Bio 723

Scientific Computing for Biologists

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1 Getting your feet wet with R

1.1 Getting Acquainted with R

1.1.1 Installing R

The R website is at http://www.r-project.org/. I recommend that you spend a few minutes checking out the resources, documentation, and links on this page. Download the appropriate R installer for your computer from the Comprehensive R Archive Network (CRAN). A direct link can be found at: http://cran.stat.ucla.edu/. As of mid August 2012 the latest R release is verison 2.15.1.

The R installer will install appropriate icons under the Start Menu (Windows) or Applications Folder (OS X). On OS X it will install two icons – "R" and "R64", corresponding to 32-bit and 64-bit versions of the executable. The 64 bit version, which allows access to much larger amounts of your comptuer's RAM, is suitable for dealing with very large data sets.

1.1.2 Starting and R Interactive Session

The OSX and Windows version of R provide a simple GUI interface for using R in interactive mode. When you start up the R GUI you'll be presented with a single window, the R console. See the your textbook, The Art of R Programming (AoRP) for a discussion of the difference between R's interactive and batch modes.

1.1.3 Accessing the Help System on R

R comes with fairly extensive documentation and a simple help system. You can access HTML versions of R documentation under the Help menu in the GUI. The HTML documentation also includes information on any packages you've installed. Take a few minutes to browse through the R HTML documentation.

The help system can be invoked from the console itself using the help function or the ? operator.

- > help(length)
- > ?length
- > ?loa

What if you don't know the name of the function you want? You can use the help. search() function.

> help.search("log")

In this case help.search("log") returns all the functions with the string 'log' in them. For more on help.search type ?help.search. Other useful help related functions include apropos() and example().

1.1.4 Navigating Directories in R

When you start the R environment your 'working directory' (i.e. the directory on your computer's file system that R currently 'sees') defaults to a specific directory. On Windows this is usually the same directory that R is installed in, on OS X it is typically your home directory. Here are examples showing how you can get information about your working directory and change your working directory.

```
> getwd()
[1] "/Users/pmagwene"
> setwd("/Users")
> getwd()
[1] "/Users"
```

Note that on Windows you can change your working directory by using the Change dir... item under the File menu, while the corresponding item is found under the Misc menu on OS X.

To get a list of the file in your current working directory use the list.files() function.

```
> list.files()
[1] "Shared" "pmagwene"
```

1.1.5 Using R as a Calculator

The simplest way to use R is as a fancy calculator.

```
> 10 + 2 # addition
[1] 12
> 10 - 2 # subtraction
[1] 8
> 10 * 2 # multiplication
[1] 20
> 10 / 2 # division
[1] 5
> 10 ^ 2 # exponentiation
[1] 100
> 10 ** 2 # alternate exponentiation
[1] 100
> sqrt(10) # square root
[1] 3.162278
> 10 ^ 0.5 # same as square root
Γ17 3.162278
> exp(1) # exponential function
Γ17 2.718282
```

```
> 3.14 * 2.5^2
[1] 19.625
> pi * 2.5^2 # R knows about some constants such as Pi
[1] 19.63495
> cos(pi/3)
[1] 0.5
> sin(pi/3)
[1] 0.8660254
> log(10)
Γ17 2.302585
> log10(10) # log base 10
Γ17 1
> log2(10) # log base 2
[1] 3.321928
> (10 + 2)/(4-5)
[1] -12
> (10 + 2)/4-5 # compare the answer to the above
Γ11 -2
```

Be aware that certain operators have precedence over others. For example multiplication and division have higher precedence than addition and subtraction. Use parentheses to disambiguate potentially confusing statements.

```
> sqrt(pi)
[1] 1.772454
> sqrt(-1)
[1] NaN
Warning message:
NaNs produced in: sqrt(-1)
> sqrt(-1+0i)
[1] 0+1i
```

What happened when you tried to calculate sqrt(-1)?, -1 is treated as a real number and since square roots are undefined for the negative reals, R produced a warning message and returned a special value called NaN (Not a Number). Note that square roots of negative complex numbers are well defined so sqrt(-1+0i) works fine.

```
> 1/0
[1] Inf
```

Division by zero produces an object that represents infinite numbers.

1.1.6 Comparison Operators

You've already been introduced to the most commonly used arithmetic operators. Also useful are the comparison operators:

```
> 10 < 9  # less than
[1] FALSE
> 10 > 9  # greater than
[1] TRUE
```

```
> 10 <= (5 * 2) # less than or equal to
[1] TRUE
> 10 >= pi # greater than or equal to
[1] TRUE
> 10 == 10 # equals
[1] TRUE
> 10 != 10 # does not equal
[1] FALSE
> 10 == (sqrt(10)^2) # Surprised by the result? See below.
[1] FALSE
> 4 == (sqrt(4)^2) # Even more confused?
[1] TRUE
```

Comparisons return boolean values. Be careful to distinguish between == (tests equality) and = (the alternative assignment operator equivalent to <-).

How about the last two statement comparing two values to the square of their square roots? Mathematically we know that both $(\sqrt{10})^2=10$ and $(\sqrt{4})^2=4$ are true statements. Why does R tell us the first statement is false? What we're running into here are the limits of computer precision. A computer can't represent $\sqrt{10}$ exactly, whereas $\sqrt{4}$ can be exactly represented. Precision in numerical computing is a complex subject and beyond the scope of this course. Later in the course we'll discuss some ways of implementing sanity checks to avoid situations like that illustrated above.

1.1.7 Working with Vectors in R

Vectors are the core data structure in R. Vectors store an ordered list of items all of the same type. Learning to compute effectively with vectors and one of the keys to efficient R programming. Vectors in R always have a length (accessed with the length() function) and a type (accessed with the typeof() function).

The simplest way to create a vector at the interactive prompt is to use the c() function, which is short hand for 'combine' or 'concatenate'.

```
> x <- c(2,4,6,8)
[1] "double"
> length(x)
[1] 4
> y <- c('joe','bob','fred')
> typeof(y)
[1] "character"
> length(y)
[1] 3
> z <- c() # empty vector
> length(z)
[1] 0
> typeof(z)
[1] "NULL"
```

You can also use c() to concatenate two or more vectors together.

```
> v <- c(1,3,5,7)
> w <- c(-1, -2, -3)
> vwx <- c(v,w,x)
> vwx
[1] 1 3 5 7 -1 -2 -3 2 4 6 8
```

Vector Arithmetic and Comparison

The basic R arithmetic operations work on vectors as well as on single numbers (in fact single numbers *are* vectors).

```
> x < -c(2, 4, 6, 8, 10)
> x * 2
[1] 4 8 12 16 20
> x * pi
[1] 6.283185 12.566371 18.849556 25.132741 31.415927
> y < -c(0, 1, 3, 5, 9)
> X + Y
[1] 2 5 9 13 19
> x * y
[1] 0 4 18 40 90
> x/y
[1]
        Inf 4.000000 2.000000 1.600000 1.111111
> z < -c(1, 4, 7, 11)
> X + Z
Γ17 3 8 13 19 11
Warning message:
longer object length
       is not a multiple of shorter object length in: x + z
```

When vectors are not of the same length R 'recycles' the elements of the shorter vector to make the lengths conform. In the example above z was treated as if it was the vector (1, 4, 7, 11, 1).

The comparison operators also work on vectors as shown below. Comparisons involving vectors return vectors of booleans.

```
> x > 5
[1] FALSE FALSE TRUE TRUE
> x != 4
[1] TRUE FALSE TRUE TRUE TRUE
```

If you try and apply arithmetic operations to non-numeric vectors, R will warn you of the error of your ways:

```
> w <- c('foo', 'bar', 'baz', 'qux')
> w**2
Error in w^2 : non-numeric argument to binary operator
```

Note, however that the comparison operators can work with non-numeric vectors. The results you get will depend on the type of the elements in the vector.

```
> w == 'bar'
[1] FALSE TRUE FALSE FALSE
> w < 'cat'
[1] FALSE TRUE TRUE FALSE</pre>
```

Indexing Vectors

For a vector of length n, we can access the elements by the indices $1 \dots n$. We say that R vectors (and other data structures like lists) are 'one-indexed'. Many other programming languages, such as Python, C, and Java, use zero-indexing where the elements of a data structure are accessed by the indices $0 \dots n-1$. Indexing errors are a common source of bugs. When moving back and forth between different programming languages keep the appropriate indexing straight!

Trying to access an element beyond these limits returns a special constant called NA (Not Available) that indicates missing or non-existent values.

```
> x <- c(2, 4, 6, 8, 10)
> length(x)
[1] 5
> x[1]
[1] 2
> x[4]
[1] 8
> x[6]
[1] NA
> x[-1]
[1] 4 6 8 10
> x[c(3,5)]
[1] 6 10
```

Negative indices are used to exclude particular elements. x[-1] returns all elements of x except the first. You can get multiple elements of a vector by indexing by another vector. In the example above x[c(3,5)] returns the third and fifth element of x.

Combining Indexing and Comparison

A very powerful feature of R is the ability to combine the comparison operators with indexing. This facilitates data filtering and subsetting. Some examples:

```
> x <- c(2, 4, 6, 8, 10)

> x[x > 5]

[1] 6 8 10

> x[x < 4 | x > 6]

[1] 2 8 10
```

In the first example we retrieved all the elements of x that are larger than 5 (read as 'x where x is greater than 5'). In the second example we retrieved those elements of x that were smaller than four *or* greater than six. The symbol | is the 'logical or' operator. Other logical operators include & ('logical and' or 'intersection') and !

(negation). Combining indexing and comparison is a powerful concept and one you'll probably find useful for analyzing your own data.

Generating Regular Sequences

Creating sequences of numbers that are separated by a specified value or that follow a particular patterns turns out to be a common task in programming. R has some built-in operators and functions to simplify this task.

```
> s <- 1:10
> s
[1] 1 2 3 4 5 6 7 8 9 10
> s <- 10:1
> s
[1] 10 9 8 7 6 5 4 3 2 1
> s <- seq(0.5,1.5,by=0.1)
> s
[1] 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
# 'by' is the 3rd argument so you don't have to specify it
> s <- seq(0.5, 1.5, 0.33)
> s
[1] 0.50 0.83 1.16 1.49
```

rep() is another way to generate patterned data.

```
> rep(c("Male","Female"),3)
[1] "Male"    "Female" "Male"    "Female"
> rep(c(T,T, F),2)
[1] TRUE TRUE FALSE TRUE TRUE FALSE
```

1.1.8 Some Useful Functions

You've already seem a number of functions (c(), length(), sin(), log, length(), etc). Functions are called by invoking the function name followed by parentheses containing zero or more *arguments* to the function. Arguments can include the data the function operates on as well as settings for function parameter values. We'll discuss function arguments in greater detail below.

Creating longer vectors

For vectors of more than 10 or so elements it gets tiresome and error prone to create vectors using c(). For medium length vectors the scan() function is very useful.

```
> test.scores <- scan()
1: 98 92 78 65 52 59 75 77 84 31 83 72 59 69 71 66
17:
Read 16 items
> test.scores
[1] 98 92 78 65 52 59 75 77 84 31 83 72 59 69 71 66
```

When you invoke scan() without any arguments the function will read in a list of values separated by white space (usually spaces or tabs). Values are read until scan() encounters a blank line or the end of file (EOF) signal (platform dependent). We'll see how to read in data from files below.

Note that we created a variable with the name test.scores. If you have previous programming experience you might be surprised that this works. Unlike most languages, R allows you to use periods in variable names. Descriptive variable names generally improve readability but they can also become cumbersome (e.g. my.long. and.obnoxious.variable.name). As a general rule of thumb use short variable names when working at the interpreter and more descriptive variable names in functions.

Useful Numerical Functions

Let's introduce some additional numerical functions that are useful for operating on vectors.

```
> sum(test.scores)
[1] 1131
> min(test.scores)
[1] 31
> max(test.scores)
[1] 98
> range(test.scores) # min,max returned as a vec of len 2
[1] 31 98
> sorted.scores <- sort(test.scores)
> sorted.scores
[1] 31 52 59 59 65 66 69 71 72 75 77 78 83 84 92 98
> w <- c(-1, 2, -3, 3)
> abs(w) # absolute value function
```

1.1.9 Function Arguments in R

Function arguments can specify the data that a function operates on or parameters that the function uses. Some arguments are required, while others are optional and are assigned default values if not specified.

Take for example the log() function. If you examine the help file for the log() function (type ?log now) you'll see that it takes two arguments, referred to as 'x' and 'base'. The argument x represents the numeric vector you pass to the function and is a required argument (see what happens when you type log() without giving an argument). The argument base is optional. By default the value of base is e=2.71828... Therefore by default the log() function returns natural logarithms. If you want logarithms to a different base you can change the base argument as in the following examples:

```
> log(2) # log of 2, base e
[1] 0.6931472
> log(2,2) # log of 2, base 2
```

```
[1] 1 > log(2, 4) # log of 2, base 4 [1] 0.5
```

Because base 2 and base 10 logarithms are fairly commonly used, there are convenient aliases for calling log with these bases.

```
> log2(8)
[1] 3
> log10(100)
[1] 2
```

1.1.10 Lists in R

R lists are like vectors, but unlike a vector where all the elements are of the same type, the elements of a list can have arbitrary types (even other lists).

```
> 1 <- list('Bob', pi, 10, c(2,4,6,8))
```

Indexing of lists is different than indexing of vectors. Double brackets (x[[i]]) return the element at index i, single bracket return a list containing the element at index i.

```
> |[1] # single brackets
[[1]]
[1] "Bob"

> |[[1]] # double brackets
[1] "Bob"
> typeof(|[1])
[1] "list"
> typeof(|[[1]])
[1] "character"
```

The elements of a list can be given names, and those names objects can be accessed using the \$ operator. You can retrieve the names associated with a list using the names() function.

```
> 1 <- list(name='Bob', age=27, years.in.school=10)
> 1
$name
[1] "Bob"

$age
[1] 27

$years.in.school
[1] 10
> 1$years.in.school
[1] 10
> 1$name
```

```
[1] "Bob"
> names(1)
[1] "name" "age" "years.in.school"
```

1.1.11 Simple Input in R

The c() and scan() functions are fine for creating small to medium vectors at the interpreter, but eventually you'll want to start manipulating larger collections of data. There are a variety of functions in R for retrieving data from files.

The most convenient file format to work with are tab delimited text files. Text files have the advantage that they are human readable and are easily shared across different platforms. If you get in the habit of archiving data as text files you'll never find yourself in a situation where you're unable to retrieve important data because the binary data format has changed between versions of a program.

1.1.12 Using scan() to input data

scan() itself can be used to read data out of a file. Download the file algae.txt from the class website and try the following (after changing your working directory):

One of the things to be aware of when using scan() is that if the data type contained in the file can not be coerced to doubles than you must specify the data type using the what argument. The what argument is also used to enable the use of scan() with columnar data. Download algae2.txt and try the following:

```
[1] 0.530 0.183 0.603 0.994 0.708 0.006 0.867 0.059 0.349 0.699 0.983 [12] 0.100
```

Use help to learn more about scan().

1.1.13 Using read.table() to input data

read.table() (and it's derivates - see the help file) provides a more convenient interface for reading tabular data. Download the turtles.txt data set from the class wiki. The data in turtles.txt are a set of linear measurements representing dimensions of the carapace (upper shell) of painted turtles (*Chrysemys picta*), as reported in Jolicoeur and Mosimmann, 1960; Growth 24: 339-354.

Using the file turtles.txt:

```
> turtles <- read.table('turtles.txt', header=T)
> turtles
   sex length width height
1
    f
          98
                 81
                        38
2
          103
                 84
                        38
     f
          103
                 86
                        42
  # output truncated
> names(turtles)
            "length" "width" "height"
[1] "sex"
> length(turtles)
[1] 4
> length(turtles$sex)
[1] 48
```

What kind of data structure is turtles? What happens when you call the read.table() function without specifying the argument header=T?

You'll be using the read.table()}function frequently. Spend some time reading the documentation and playing around with different argument values (for example, try and figure out how to specify different column names on input).

Note: read.table() is more convenient but scan() is more efficient for large files. See the R documentation for more info.

1.1.14 Basic Statistical Functions in R

There are a wealth of statistical functions built into R. Let's start to put these to use. If you wanted to know the mean carapace width of turtles in your sample you could calculate this simply as follows:

```
> sum(turtles$width)/length(turtles$width)
[1] 95.4375
```

Of course R has a built in mean() function.

```
mean(turtles$width) [1] 95.4375
```

One of the advantages of the built in mean() function is that it knows how to operate on lists as well as vectors:

```
> mean(turtles)
    sex length width height
    NA 124.68750 95.43750 46.33333
Warning message:
argument is not numeric or logical: returning NA in: mean.default(X[[1]],
    ...)
```

Can you figure out why the above produced a warning message? Let's take a look at some more standard statistical functions:

```
> min(turtles$width)
Γ17 74
> max(turtles$width)
[1] 132
> range(turtles$width)
[1] 74 132
> median(turtles$width)
Γ17 93
> summary(turtles$width)
   Min. 1st Qu.
                Median Mean 3rd Qu.
                                          Max.
 74.00 86.00
                 93.00
                         95.44 102.00
                                        132.00
> var(turtles$width) # variance
Γ17 160.6769
> sd(turtles$width) # standard deviation
[1] 12.67584
```

1.2 Exploring Univariate Distributions in R

1.2.1 Histograms

One of the most common ways to examine a the distribution of observations for a single variable is to use a histogram. The hist() function creates simple histograms in R.

```
hist(turtles$length) # create histogram with fxn defaults?hist # check out the documentation on hist
```

Note that by default the hist() function plots the frequencies in each bin. If you want the probability densities instead set the argument freq=FALSE.

```
> hist(turtles$length,freq=F) # y-axis gives probability density
```

Here's some other ways to fine tune a histogram in R.

```
> hist(turtles$length, breaks=12) # use 12 bins
> mybreaks = seq(85,185,8)
> hist(turtles$length, breaks=mybreaks) # specify bin boundaries
> hist(turtles$length, breaks=mybreaks, col='red') # fill the bins with red
```

1.2.2 Density Plots

One of the problems with histograms is that they can be very sensitive to the size of the bins and the break points used. You probably noticed that in the example above as we changes the number of bins and the breakpoints to generate the histograms for the turtles\$length variable. This is due to the discretization inherent in a histogram. A 'density plot' or 'density trace' is a continuous estimate of a probability distribution from a set of observations. Because it is continuous it doesn't suffer from the same sensitivity to bin sizes and break points. One way to think about a density plot is as the histogram you'd get if you averaged many individual histograms each with slightly different breakpoints.

```
> d <- density(turtles$length)
> plot(d)
```

A density plot isn't entirely parameter free – the parameter you should be most aware of is the 'smoothing bandwidth'.

```
> d <- density(turtles$length) # let R pick the bandwidth
> plot(d,ylim=c(0,0.020)) # gives ourselves some extra headroom on y-axis
> d2 <- density(turtles$length, bw=5) # specify bandwidth
> lines(d2, col='red') # use lines to draw over previous plot
```

The bandwidth determines the standard deviation of the 'kernel' that is used to calculate the density plot. There are a number of different types of kernels you can use; a Gaussian kernel is the R default and is the most common choice. In the example above, R picked a bandwidth of 8.5 (the black line in our plot). When we specified a smaller bandwith of 5, the resulting density plot (red) is less smooth. There exists a statistical literature on picking 'optimum' kernel sizes. In general, larger data sets support the use of smaller kernels. See the R documentation for more info on the density() function and references to the literature on density estimators.

The lattice package is an R library that makes it easier to create graphics that show conditional distributions. Here's how to create a simple density plot using the lattice package.

```
> library(lattice)
> densityplot(turtles$length) # densityplot defined in lattice
```

Notice how by default the lattice package also drew points representing the observations along the x-axis. These points have been 'jittered' meaning they've been randomly shifted by a small amount so that overlapping points don't completely hide each other. We could have produced a similar plot, without the lattice package, as so:

```
> d <- density(turtles$length)
> plot(d)
> nobs <- length(turtles$length)
> points(jitter(turtles$length), rep(0,nobs))
```

Notice that in our version we only jittered the points along the x-axis. You can also combine a histogram and density trace, like so:

```
> hist(turtles$length, 10, xlab='Carapace Length (mm)',freq=F)
```

```
> d <- density(turtles$length)
> lines(d, col='red', lwd=2) # red lines, with pixel width 2
```

Notice the use of the freq=F argument to scale the histogram bars in terms of probability density.

Finally, let's some of the features of lattice to produce density plots for the 'length' variable of the turtle data set, conditional on sex of the specimen.

```
> densityplot(~length | sex, data = turtles)
```

There are a number of new concepts here. The first is that we used what is called a 'formula' to specify what to plot. In this case the formula can be read as 'length conditional on sex'. We'll be using formulas in several other contexts and we discuss them at greater length below. The data argument allows us to specify a data frame or list so that we don't always have to write arguments like turtles\$length or turtles\$sex which can get a bit tedious.

1.2.3 Box Plots

Another common tool for depicting a univariate distribution is a 'box plot' (sometimes called a box-and-whisker plot). A standard box plot depicts five useful features of a set of observations: the median (center most line), the upper and lower quartiles (top and bottom of the box), and the minimum and maximum observations (ends of the whiskers).

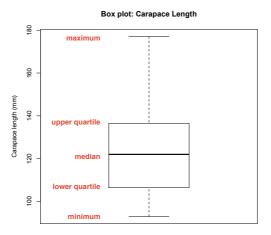


Figure 1.1: A box plot represents a five number summary of a set of observations.

There are many variants on box plots, particularly with respect to the 'whiskers'. It's always a good idea to be explicit about what a box plot you've created depicts.

Here's how to create box plots using the standard R functions as well as the lattice package:

> boxplot(turtles\$length)

- > boxplot(turtles\$length, col='darkred', horizontal=T) # horizontal version
- > title(main = 'Box plot: Carapace Length', ylab = 'Carapace length (mm)')
- > bwplot(~length,data=turtles) # using the bwplot function from lattice

Note how we used the title() function to change the axis labels and add a plot title.

Historical note - The box plot is one of many inventions of the statistician John W. Tukey. Tukey made many contributions to the field of statistics and computer science, particularly in the areas of graphical representations of data and exploratory data analysis.

1.2.4 Bean Plots

My personal favorite way to depict univariate distributions is called a 'beanplot'. Beanplots combine features of density plots and boxplots and provide information rich graphical summaries of single variables. The standard features in a beanplot include the individual observations (depicted as lines), the density trace estimated from the observations, the mean of the observations, and in the case of multiple beanplots an overall mean.



Figure 1.2: Beanplots combine features of density and box plots.

The beanplot package is not installed by default. To download it and install it use the R package installer under the Packages & Data menu. If this is the first time you use the package installer you'll have to choose a CRAN repository from which to download package info (I recommend you pick one in the US). Once you've done so you can search for 'beanplot' from the Package Installer window. You should also check the 'install dependencies' check box.

Once the beanplot package has been installed check out the examples to see some of the capabilities:

> library(beanplot)

Note the use of the library() function to make the functions in the beanplot library available for use. Here's some examples of using the beanplot function with the turtle data set:

```
> beanplot(turtles$length) # note the message about log='y'
> beanplot(turtles$length, log='') # DON'T do the automatic log transform
> beanplot(turtles$length, log='', col=c('white','blue','blue','red'))
```

In the final version we specified colors for the parts of the beanplot. See the explanation of the col argument int he beanplot function for details.

We can also compare the carapace length variable for male and female turtles.

```
> beanplot(length ~ sex, data = turtles, col=list(c('red'),c('black')),
names = c('females','males'),xlab='Sex', ylab='Caparace length (mm)')
```

Note the use of the formula notation to compare the carapace length variable for males and females. Note the use of the list argument to col, and the use of vectors within the list to specify the colors for female and male beauplots.

There is also a asymmetrical version of the beanplot which can be used to more directly compare distributions between two groups. This can be specified by using the argument side='both' to the beanplot function.

```
> beanplot(length~sex, data=turtles, col=list(c('red'),c('black')),names=c(
    'females','males'),xlab='Sex', ylab='Carapace length (mm)',side='both')
```

Plots like this one are very convenient for comparing distributions between samples grouped by treatment, sex, species, etc.

We can also create a beauplot with multiple variables in the same plot if the variables are measured on the same scale.

```
> beanplot(turtles$length, turtles$width, turtles$height, log='',
names=c('length','width','height'), ylab='carapace dimensions (mm)')
```

1.2.5 Demo Plots in R

To get a sense of some of the graphical power of R try the demo() function:

```
> demo(graphics)
```

2 Vector Operations and Exploring Bivariate Relationships in R

2.1 Vector Operations in R

As you saw last week R vectors support basic arithmetic operations that correspond to the same operations on geometric vectors. For example:

R also has an operator for the dot product, denoted %*%. This operator also designates matrix multiplication, which we will discuss next week. By default this operator returns an object of the R matrix class. If you want a scalar (or the R equivalent of a scalar, i.e. a vector of length 1) you need to use the drop() function.

```
> z <- x %*% x
> class(z)  # note use of class() function
[1] "matrix"
> z
      [,1]
[1,] 1240
> drop(z)
[1] 1240
```

In lecture we saw that many useful geometric properties of vectors could be expressed in the form of dot products. Let's start with some two-dimensional vectors where the geometry is easy to visualize:

```
> a <- c(1, 0) # the point (1,0)
> b <- c(0, 1) # the point (0,1)
```

Now let's draw our vectors:

```
# create empty plot w/specified x- and y- limits
# the 'asp=1' argument maintains the scaling of the x- and y-axes
# so that units are equivalent for both axes (i.e. squares remain squares)
> plot(c(-2,2),c(-1,2),type='n', asp=1)

# draw an arrow from origin (0,0) to x,y coordinates of vector "a"
# the length argument changes the size of the arrowhead
# use the R help to read more about the arrows function
> arrows(0, 0, a[1], a[2], length=0.1)

# and now for the vector "b"
> arrows(0, 0, b[1], b[2], length=0.1)
```

You should now have a figure that looks like the one below: Let's see what the dot

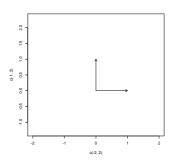


Figure 2.1: A simple vector figure.

product can tell us about these vectors. First recall that we can calculate the length of a vector as the square-root of the dot product of the vector with itself $(|\vec{a}|^2 = \vec{a} \cdot \vec{a})$

```
> len.a <- drop(sqrt(a %*% a))
> len.a
[1] 1
> len.b <- drop(sqrt(b %*% b))</pre>
```

How about the angle between a and b?

```
> dot.ab <- a %*% b
> dot.ab
      [,1]
[1,] 0
> cos.ab <- (a %*% b)/(len.a * len.b)
> cos.ab
      [,1]
[1,] 0
```

A key point to remember dot product of two vectors is zero if, and only if, they are orthogonal to each other (regardless of their dimension).

Assignment 2.1

```
In R, use the dot product operator and the acos() function to calculate the angle (in radians) between the vectors x = [-3, -3, -1, -1, 0, 0, 1, 2, 2, 3] and y = [-8, -5, -3, 0, -1, 0, 5, 1, 6, 5].
```

2.2 Writing Functions in R

So far we've been mostly using R's built in functions. However the power of a true programming language is the ability to write your own functions.

The general form of an R function is as follows:

```
funcname <- function(arg1, arg2) {
  # one or more expressions
  # last expression is the object returned
  # or you can explicitly return an object
}</pre>
```

To make this concrete, here's an example where we define a function in the interpreter and then put it to use:

```
> myfunc <- function(x,y){
+ # don't type the '+' symbols, these show continuation lines
+ x^2 + y^2
+ }
> a <- 1:5
> b <- 6:10
> a
[1] 1 2 3 4 5
> b
[1] 6 7 8 9 10
> myfunc(a,b)
[1] 37 53 73 97 125
> myfunc
function(x,y){
    x^2 + y^2
}
```

If you type a function name without parentheses R shows you the function's definition. This works for built-in functions as well (thought sometimes these functions are defined in C code in which case R will tell you that the function is a '.Primitive').

2.2.1 Putting R functions in Scripts

When you define a function at the interactive prompt and then close the interpreter your function definition will be lost. The simple way around this is to define your R functions in a script that you can than access at any time.

Choose File > New Script (or File > New Document in OS X) in the R GUI . This will bring up a blank editor window. Enter your function into the editor and save the source file in your R working directory with a name like vecgeom.R.

```
# functions defined in vecgeom.R

veclength <- function(x) {
    # Given a numeric vector, returns length of that vector
    sqrt(drop(x %*% x))
}

unitvector <- function(x) {
    # Return a unit vector in the same direction as x
    x/veclength(x)
}

vec.cos <- function(x,y) {
    # Calculate the cos of the angle between vectors x and y
    len.x <- veclength(x)
    len.y <- veclength(y)
    return( (x %*% y)/(len.x * len.y) )
}</pre>
```

There are two functions defined above, one of which calls the other. Both take single vector arguments. At this point there is no error checking to insure that the argument is reasonable but R's built in error handling will do just fine for now.

Once your functions are in a script file you can make them accessible by using the source() function (See also the File > Source R code... menu item in the R GUI):

```
> source("vecgeom.R")
> x <- c(1,0.4)
> veclength(x)
[1] 1.077033
> ux <- unitvector(x)
> ux
[1] 0.9284767 0.3713907
> veclength(ux)
[1] 1
```

Let's also add the following function to vecgeom.R to aid in visualizaing 2D vectors:

```
draw.vectors <- function(a, b, colors=c('red', 'blue'), clear.plot=TRUE){
    # figure out the limits such that the origin and the vector
    # end points are all included in the plot
    xhi <- max(0, a[1], b[1])
    xlo <- min(0, a[1], b[1])
    yhi <- max(0, a[2], b[2])
    ylo <- min(0, a[2], b[2])

xlims <- c(xlo, xhi)*1.10 # give a little breathing space around</pre>
```

```
vectors
ylims <- c(ylo, yhi)*1.10

if (clear.plot){
    plot(xlims, ylims, type='n', asp=1, xlab="x-coord", ylab="y-coord")
}
arrows(θ, θ, a[1], a[2], length=θ.1, col=colors[1])
arrows(θ, θ, b[1], b[2], length=θ.1, col=colors[2])
}</pre>
```

You can use this new function as follows:

```
# you need to source the file everytime you change it
> source("/Users/pmagwene/Downloads/vecgeom.R")
> x <- c(1,0.4)
> y <- c(0.2, 0.8)
> draw.vectors(x,y) # draw the original vectors
```

The resulting figure should resemble the one below.

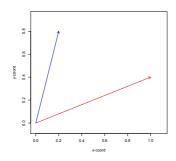


Figure 2.2: Another vector figure.

Notice that we included a clear.plot argument in our draw.vectors function. I included this so we could add additional vectors to our plot, without overwriting the old vectors, as demonstrated below:

```
# draw the unit vectors that point in the same directors as the original
    vectors
> ux <- unitvector(x)
> uy <- unitvector(y)
> draw.vectors(ux, uy, colors=c('black', 'green'), clear.plot=F)
```

Assignment 2.2

Write a function in R that takes two vectors, \vec{x} and \vec{y} , and returns a list containing the projection of \vec{y} on \vec{x} and the component of \vec{y} in \vec{x} :

$$P_{\vec{x}}(\vec{y}) = \left(\frac{\vec{x} \cdot \vec{y}}{|\vec{x}|}\right) \frac{\vec{x}}{|\vec{x}|}$$

and

$$C_{\vec{x}}(\vec{y}) = \frac{\vec{x} \cdot \vec{y}}{|\vec{x}|}$$

2.3 Exploring Bivariate Relationships in R

Let's use a dataset called iris (included in the standard R distribution) to explore bivariate relationships between variables. This data set was made famous by R. A. Fisher who used it to illustrate many of the fundamental statistical methods he developed. The data set consists of four morphometric measurements for specimens from three different iris species. Use the R help to read about the iris data set (?iris). We'll be using this data set repeatedly in future weeks so familiarize yourself with it.

```
> ?iris
> names(iris)
[1] "Sepal.Length" "Sepal.Width" "Petal.Length" "Petal.Width"
[5] "Species"
> unique(iris$Species)
[1] setosa versicolor virginica
Levels: setosa versicolor virginica
> dim(iris)
[1] 150 5
```

For now let's just work with the *I. setosa* specimens. Read the help file for subset().

```
> setosa <- subset(iris, Species == 'setosa', select = -Species)
> dim(setosa)
[1] 50 4
> names(setosa)
[1] "Sepal.Length" "Sepal.Width" "Petal.Length" "Petal.Width"
```

Notice how we used the select argument to subset() in order to drop the Species column. Let's explore the setosa subset with some graphs.

```
> plot(setosa$Sepal.Length, setosa$Sepal.Width)
> plot(setosa$Sepal.Width ~ setosa$Sepal.Length)
```

Did you notice what is different between the two versions above? You can also use the data argument with plot, like so:

```
> plot(Sepal.Width ~ Sepal.Length, data = setosa)
```

The xyplot() function from the lattice package does pretty much the same thing:

```
> library(lattice)
> xyplot(Sepal.Width ~ Sepal.Length, data = setosa)
```

Let's also explore a number of the other bivariate relationships in this data set:

```
# an alternate way to generate such a plot, using the data argument to
    specify where the variables are defined
> plot(Petal.Length ~ Sepal.Length, data = setosa)

# same form as the first plot, but changing the character used for the plot
    using the 'pch' argument. The 'cex' argument increases the size of the
    characters by the specified factor (1.5x in this case)
> plot(setosa$Sepal.Length, setosa$Petal.Width, pch = 20, cex=1.5)
```

Often times it's useful to look at many bivariate relationships simultaneously. The pairs() function allows you to do this:

> pairs(setosa)

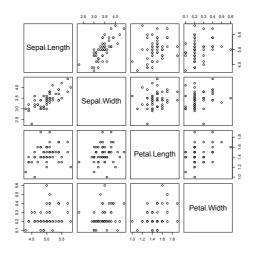


Figure 2.3: Output of the pairs() function for the *I. setosa* specimens in the iris dataset.

Let's return to our use of the dot product to explore the relationship between variables. First let's add a function to vecgeom.R to calculate the cosine of the angle between to vectors.

```
# add to vecgeom.R

vec.cos <- function(x,y) {
    # Calculate the cos of the angle between vectors x and y
    len.x <- veclength(x)
    len.y <- veclength(y)</pre>
```

```
return( (x %*% y)/(len.x * len.y) )
}
```

We can then use this function to examine the relationships between the variables in the setosa dataset. First we'll center the setosa dataset using the <code>scale()</code> function. <code>scale()</code> has two logical arguments center and <code>scale</code>. By default both are TRUE which will center <code>and</code> scale the variables. But for now we just want to center the data. <code>scale()</code> returns a matrix object sow we use the <code>data.frame</code> function to cast the object back to a data frame.

```
> source("/Users/pmagwene/Downloads/vecgeom.R")
> ctrd <- scale(setosa,center=T,scale=F)</pre>
> class(ctrd)
[1] "matrix"
> names(ctrd)
NULL
> ctrd <- data.frame(scale(setosa,center=T,scale=F))</pre>
> class(ctrd)
[1] "data.frame"
> names(ctrd)
[1] "Sepal.Length" "Sepal.Width" "Petal.Length" "Petal.Width"
> vec.cos(ctrd$Sepal.Length, ctrd$Sepal.Width)
[1,] 0.7425467
> vec.cos(ctrd$Sepal.Length, ctrd$Petal.Length)
          [,1]
[1,] 0.2671758
> vec.cos(ctrd$Sepal.Length, ctrd$Petal.Width)
          [,1]
[1,] 0.2780984
```

Consider the values above in the context of the scatter plots you generated with the pairs() function; and then recall that for mean-centered variables, $\operatorname{cor}(X,Y) = r_{XY} = \cos\theta = \frac{\bar{x} \cdot \bar{y}}{|\bar{x}||\bar{y}|}$. So our vec.cos() function, when applied to centered data, is equivalent to calculating the correlation between x and y. Let's confirm this using the built in $\operatorname{cor}()$ function in R:

```
> cor(setosa$Sepal.Length, setosa$Sepal.Width)
[1] 0.7425467
> cor(setosa) # called like this will calculate all pairwise correlations
            Sepal.Length Sepal.Width Petal.Length Petal.Width
Sepal.Length
               1.0000000
                           0.7425467
                                       0.2671758
                                                   0.2780984
Sepal.Width
               0.7425467
                           1.0000000
                                       0.1777000
                                                   0.2327520
               0.2671758
Petal.Length
                           0.1777000
                                       1.0000000
                                                   0.3316300
Petal.Width
               0.2780984
                           0.2327520
                                       0.3316300
                                                   1.0000000
```

2.3.1 Bivariate Regression in R

R has a flexible built in function, Im() for fitting linear models. Bivariate regression is the simplest case of a linear model.

```
> setosa.lm <- lm(Sepal.Width ~ Sepal.Length, data=setosa)</pre>
> class(setosa.lm)
[1] "lm"
> names(setosa.lm)
 [1] "coefficients"
                     "residuals"
                                     "effects"
                                                     "rank"
 [5] "fitted.values" "assign"
                                     "ar"
                                                     "df.residual"
 [9] "xlevels"
                                                     "model"
                     "call"
                                     "terms"
> summary(setosa.lm)
Call:
lm(formula = Sepal.Width ~ Sepal.Length, data = setosa)
Residuals:
     Min
               10
                    Median
                                 30
                                         Max
-0.72394 -0.18273 -0.00306 0.15738 0.51709
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)
             -0.5694
                          0.5217 -1.091
                                            0.281
                          0.1040 7.681 6.71e-10 ***
Sepal.Length
               0.7985
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 ''. 0.1 ''
Residual standard error: 0.2565 on 48 degrees of freedom
Multiple R-squared: 0.5514, Adjusted R-squared: 0.542
F-statistic: 58.99 on 1 and 48 DF, p-value: 6.71e-10
```

As demonstrated above, the summary() function spits out key diagnostic information about the model we fit. Now let's create a plot illustrating the fit of the model.

```
> plot(Sepal.Width ~ Sepal.Length, data=setosa, xlab="Sepal Length (cm)",
    ylab="Sepal Width (cm)", main="Iris setosa")
> abline(setosa.lm, col='red', lwd=2, lty=2) # see ?par for info about lwd
    and lty
```

Your output should resemble the figure below. Note the use of the function abline() to plot the regression line. Calling plot() with an object of class lm shows a series of diagnostic plots. Try this yourself.

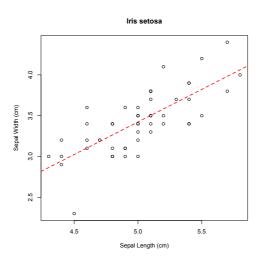


Figure 2.4: Linear regression of Sepal Width on Sepal Length for *I. setosa*.

Assignment 2.3

Write your own regression function (i.e. your code shouldn't refer to the built in regression functions) for mean centered vectors in R. The function will take as it's input two vectors, \vec{x} and \vec{y} . The function should return:

- 1. a list containing the mean-centered versions of these vectors
- 2. the regression coefficient b in the mean centered regression equation $\vec{\hat{y}} = b\vec{x}$
- 3. the coefficient of determination, R^2

Demonstrate your regression function by using it to carry out regressions of Sepal.Length on Sepal.Width separately for the 'versicolor' and 'virginica' specimens from the iris data set (again, subset() is your friend). Include plots in which you use the plot() and abline () functions to illustrate your calculated regression line.