Bayesian Analysis: Practical Sessions

Session 0: Probability and simulations**.**

Goals:

- draw probability distributions
- compute probabilities through simulations

Exercise 0.1. Monty Hall problem. Imagine you are in a game show. The presenter, whose name is Monty Hall, shows you three doors. Behind one of the doors there's a car. Behind the other two doors there is a goat. You have to choose one door, then you receive the prize behind it. Hence, you can win a car or a goat. But before the presenter opens the door that you have chosen, he opens one of the others two doors which hide a goat, because the presenter knows where the car is. Hence, after that, there are two closed doors, behind one door there's the car and behind the other closed door there's a goat. And now the presenter asks you if you want to change your chosen door for the other closed door. The question is: should you change your chosen door by the other closed door?, or should you remain in your first option?, or it doesn't matter. For example, you choose the door number 1 and the presenter opens the door number 3, which has a goat. Then the presenter tells you "do you want to change your choice?", it means if you want choose the door number 2 instead of door number 1. Obviously, you want to choose the option, change or not change, which gives you the highest probability to win the car. Implement an R program in order to answer this dilemma using simulations.

Exercise 0.2. A Coin, a blue dice and a special red dice. Suppose: *C* is a random variable that gets 1 if you get head after tossing a coin or 0 if you get tail, *B* is a random variable that gets the value of a blue dice after rolling, and *R* is a random variable that gets the value of a red dice where the probability of getting an odd number is twice as high as getting an even number. Now we define a new variable *Z* as:

$$Z = C*(B+R).$$

Using simulations:

a) Draw the probability distribution of *Z*.
b) Compute the probability that *Z* gets values higher than 1.

Exercise 0.3. A Coin and a dice. Suppose: *C* is a random variable that gets 1 if you get head after tossing a coin or 0 if you get tail, *B* is a random variable that gets the value of a dice after rolling. Now we define a new variable *Z* as:

$$Z = C*B^2.$$

Using simulations:

a) Draw the probability distribution of *Z*.
b) Compute the probability that *Z* gets values higher than 1.