

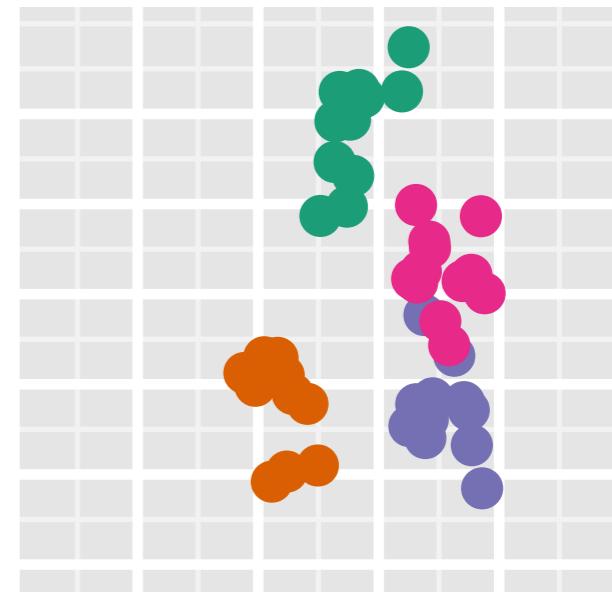
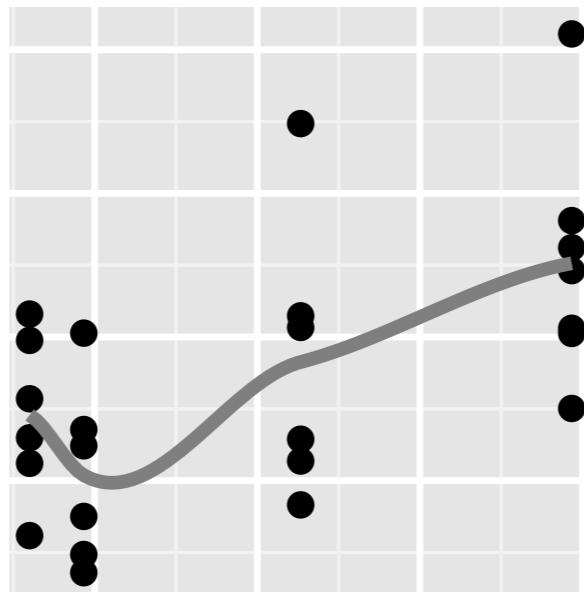
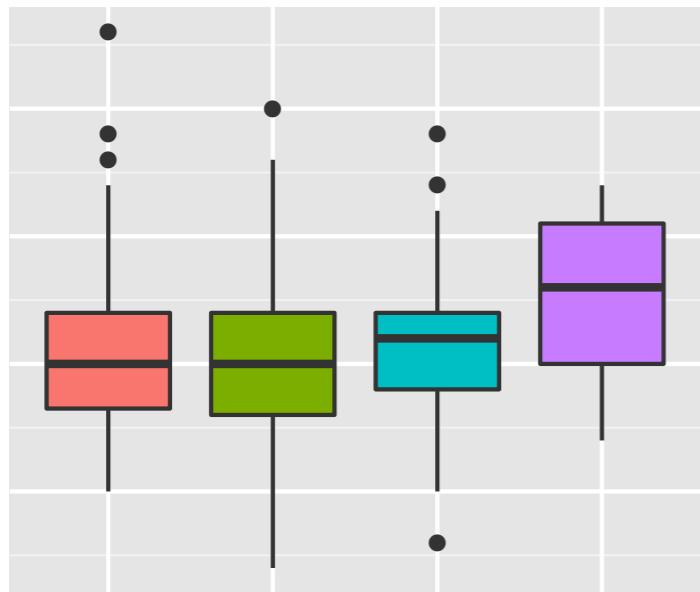


Statistics on Street Corners

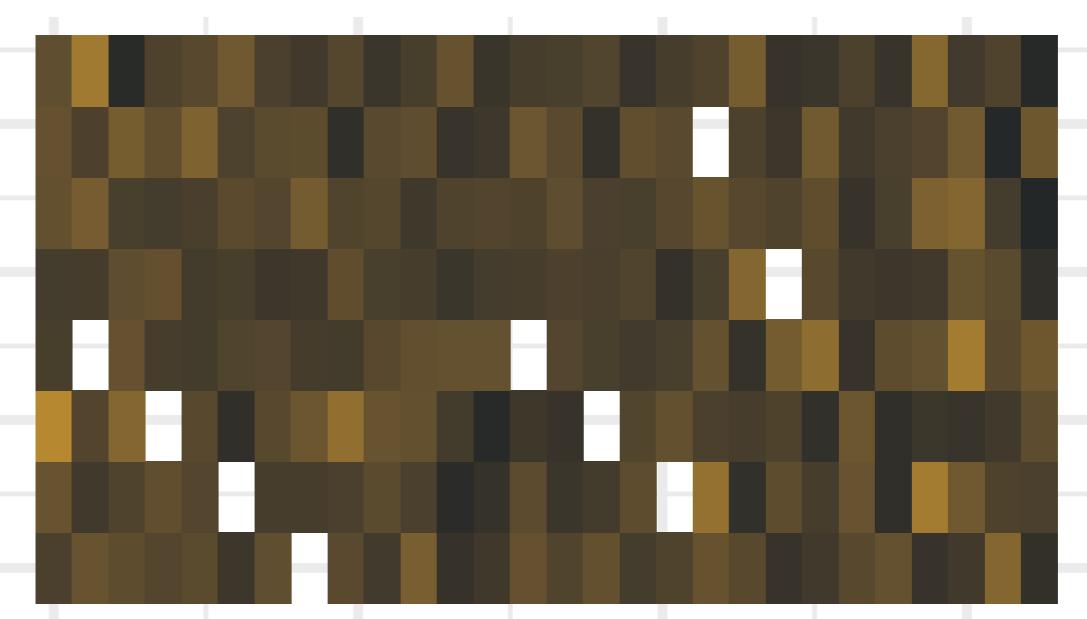
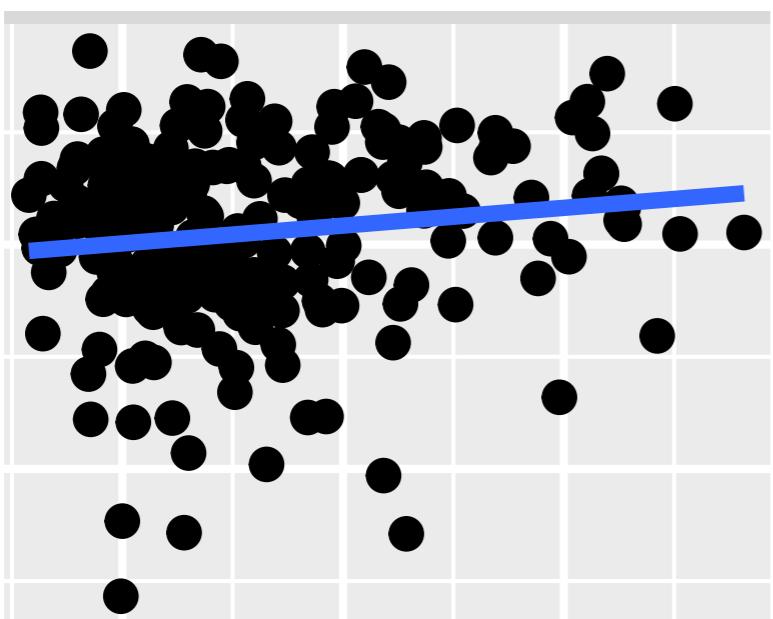
Using Crowd-Sourcing to Conduct Statistical Inference with
Exploratory Data Analysis

Di Cook
Econometrics and Business Statistics,
Monash University

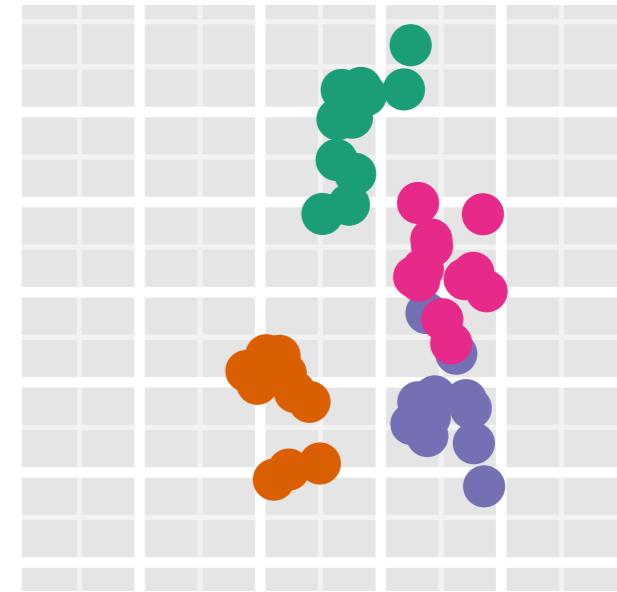
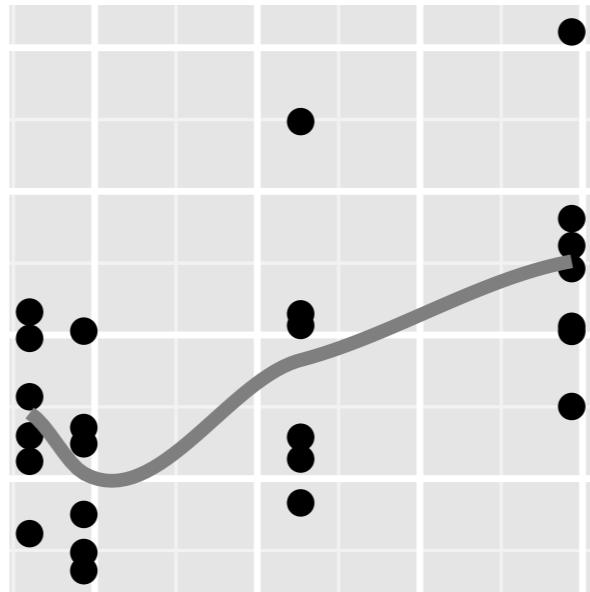
Joint work with Heike Hofmann, Mahbub Majumder,
Niladri Roy Chowdhury, Hadley Wickham, Andreas Buja, Debby
Swayne, Eun-kyung Lee, Yifan Zhao, Tengfei Yin, Lendie Follett,
Eric Hare, Adam Loy, Susan Vanderplas, Nathaniel Tomasetti



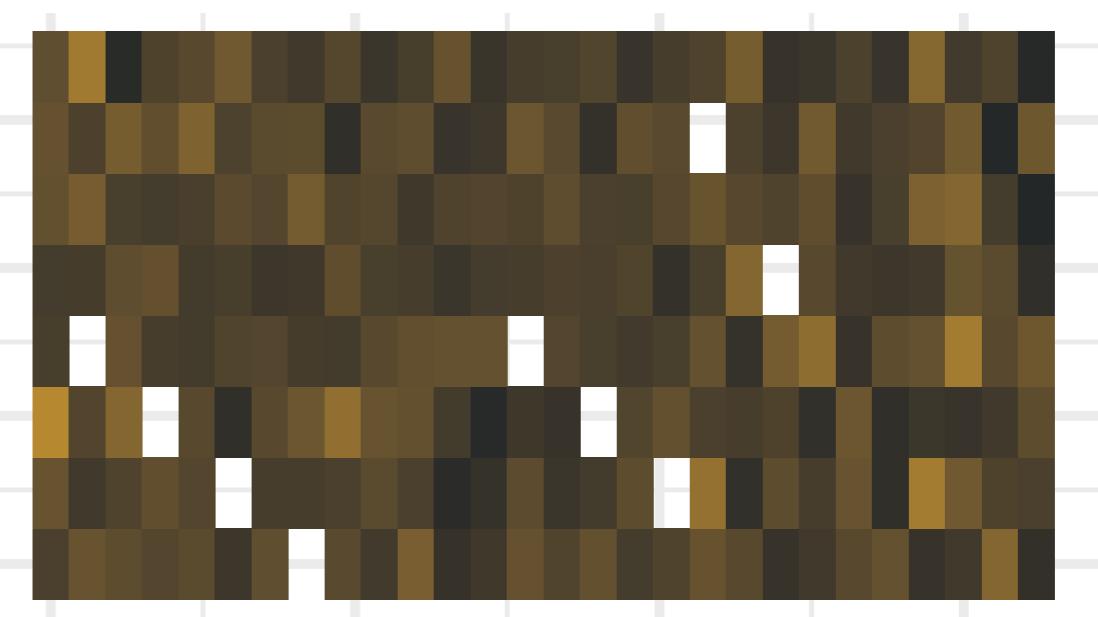
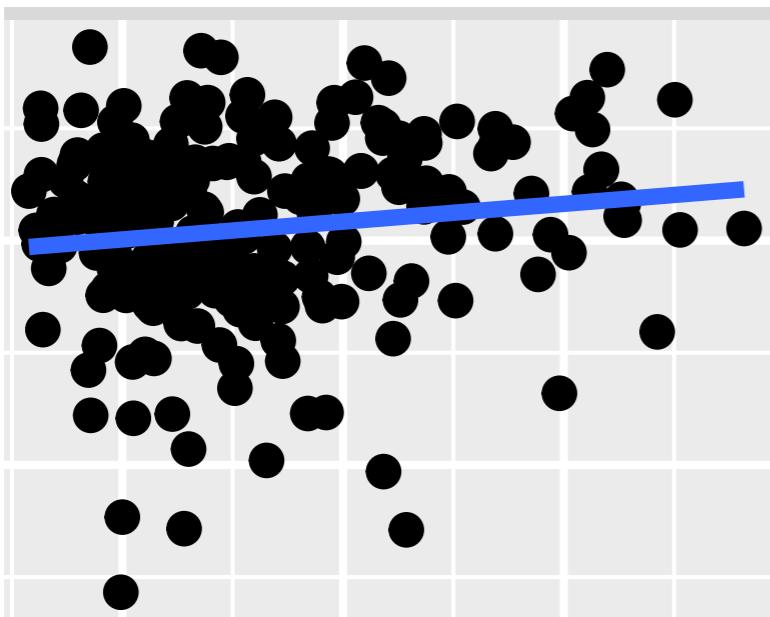
What do you see?



Purple group
has generally
bigger values
than the others



What do you see?

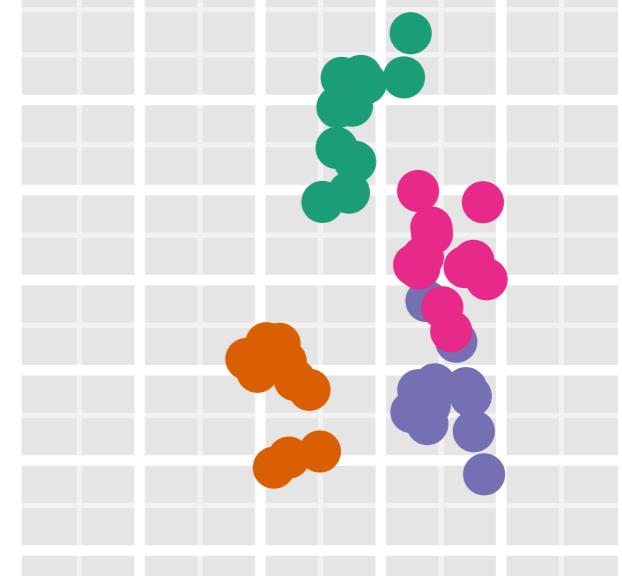


-

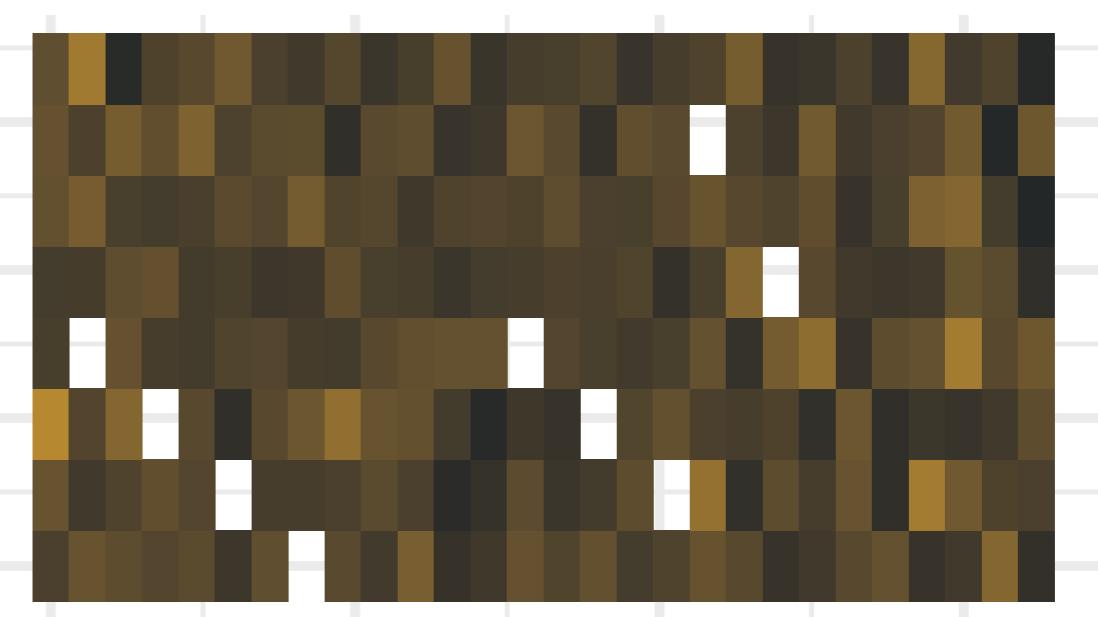
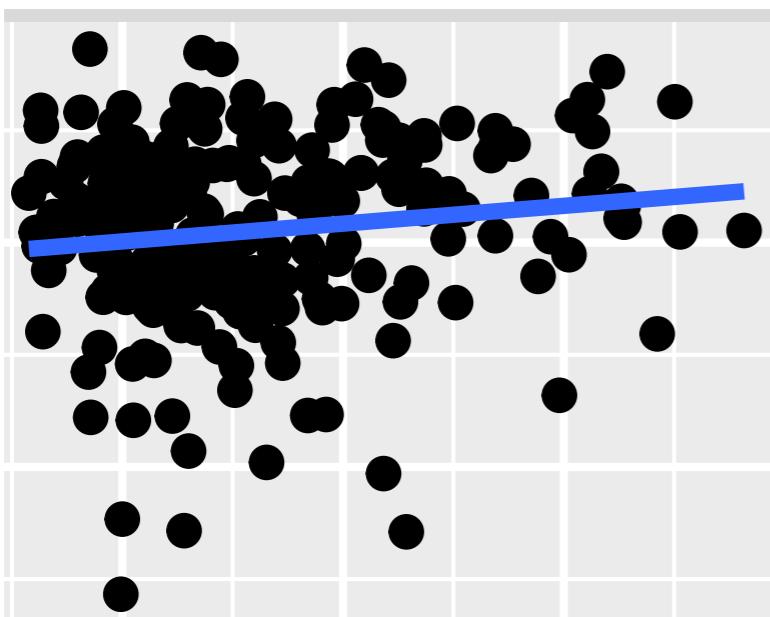
Purple group has generally bigger values than the others

-
- ;

Relationship between the two variables is positive



What do you see?

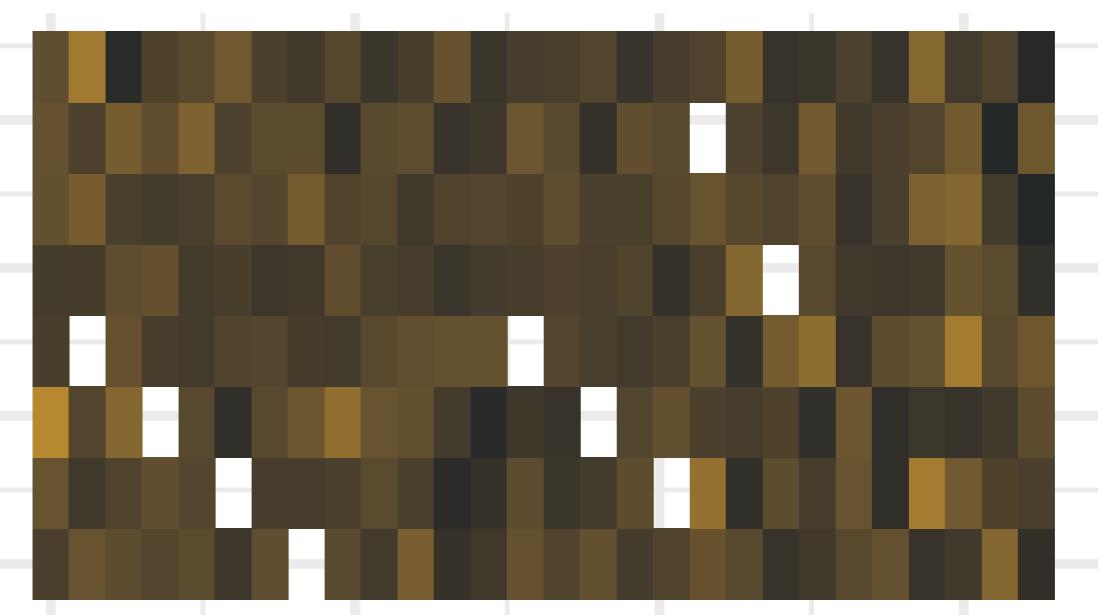
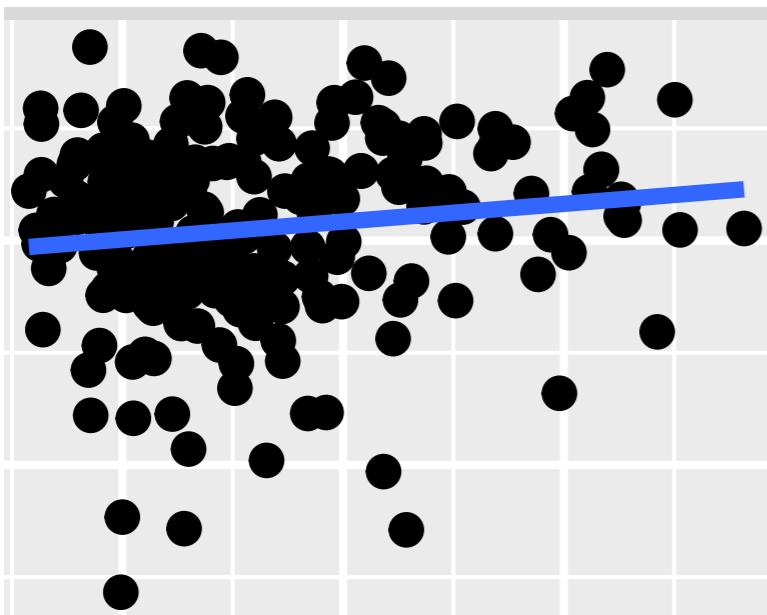


- Purple group has generally bigger values than the others

- Relationship between the two variables is positive

- Four separated groups, green and orange different from the other two

What do you see?

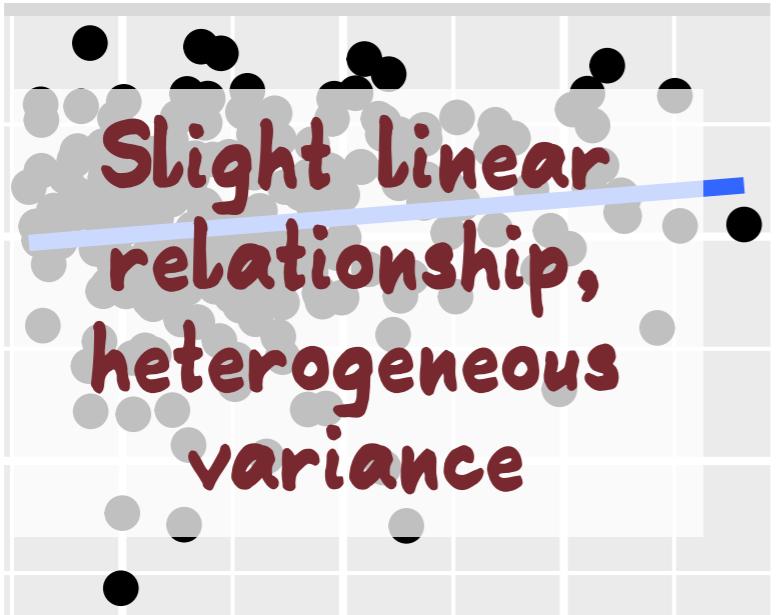


- Purple group has generally bigger values than the others

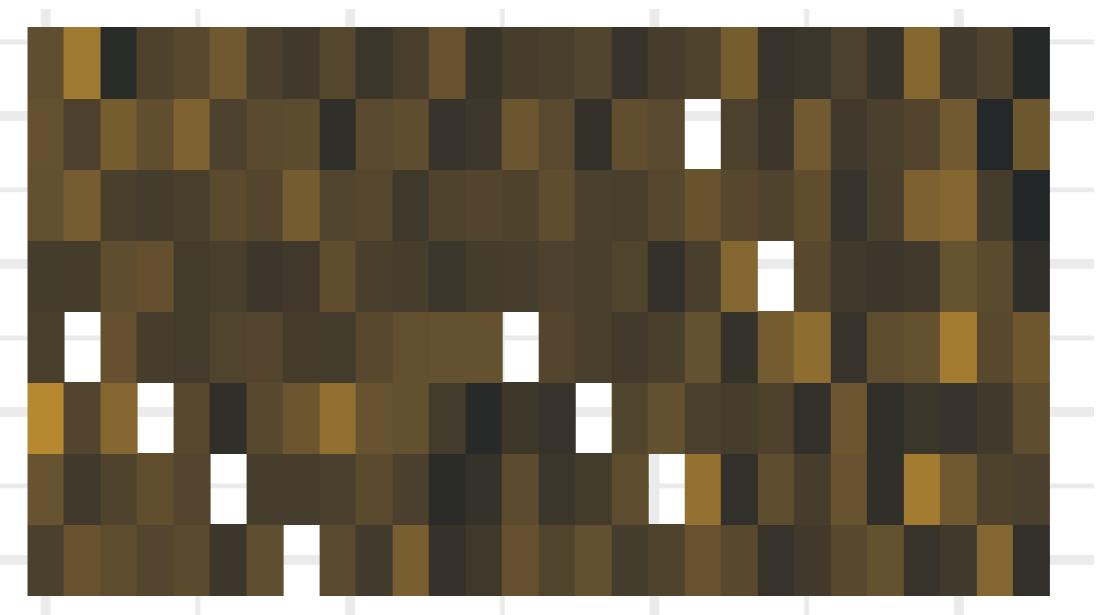
- Relationship between the two variables is positive

- Four separated groups, green and orange different from the other two

What do you see?



Slight linear relationship, heterogeneous variance



- Purple group has generally bigger values than the others

- Relationship between the two variables is positive

- Four separated groups, green and orange different from the other two

What do you see?

Slight linear relationship, heterogeneous variance

A scatter plot with a light gray grid. It contains many small gray dots and a few larger black dots. A blue line of best fit shows a slight upward trend.

Possibly spatial dependence

A pixelated heatmap with a grid of brown, tan, and white squares. A white rectangular box highlights a central area of the grid.

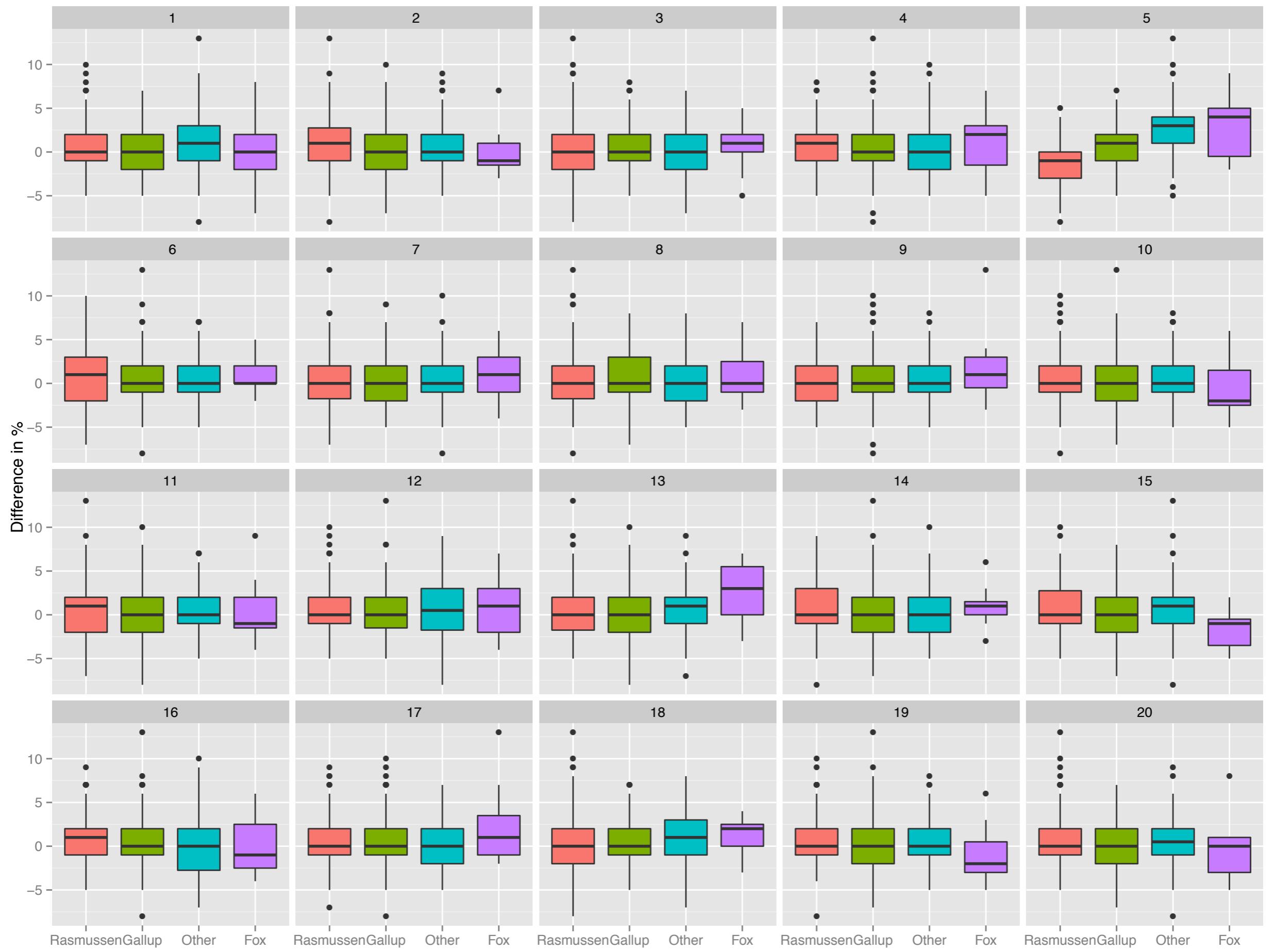
Statistical graphics

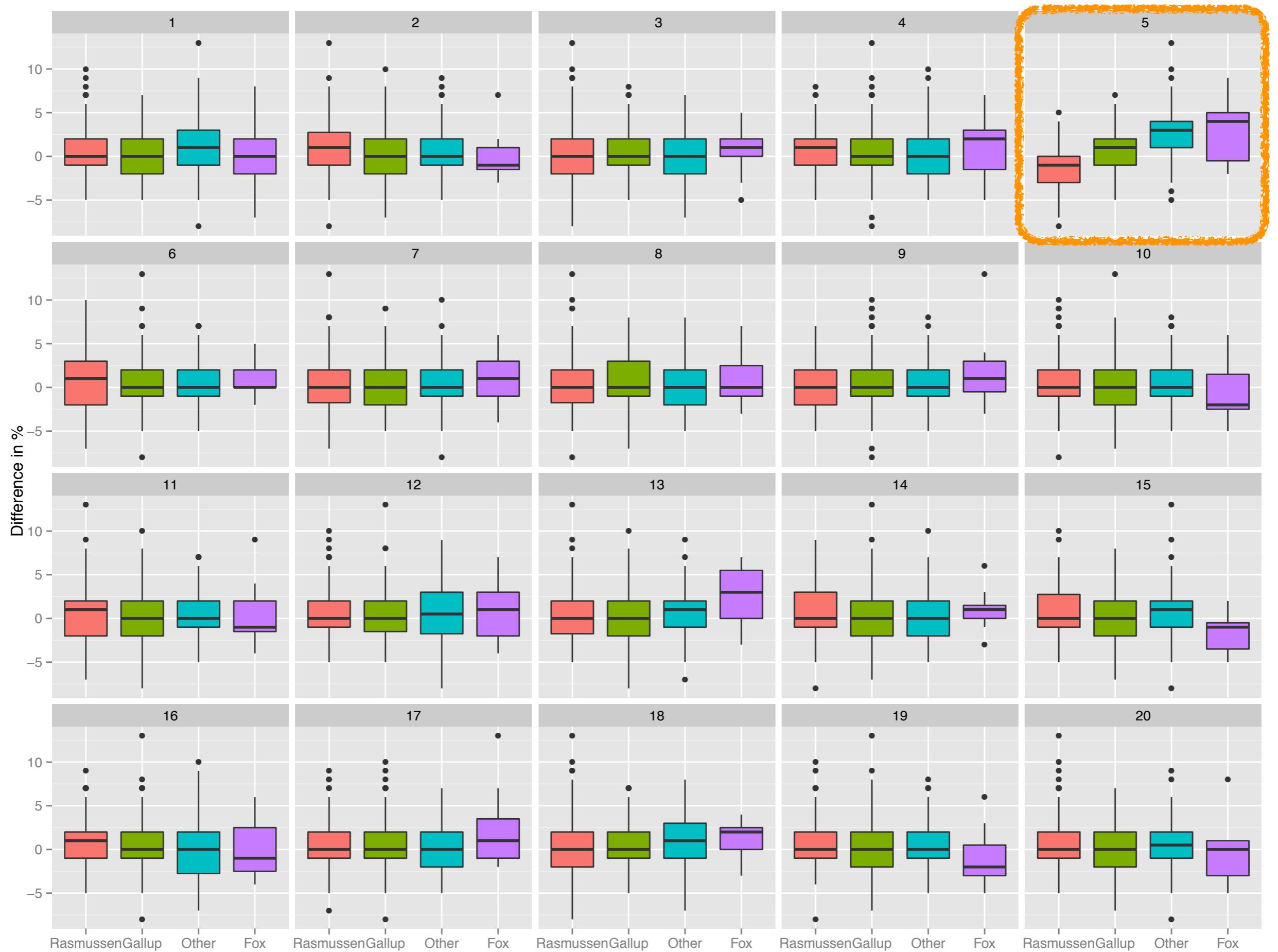
- Data plots are integral to our field
- Plots encourage curiosity and discovery when most of statistics is about being skeptical.
- This work bridges the gap between inferential and exploratory statistics, permitting curiosity with skepticism.

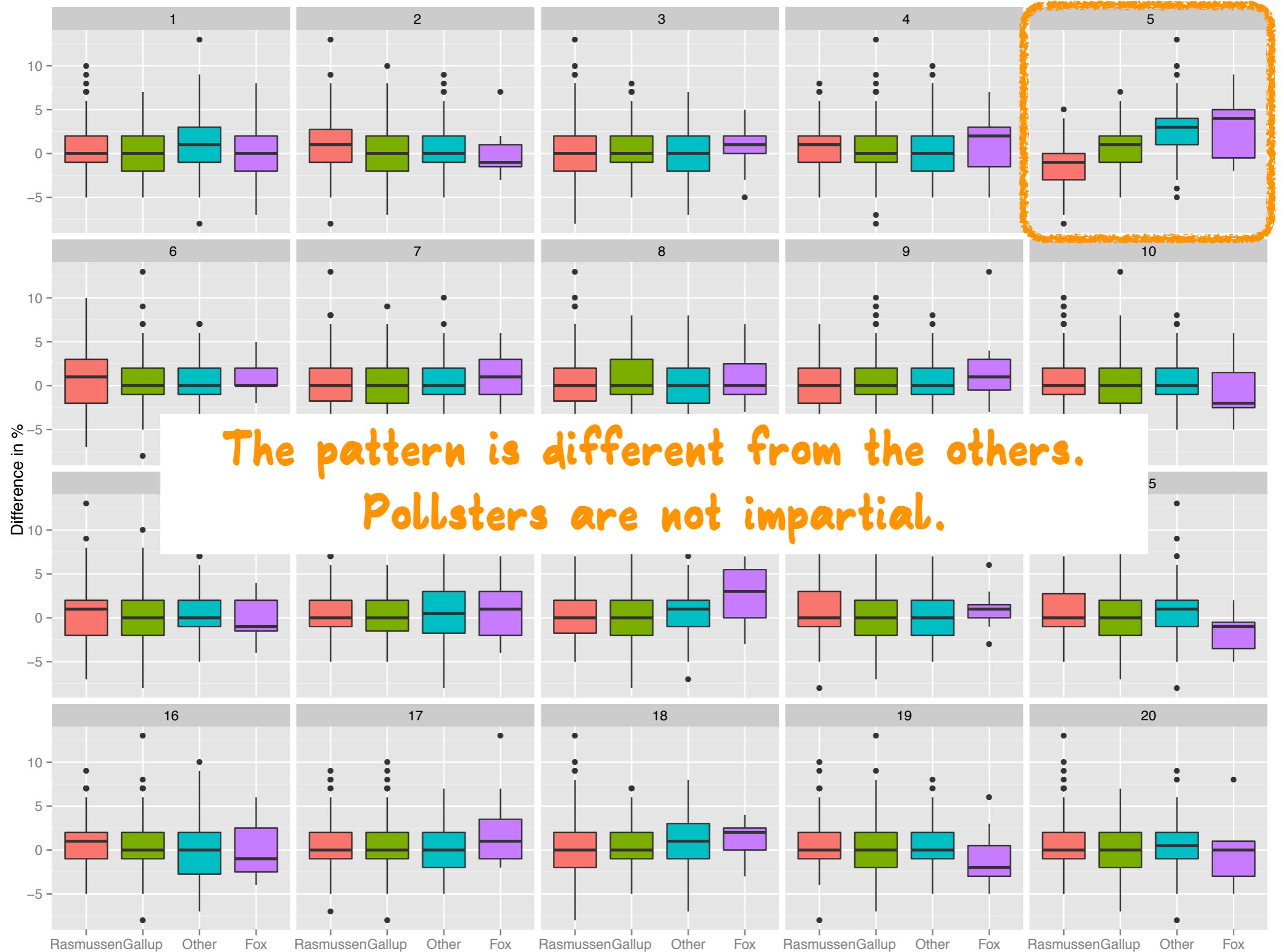
Can we believe what we see?
How can we see what we don't see?

Idea

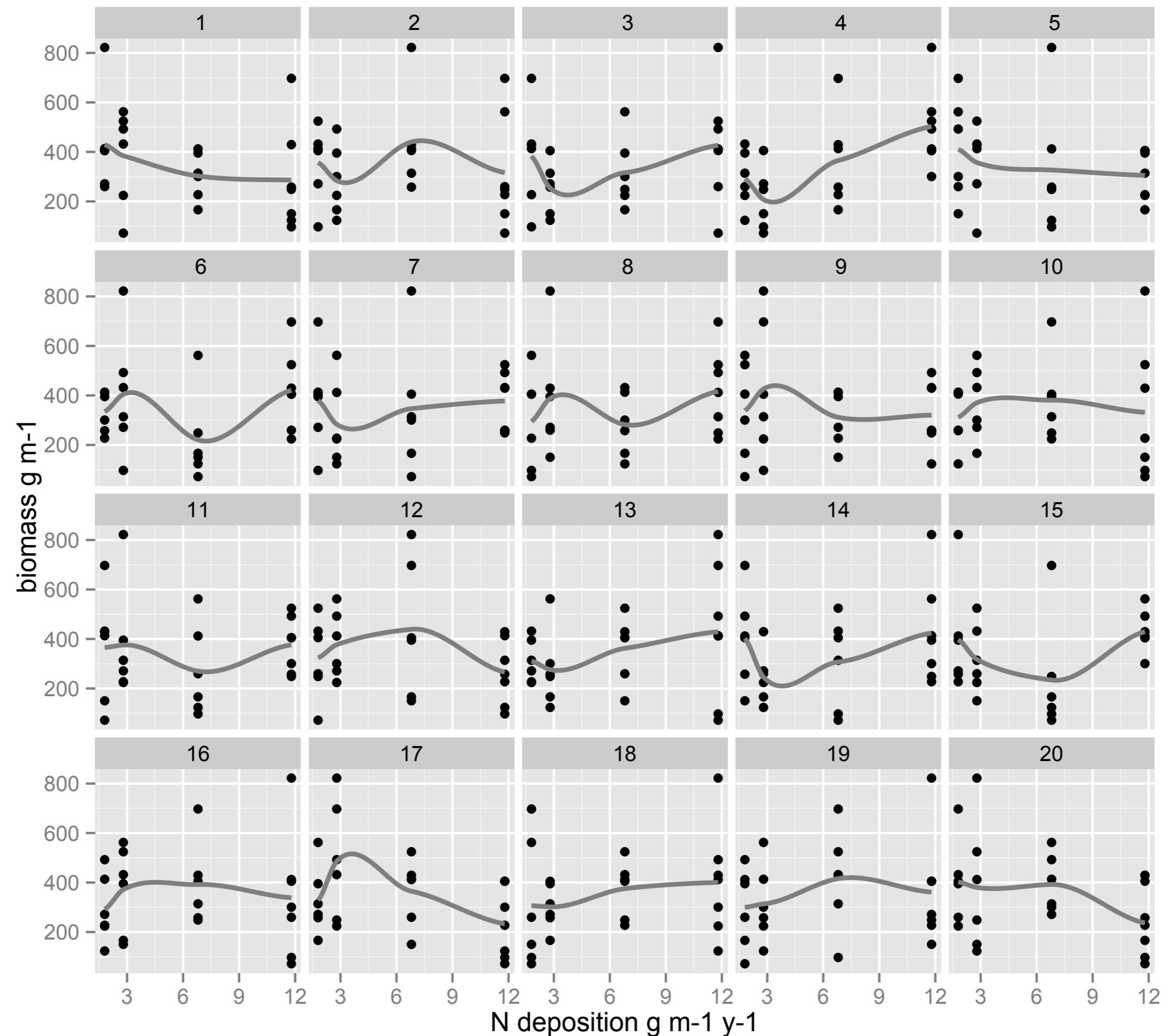
Instead of looking at the data plot in isolation, place it in the context of plots of data where there is nothing happening.

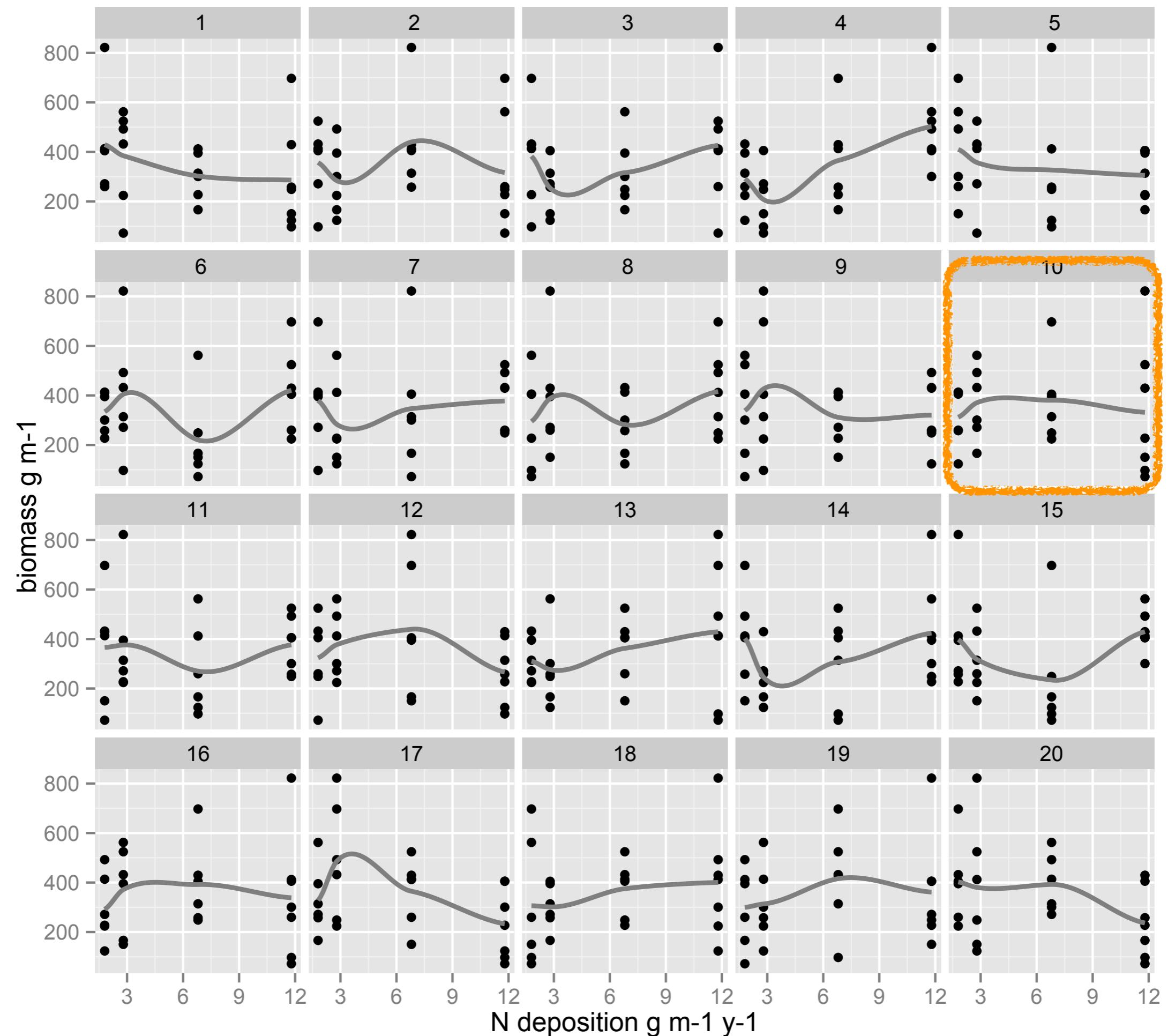


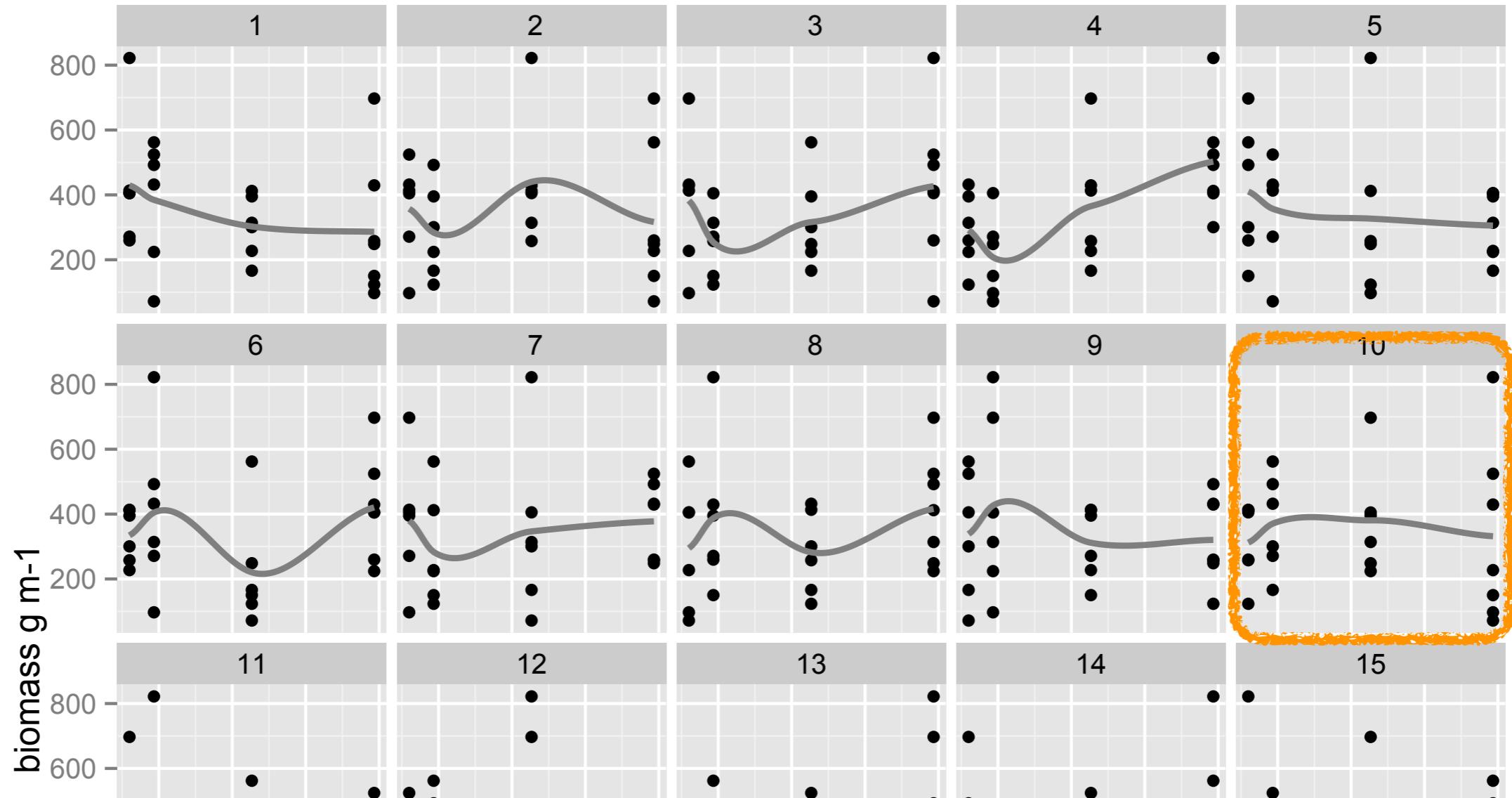




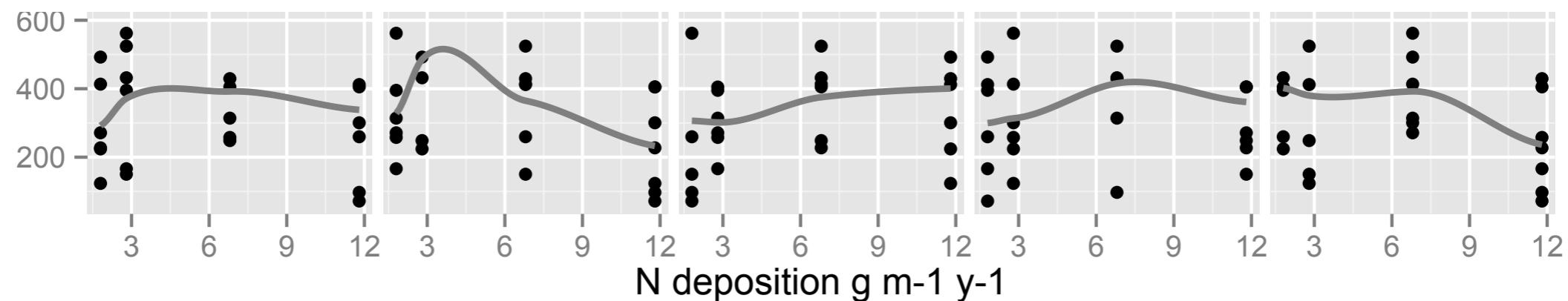
The pattern is different from the others.
Pollsters are not impartial.

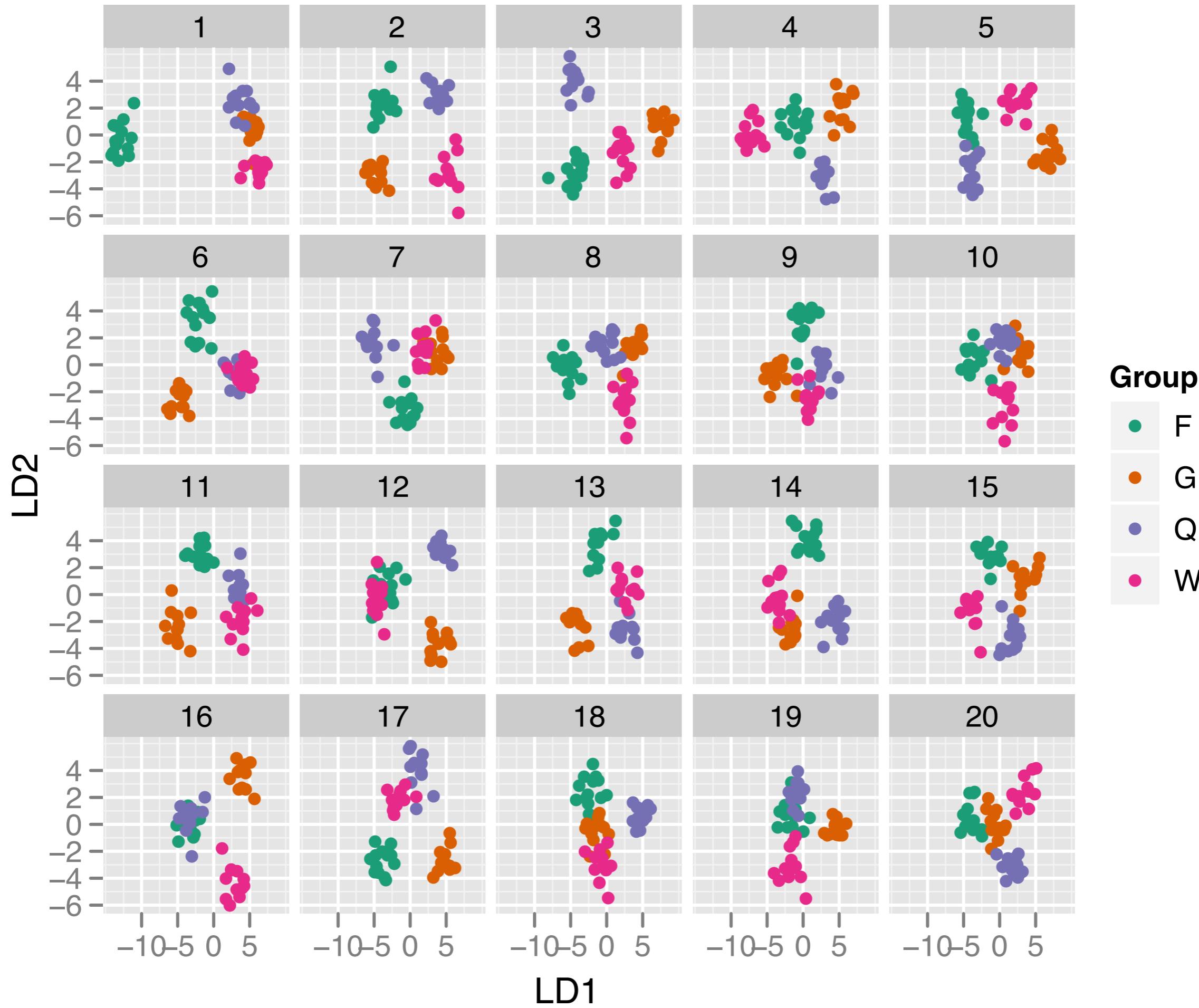






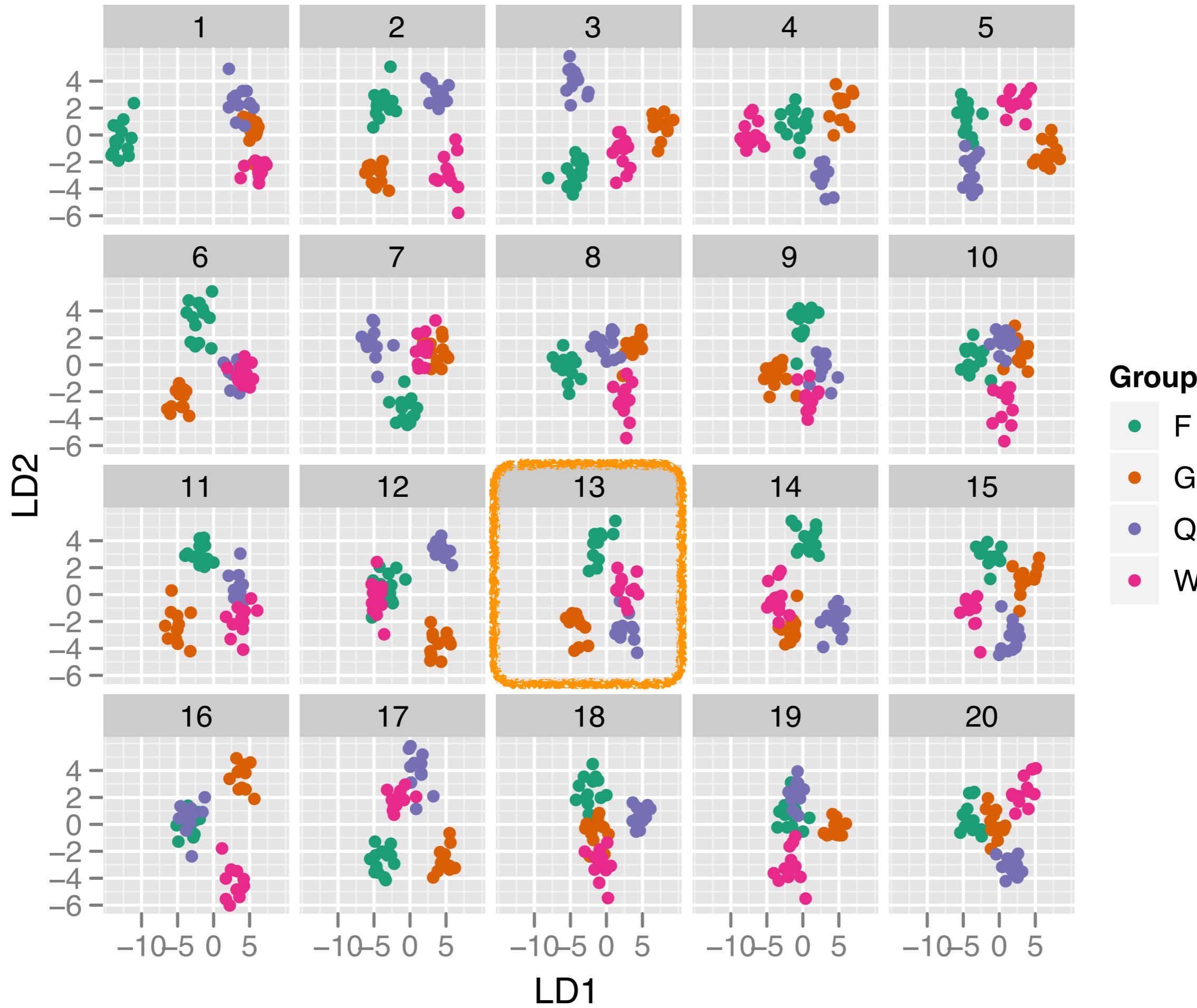
The pattern is not any more different than in other plots. The plot I showed earlier is actually a NULL plot, where I know the pattern is due to randomness.





Group

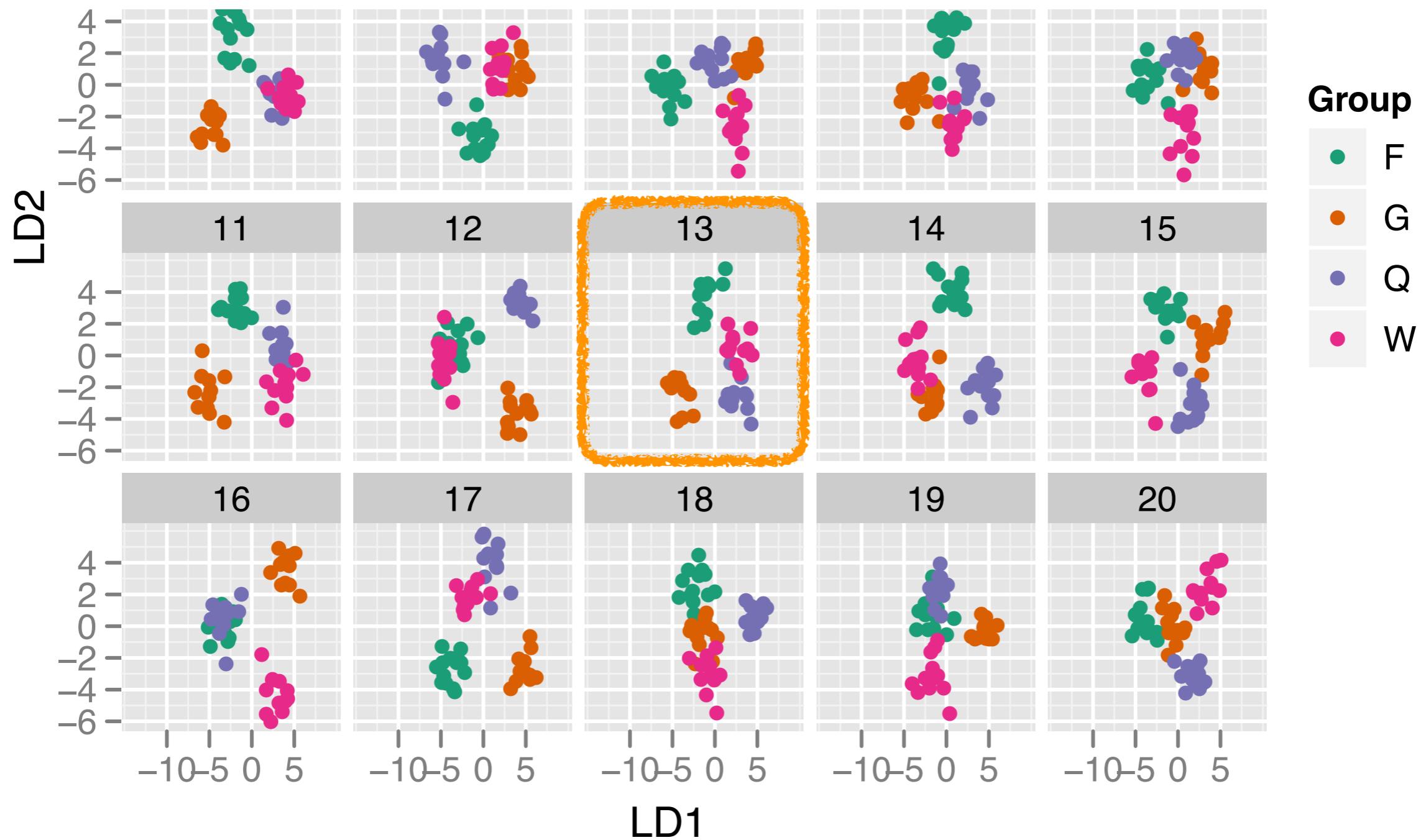
- F
- G
- Q
- W

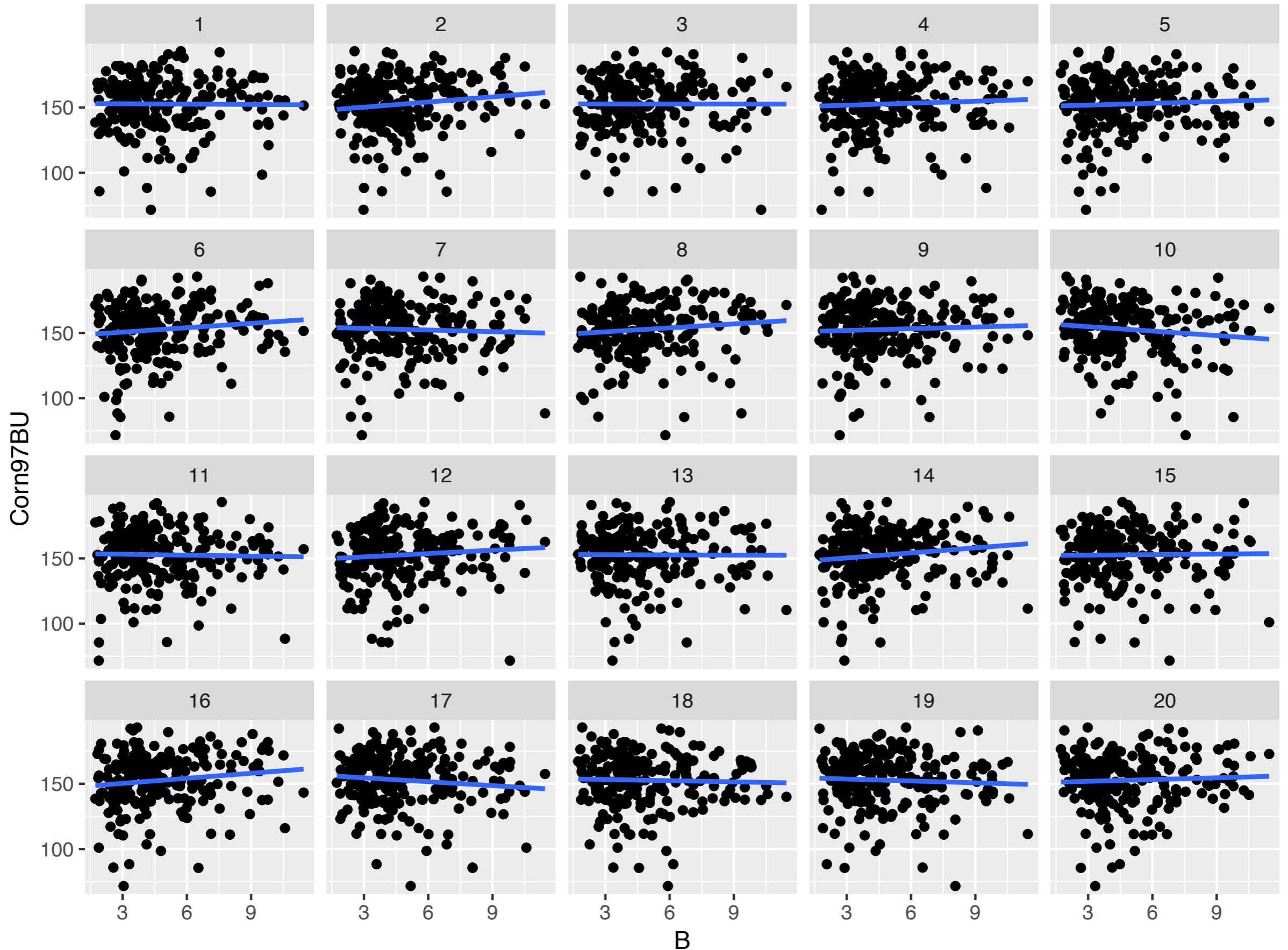


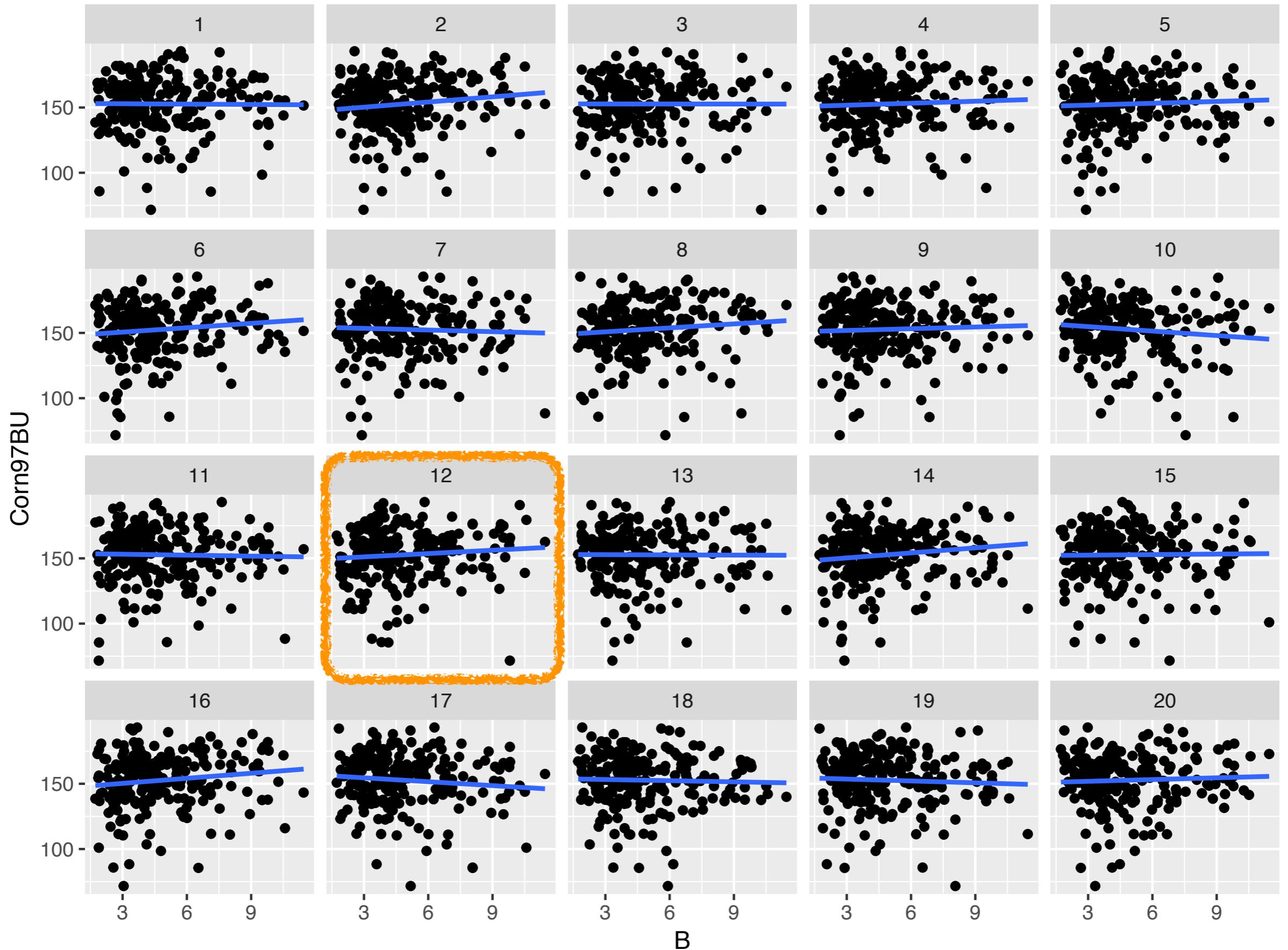
Group

●	F
●	G
●	Q
●	W

The groups are no more separated than in other plots.
The separation in the data is purely by chance, actually
because it is a projection from EMPTY high-dimensional
space, there are many ways to find gaps.

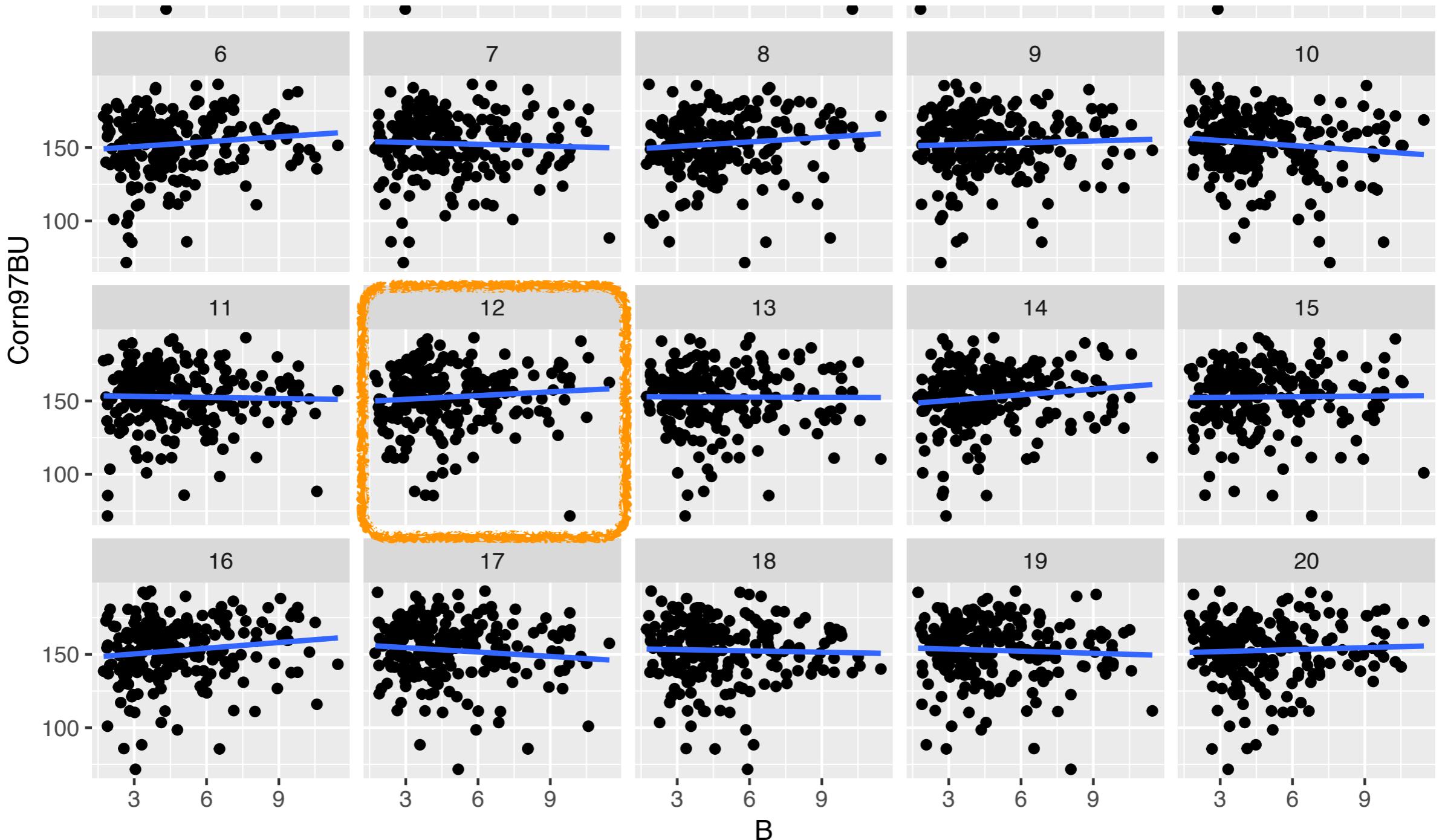


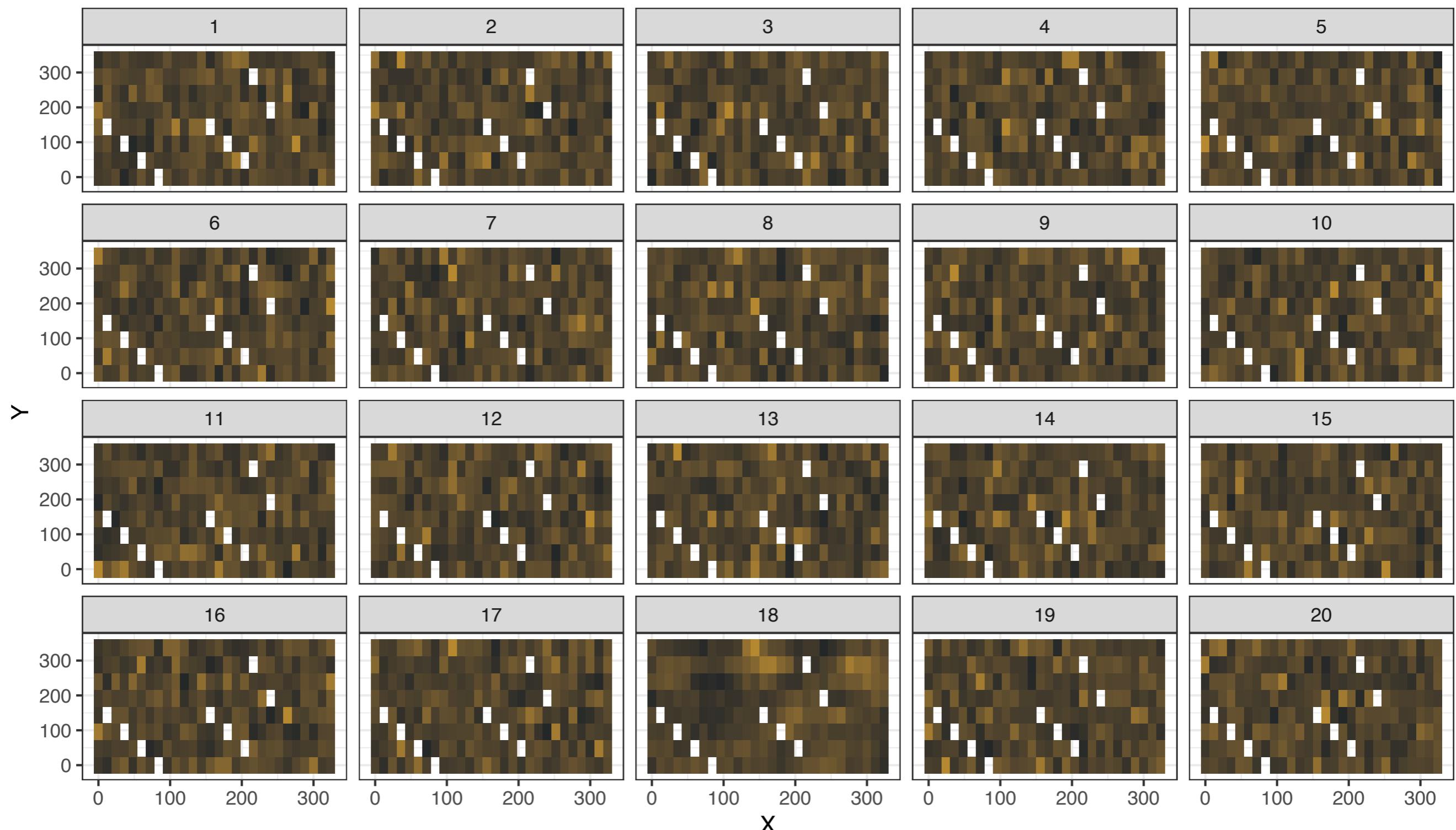


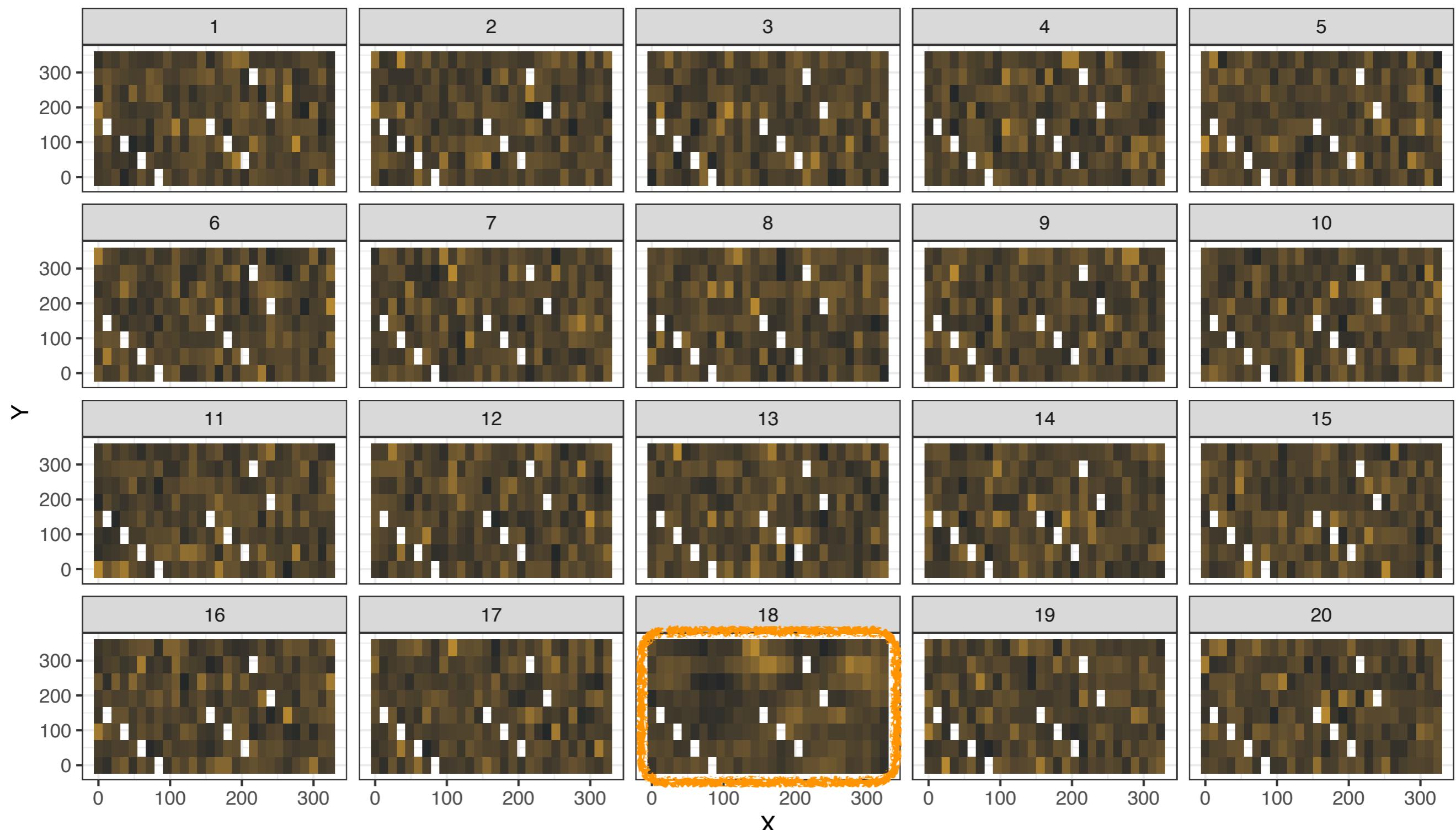


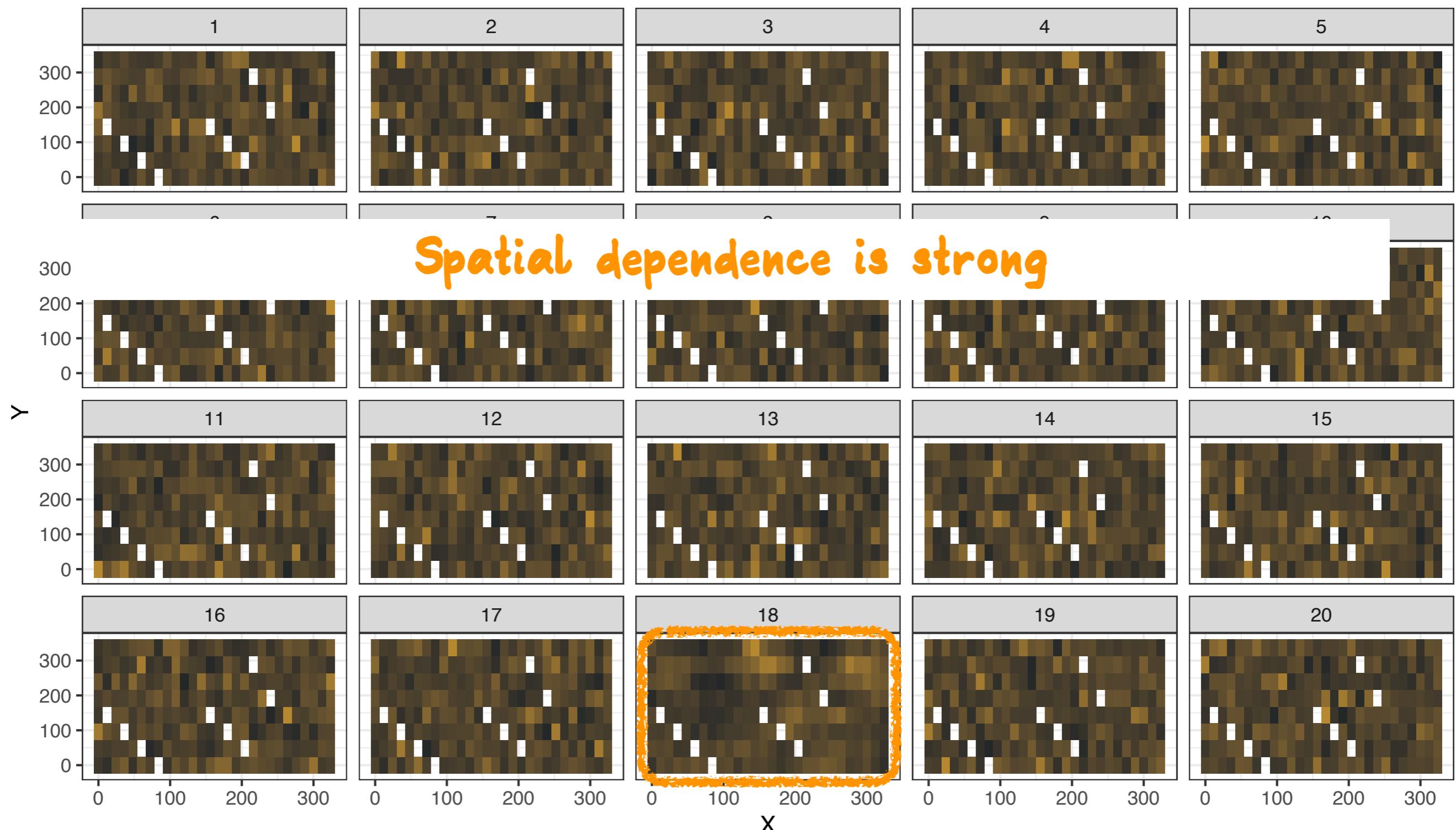


There is association between yield and boron, due to a single outlier.









Spatial dependence is strong

Outline

- ❖ Protocols, computing p-values and power
- ❖ Fitting into the tidy verse
- ❖ Validation studies
- ❖ Applications
- ❖ How you can use these tools

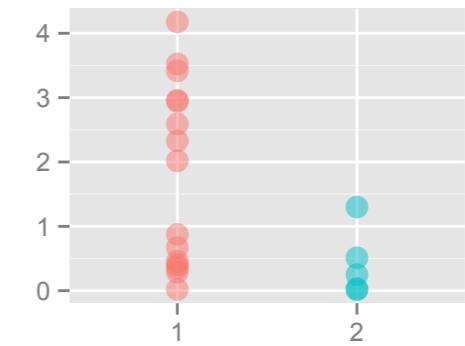
Protocols

- ❖ Rorschach: Show many pictures of data with “nothing” happening, pictures from a null distribution
- ❖ Lineup: Embed the plot of the data among plots of data generated from the null distribution

Source: Buja et al (2009) RSPT(A)

Protocols

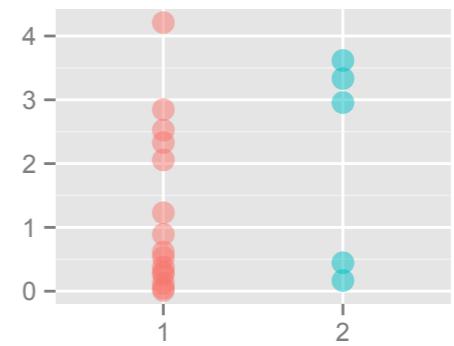
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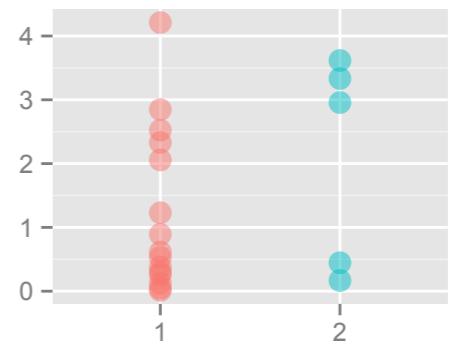
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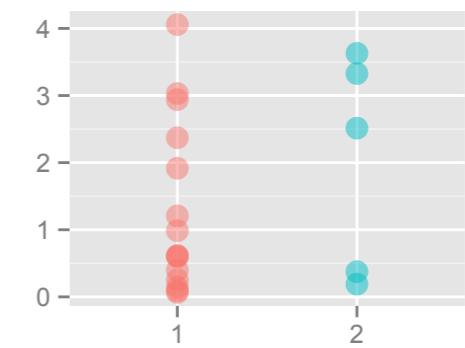
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Protocols

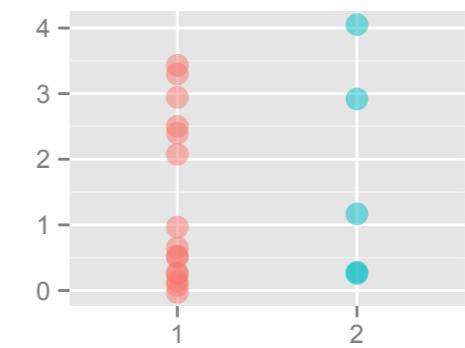
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Protocols

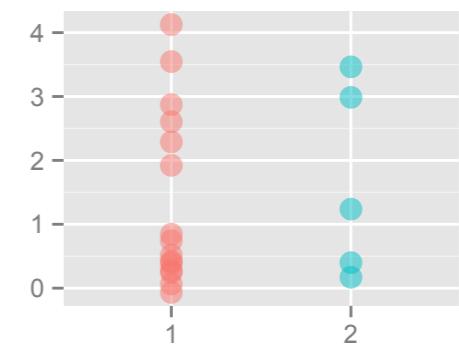
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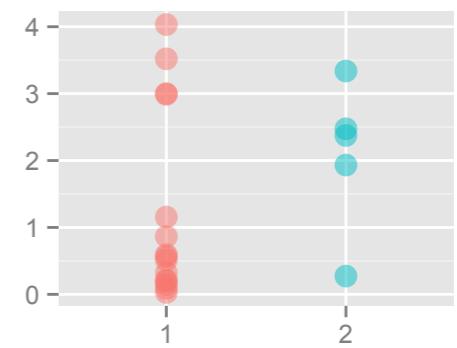
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Protocols

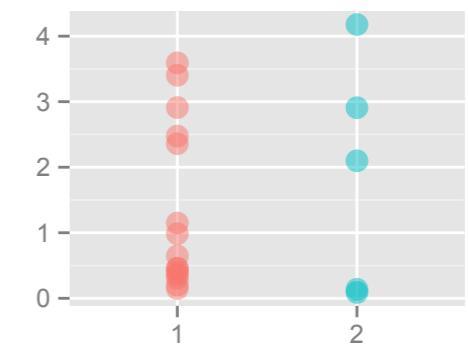
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Protocols

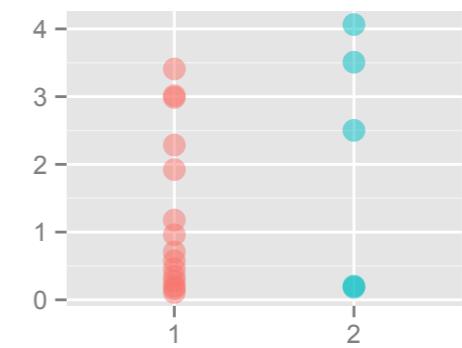
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Protocols

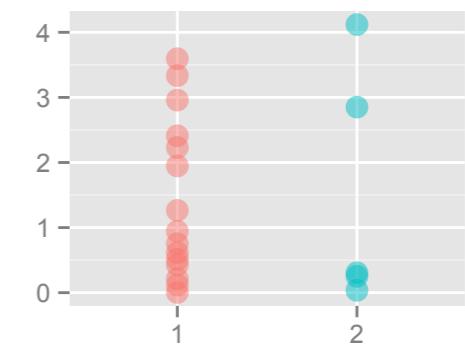
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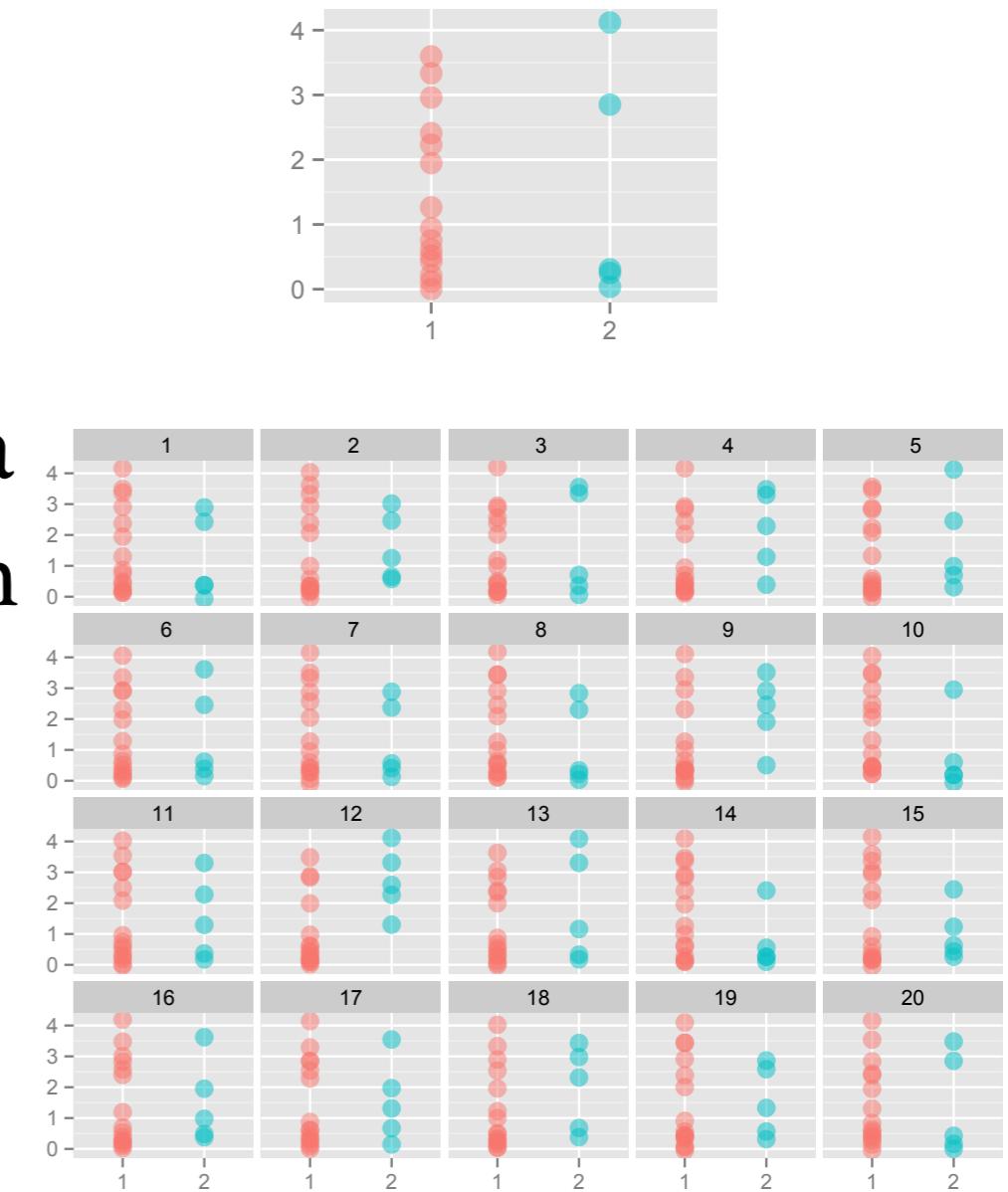


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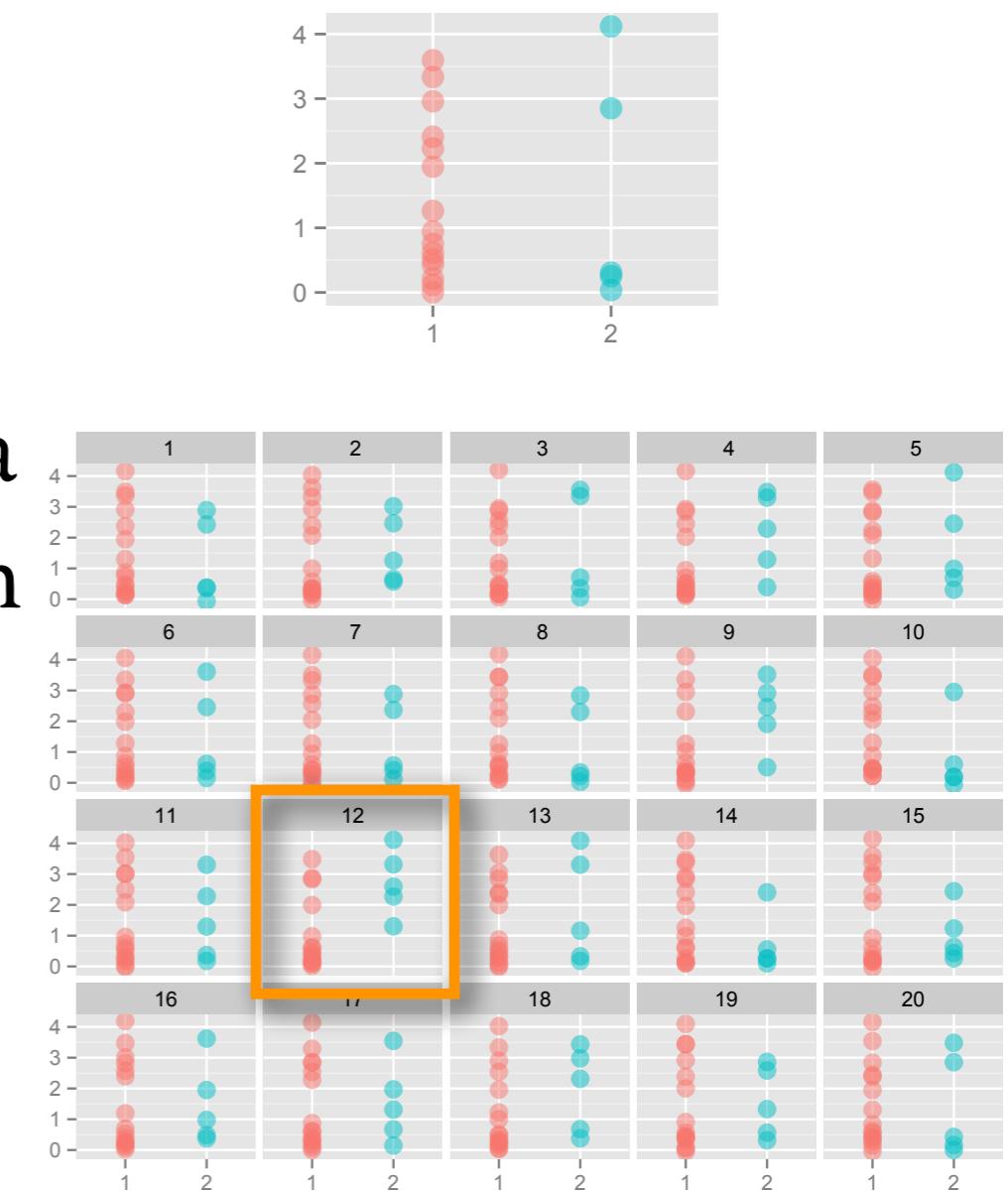
Source: Buja et al (2009) RSPT(A)

Protocols

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Data plot

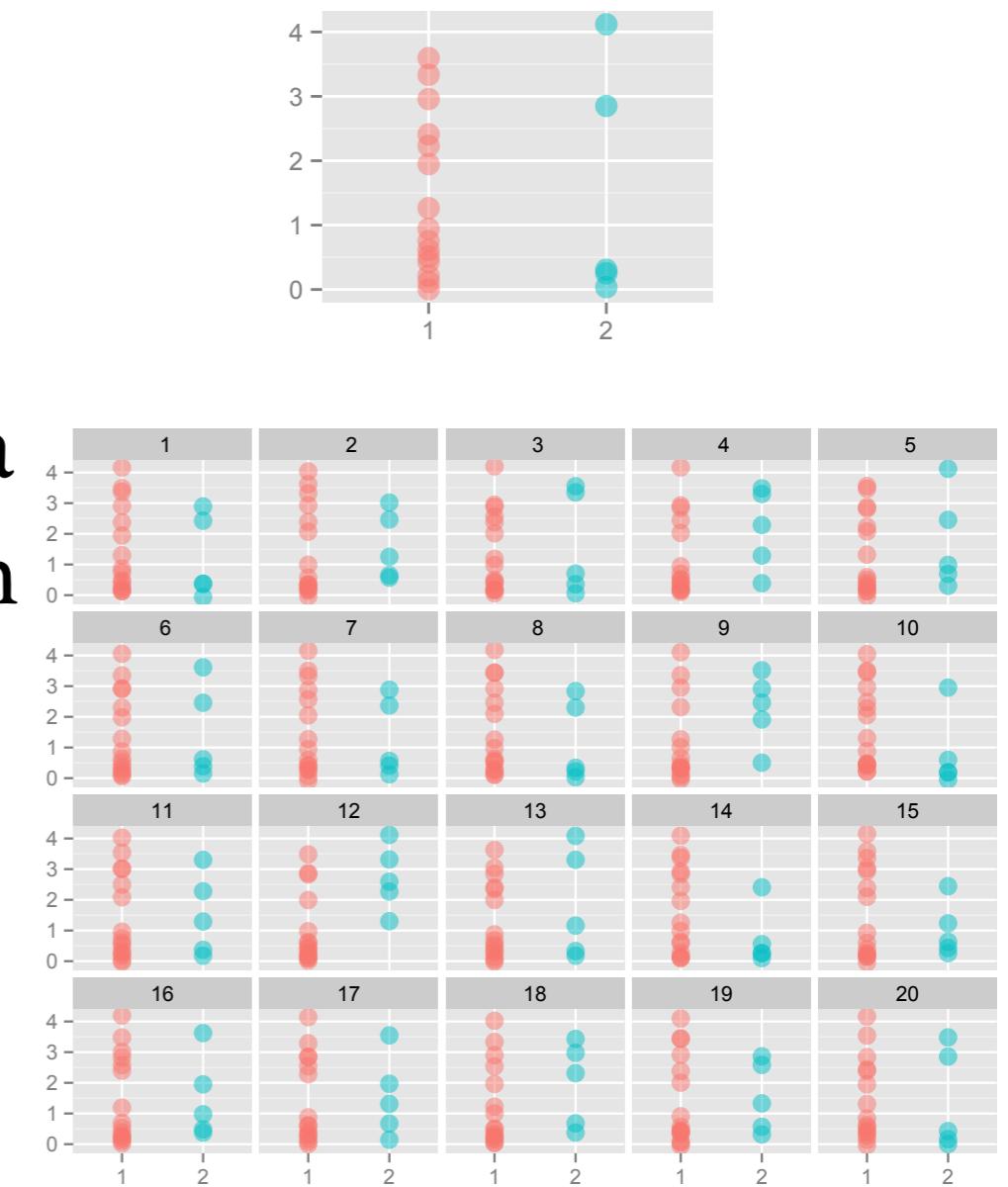


Source: Buja et al (2009) RSPT(A)

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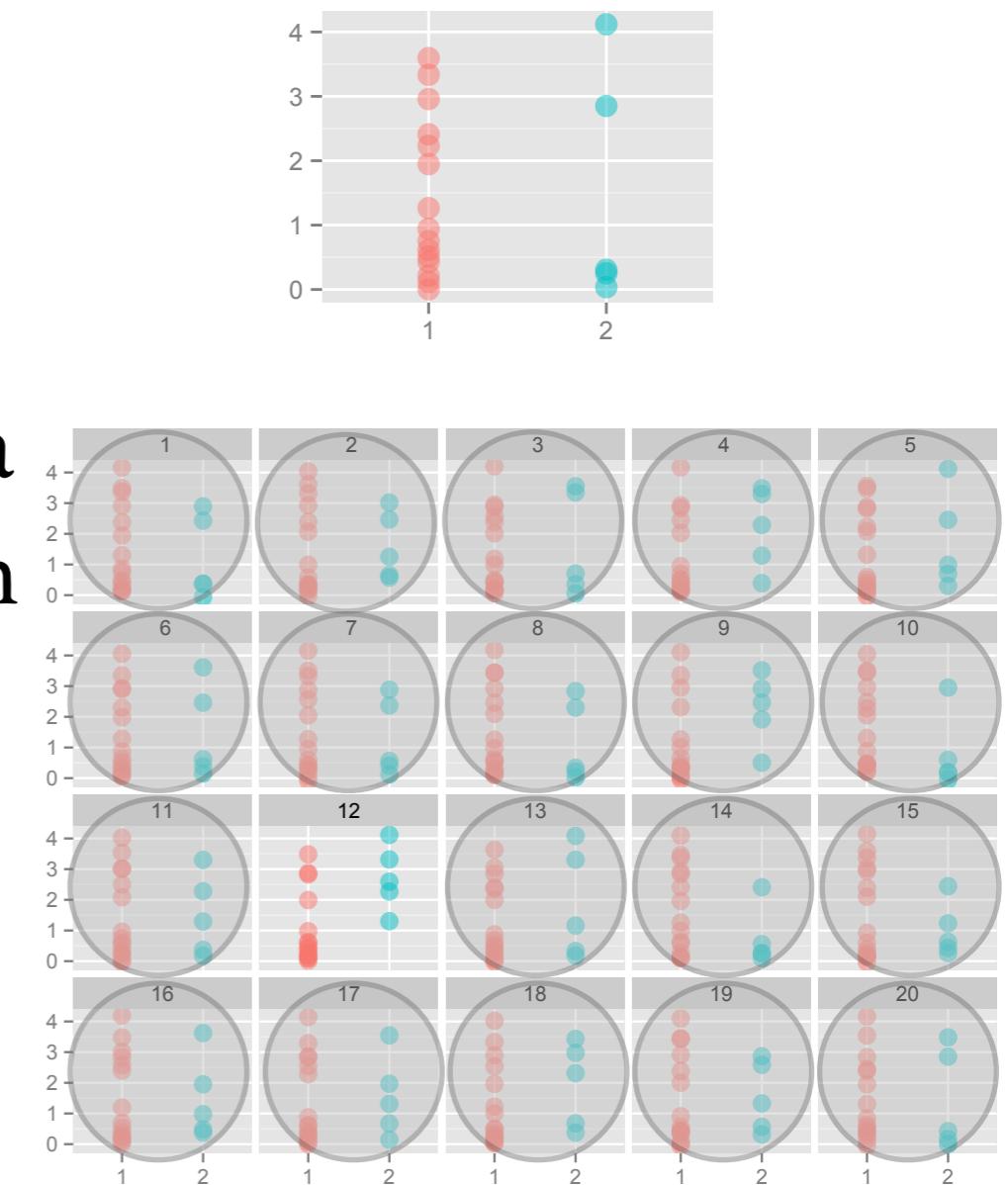
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Protocols

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Null plots

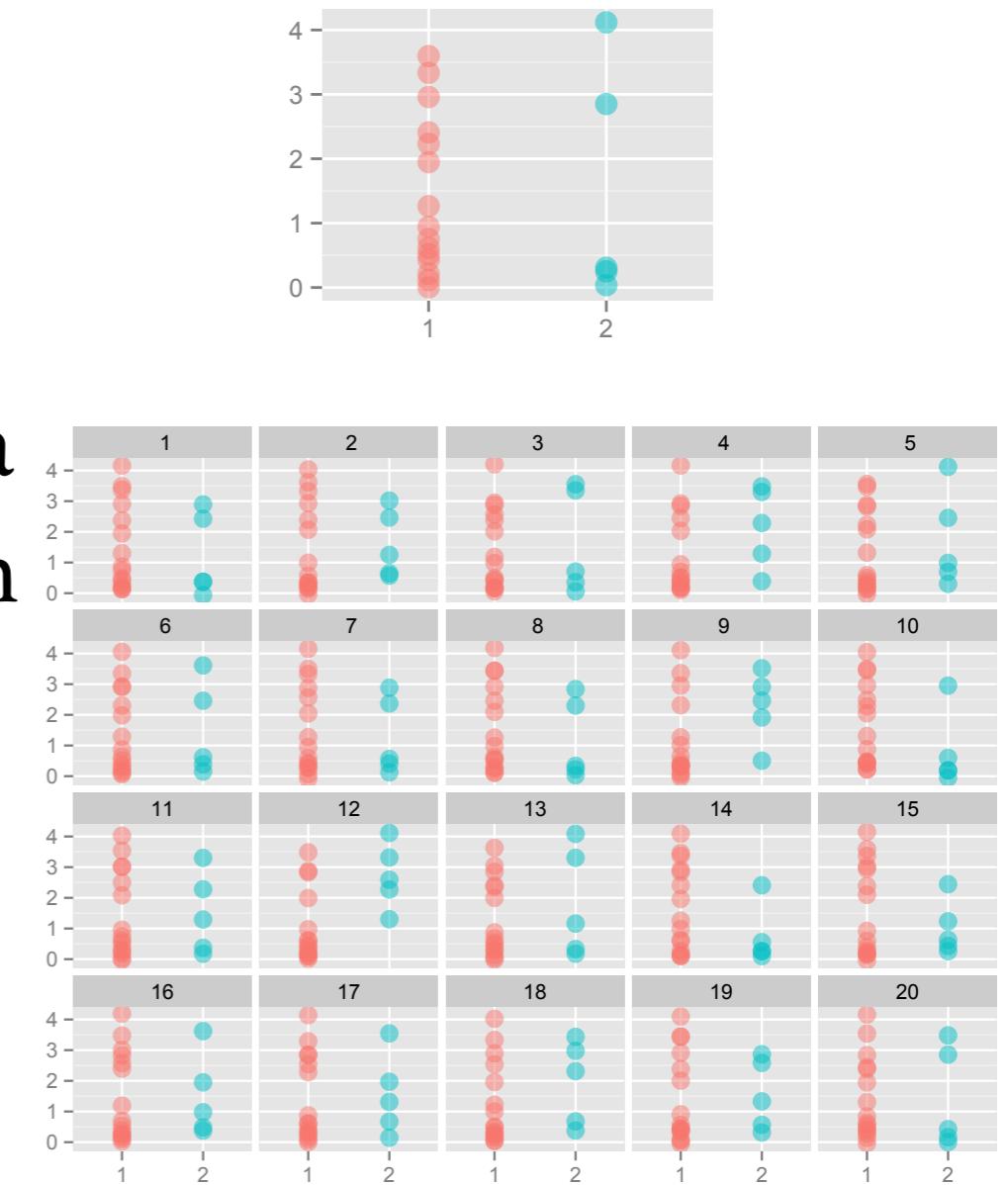


Source: Buja et al (2009) RSPT(A)

Protocols

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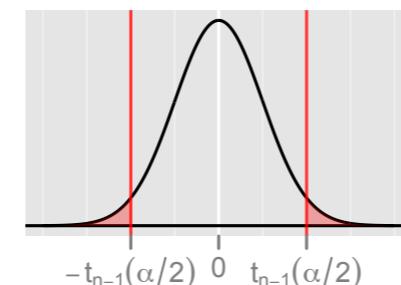
Hypothesis testing

Mathematical Inference

Hypothesis $H_0 : \mu_1 = \mu_2$ vs $H_a : \mu_1 \neq \mu_2$

Test Statistic $T(y) = \frac{\bar{y}_1 - \bar{y}_2}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Sampling Distribution $f_{T(y)}(t);$

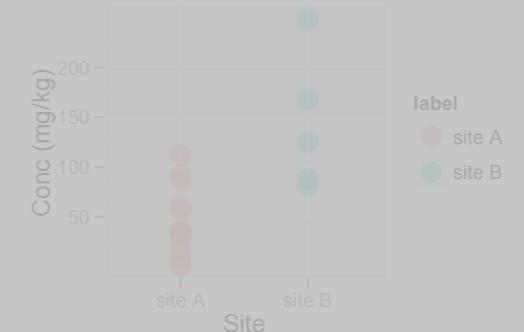


Reject H_0 if

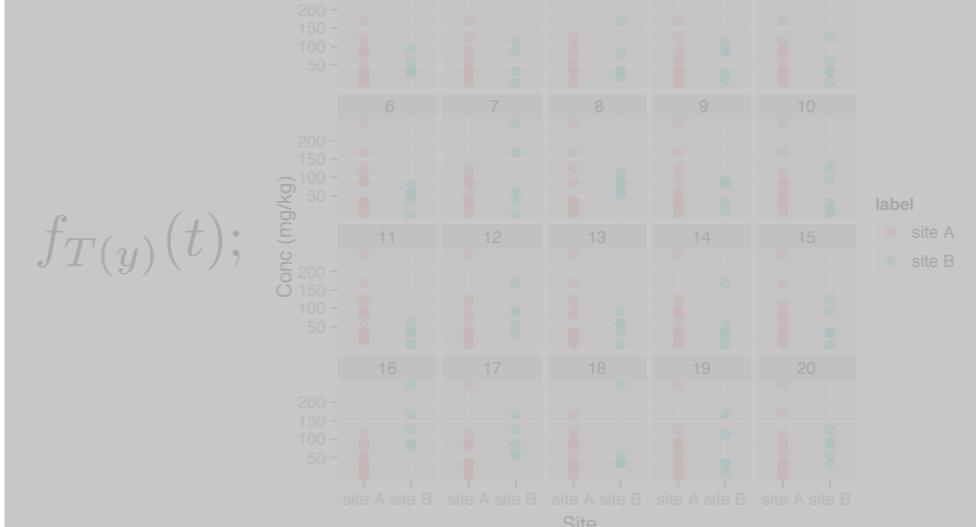
observed T is extreme

Visual Inference

$H_0 : \mu_1 = \mu_2$ vs $H_a : \mu_1 \neq \mu_2$

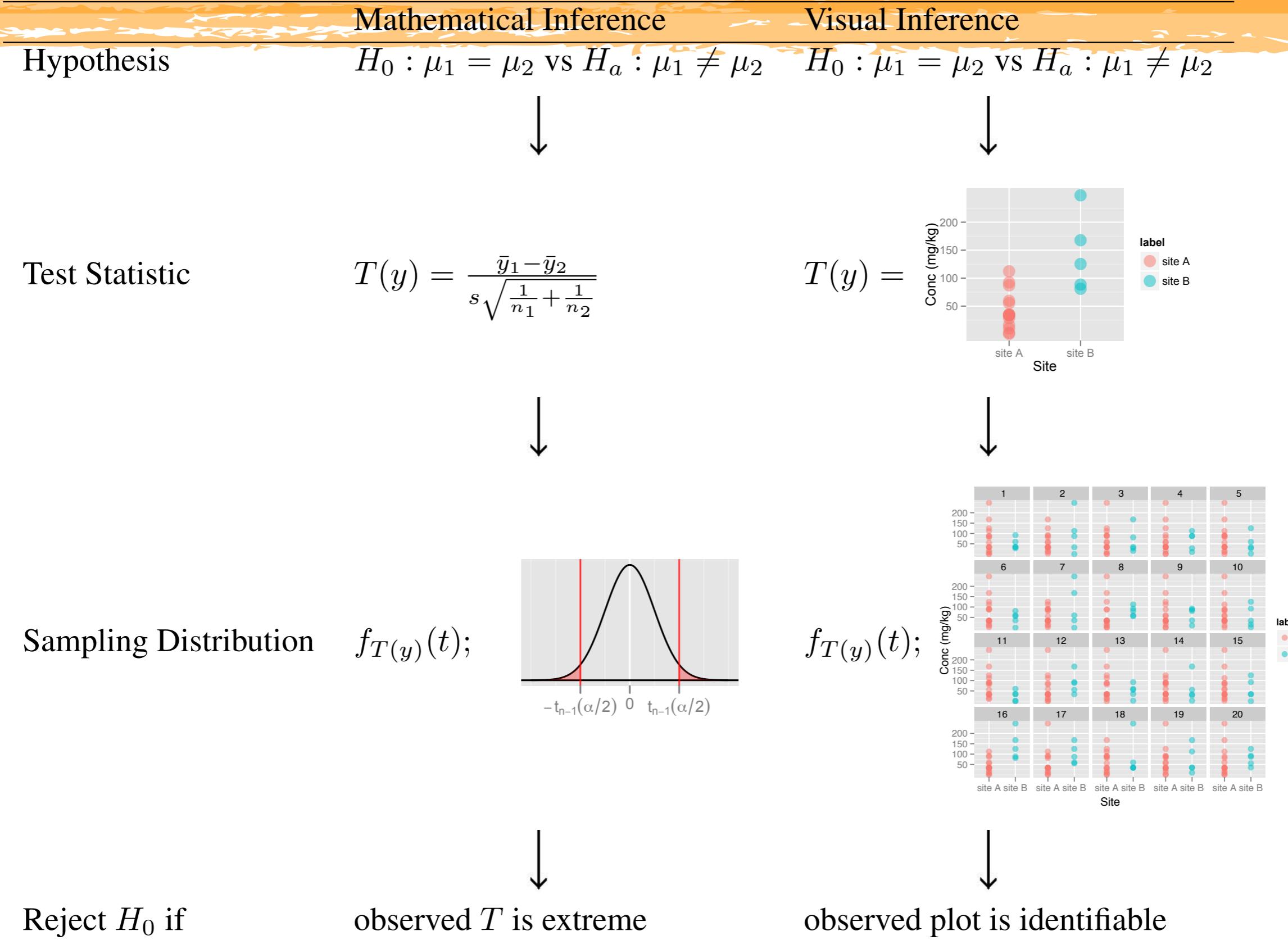


$T(y) =$



observed plot is identifiable

Hypothesis testing



Plots as statistics

```
ggplot(data = <DATA>) +  
  <GEOM_FUNCTION>(  
    mapping = aes(<MAPPINGS>),  
    stat = <STAT>,  
    position = <POSITION>  
  ) +  
  <COORDINATE_FUNCTION> +  
  <FACET_FUNCTION>
```

Plots as statistics

```
ggplot(data = <DATA>) +  
  <GEOM_FUNCTION>(  
    mapping = aes(<MAPPINGS>),  
    stat = <STAT>,  
    position = <POSITION>  
  ) +  
  <COORDINATE_FUNCTION> +  
  <FACET_FUNCTION>
```

Connection between
variables and
graphical elements

data: tidy_data

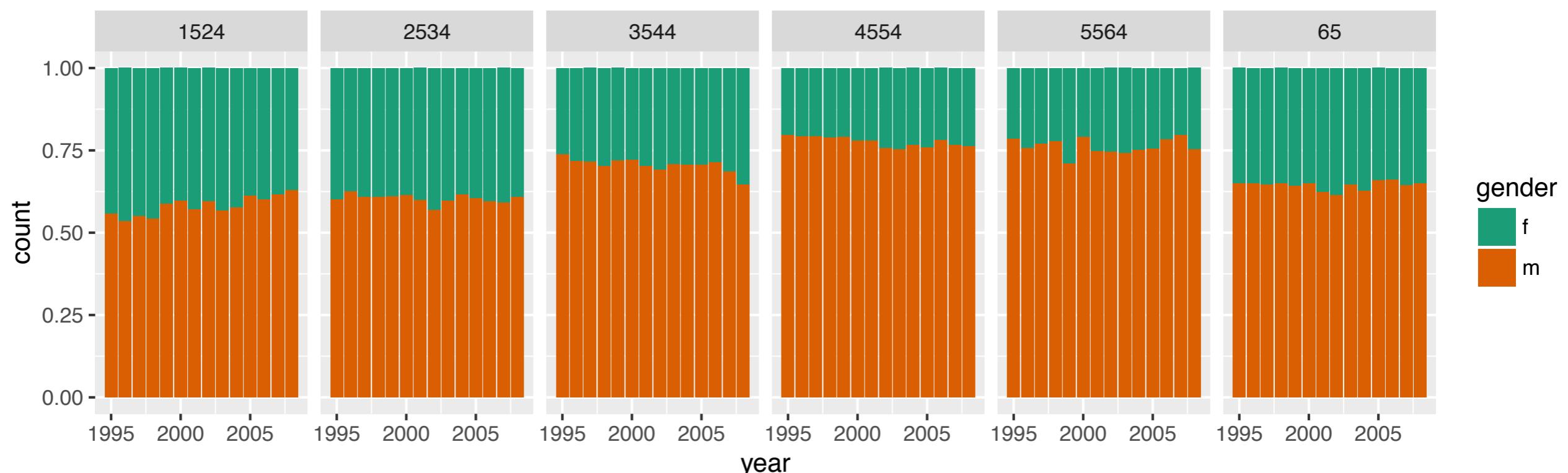
layer:

mapping: x = year,
y = count, fill = gender

geom: fill-bar

facet: age

100% charts



data: tidy_data

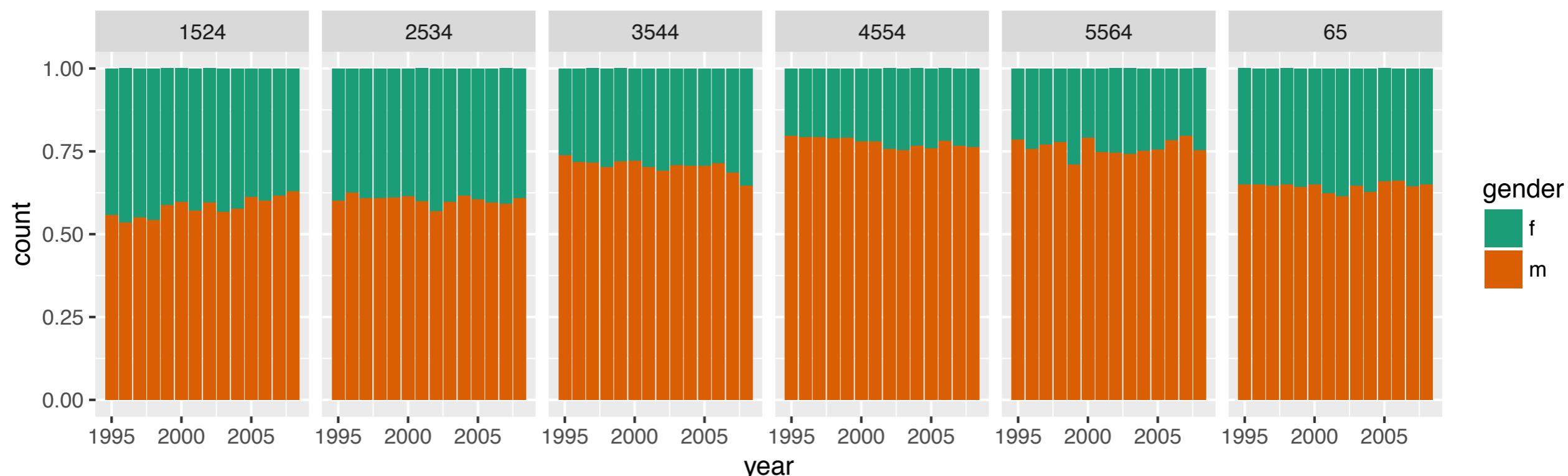
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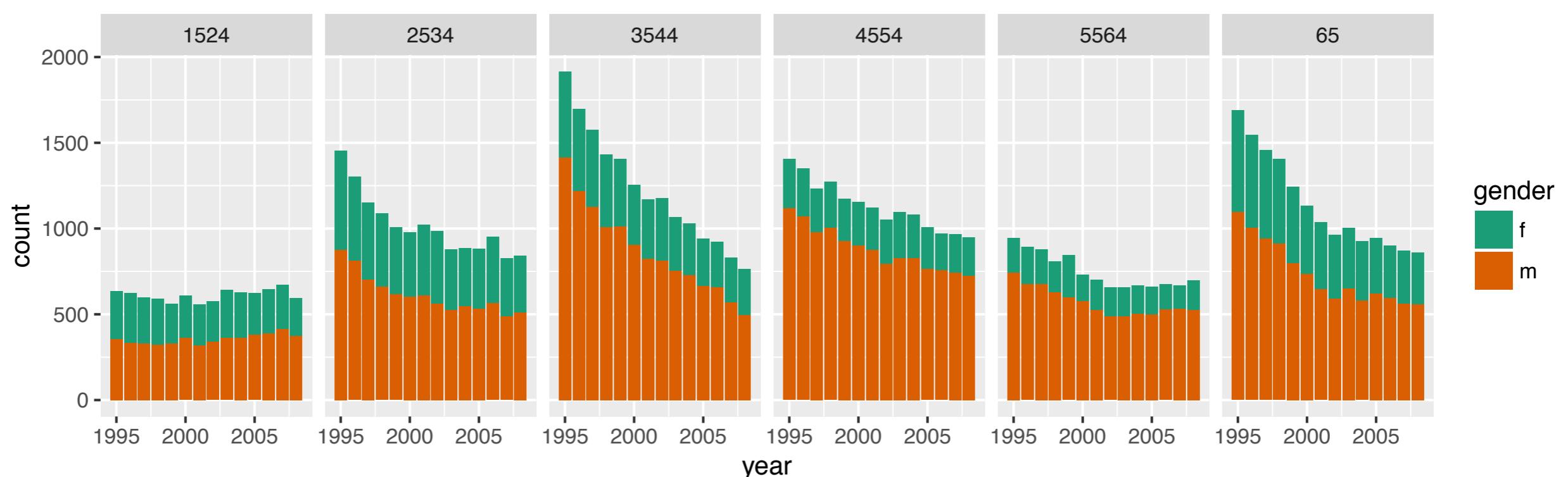
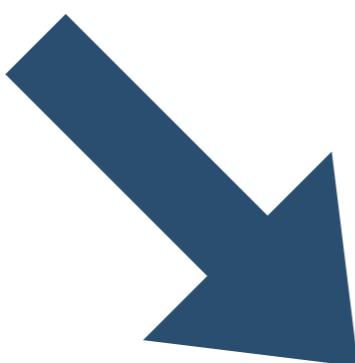
layer:

mapping: x = year,
y = count, fill = gender

geom: bar

facet: age

stacked barcharts



data: tidy_data

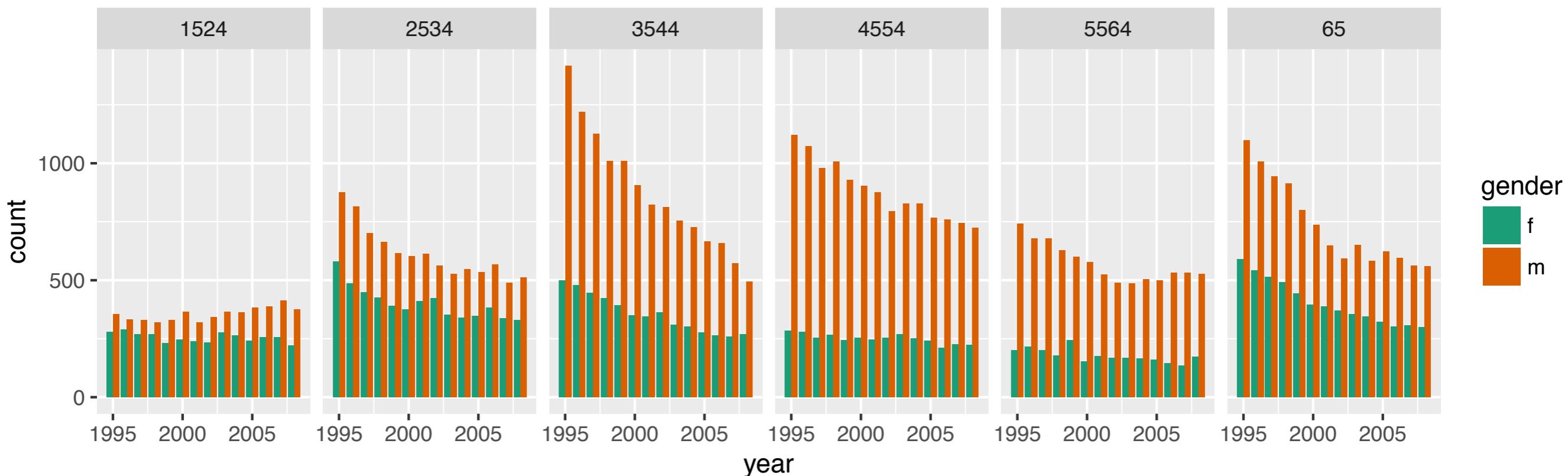
layer:

mapping: x = year,
y = count, fill = gender

geom: dodge-bar

facet: age

side-by-side bar charts



data: tidy_data

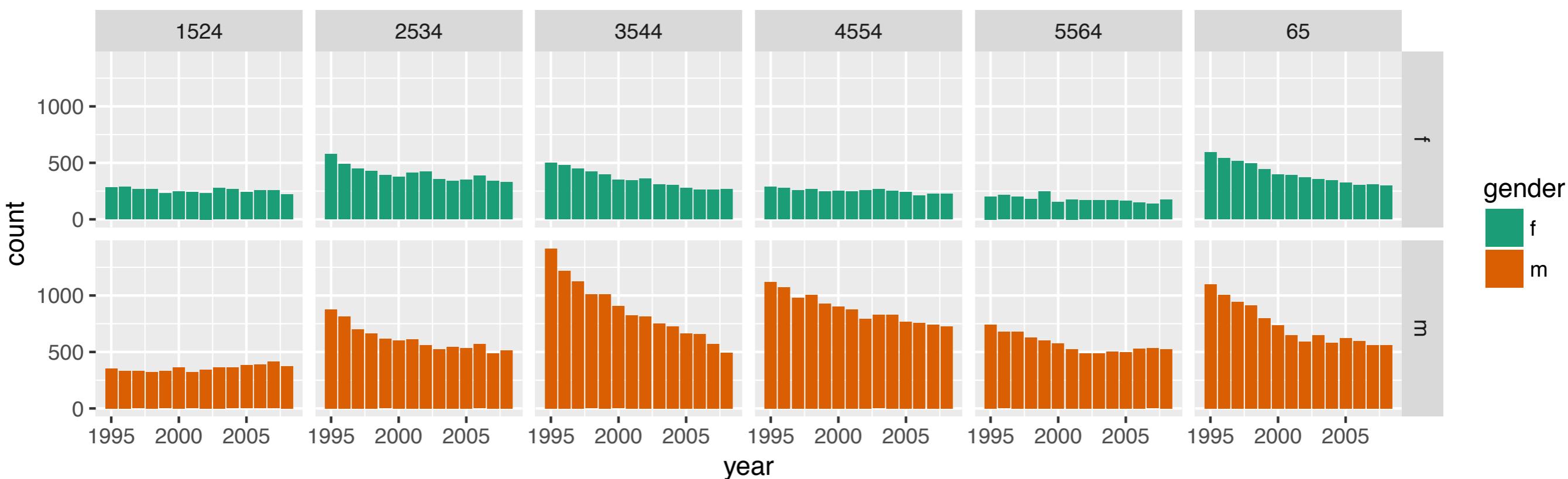
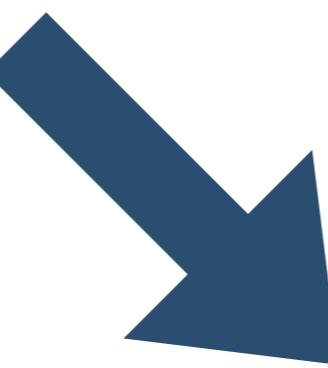
layer:

mapping: x = year,
y = count, fill = gender

geom: bar

facet: gender~age

bar charts



data: tidy_data

layer:

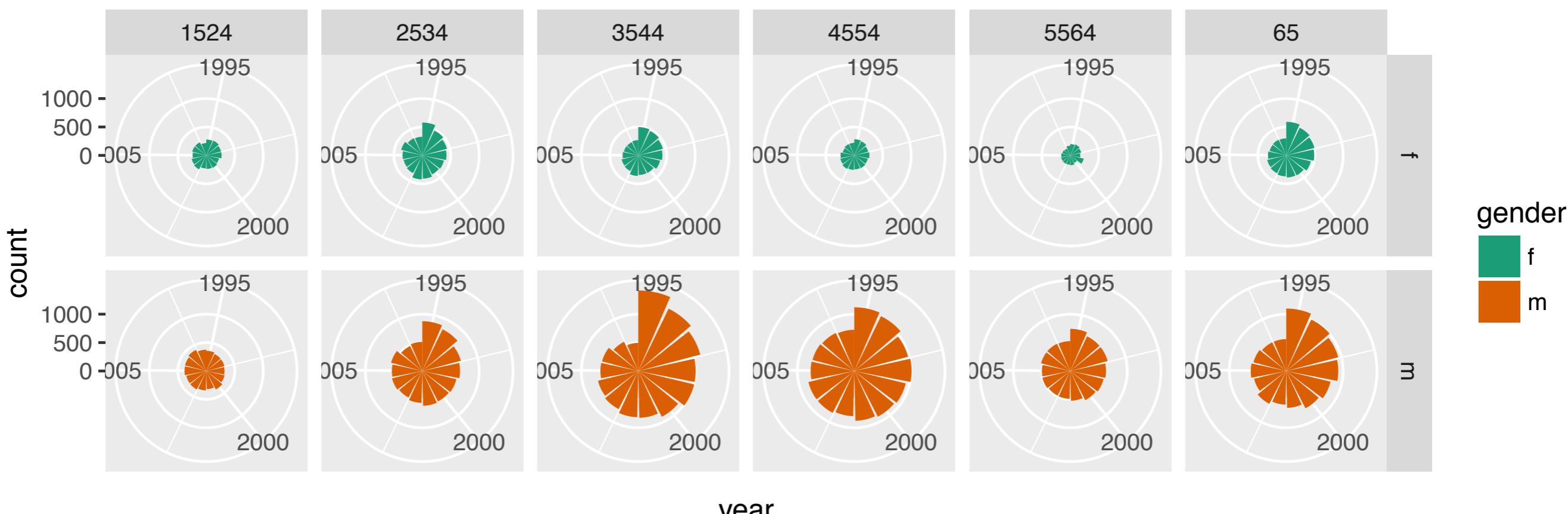
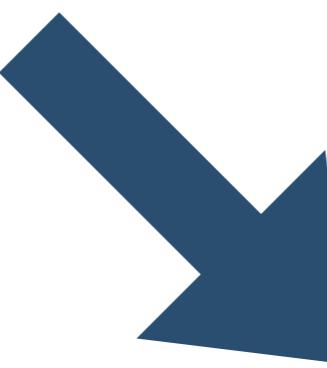
mapping: x = year,
y = count, fill = gender

geom: bar

facet: gender~age

coord: polar

rose plots



data: tidy_data

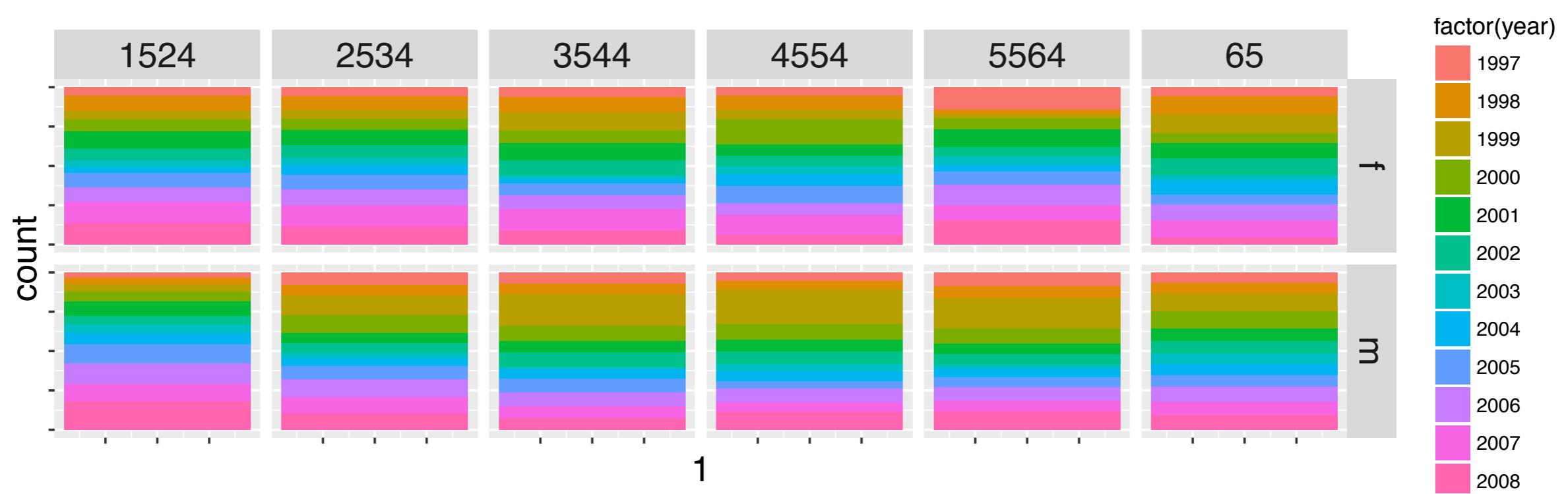
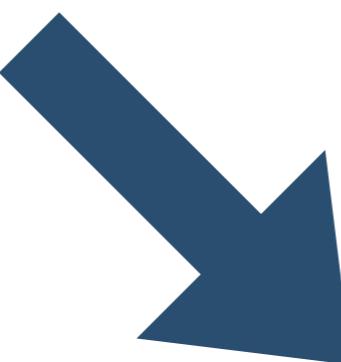
layer:

mapping: x = 1,
y = count, fill = year

geom: fill-bar

facet: gender~age

100% charts



data: tidy_data

layer:

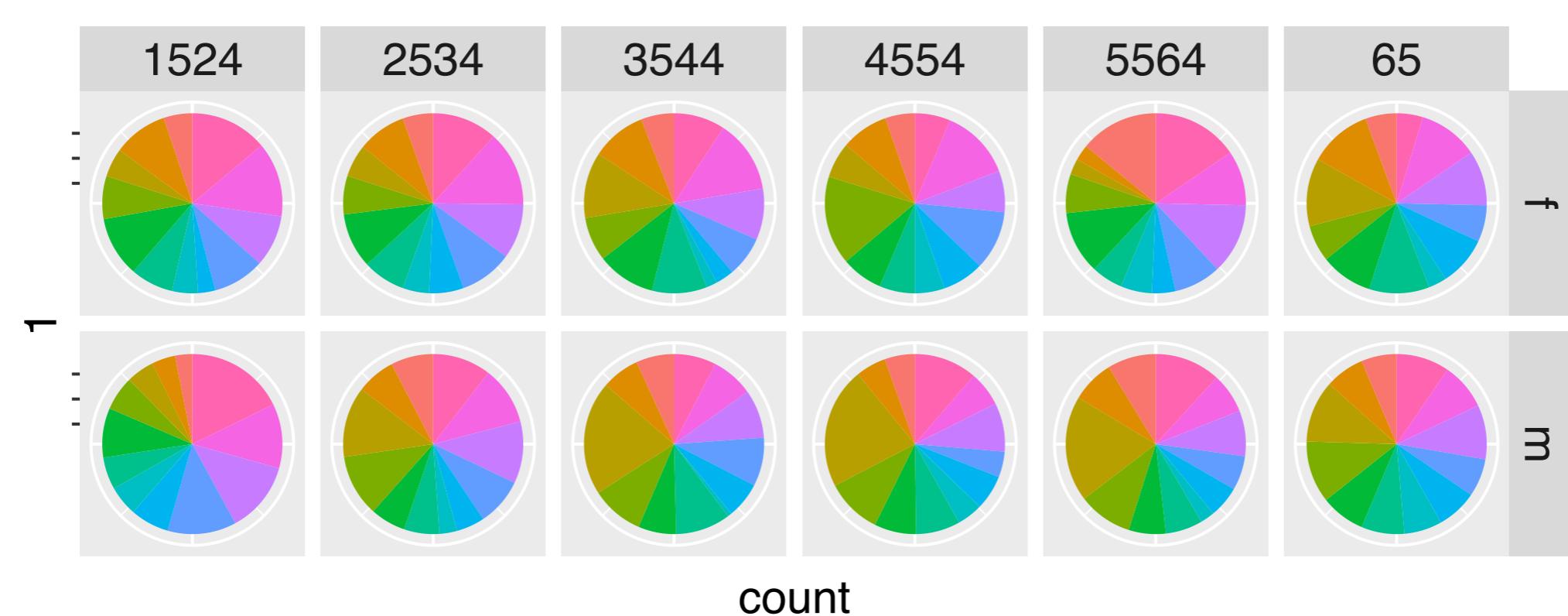
mapping: x = 1,
y = count, fill = year

geom: fill-bar

facet: gender~age

coord: polar

pie charts





<http://knowyourmeme.com/memes/morphing/photos/sort/favorites>



HELEN
GREEN

Key components

- ➊ A plot is a test statistic, adding the data is the observed test statistic
- ➋ Type of plot used indicates null/alternative hypothesis, eg scatterplot suggests null hypothesis “no association between two variables”
- ➌ Null hypothesis determines null generating mechanism
- ➍ Generate draws from the null, plot, compare with data plot using an uninvolved observer
- ➎ Data plot detected equivalent to rejection of null, it is extreme relative to the sampling distribution

Significance

- What is the p -value?
- For one observer, the probability of randomly selecting the data plot is $1/m$, where m is the number of plots in the lineup.
- With multiple observers, the p -value is estimated by

$$P(X \geq x) = 1 - \text{Binom}_{K,1/m}(x-1) = \sum_{i=x}^K \binom{K}{i} \left(\frac{1}{m}\right)^i \left(\frac{m-1}{m}\right)^{K-i}$$

Source: Majumder et al (2013) JASA

Significance

- What is the p -value?
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Number of independent observers

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Number of observers choosing data plot

Source: Majumder et al (2013) JASA

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Computing p -values

Scenario: same data plot and null plots, multiple observers

Simulation:

1. Sample $p_{\text{data}} \sim U[0,1]$ (why uniform? Murdoch et al (2008) P-Values are Random Variables, TAS 62, 242-245)
2. Sample $(m-1)$ null p -values, $p_j \sim U[0,1], j=1, \dots, m$
3. Compute 'strength' of data plot relative to other plots:
$$q = (1-p_{\text{data}})/\sum_j(1-p_j)$$
4. Simulate number of data plot picks $X \sim Bin(K, q)$
5. Repeat steps 1-4, N times to obtain distribution of X
6. Examine where observed X is placed in the distribution of X 's generated assuming the null hypothesis is true.

Computing p -values

Scenario: same data plot and null plots, multiple observers

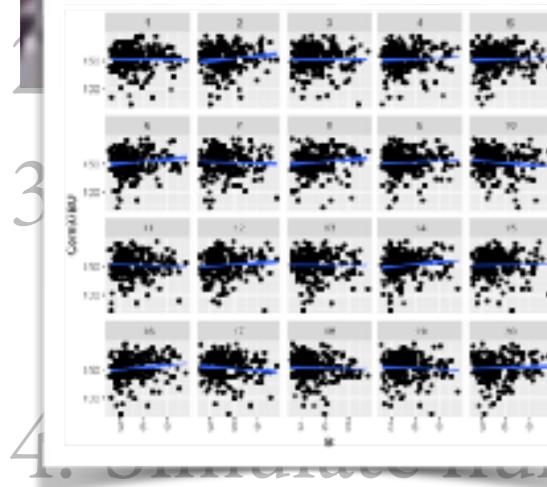
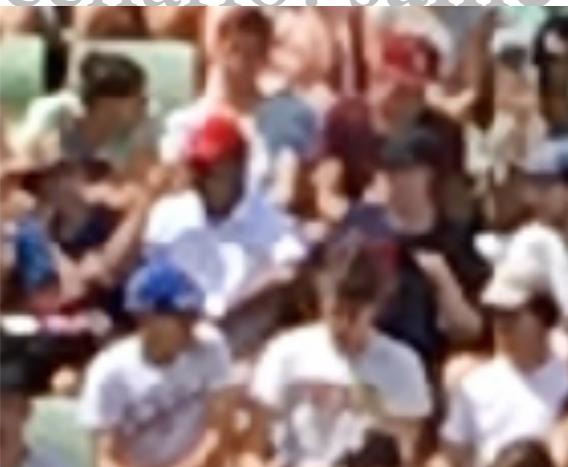
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Computing p -values

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S



$$U[0,1]$$

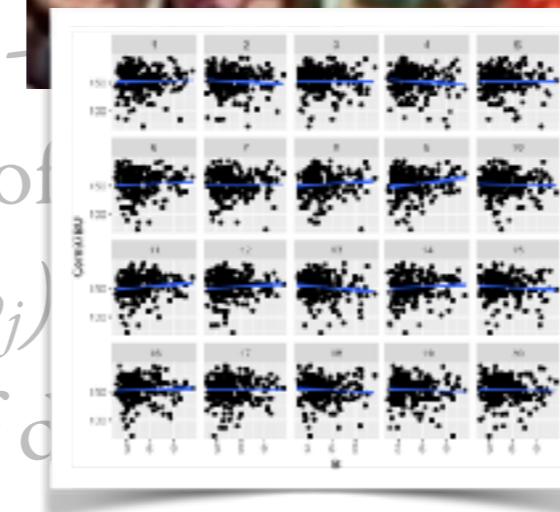
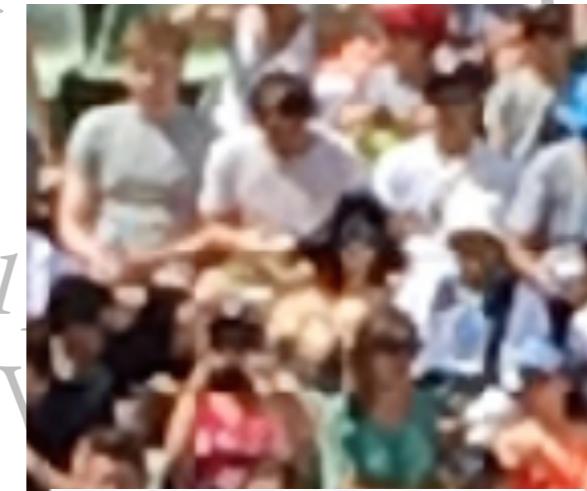
random V

null p -

length'

$$\sum j(1-p_j)$$

number of d



$$U[0,1]$$

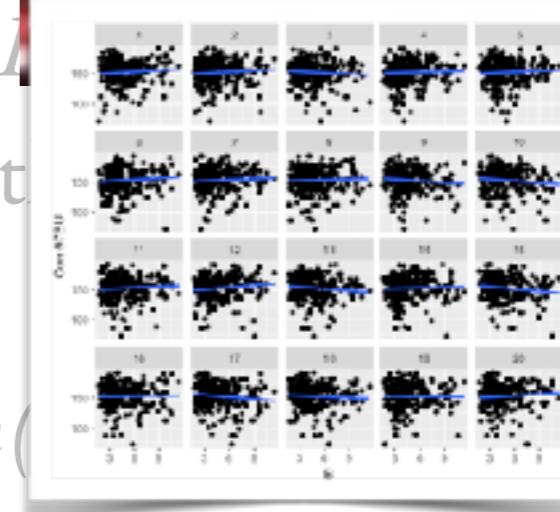
Murdo

$$2, 242-2$$

, 1], j=1

ive to ot

$$X \sim Bin($$



$$U[0,1]$$

P-

1,

$$1, 1)$$

ive to ot

5. Repeat steps 1-4, N times to obtain distribution of X
6. Examine where observed X is placed in the distribution of X 's generated assuming the null hypothesis is true.
Simulate picks from Binomial, with K observers all looking at the same lineup

Power of the Test

- ❖ Power is used to compare the performance of tests in statistics.

Definition 2.6. The *power* of a visual test, V_θ , is defined as the probability to reject the null hypothesis for a given parameter value θ :

$$\text{Power}_V(\theta) = \Pr(\text{Reject } H_0 \mid \theta).$$

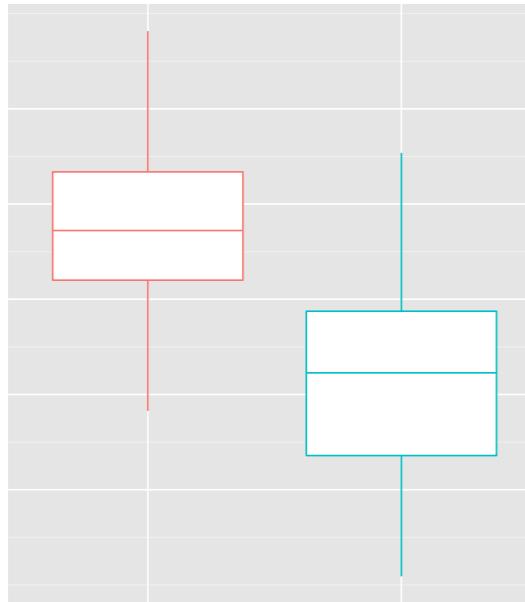
- ❖ Estimate by the proportion of observers picking the data plot.

Source: Majumder et al (2013) JASA

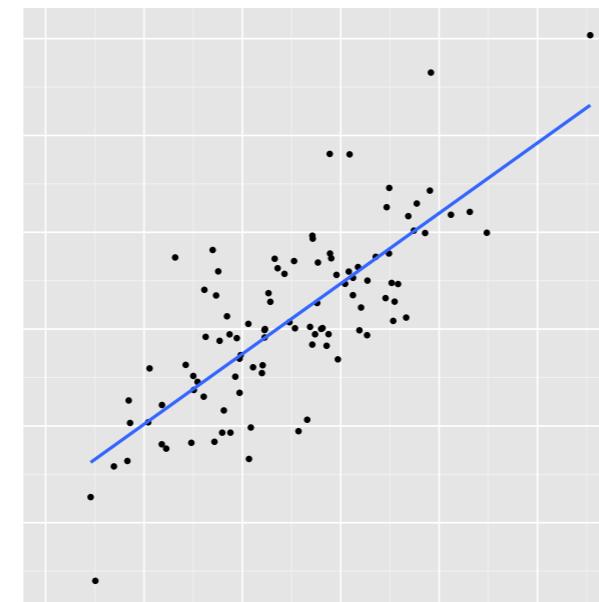
Validation experiments

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_1 X_{i2} + \beta_3 X_{i1} X_{i2} + \cdots + \epsilon_i$$

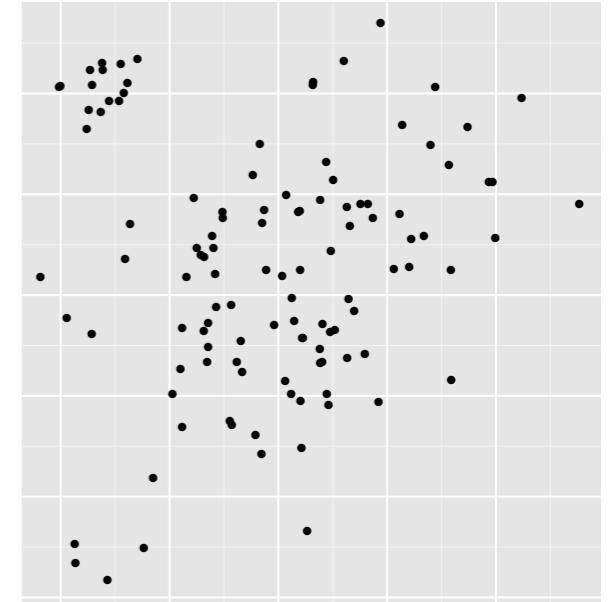
$$H_0 : \beta_k = 0 \text{ vs } H_1 : \beta_k \neq 0$$



Categorical X



Quantitative X

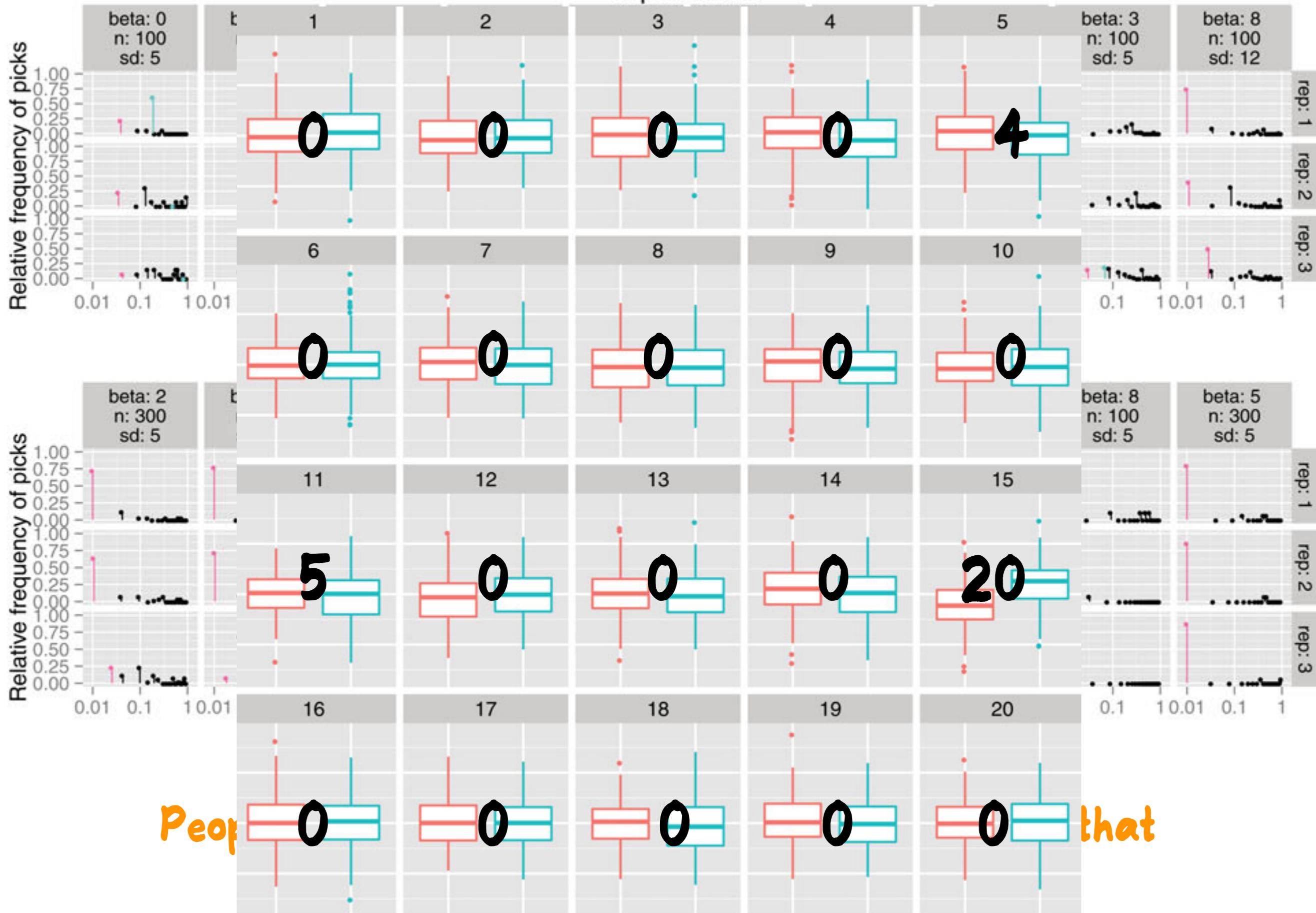


Contamination

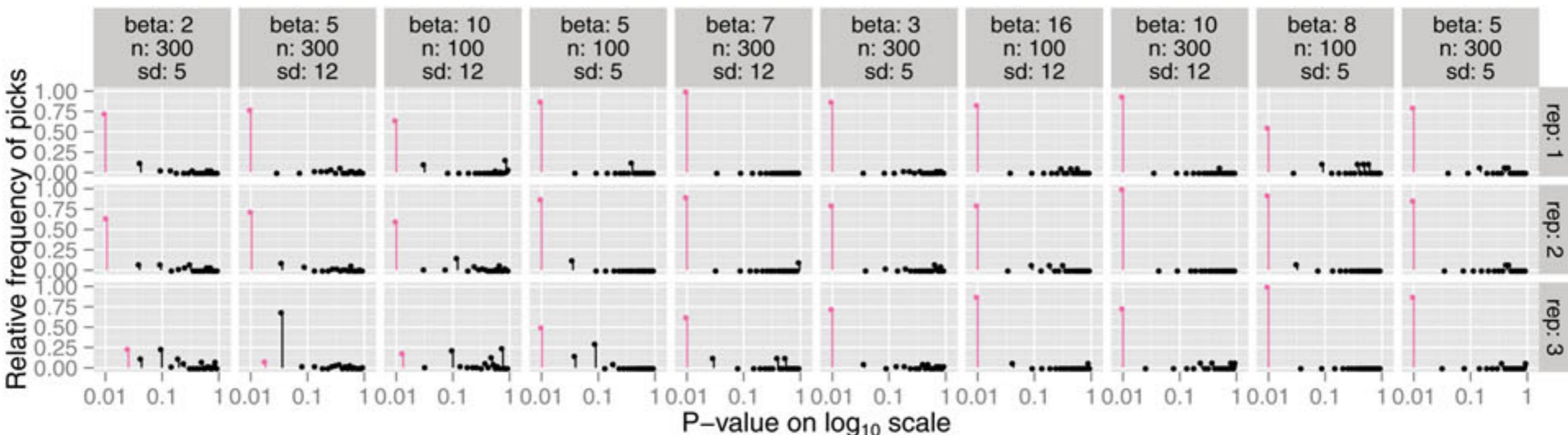
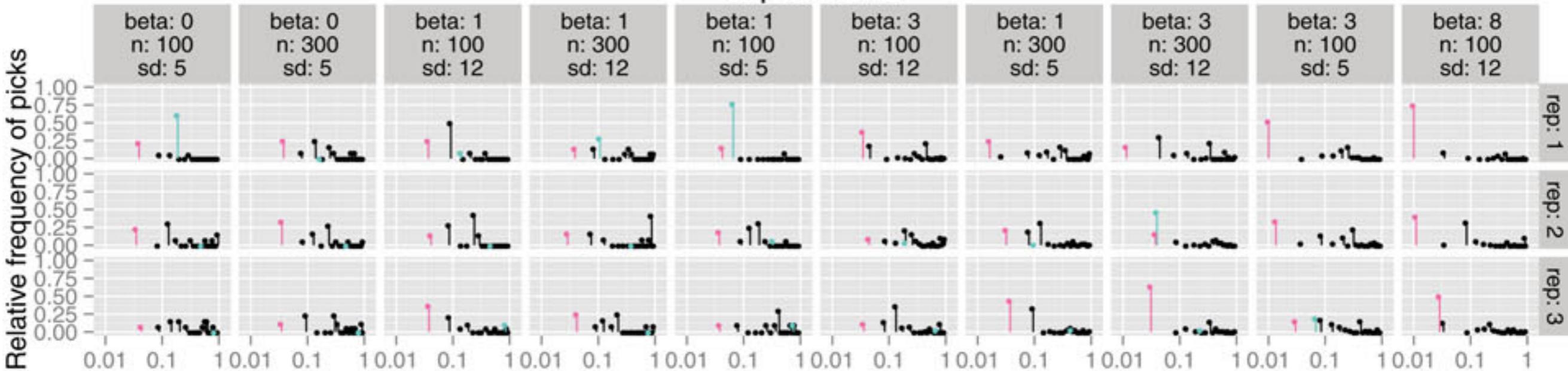
Results

- ➊ People's eyes operate similarly to classical test statistics.
- ➋ The detection of data plot is related to effect size, yielding results closer to practical significance. Less sensitive to sample size. (NOTE: We do not expect people to see statistical significance.)
- ➌ The more observers the more powerful the procedures.
- ➍ Education, gender and age do not affect detection rate

Experiment 1



Experiment 1



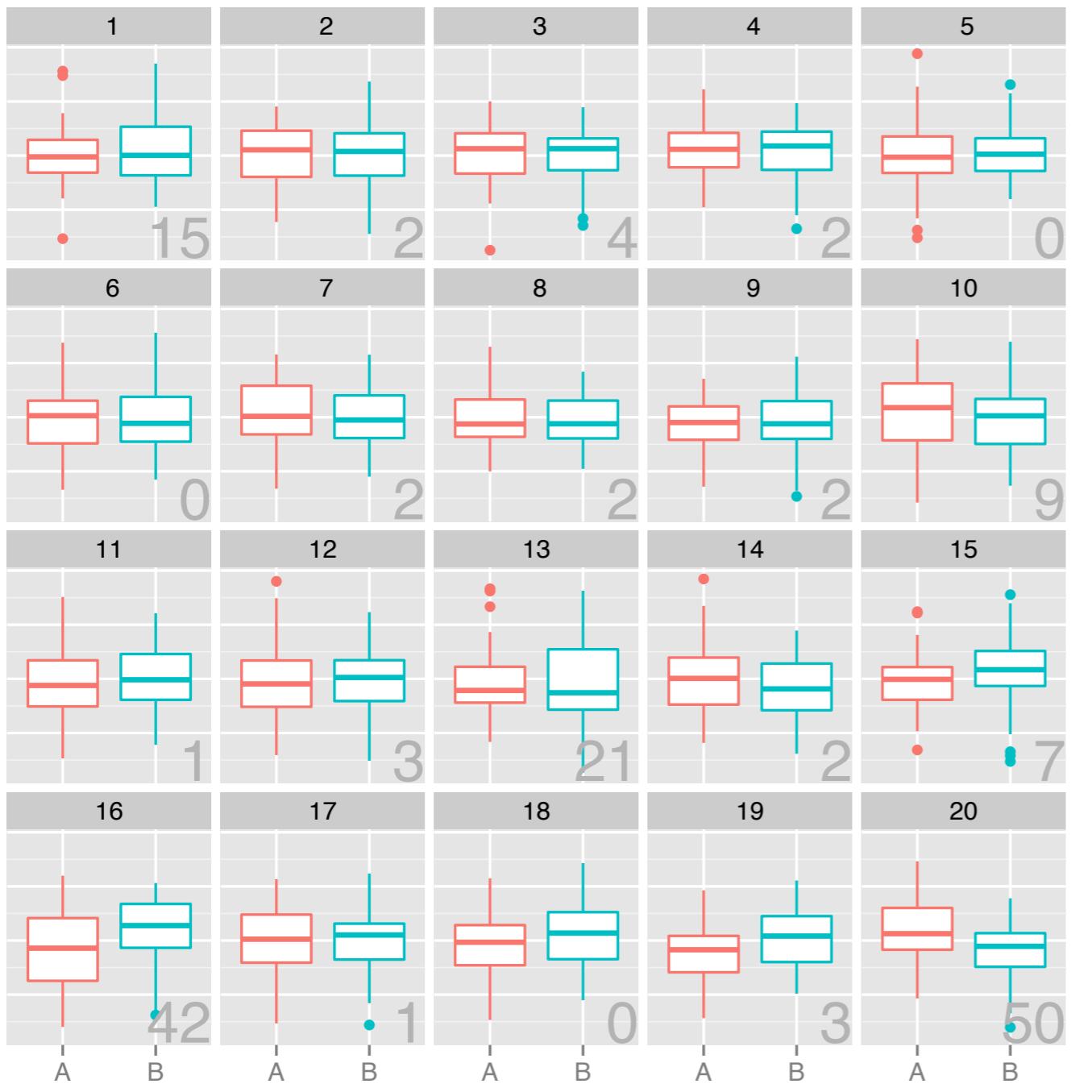
People tend to pick the plot from the lineup that has the lowest p-value

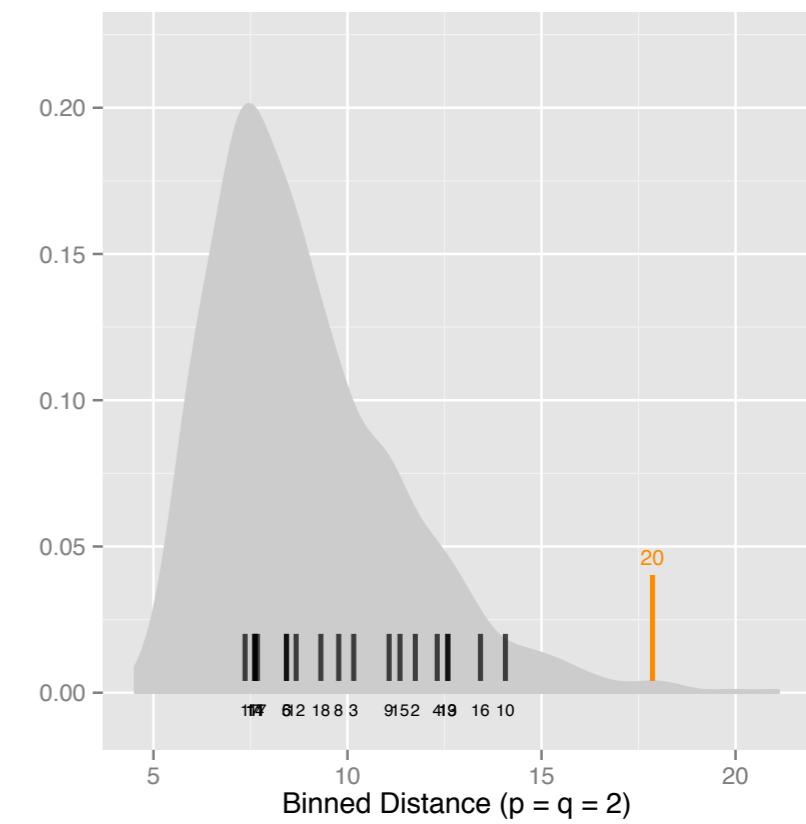
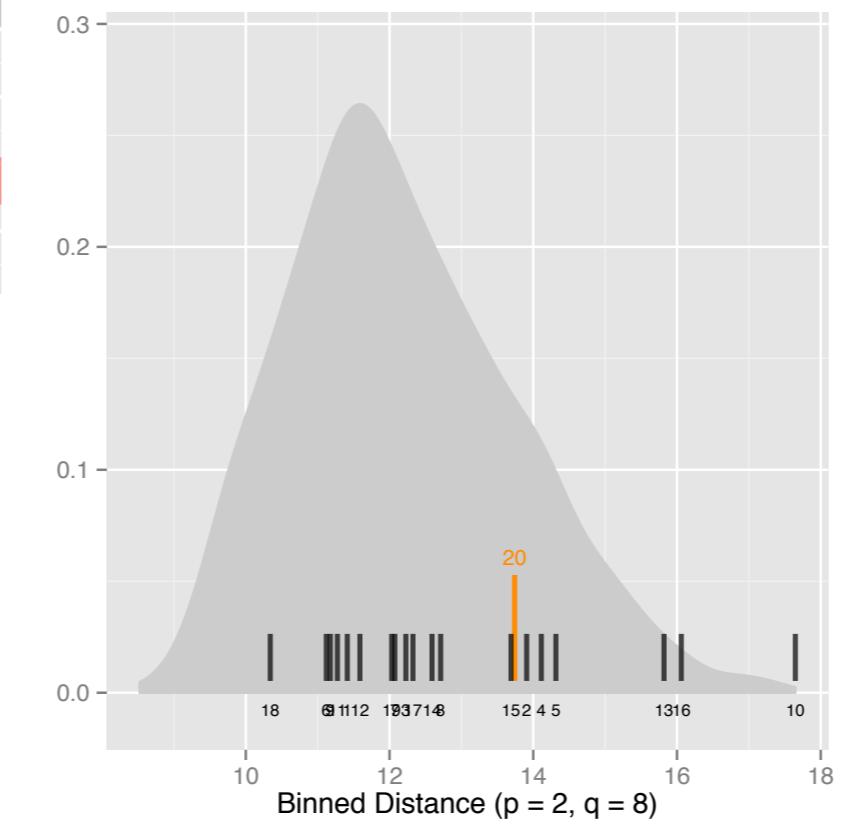
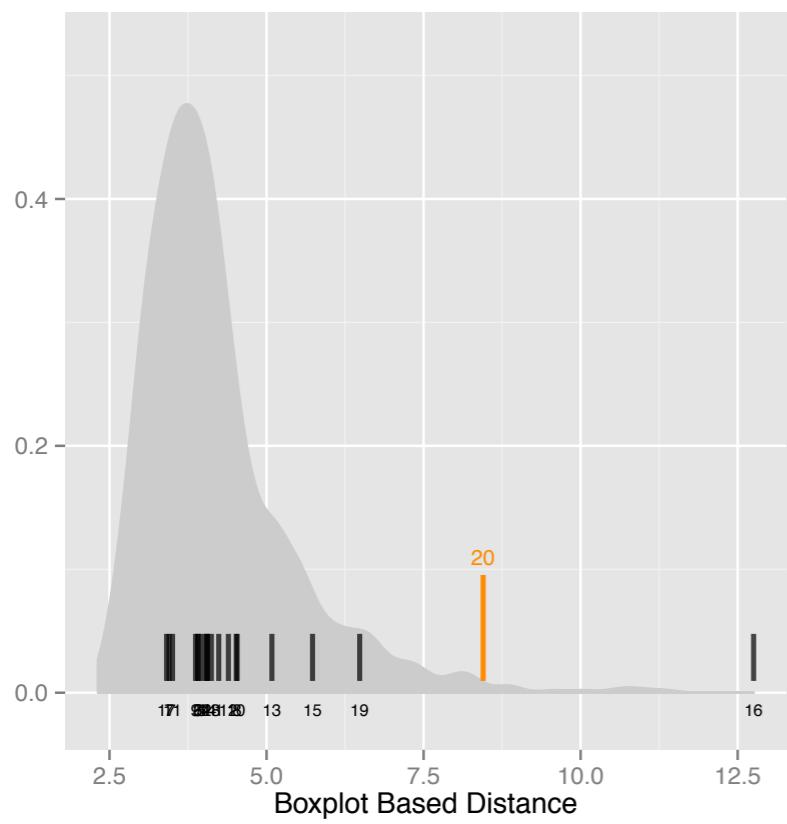
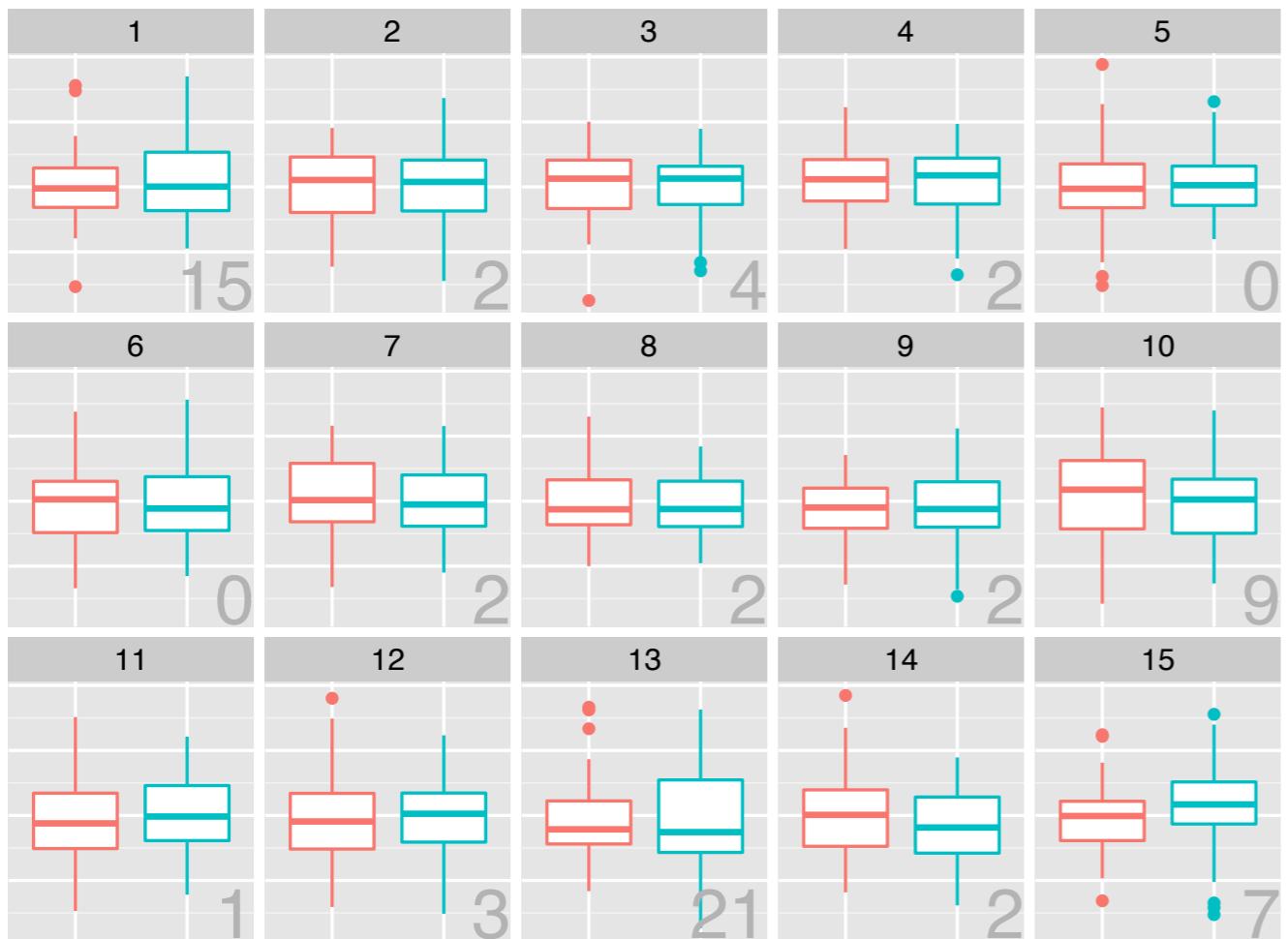
Source: Majumder et al (2013) JASA

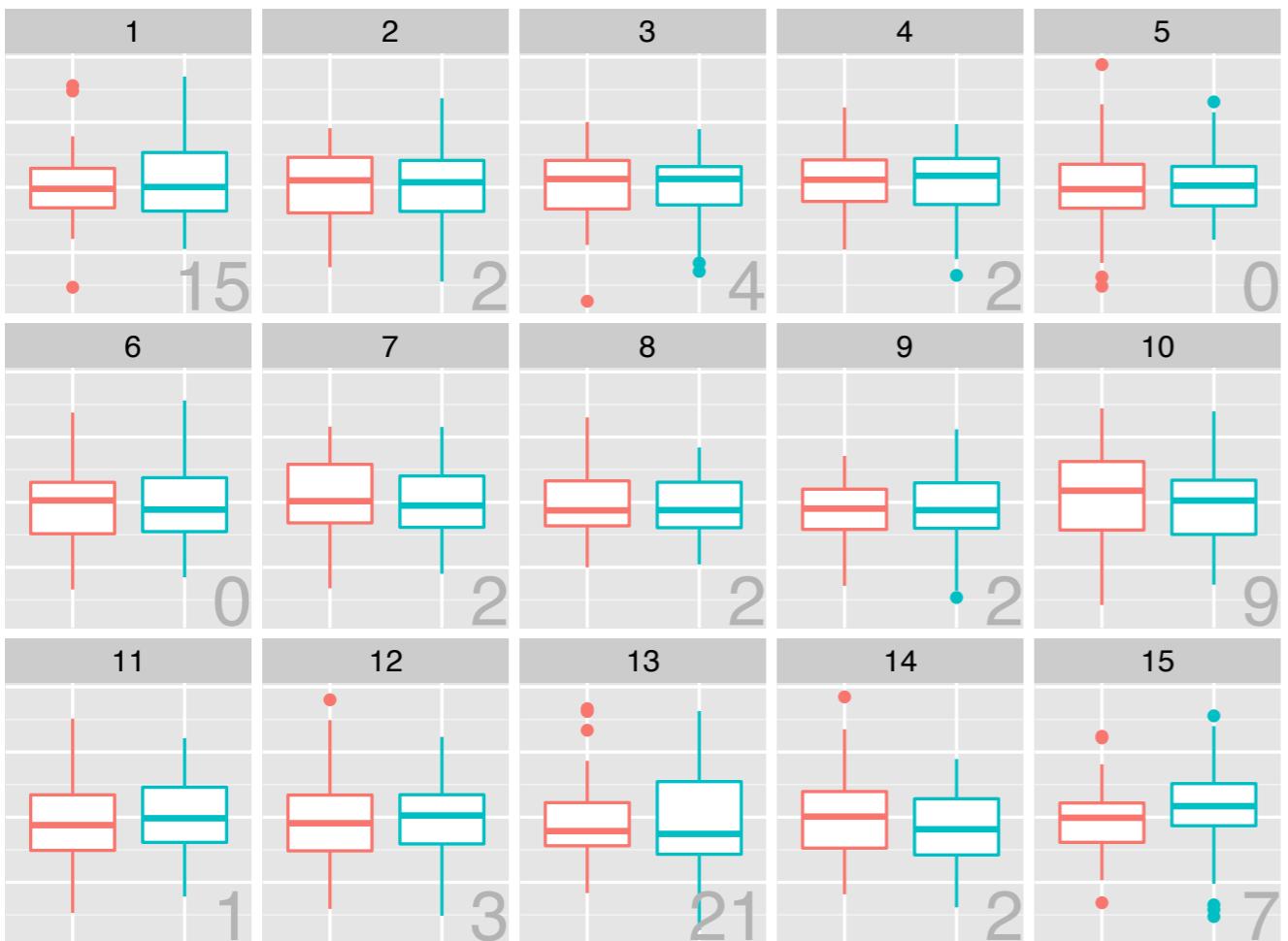
Quantify patterns?

- ➊ Can we use this process to design metrics that capture structure, so that we can automate pattern recognition?
- ➋ Measure difference between data plot and null plots
- ➌ Match these to what people choose

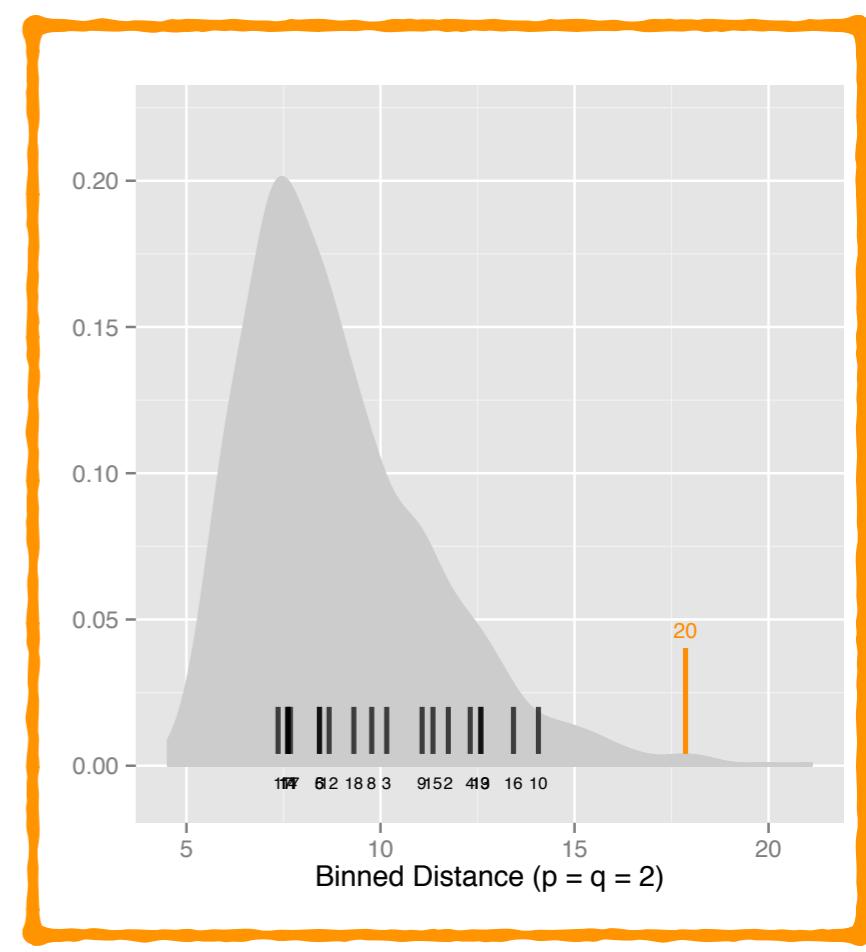
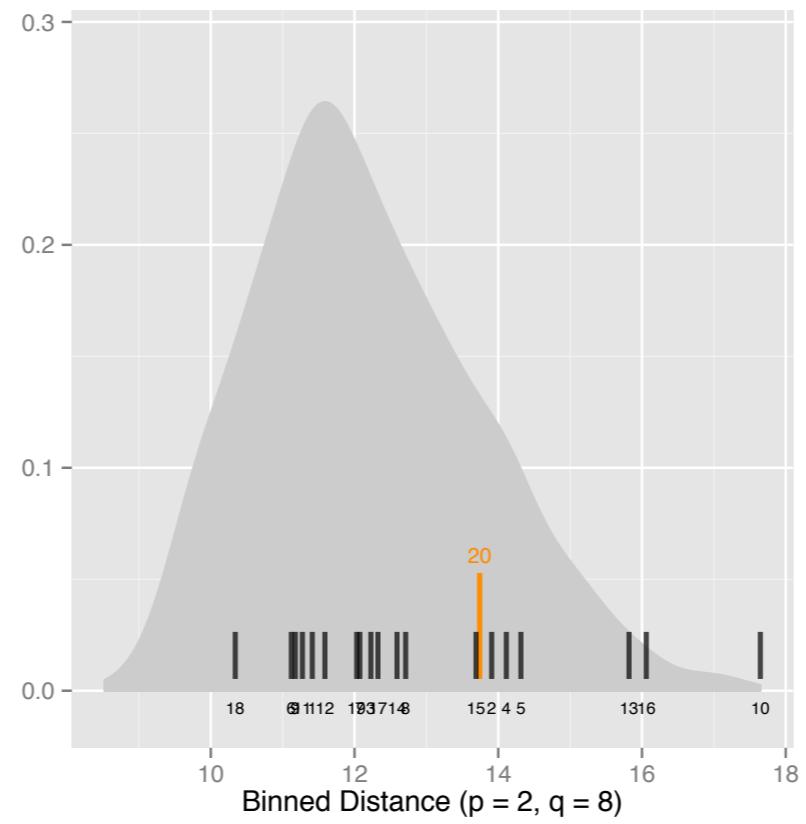
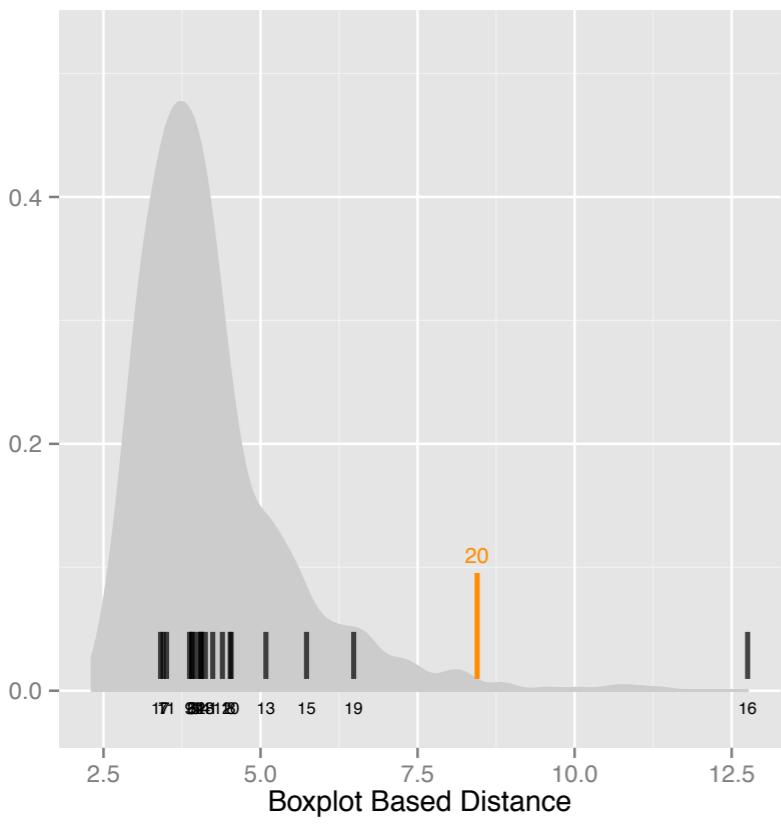
Source: Roy Chowdhury et al (2017) JCGS







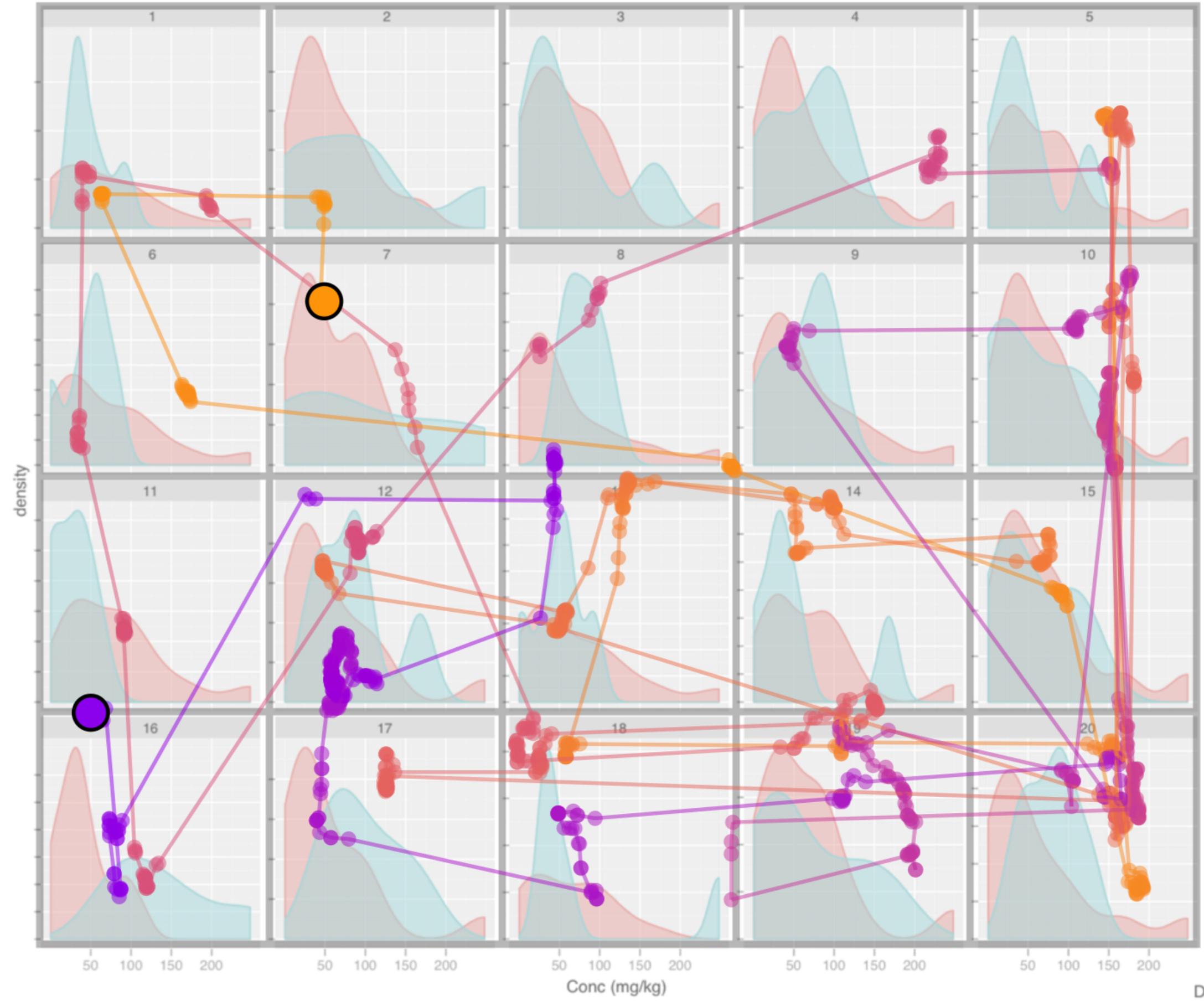
This metric best captures what people select

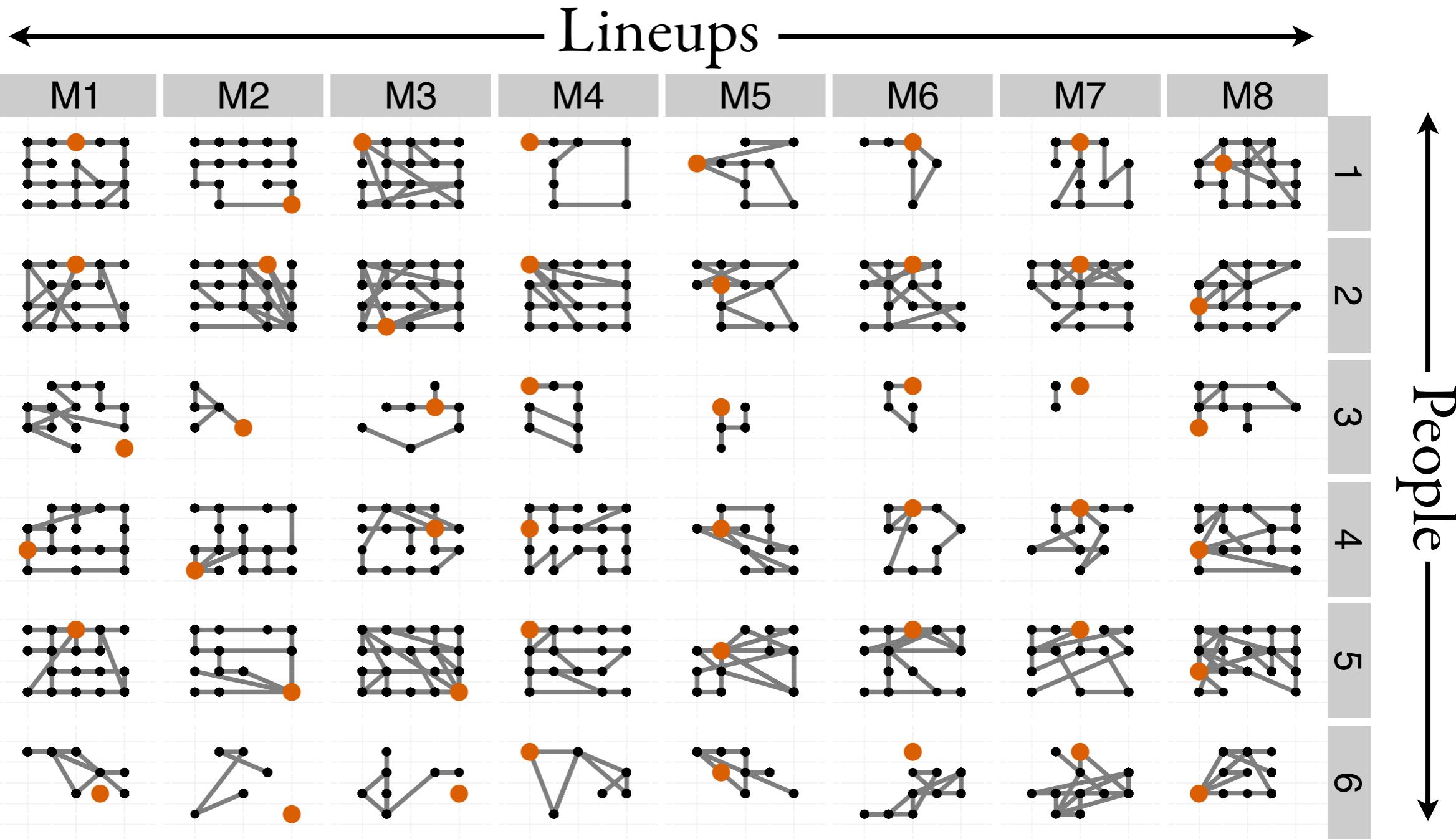


Follow-up eye tracking

- 12 lineups shown to 24 subjects
- Examined where their eyes focused in the page of plots while answering the same question as the Turk study

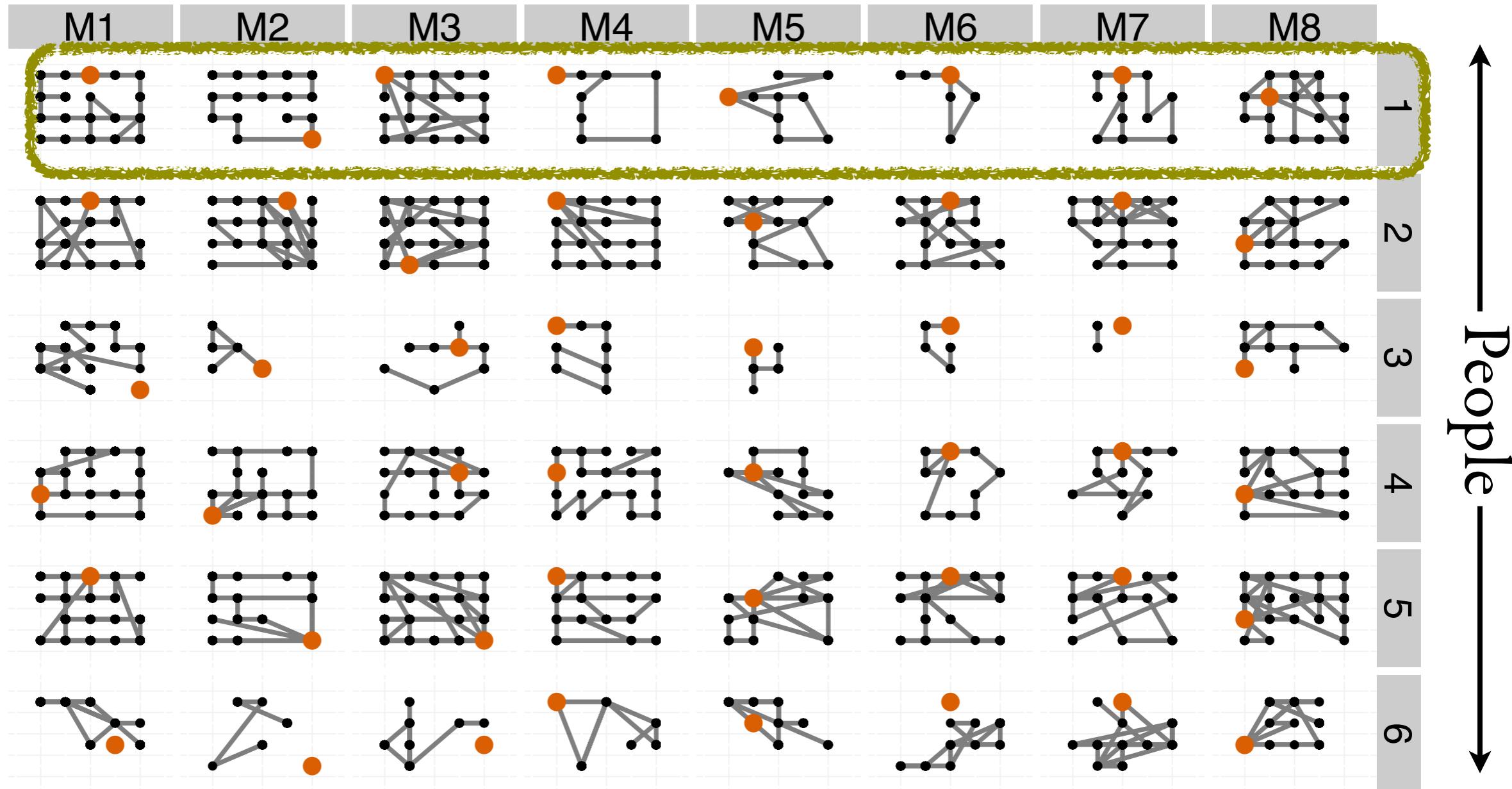
Source: Zhao et al (2013) IJITAS





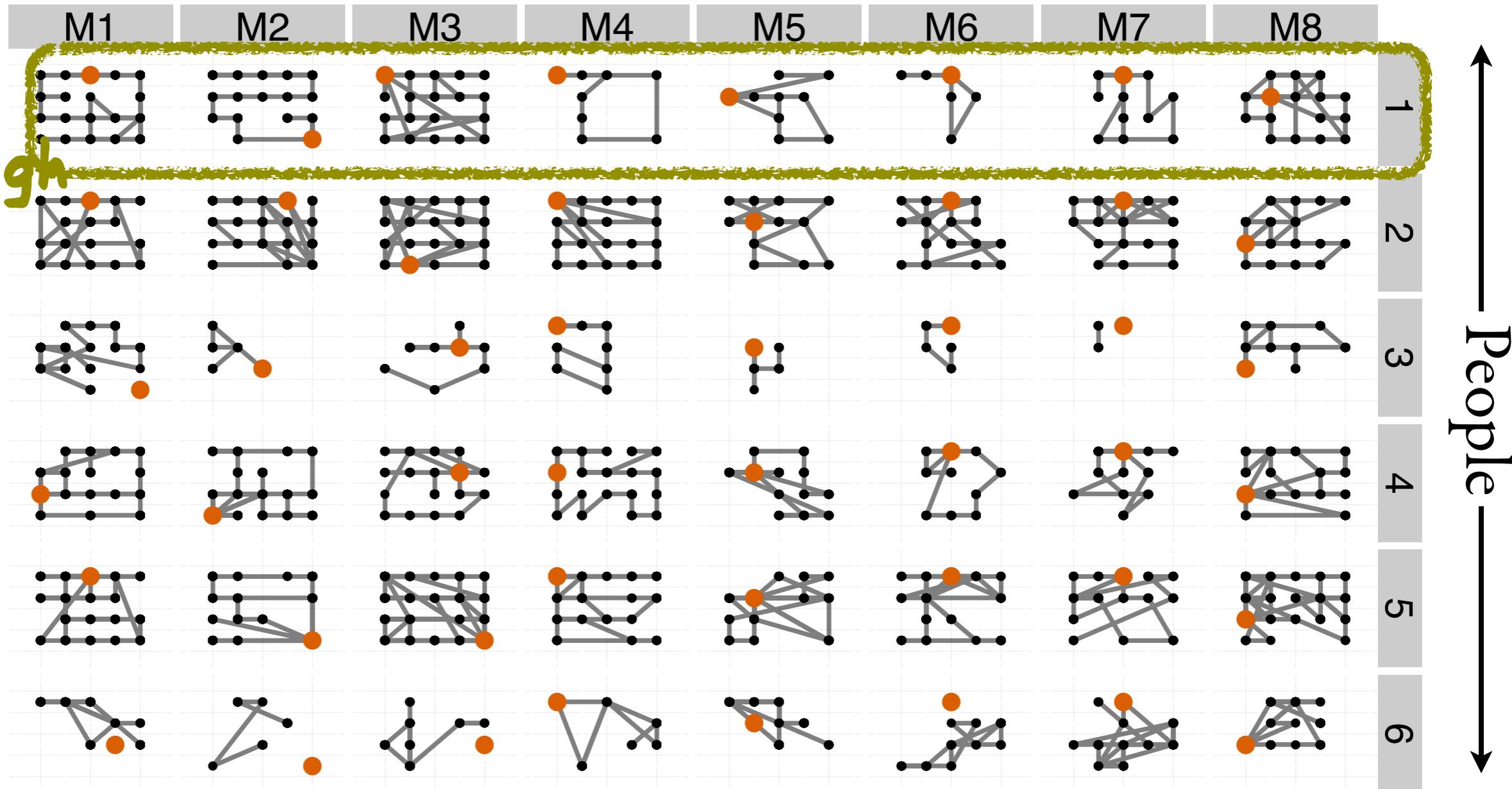
Source: Zhao et al (2013) IJITAS

Lineups



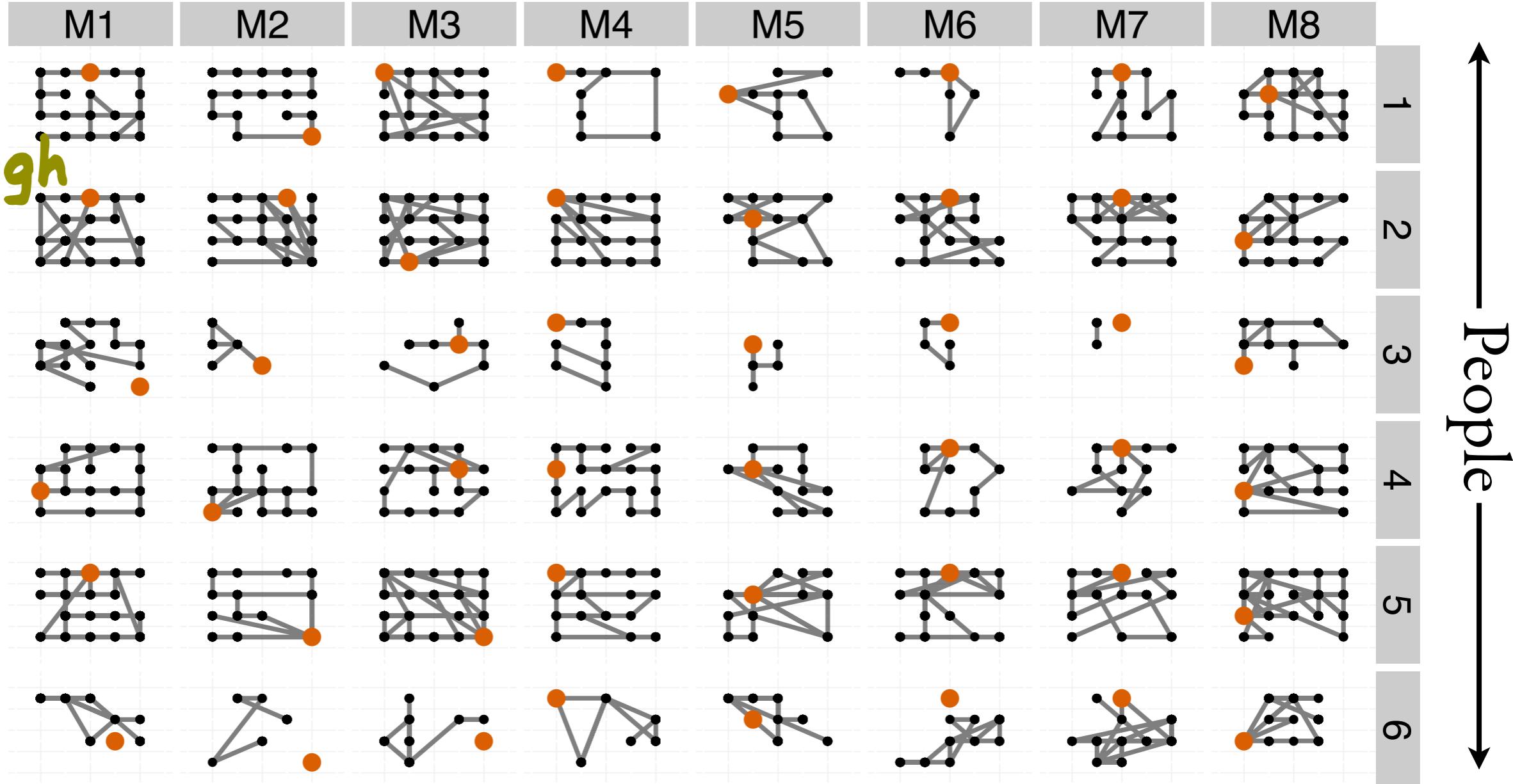
Source: Zhao et al (2013) IJITAS

Lineups

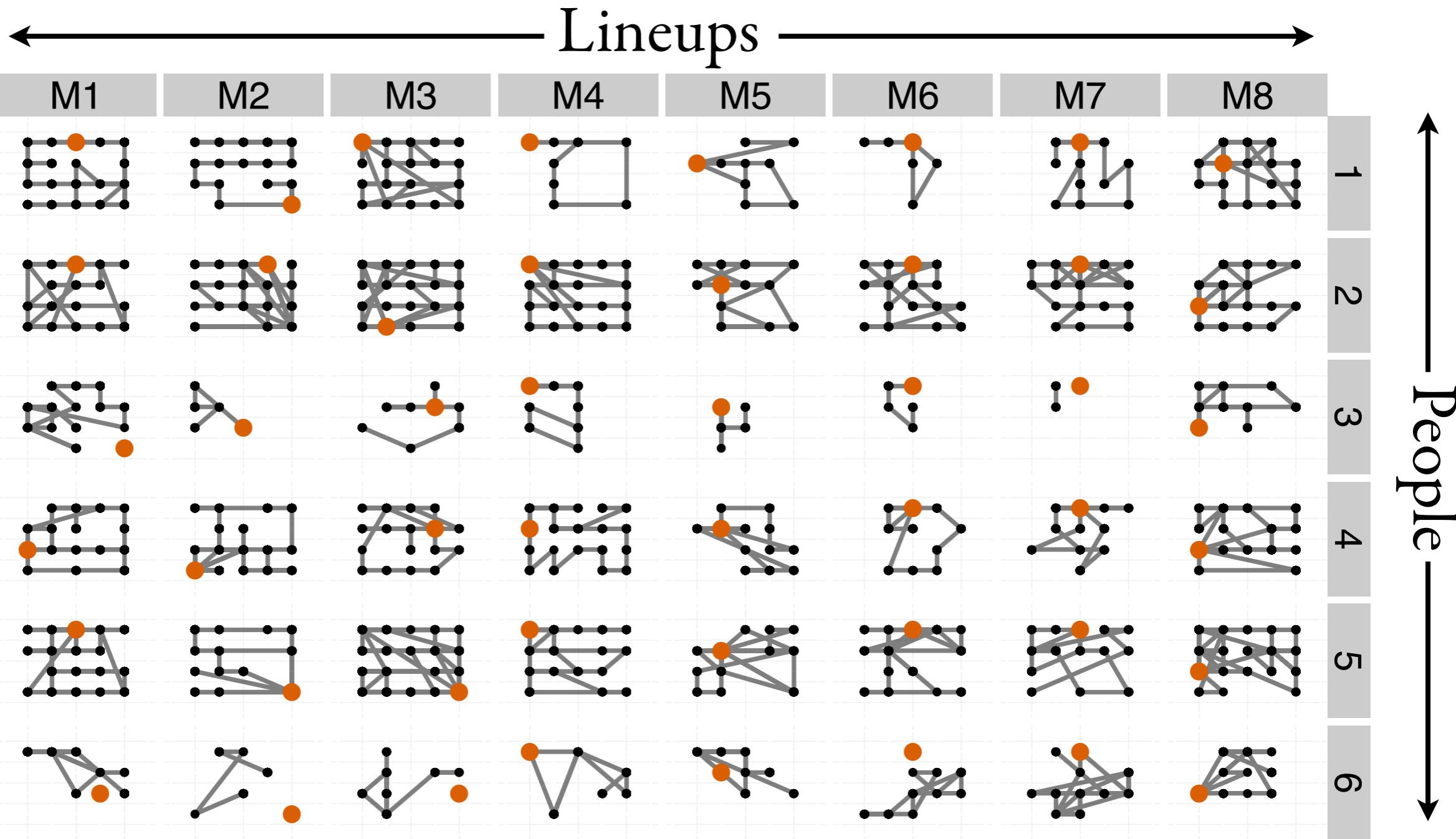


Source: Zhao et al (2013) IJITAS

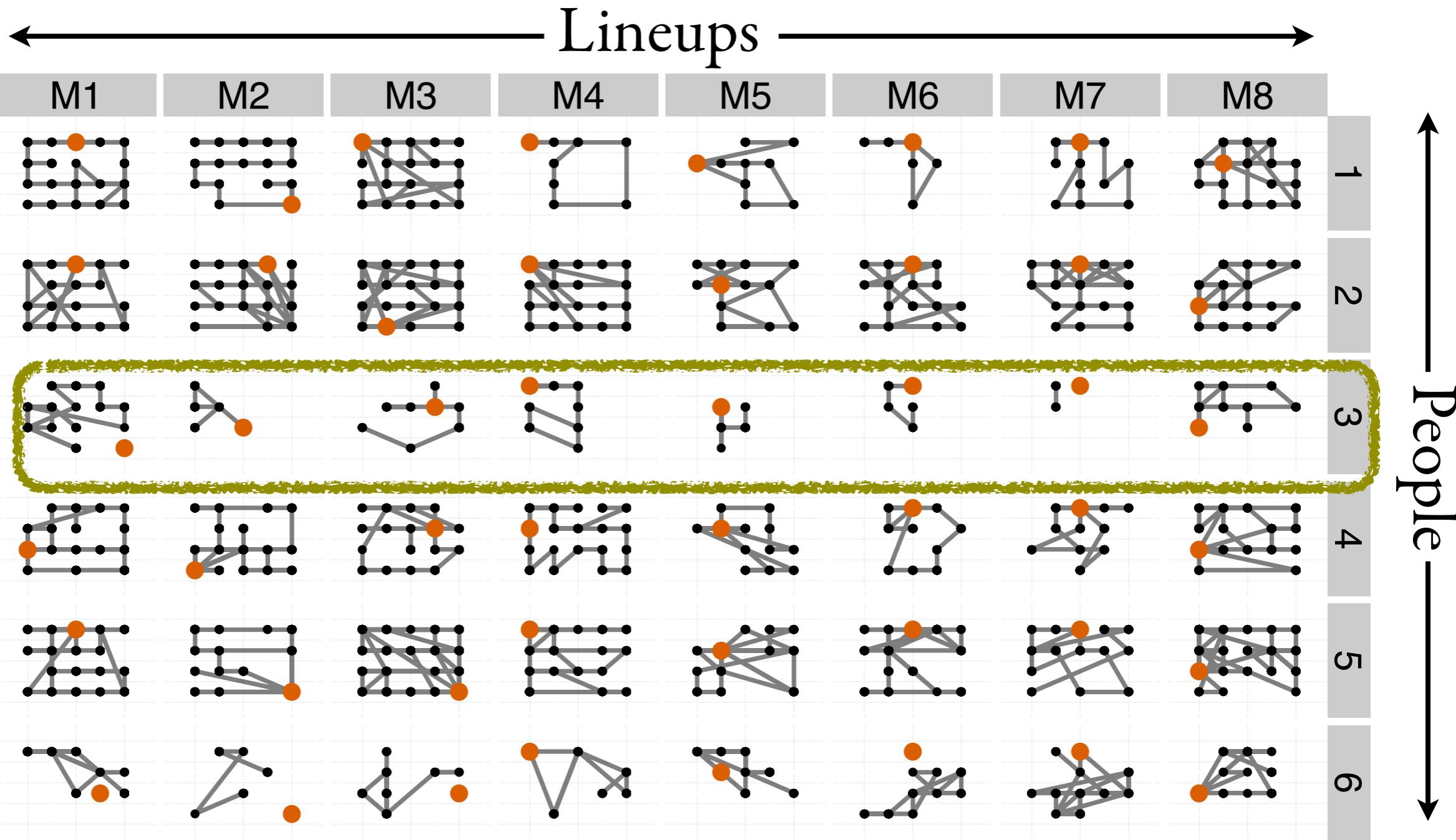
Lineups



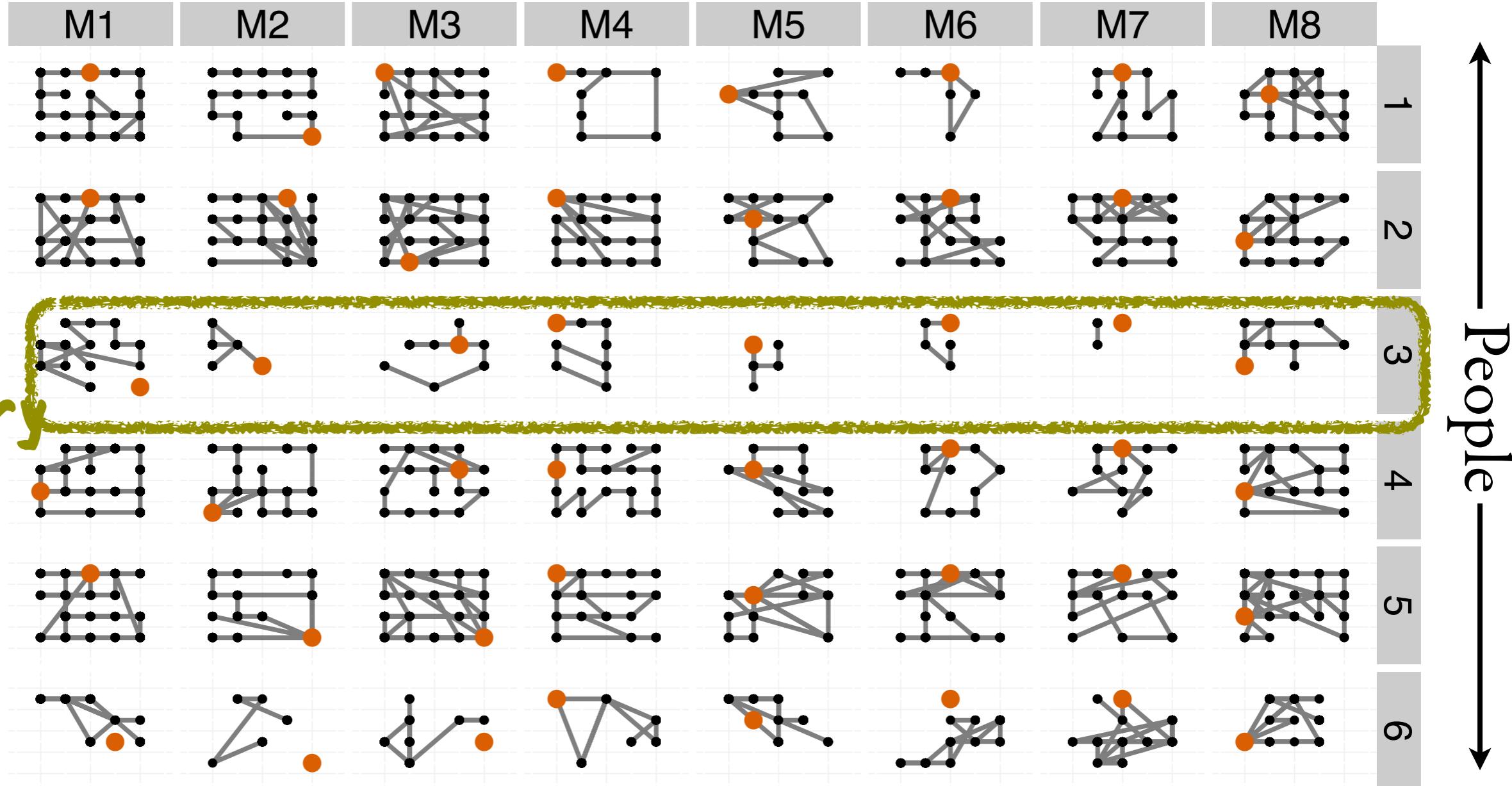
Source: Zhao et al (2013) IJITAS



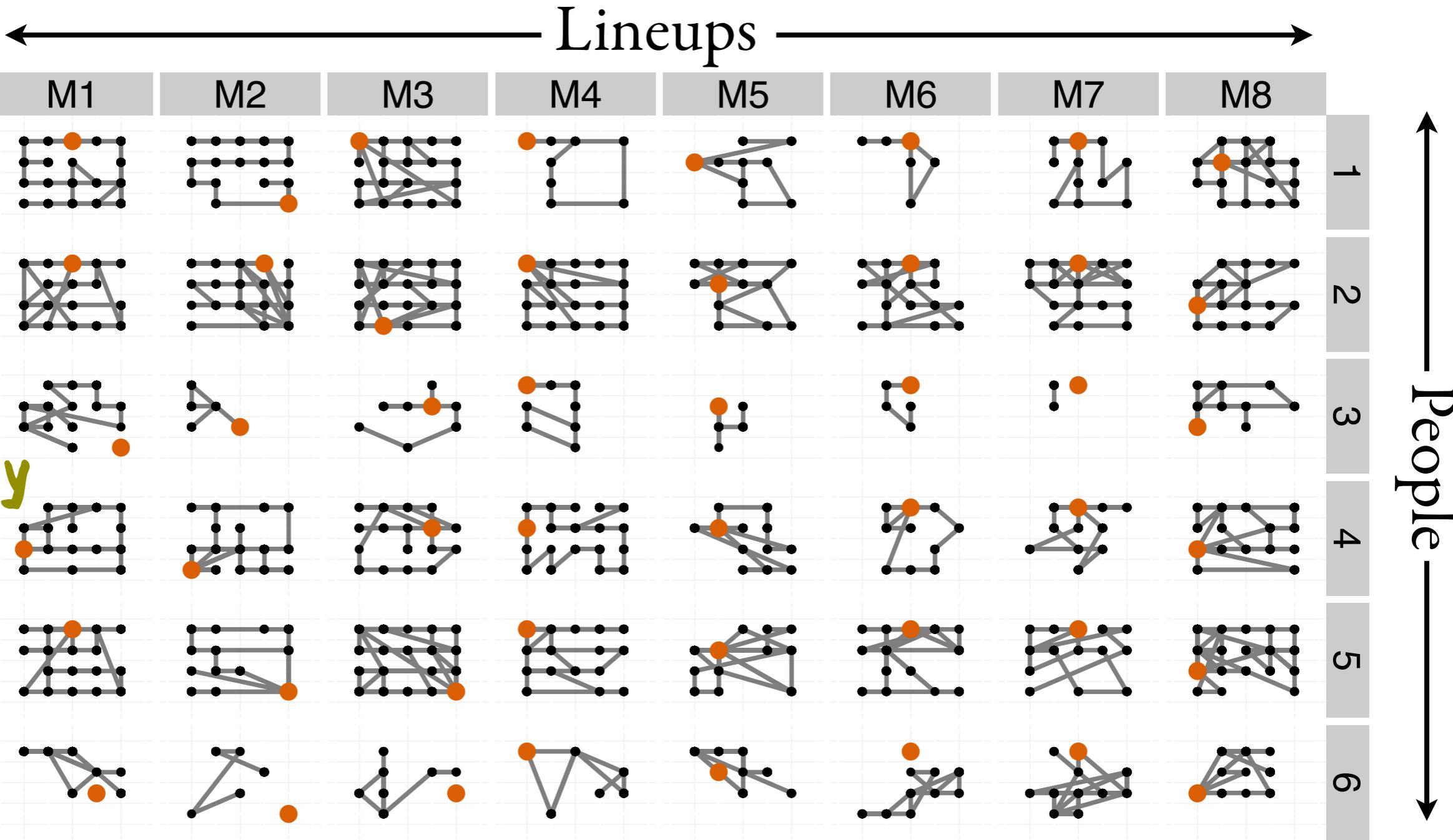
Source: Zhao et al (2013) IJITAS



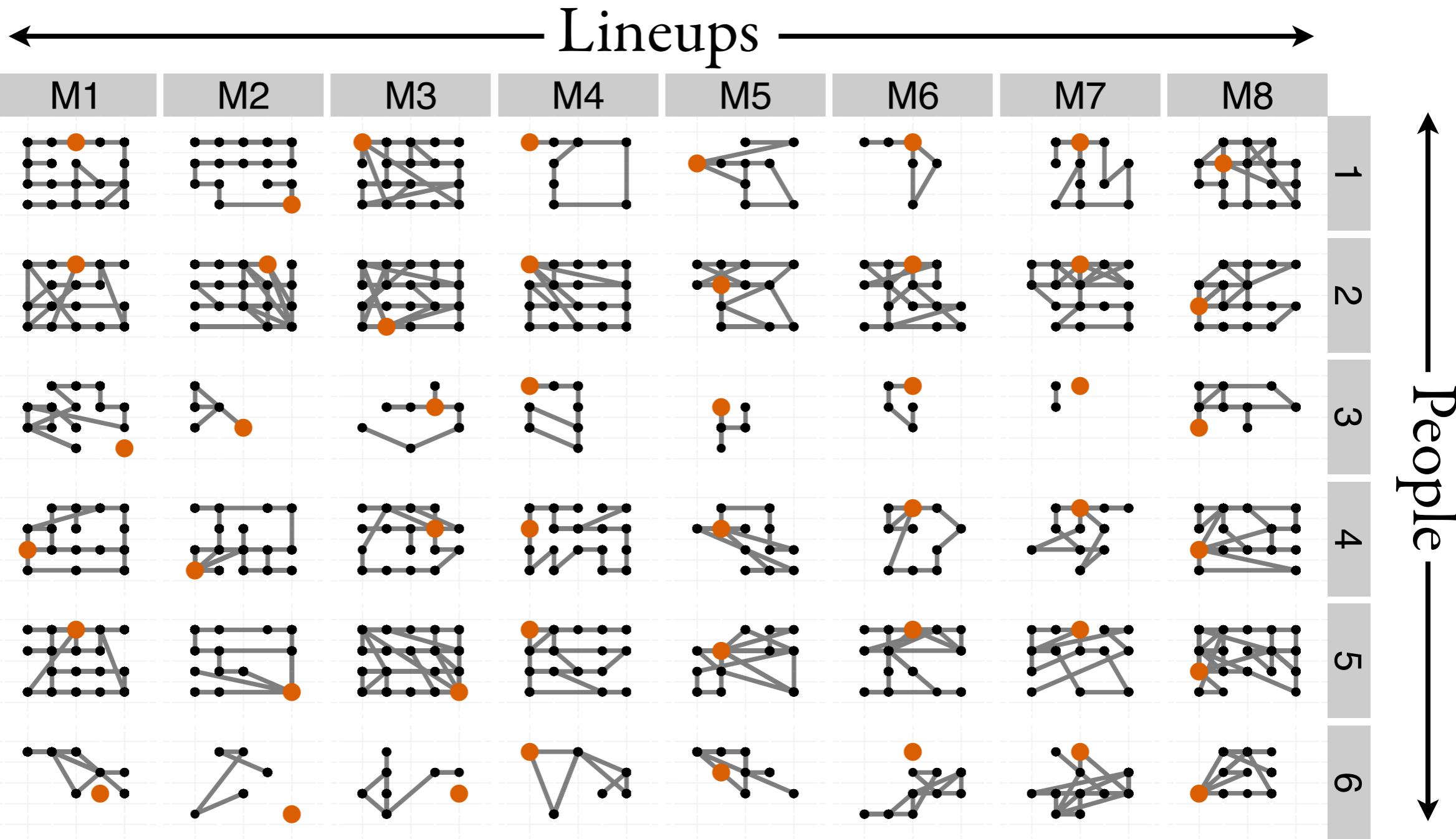
Lineups



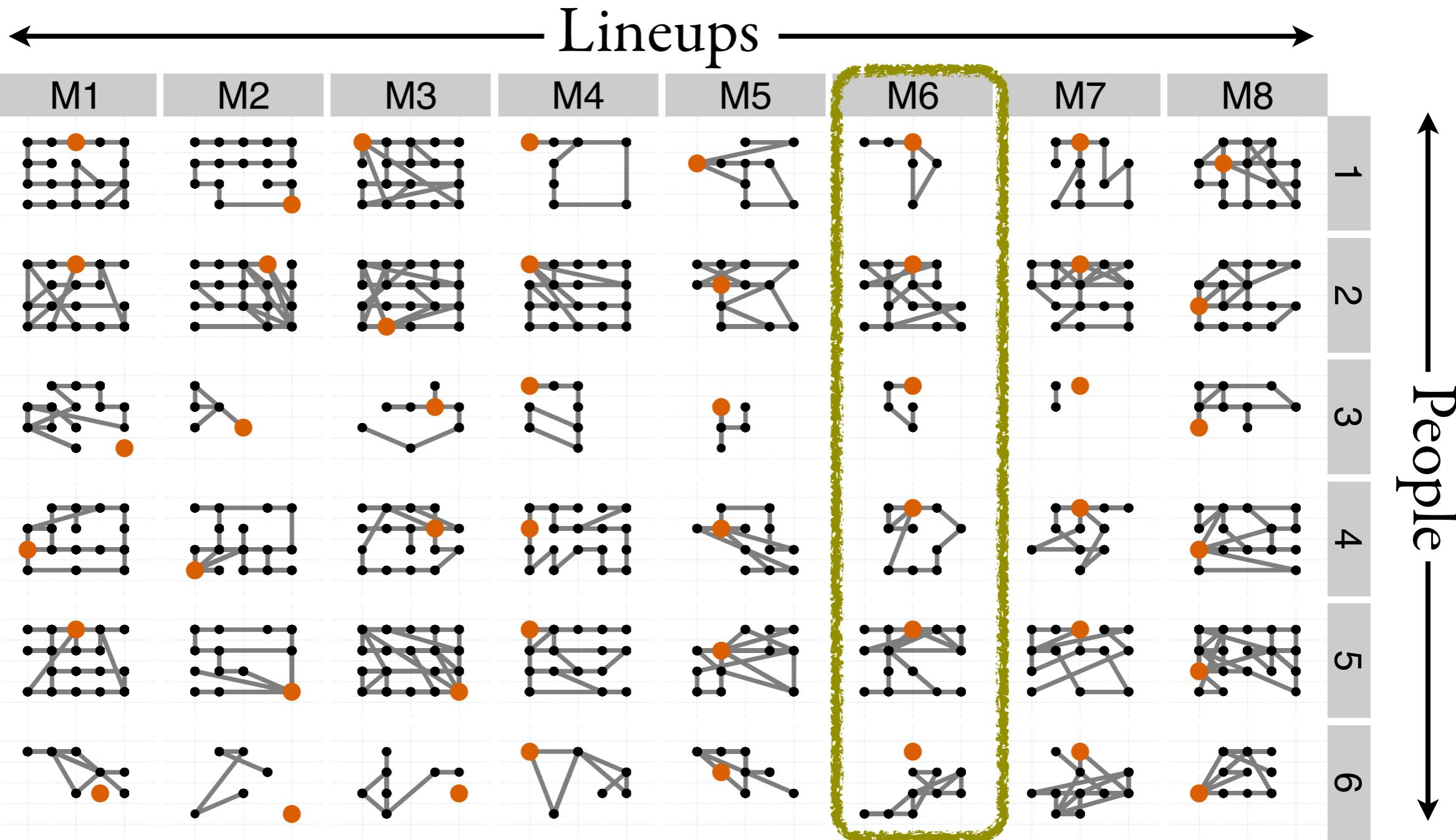
Source: Zhao et al (2013) IJITAS



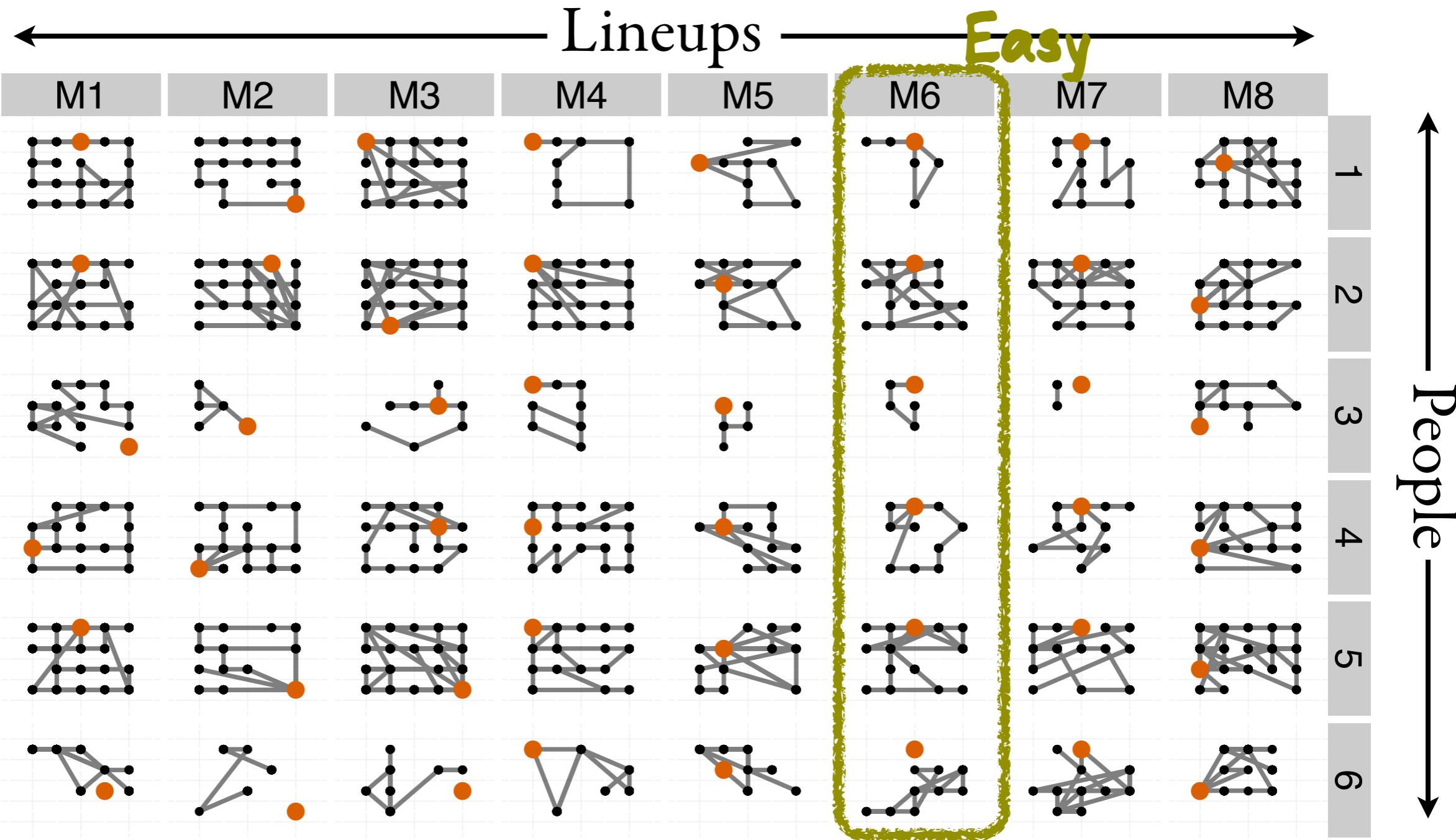
Source: Zhao et al (2013) IJITAS

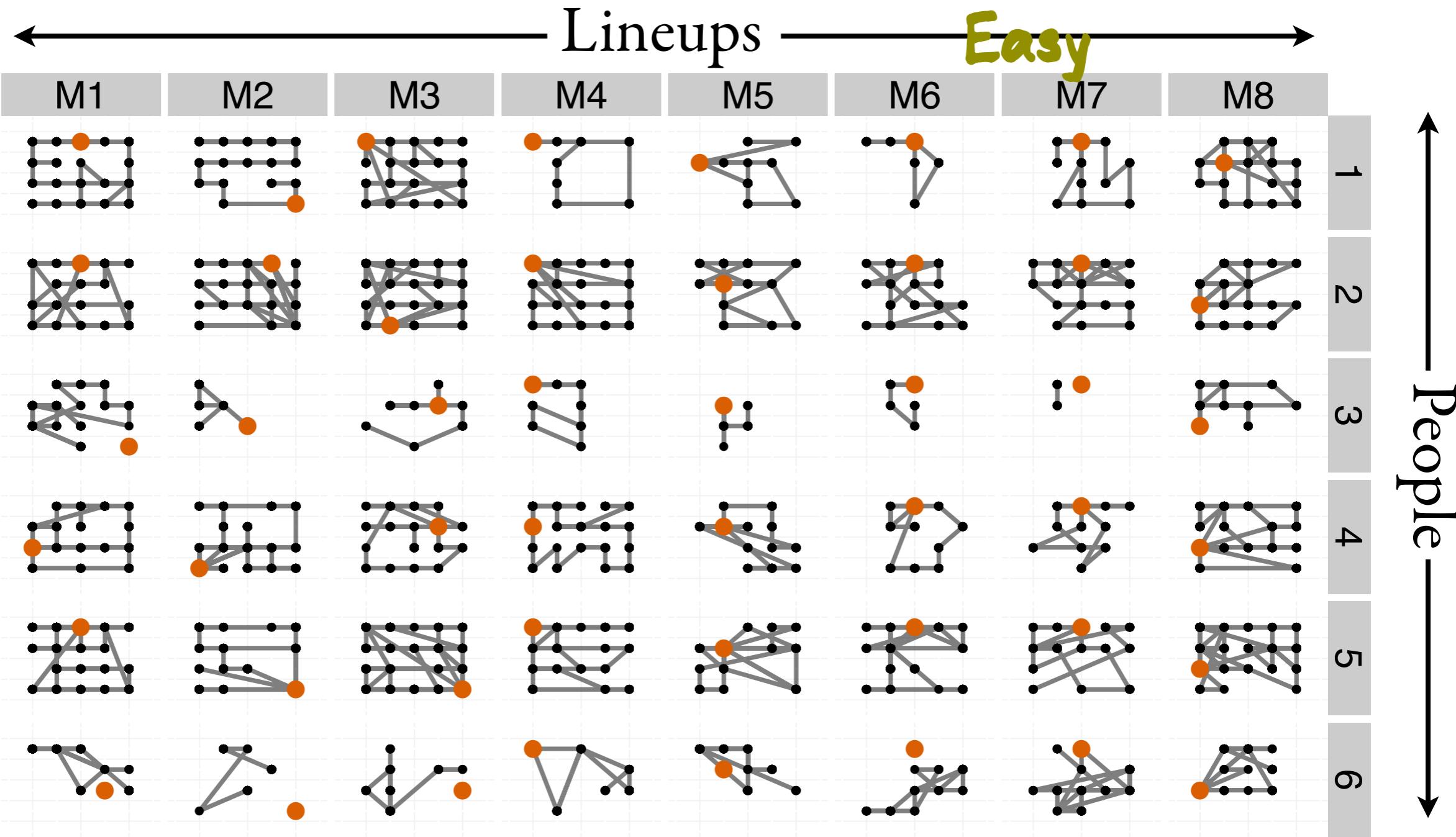


Source: Zhao et al (2013) IJITAS

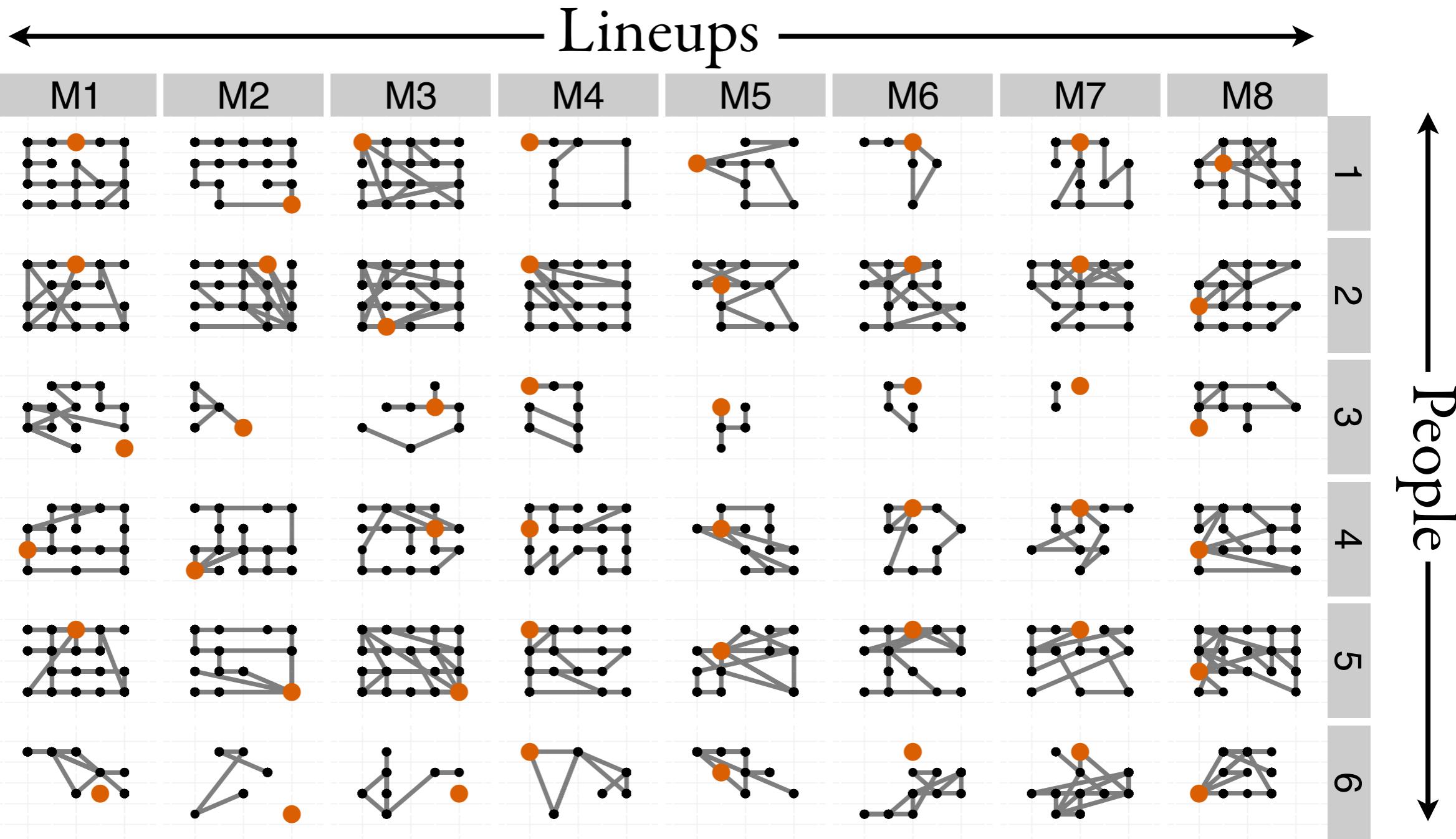


Source: Zhao et al (2013) IJITAS

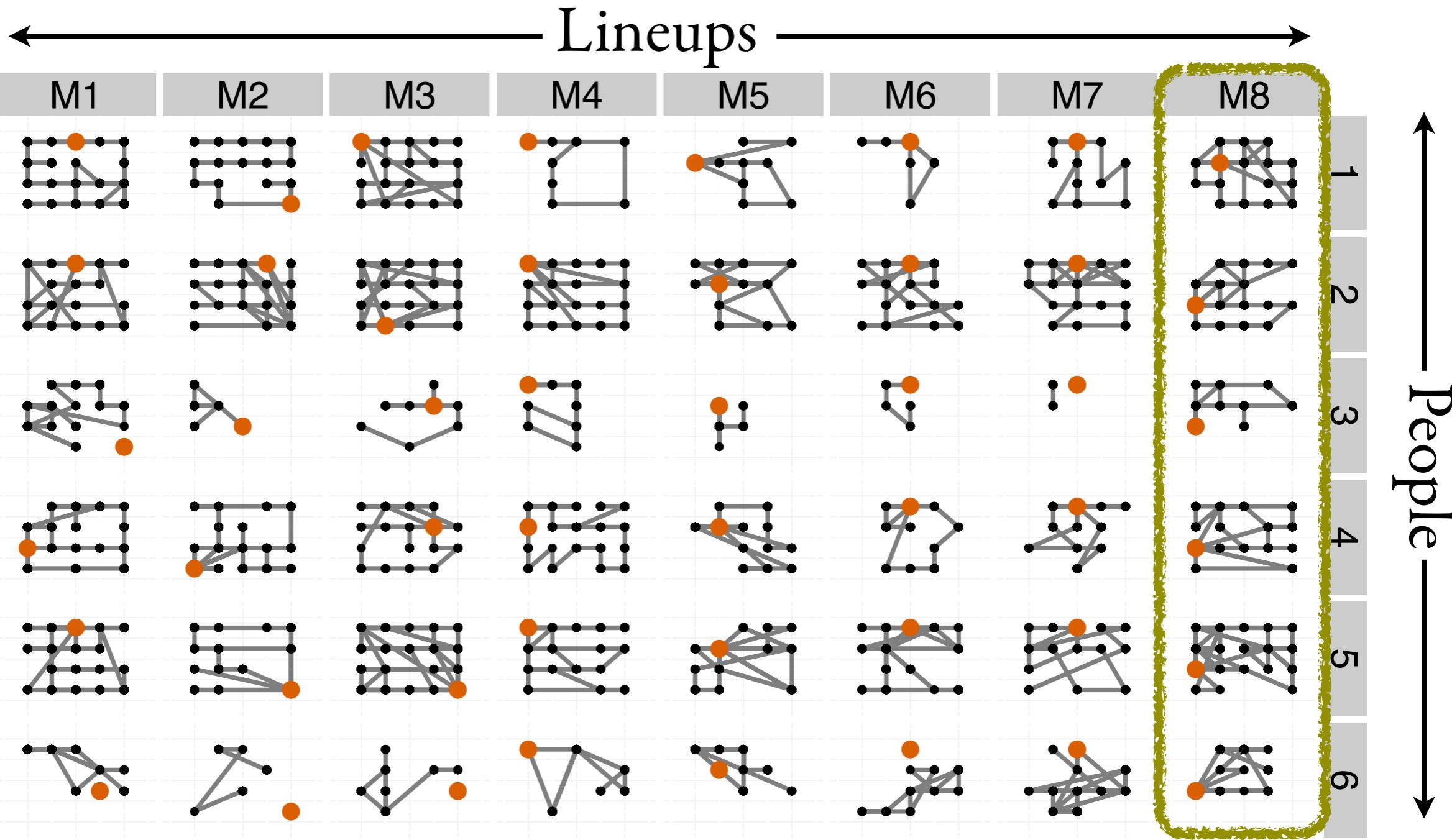




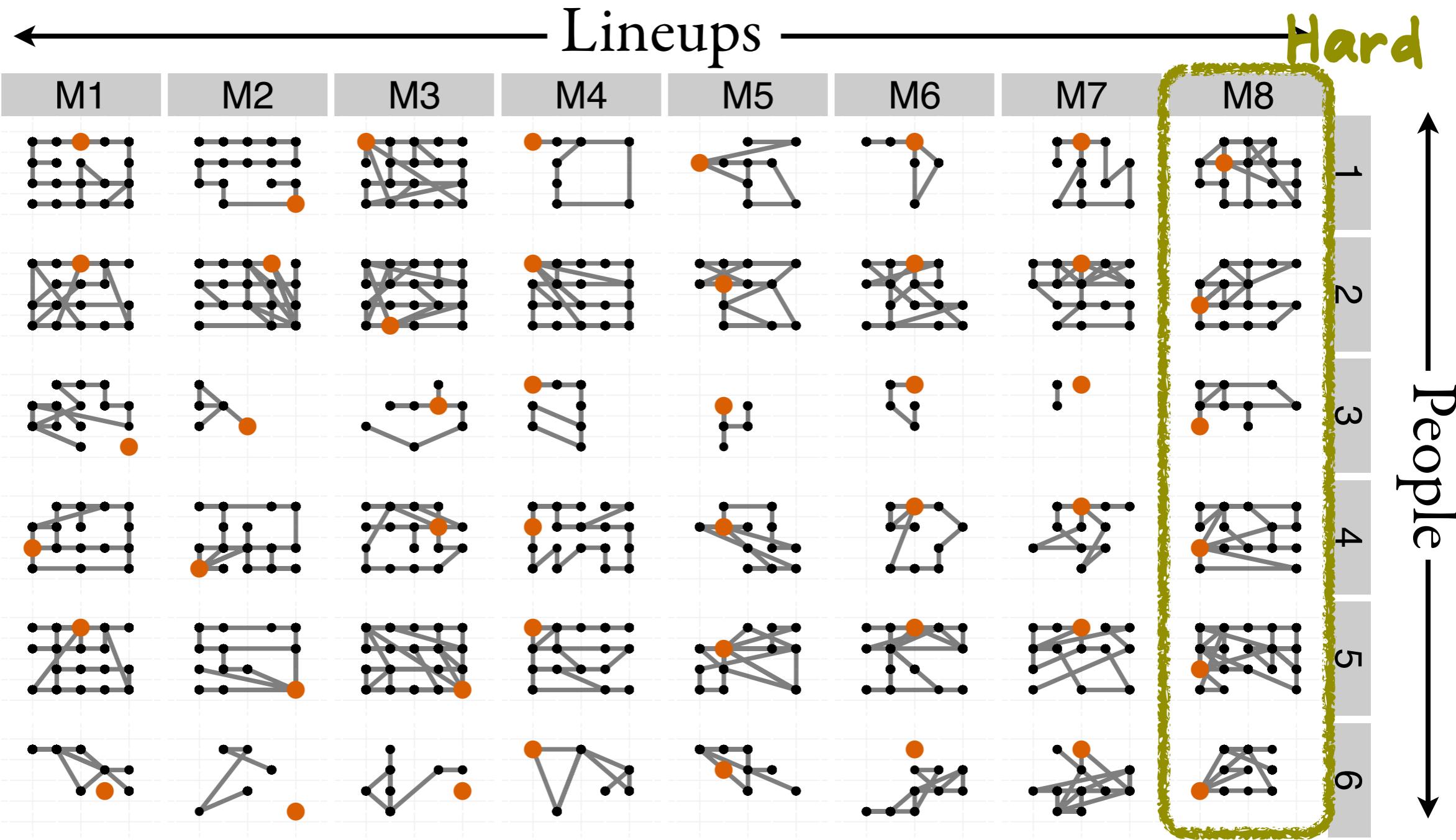
Source: Zhao et al (2013) IJITAS



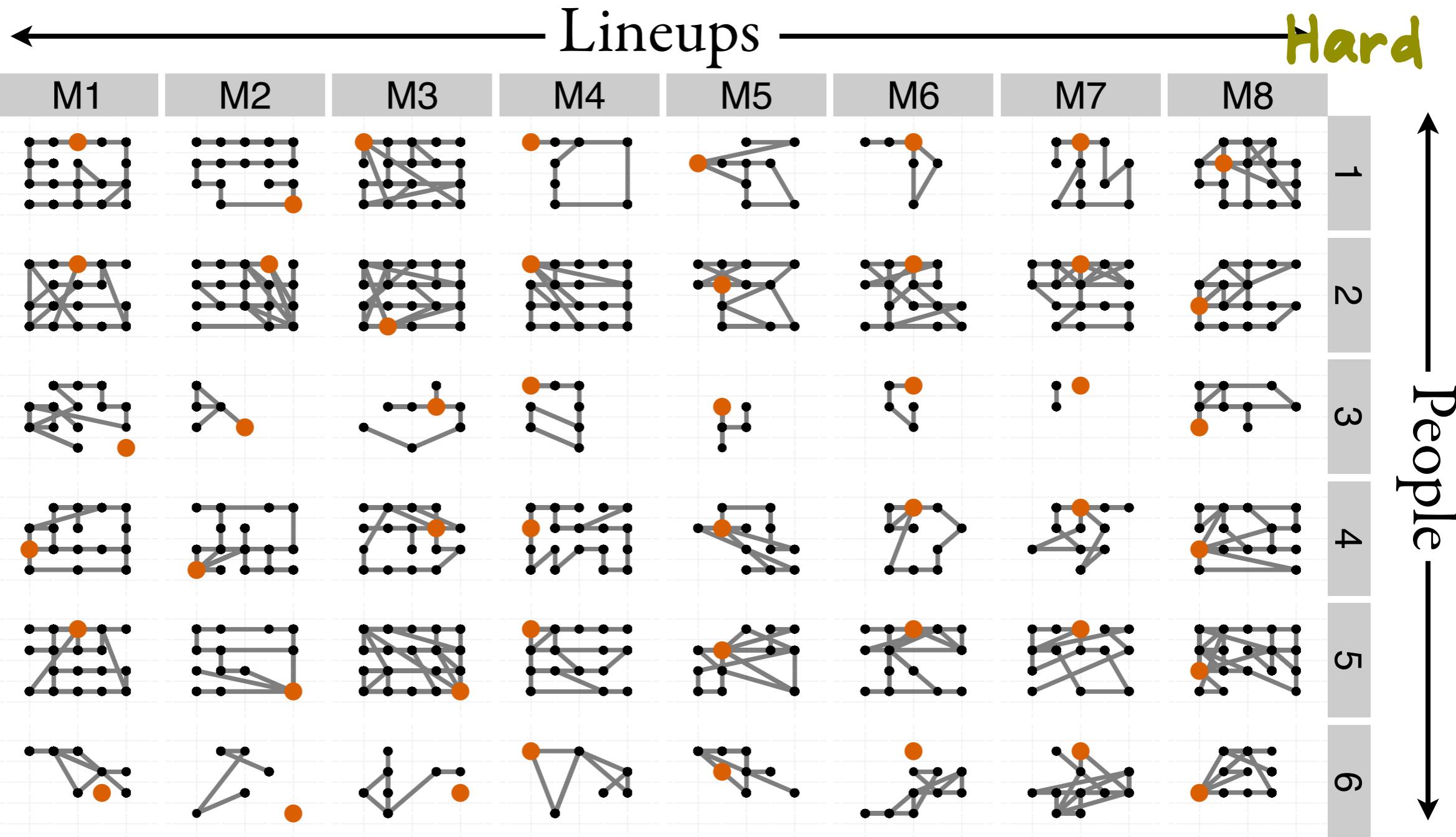
Source: Zhao et al (2013) IJITAS



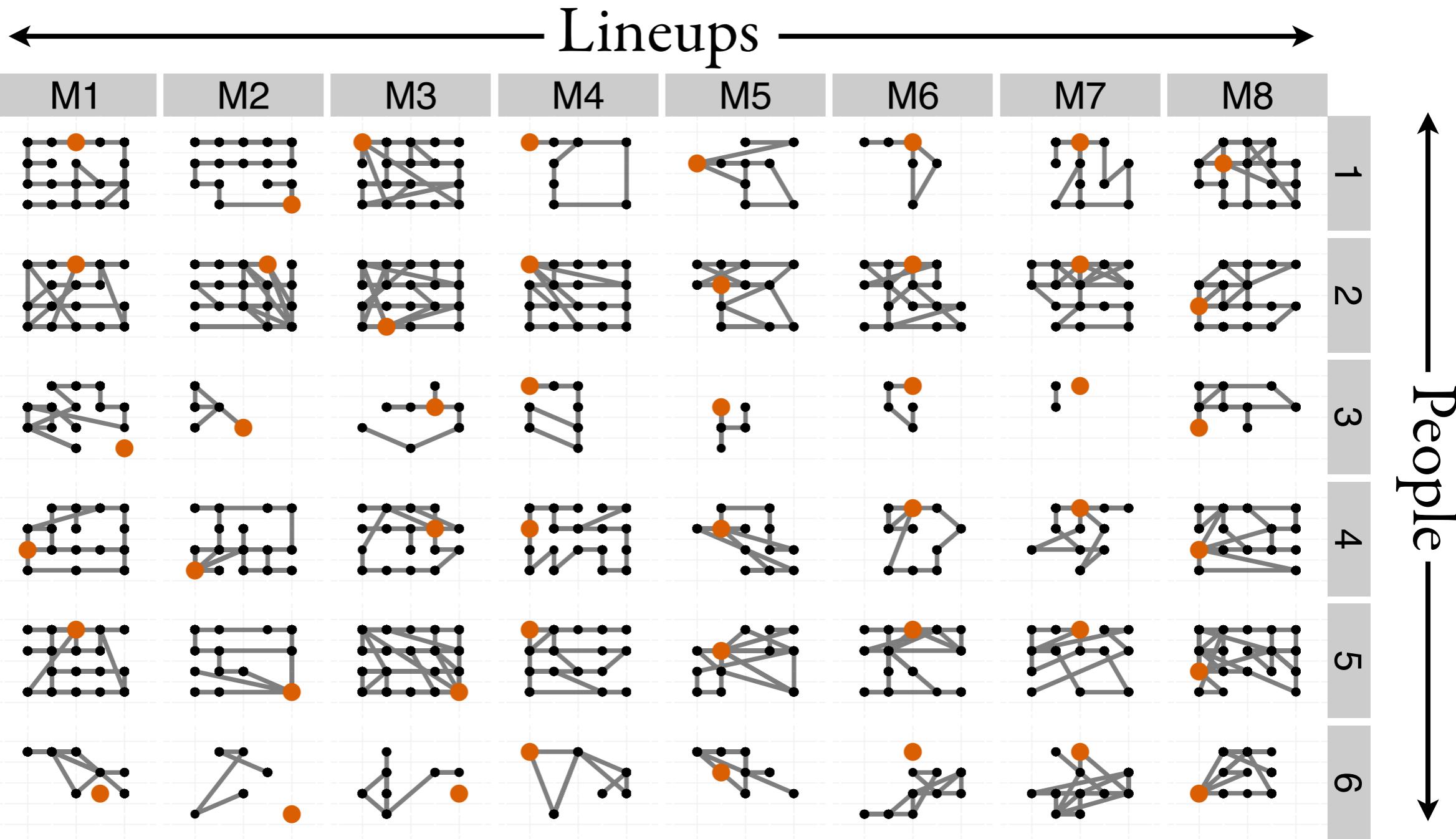
Source: Zhao et al (2013) IJITAS



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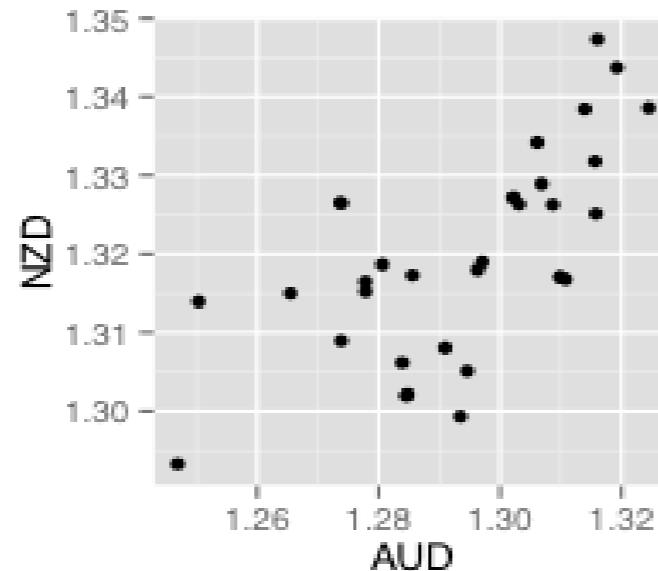


Source: Zhao et al (2013) IJITAS

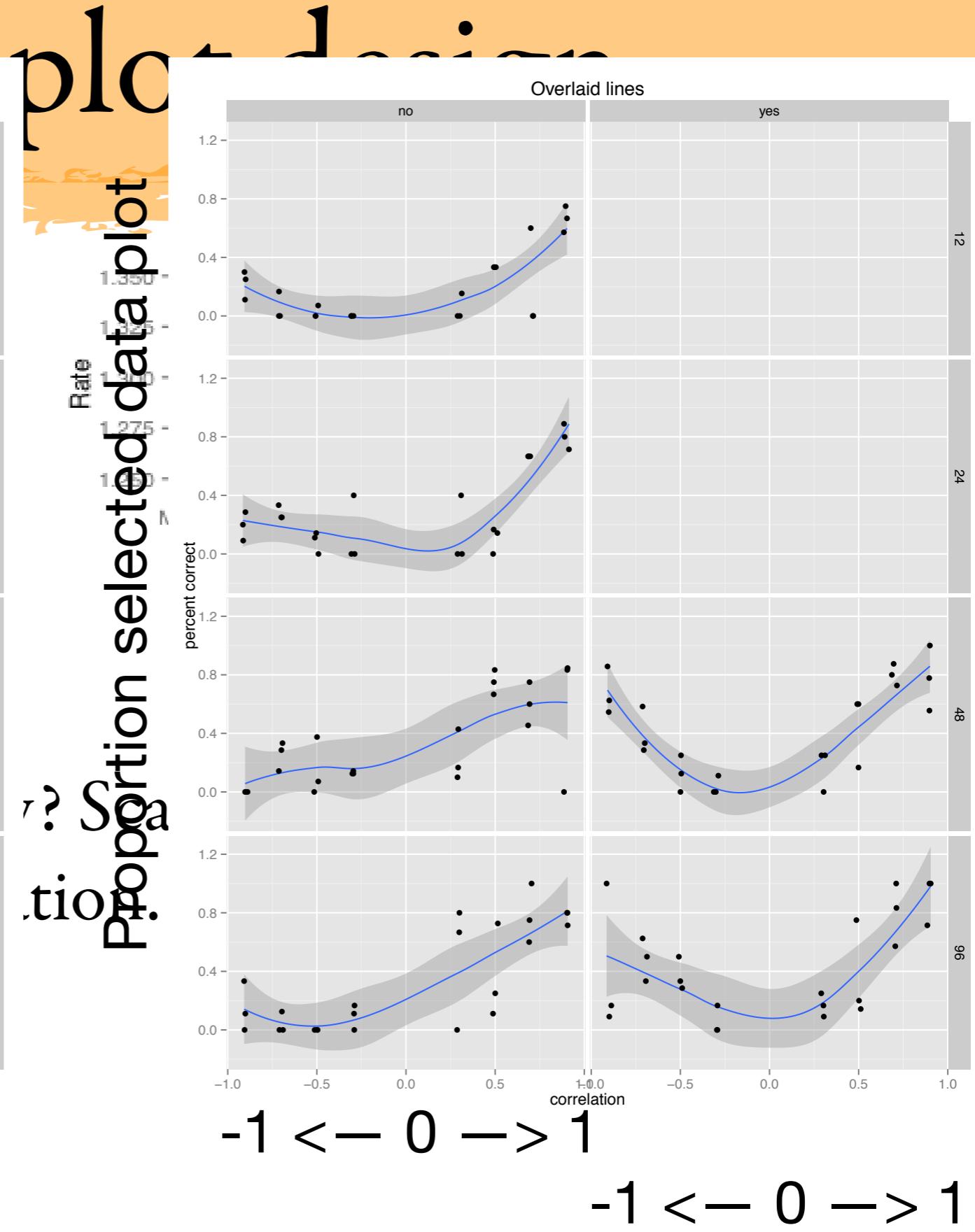
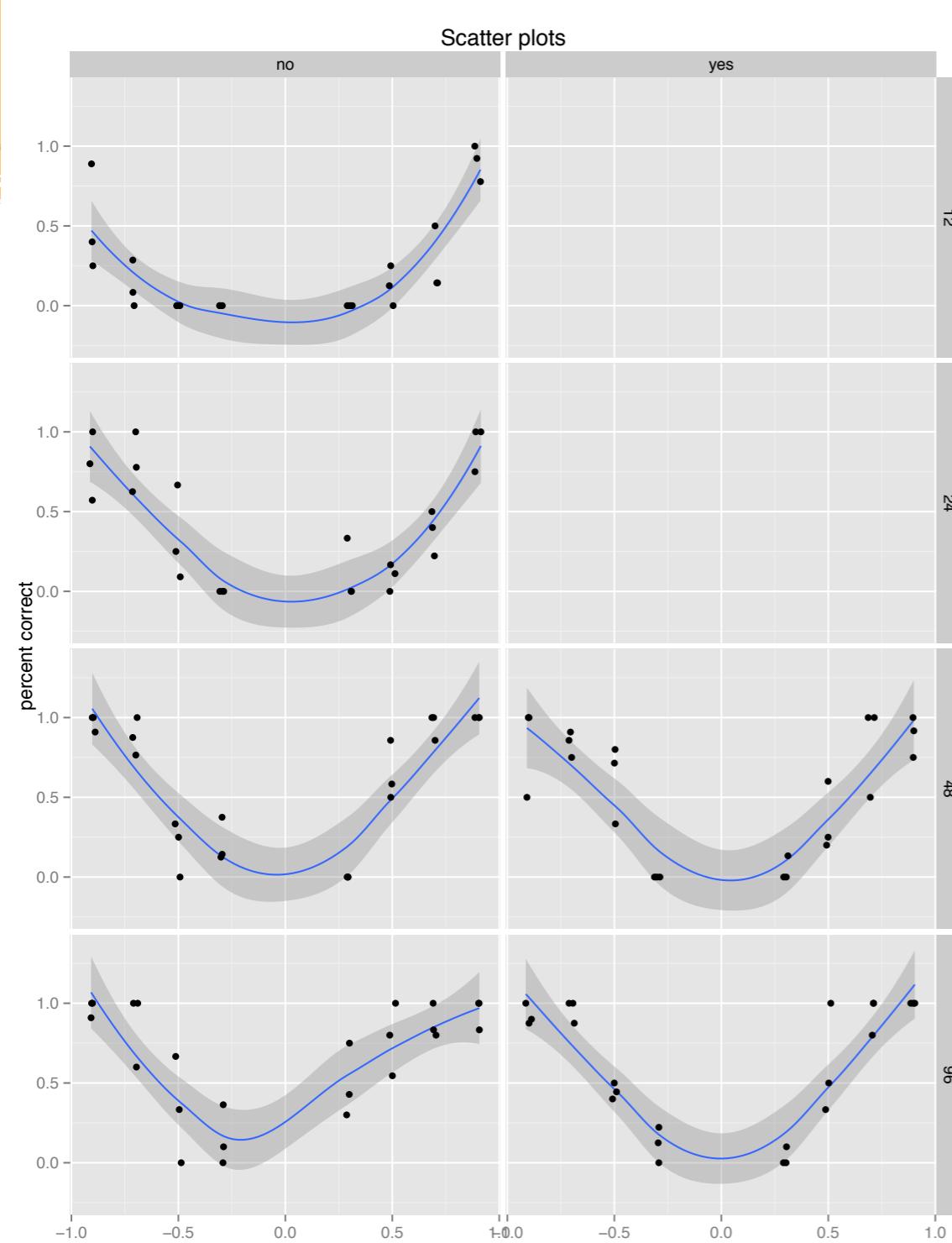
Eye-tracking

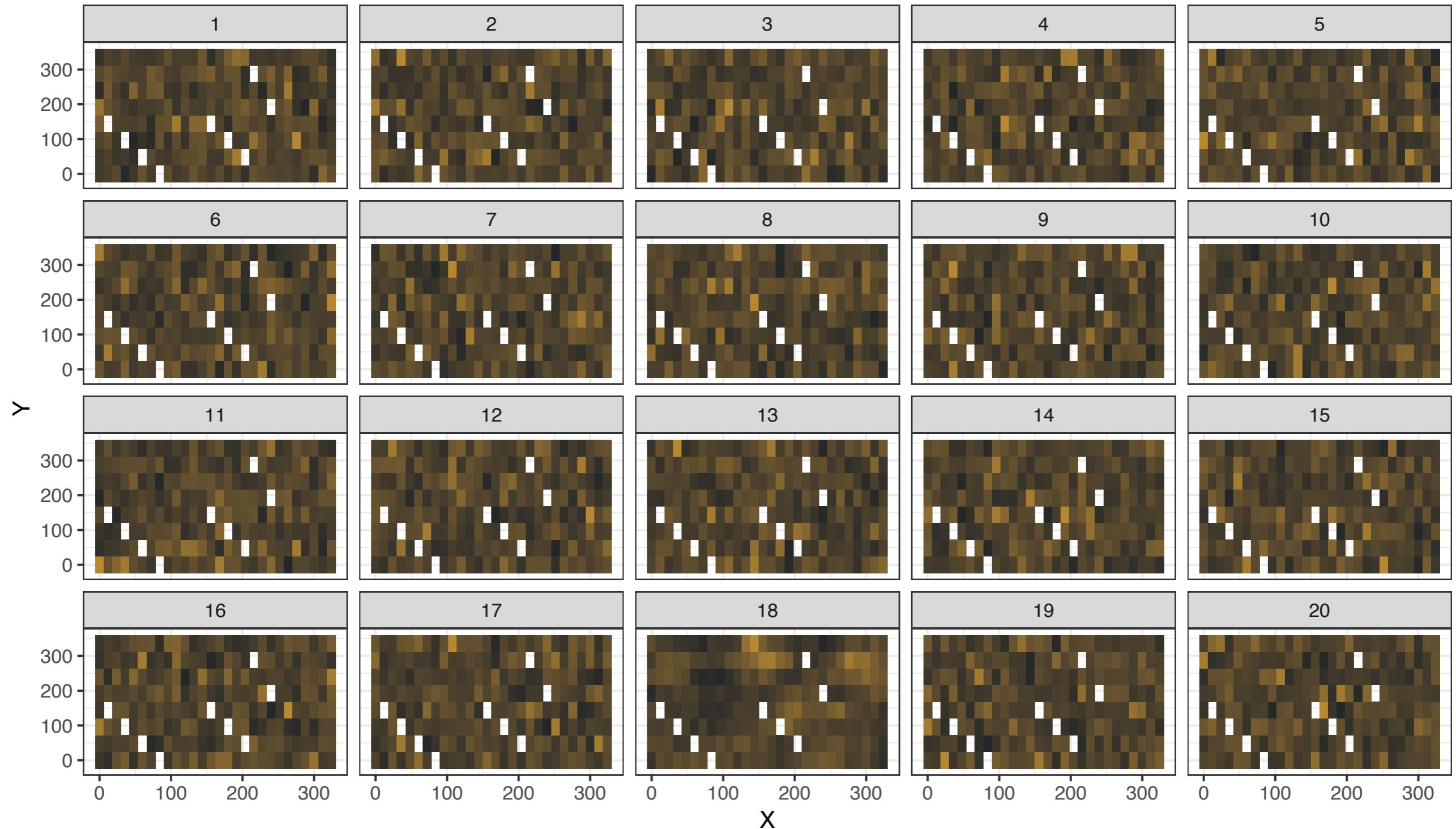
- People tended to methodically look around the lineup to find the most unusual plot
- For more difficult lineups all looked harder, and for easier lineups the looking was fast
- Some people were much faster than others in looking
- There are two stages, initial scan, and a final comparison between a few plots

Testing plot design

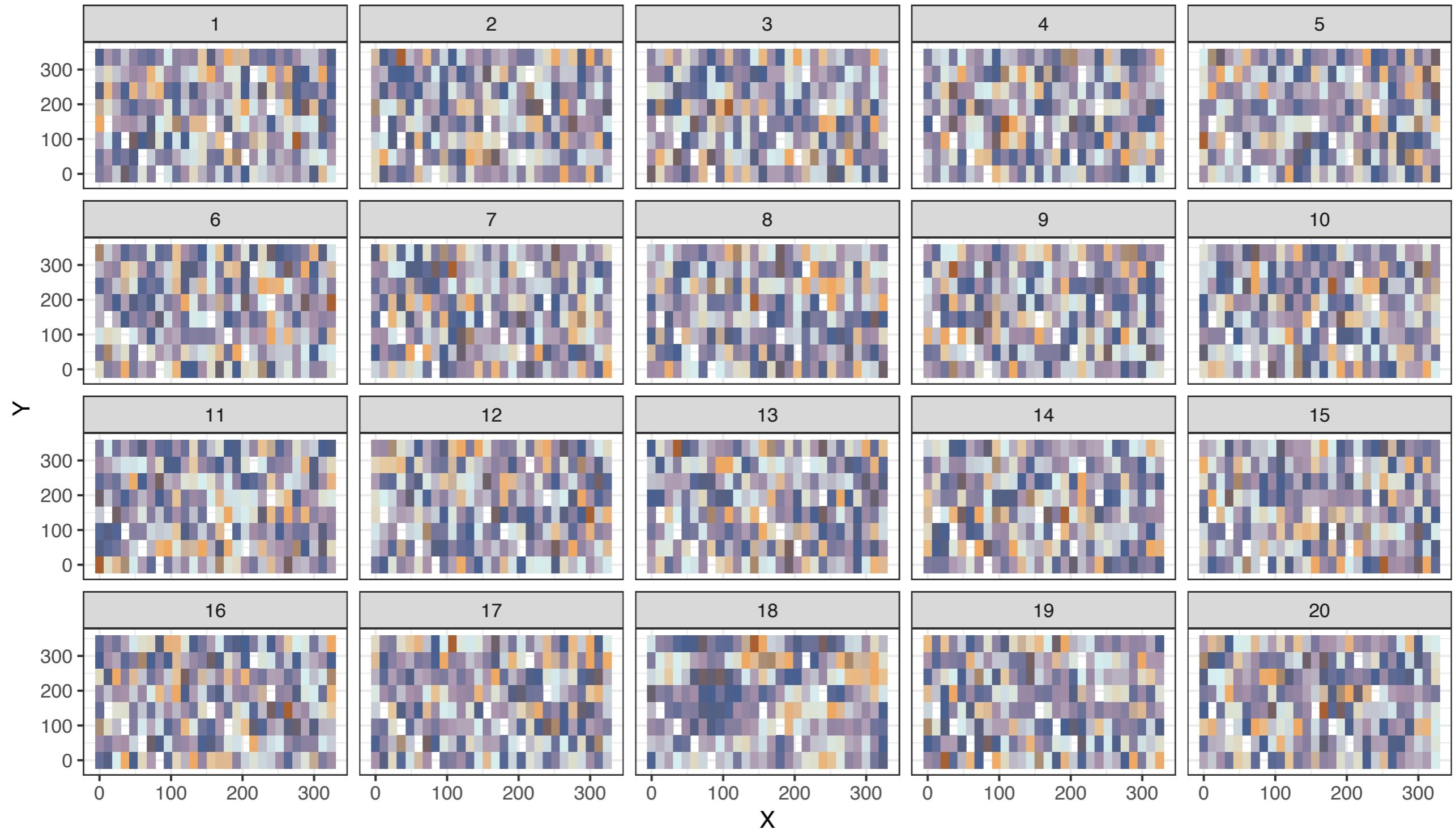


Which is the best display? Scatterplots universally better for reading association.

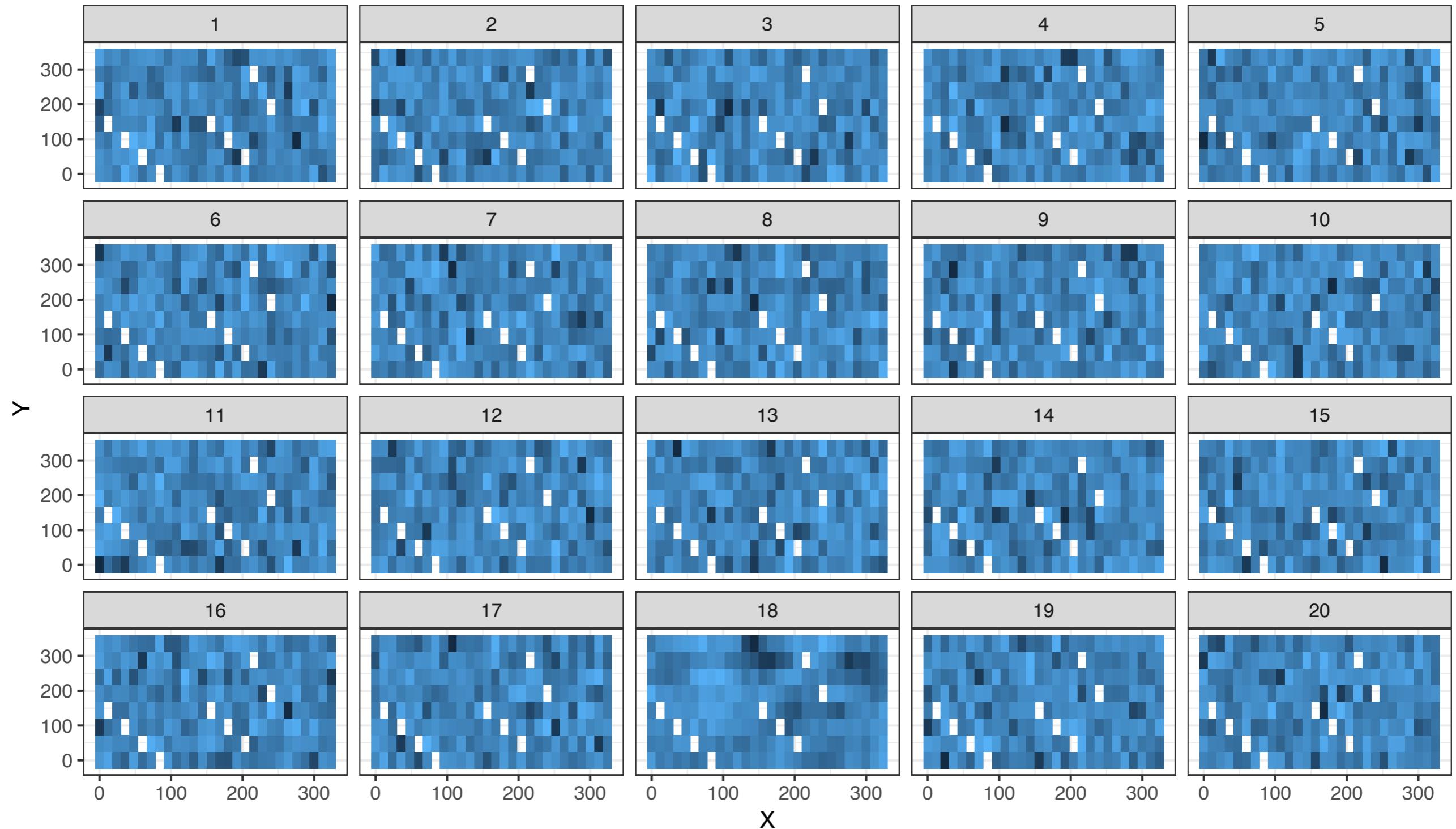




Does colour affect the ease of detection of the data plot?



Does colour affect the ease of detection of the data plot?



Does colour affect the ease of detection of the data plot?

Some foundations...

- Scott et al (1954): Generated synthetic plates to compare with real astronomical plates, acknowledged in Brillinger's (2005) Neyman lecture.
- Daniel (1976) had 40 pages of null plots for industrial applications.
- Diaconis (1983) describes ‘magical thinking’.
- Buja et al (1988) describe ‘Informal Statistical Inference’ in association with the software Dataviewer.
- Gelman (2004) simulate data from statistical models.
- Davies (2008) suggest viewing null data sets.

R Package

nullabor package on CRAN and <https://github.com/dcook/nullabor>

When you plot your data, plot it first in a lineup, so you can be the unbiased observer

Several null generating procedures included, p -value and power calculations

R Package

nullabor package on CRAN and <https://github.com/dcook/nullabor>

When you plot your data, plot it first in a lineup, so you can be the unbiased observer

```
> lineup(null_permute("Obama.Romney"),  
tracking.polls[,c(9,11)])  
> decrypt("fg0t DARA up iYzuRuYp Q")  
[1] "True data in position 5"
```

Several null generating procedures included, p -value and power calculations

Procedure

1. Organise your data into tidy form
2. Define your plots using the grammar of graphics, where variables are clearly mapped to elements of the plot
3. Specify the null-generating mechanism
4. Make your plot using a lineup function that shows the data plot among null plots
5. Is the data plot distinguishable from the nulls? If so, you can say “YOU CAN SEE ...”

Summary and future work

- This work melds inferential and exploratory statistics.
- It has been used to provide new visual diagnostics for hierarchical linear models, and an objective way to compare plot designs.
- Develop new metrics on data plots, with a plan to automate some data plot reading.
- Lots more work on p-value and power estimation needed.
- Web service to evaluate lineups

Acknowledgements

Plots produced using R package **ggplot2** by Hadley Wickham

Lineups made with R package **nullabor**

Projection pursuit (experiment 7) done using R package **tourr** by Wickham, Cook, with PDA index from Lee

National Science Foundation grant DMS 1007697

These slides are available at: <https://github.com/dicook/Biometric2017>



Heike



Andreas



Debby



Hadley



Eun-kyung



Eric



Mahbub



Tengfei



Susan



Adam



Niladri

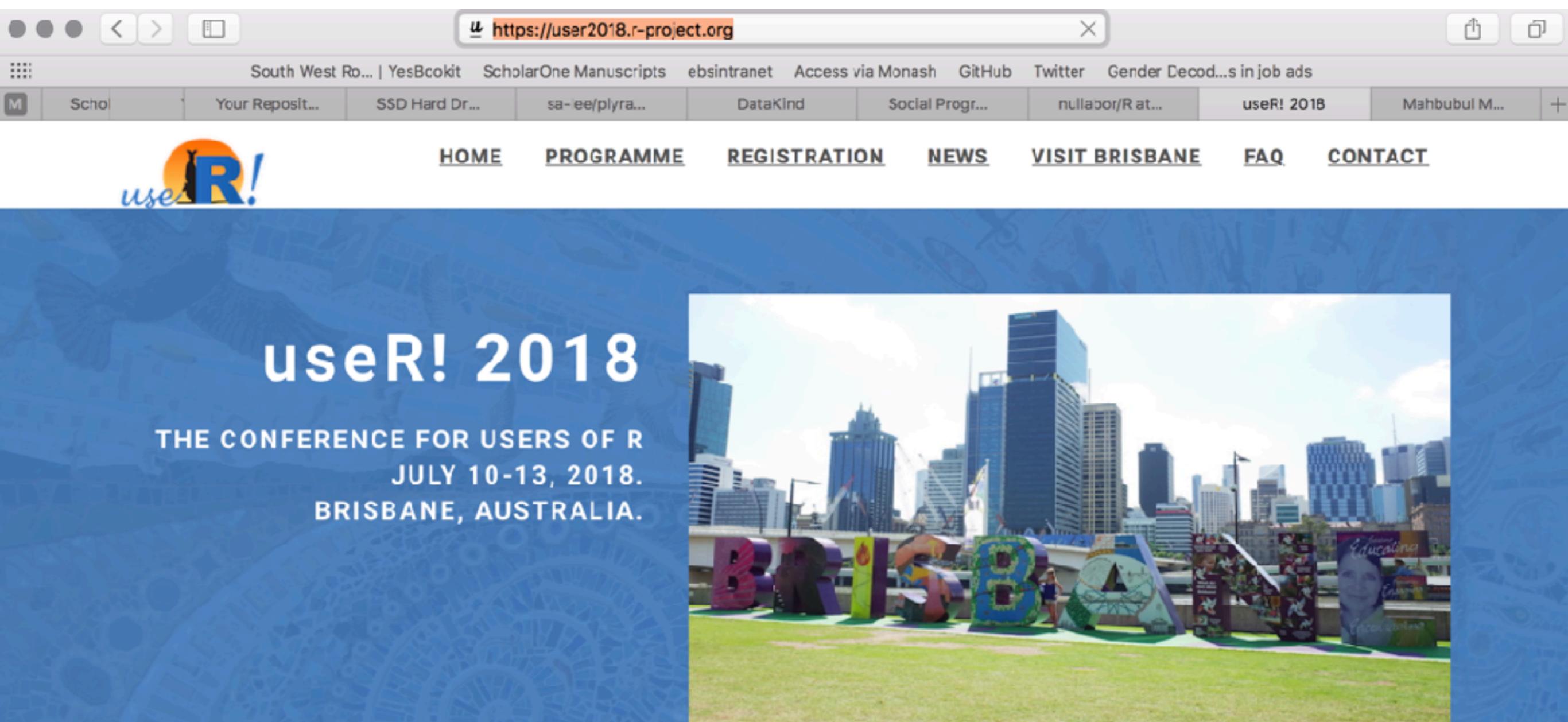


Nathaniel

useR! 2018 is in Australia next year!!!!

First time ever it has been held outside Europe and USA. Starting a trend!!!

If you want a postcard sent to you, email me,
dicook@monash.edu



The screenshot shows a web browser window with the URL <https://user2018.r-project.org> in the address bar. The page content is the official website for useR! 2018, featuring a blue background with a circular pattern. On the left, there's a logo with the word "use" in blue and "R!" in yellow with a silhouette of a person. The main title "useR! 2018" is in large white letters, followed by the subtitle "THE CONFERENCE FOR USERS OF R" and the dates "JULY 10-13, 2018." Below that is the location "BRISBANE, AUSTRALIA." To the right, there's a large image of the Brisbane city skyline with a prominent bridge, and in the foreground, there's a large, colorful sculpture of the word "BRISBANE". The top navigation bar includes links for HOME, PROGRAMME, REGISTRATION, NEWS, VISIT BRISBANE, FAQ, and CONTACT.



Australia has the best place
for conferences!

Thanks to the Biometrics by
the Border organisers for
changing the natural order of
the world, and the great
location!