

Session 0: Simulating Random Variables

ANKIT TEWARI

01-March-2018

Exercise 0.2. A Coin, a blue dice and a special red dice. Suppose: C is a random variable that gets 1 if you get head after tossing a coin or 0 if you get tail, B is a random variable that gets the value of a blue dice after rolling, and R is a random variable that gets the value of a red dice where the probability of getting an odd number is twice as high as getting an even number.

Now, as we can see C is a random variable that gets 1 if you get head after tossing a coin or 0 if you get tail. Then we can code this situation as,

$$\text{Coin} = \text{Head} \rightarrow C = 1$$

$$\text{Coin} = \text{Tail} \rightarrow C = 0$$

Now, since R is a random variable that can take the any one of the six values on rolling a dice, the sample space consists of $\{1, 2, 3, 4, 5, 6\}$. Now, we know that probability of getting an odd number is twice the probability of getting an even number. In such a case, we assign the probability to the odd numbers i.e. $\{1, 3, 5\}$ twice the probability assigned to the even numbers $\{2, 4, 6\}$. So, if the probability assigned to $\{2, 4, 6\}$ is x then the probability assigned to each of the $\{1, 3, 5\}$ is $2x$. Now, we already know that,

$$x + x + x + 2x + 2x + 2x = 1$$

which implies that,

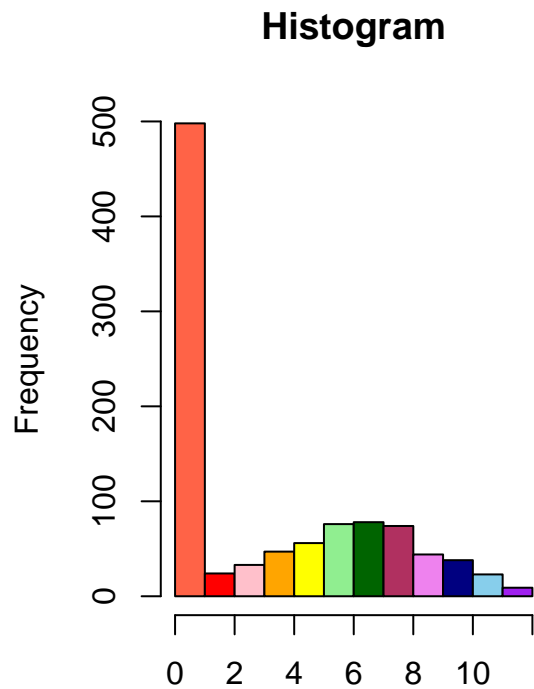
$$9x = 1 \rightarrow x = \frac{1}{9}$$

Finally, we use the information drawn from above random variables to create a new random variable as,

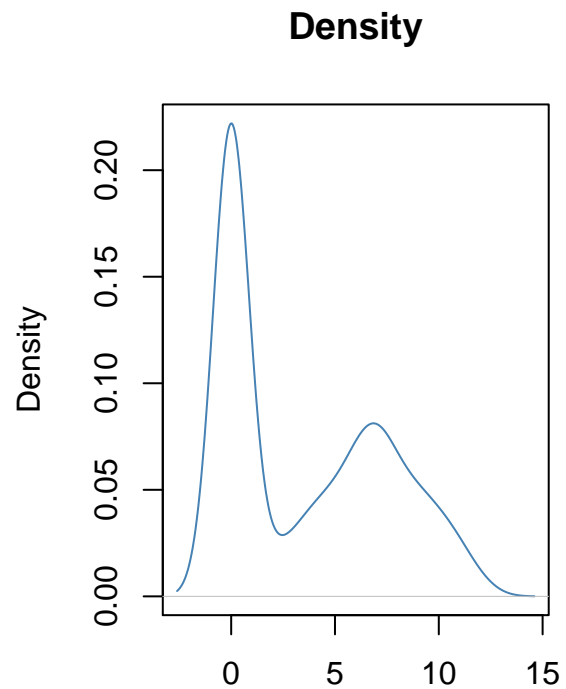
$$Z = C * (R + B)$$

The following function provides this purpose by taking a parameter n which corresponds to the number of simulations.

```
simulate_random_variable <- function(n){  
  flip <- sample(c("Heads", "Tails"), n, replace = TRUE)  
  C <- ifelse(flip == "Heads", 1, 0)  
  #plot(density(C), main = "")  
  B <- sample(1:6, n, replace=TRUE, prob = c(1/9, 1/9, 1/9, 1/9, 1/9, 1/9))  
  #plot(density(B), main = "")  
  R <- sample(1:6, n, replace=TRUE, prob = c(2/9, 1/9, 2/9, 1/9, 2/9, 1/9))  
  #plot(density(R), main = "")  
  Z <- C*(B+R)  
  return(Z)  
}  
  
par(mfrow = c(1,2))  
hist(simulate_random_variable(1000), col = c("tomato", "red", "pink", "orange", "yellow", "lightgreen",  
  main = "Histogram")  
plot(density(simulate_random_variable(1000)), col="steelblue", main = "Density")
```



`simulate_random_variable(1000)`



`N = 1000 Bandwidth = 0.8687`

Now, let us compute the probability that the random variable Z gets a value greater than 1. The following code achieves this purpose-

```
N <- 1000
Z <- simulate_random_variable(N)
sum(Z > 1)/N
```

```
## [1] 0.489
```