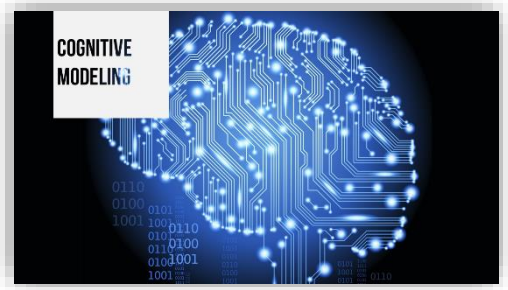


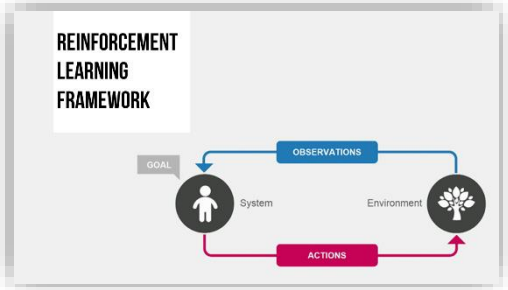


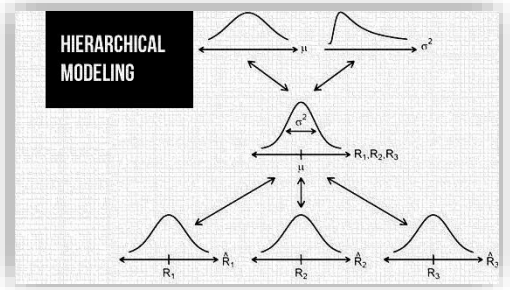


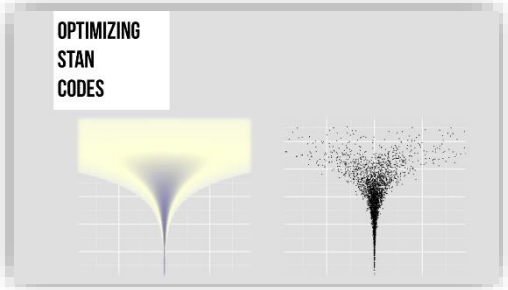


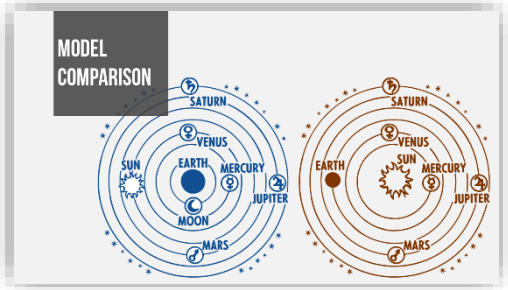


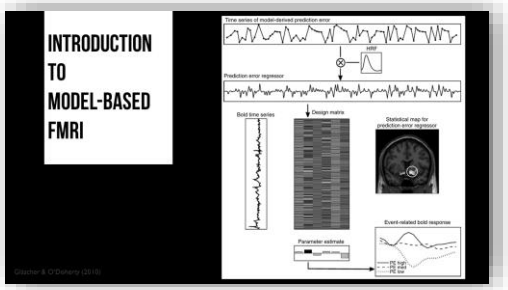


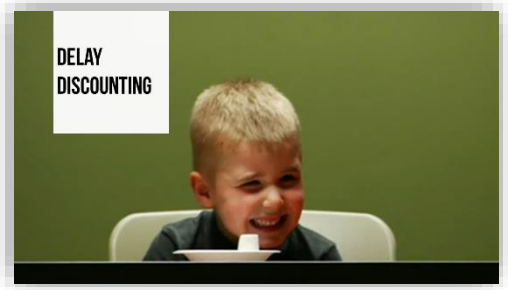












Summary of Examples/Exercises

FOLDER	TASK	MODEL
01.R_basics	NA	NA
02.binomial_globe	Globe toss	Binomial Model
03.bernoulli_coin	Coin flip	Bernoulli Model
04.regression_height	Observed weight and height	Linear regression model
05.regression_height_poly		
06.reinforcement_learning	2-armed bandit task	Simple reinforcement learning (RL) model
07.optm_rl		
08.compare_models	Probabilistic reversal learning task	Simple and fictitious RL models
09.debugging	Memory Retention	Exponential decay model
10.model_based	2-armed bandit task	Simple RL model
11.delay_discounting	Delay discounting task	Hyperbolic and exponential discounting model

After the Workshop, you...

cognitive model
statistics
computing

…are able to implement your own model

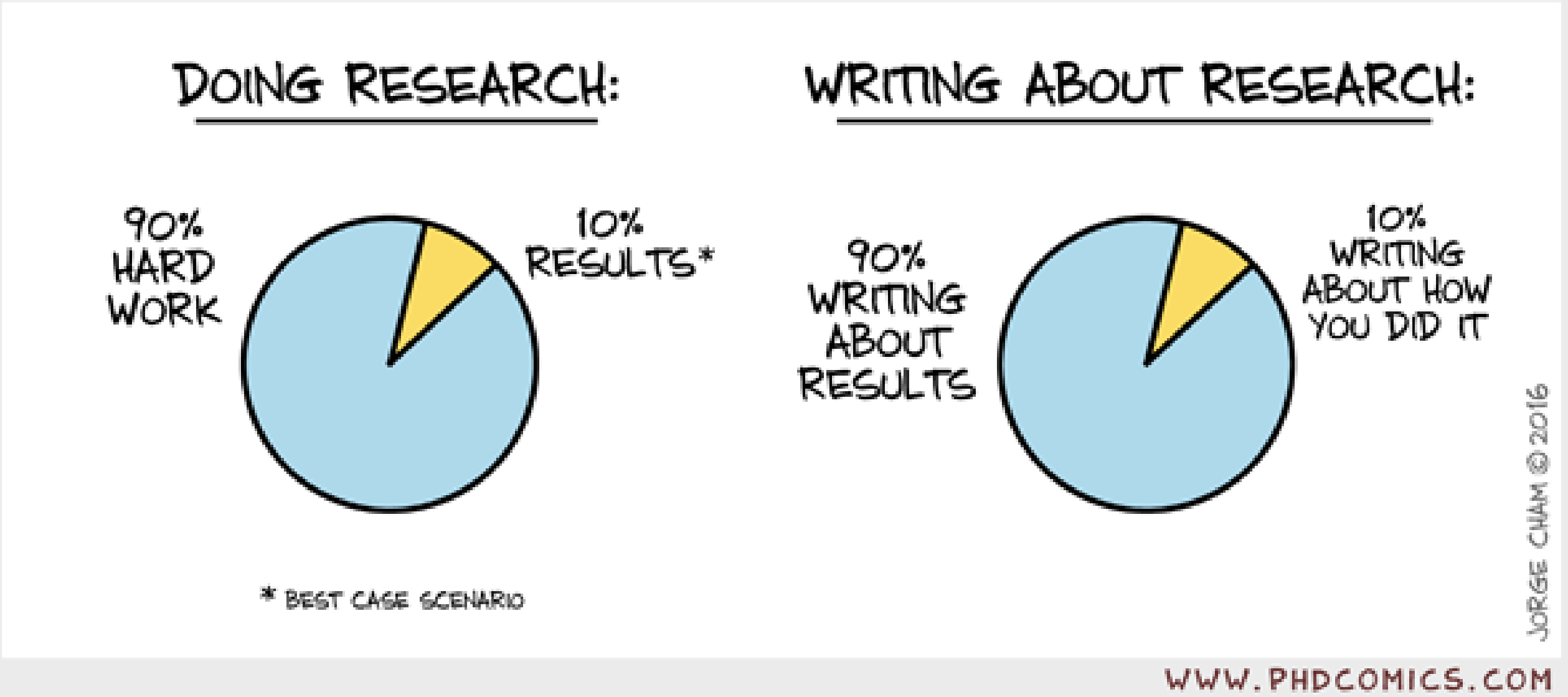
…consider the implementation of the "computational modeling" section

…feel comfortable with reading mathematical equations

…gain insightful understanding of Bayesian stats and modeling

…take it as a good start and work on it later

Remember: we are NOT modelers!


DOING RESEARCH:
90% HARD WORK
10% RESULTS*
* BEST CASE SCENARIO
WRITING ABOUT RESEARCH:
90% WRITING ABOUT RESULTS
10% WRITING ABOUT HOW YOU DID IT
JORGE CHAM © 2016
WWW.PHDCOMICS.COM

Write Your Own Tutorial Paper!

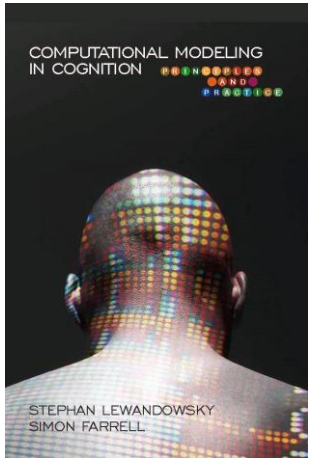
Revealing neuro-computational mechanisms of reinforcement learning and decision-making with the hBayesDM package

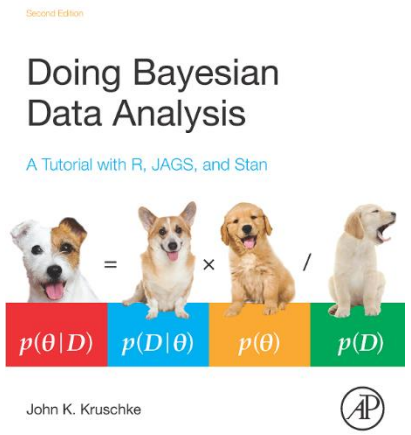
Woo-Young Ahn, Nathaniel Haines, Lei Zhang

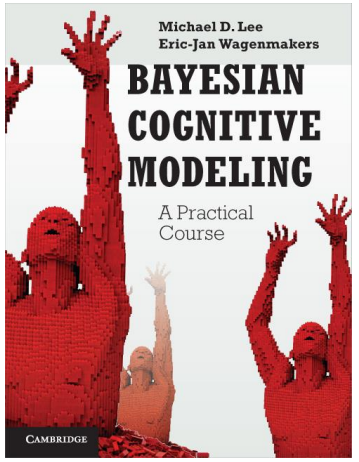
doi: http://dx.doi.org/10.1101/064287

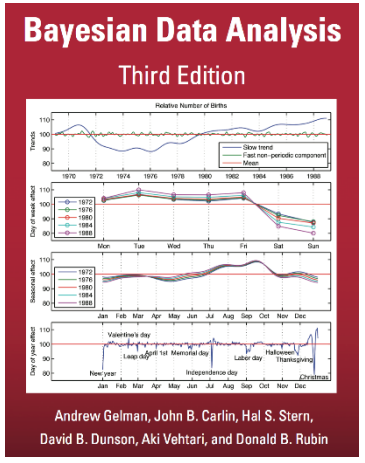
Resources

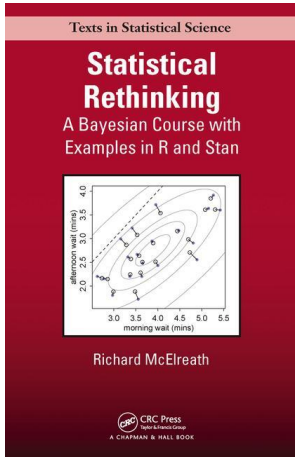
cognitive model
statistics
computing

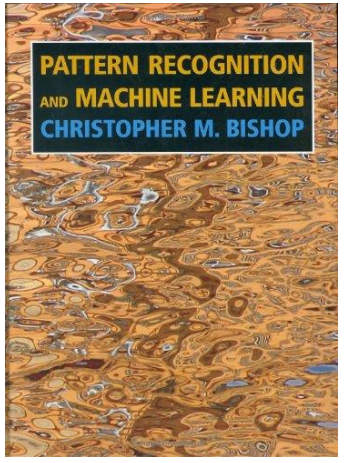












https://groups.google.com/forum/?fromgroups#!forum/stan-users


ANY
QUESTIONS
?

Happy Computing!