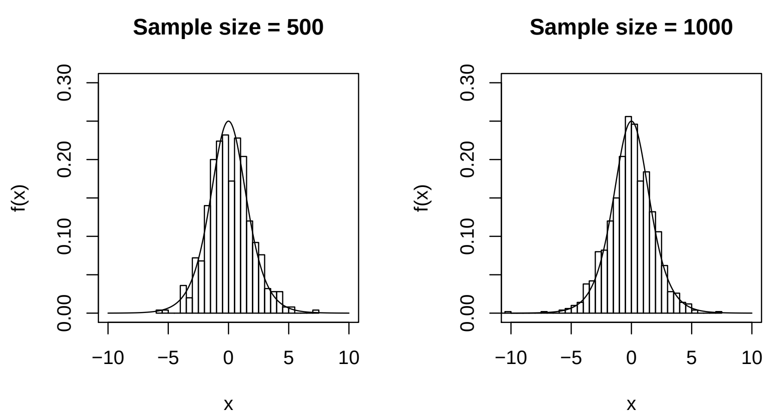
**Introduction to Bayesian Econometrics, Second Edition**
**Answers to Exercises, Chapter 5**

1. Since $f(x) = \frac{2}{a^2}x$ $(0 \leq x \leq a)$, we have $F(x) = \frac{1}{a^2}x^2$ and $F^{-1}(u) = \sqrt{a^2 u} = x$. When $a = 1$, the draws from the distribution appear in Figure 1 (the true distribution is denoted by the line overlaid on the histogram).

2. Since
$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2},$$
we have
$$F(x) = \frac{1}{1+e^{-x}},$$
$$F^{-1}(u) = -\log(\frac{1}{u} - 1) = x.$$

The following histograms show draws using the inverse CDF method.




For the general form, set $z = (x-a)/b$. Then
$$F^{-1}(u) = -\log(\frac{1}{u} - 1) = z = \frac{x-a}{b},$$

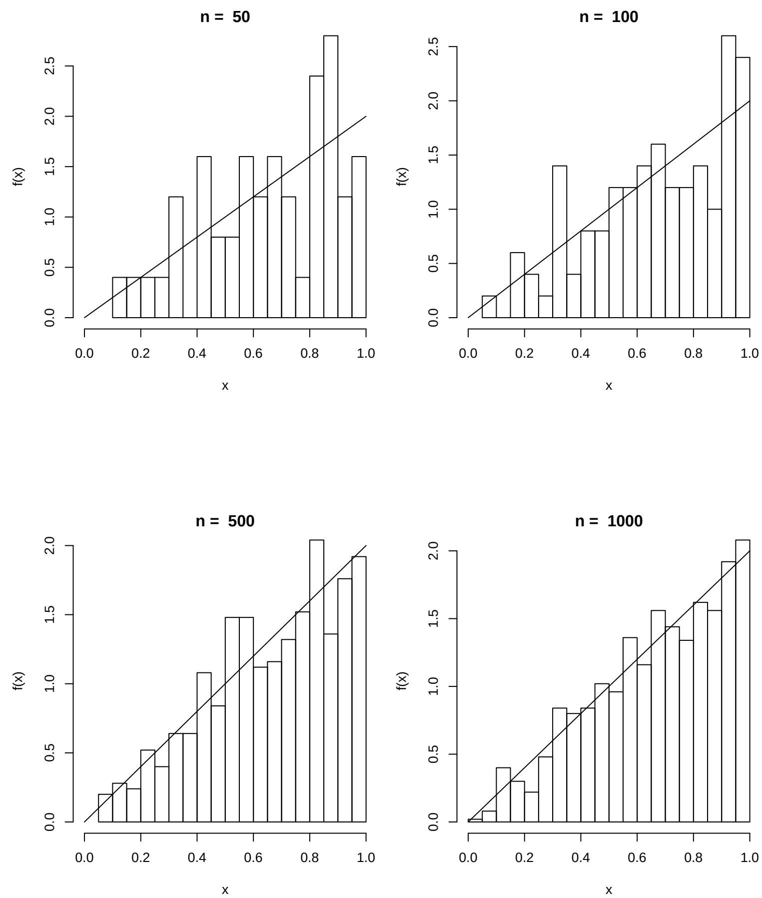


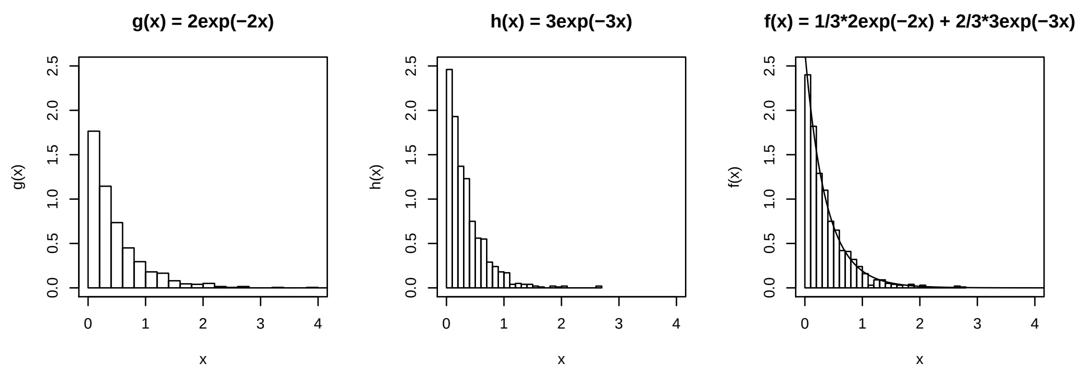

Figure 1: Exercise 5.1, $a = 1$, various $n$.

so that

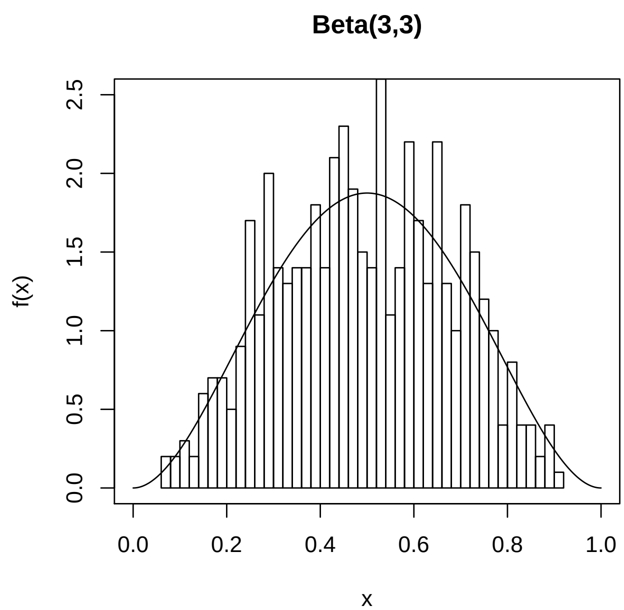
$$x = a + bz = a - b\log(\frac{1}{u} - 1).$$

3. The target density is $f(x) = \frac{1}{3}(2e^{-2x}) + \frac{2}{3}(3e^{-3x})$.

   First, we sample from $g(x) = 2e^{-2x}$ and $h(x) = 3e^{-3x}$ ( these are exponential distributions with scale parameters 1/2 and 1/3, respectively) using the inverse CDF method, with $G^{-1}(u) = -\frac{\log(1-u)}{2}$ and $H^{-1}(u) = -\frac{\log(1-u)}{3}$. We then choose a draw from $g(x)$ with a probability of 1/3 and from $h(x)$ with probability 2/3. The following histograms show $f(x)$ as a mixture of $g(x)$ and $h(x)$ The true density of $f(x)$ is overlaid in the last panel.

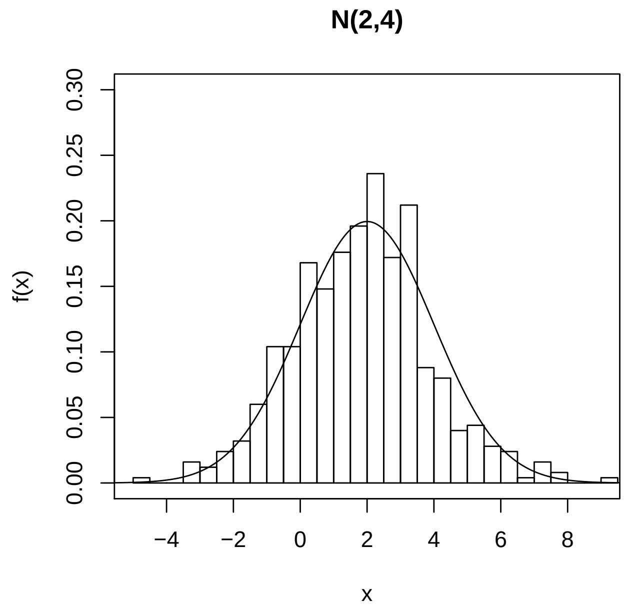



4. The true mean is $\frac{\alpha}{\alpha+\beta} = 0.500$, and the true variance is $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = 1/28 = 0.0357$. The mean and the variance of the draws in the following histogram are 0.494 and 0.0344, respectively. These are very close to the true values.




5. To draw a sample from $N(2,4)$, first sample from $N(0,1)$ using algorithm 5.4. Then transform the draws by multiplying by the standard deviation (2) and by adding the mean (2). For example, try the R code in `AnswerEx5_5.r`.

   The mean of the draws is 1.924 and the standard deviation is 4.529.




6.

$$
\begin{aligned}
P(a_1 < x \leq b_1, a_2 < x \leq b_2) &= \int_{a_1'}^{b_1'} \phi(y_1)\left[\Phi(b_2'|y_1) - \Phi(a_2'|y_1)\right]\, dy_1 \\
&= \int_{a_1'}^{b_1'} \left[\Phi(b_2'|y_1) - \Phi(a_2'|y_1)\right]\left[\Phi(b_1') - \Phi(a_1')\right] \frac{\phi(y_1)}{\Phi(b_1') - \Phi(a_1')}\, dy_1 \\
&\approx \frac{1}{G}\sum \left[\Phi(b_2'|Y_1^{(g)}) - \Phi(a_2'|Y_1^{(g)})\right]\left[\Phi(b_1') - \Phi(a_1')\right]
\end{aligned}
$$

because $\phi(y_1)/\left[\Phi(b_1') - \Phi(a_1')\right]$ is the density function of a standard normal variate truncated to $(a_1', b_1')$.

7. We apply the expression in exercise 6. First, sample $Y_1$ using the Inverse CDF method. Second, compute the probability of interest using importance sampling.

We have

$$\mu_1 = 1, \qquad \mu_2' = -0.5 + (1/1)(x_1 - 1),$$
$$\sigma_{11} = 2, \qquad \sigma_{22}' = 3 - 1/1.$$

The mean of 1000 draws is 0.299. R code may be found in `AnswerEx5_7.r`.