Dynamics of Complex Systems

Part 1:

Tools for studying system dynamics
Fitting parameters to analytic solutions

Part 2:

Multivariate Models
Simulation of continuous time

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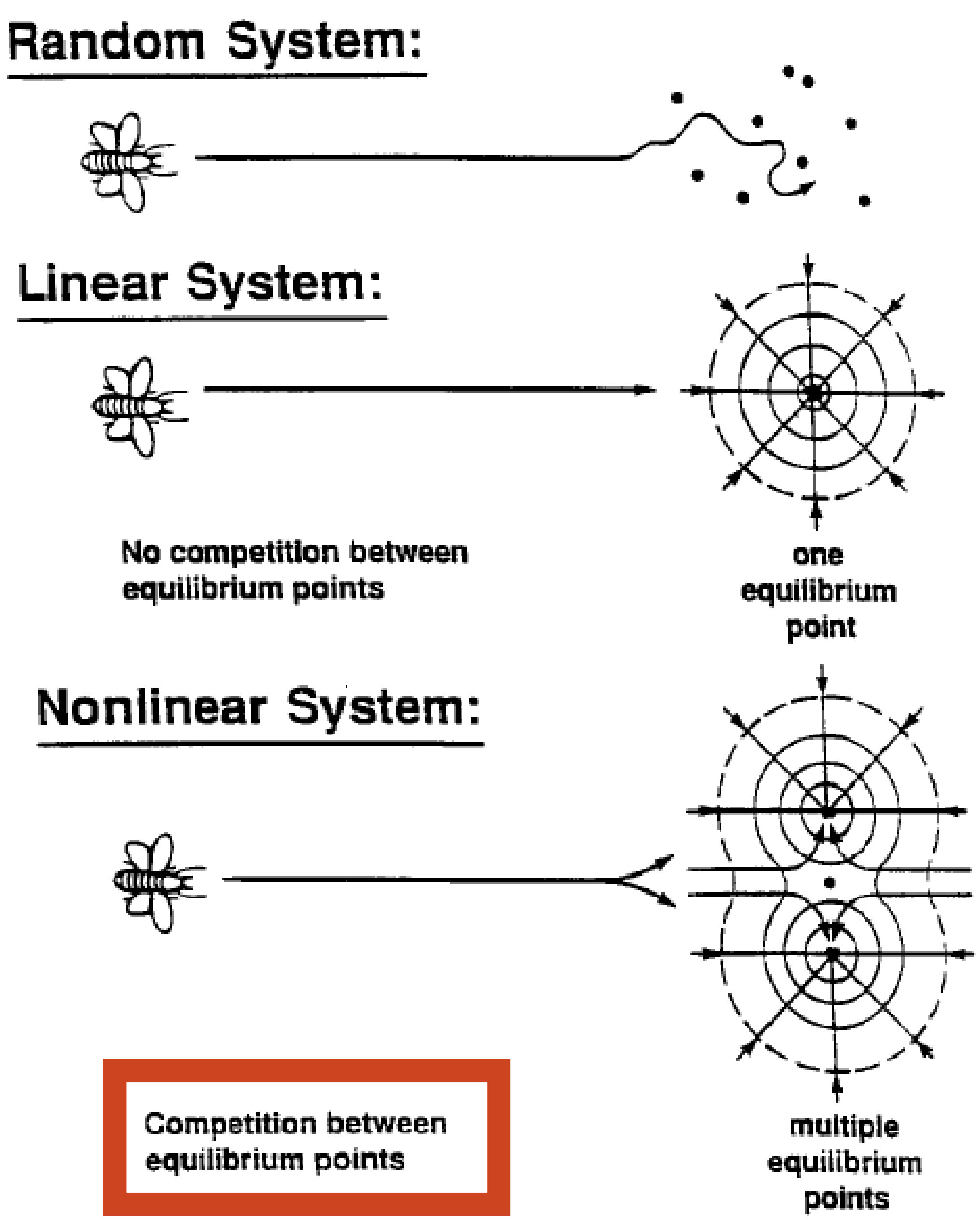
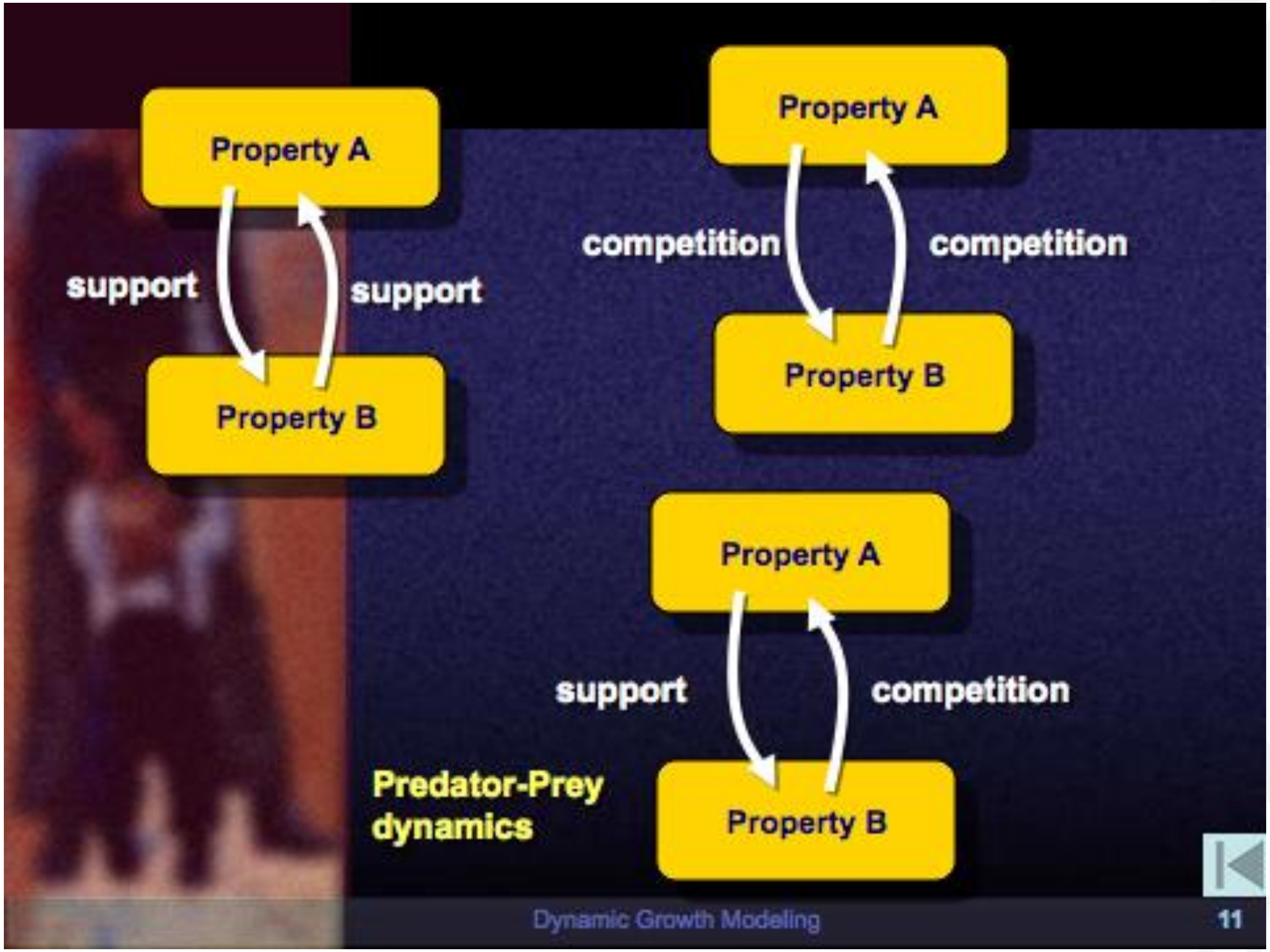
Termite cathedrals: Coupled dynamical processes



Simple Coupling Dynamics: 2D-systems



Multivariate Models... Multivariate State Space

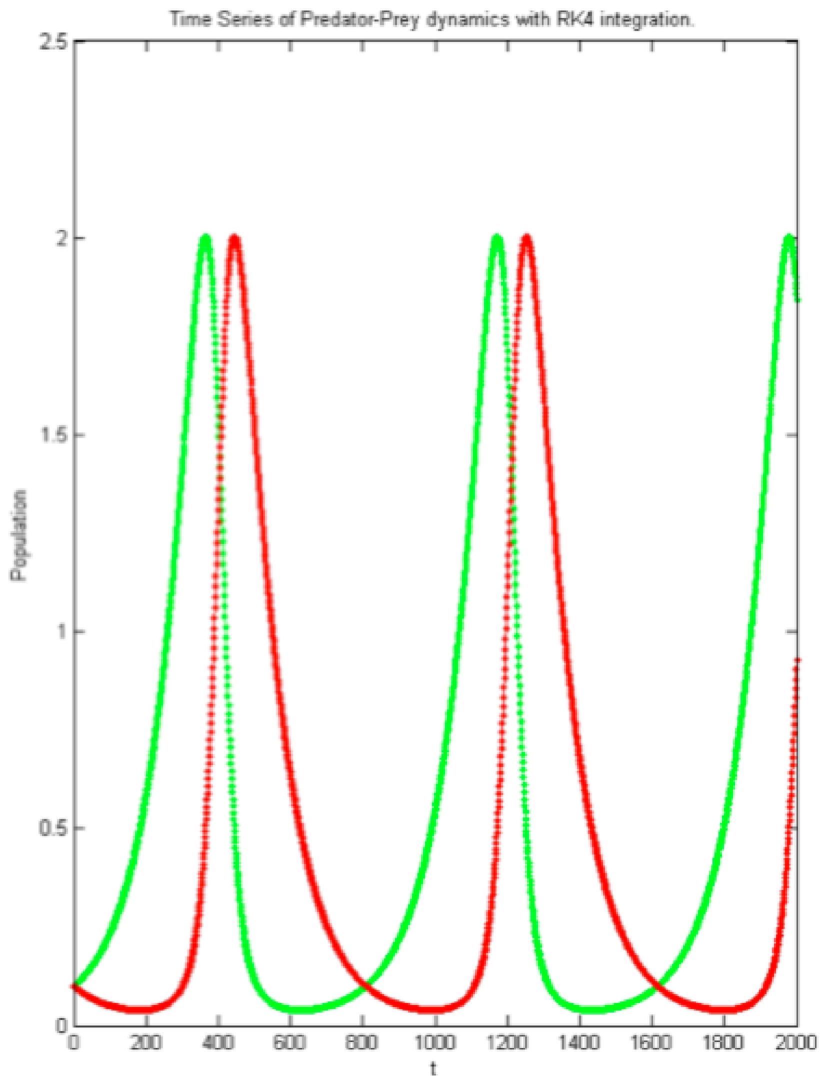
Predator-Prey model (Lotka-Volterra)

$$\boxed{\begin{aligned}\frac{dR}{dt} &= (a - b \times F) \times R,\\ \frac{dF}{dt} &= (c \times R - d) \times F.\end{aligned}}$$

A 2-D state space
2 coupled flows ~

- *R* is the ***number of rabbits*** in a year
- *F* is the ***number of foxes*** in a year

Multivariate Models... Multivariate State Space



Rabbits
Foxes

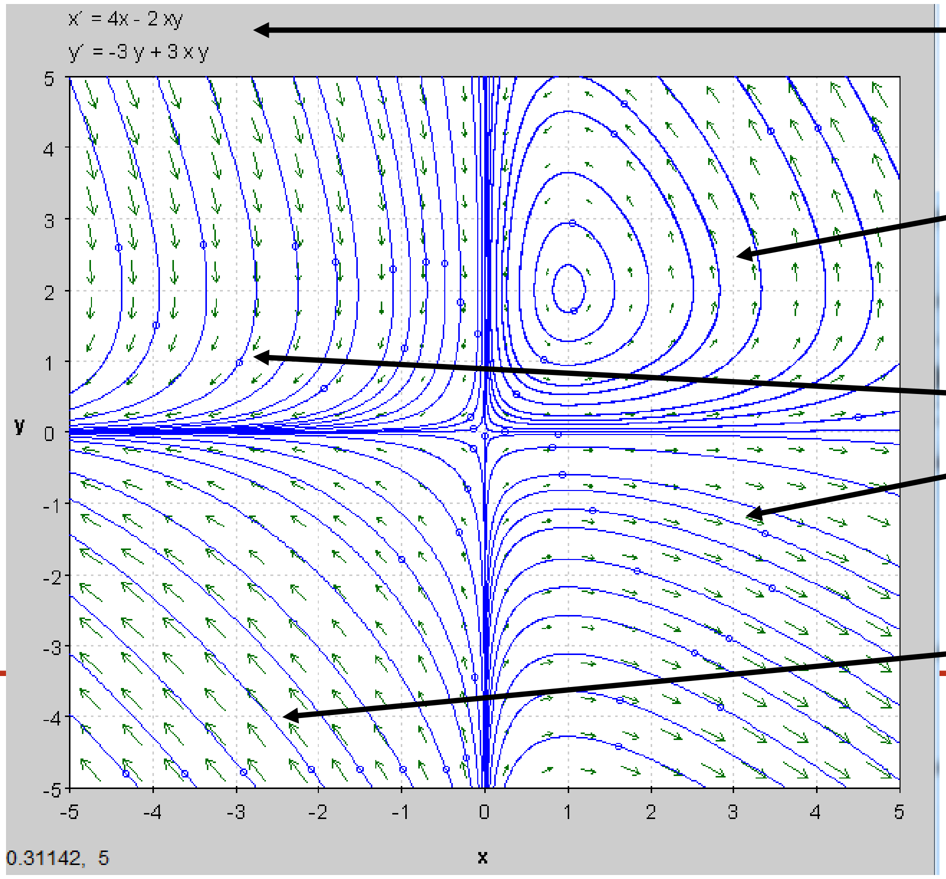
Time Series

Multivariate Models... Multivariate State Space

Coupling 'causes' a reduction in the degrees of freedom a system has available to generate its behaviour…

The system will not occupy every point in state space, just a limited set of points, an attractor state

Coupling dynamics = Interaction dynamics



Note: This is just one set of parameter settings, using different initial values

Equilibrium states only here

Here one species will 'win', initially, if food is depleted it won't last

Both <0 could be shrinking populations; probably unrealistic

Lorenz System

$$\frac{dx}{dt} = a(y - x),$$

$$\frac{dy}{dt} = x(b - z) - y,$$

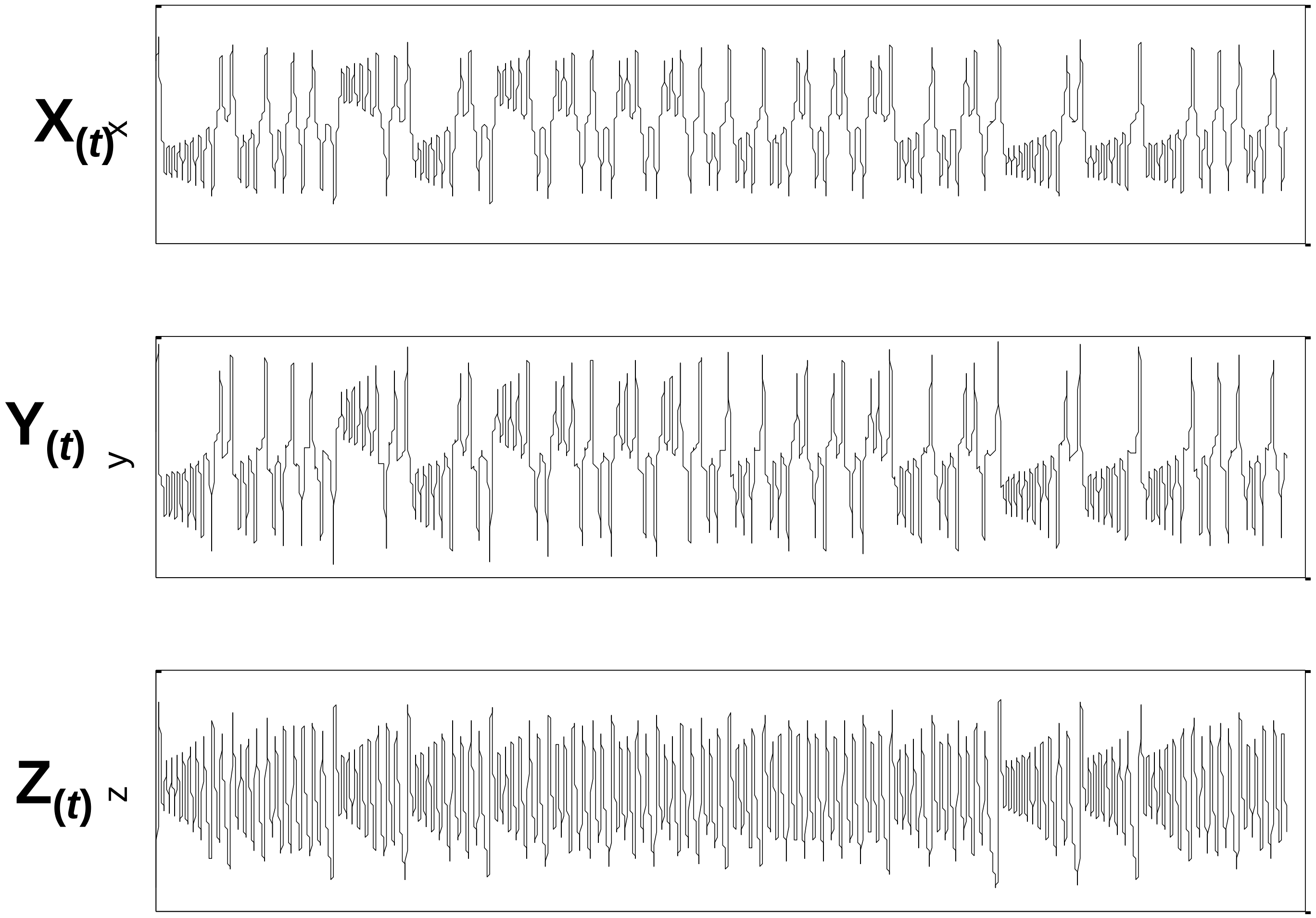
$$\frac{dz}{dt} = xy - cz.$$

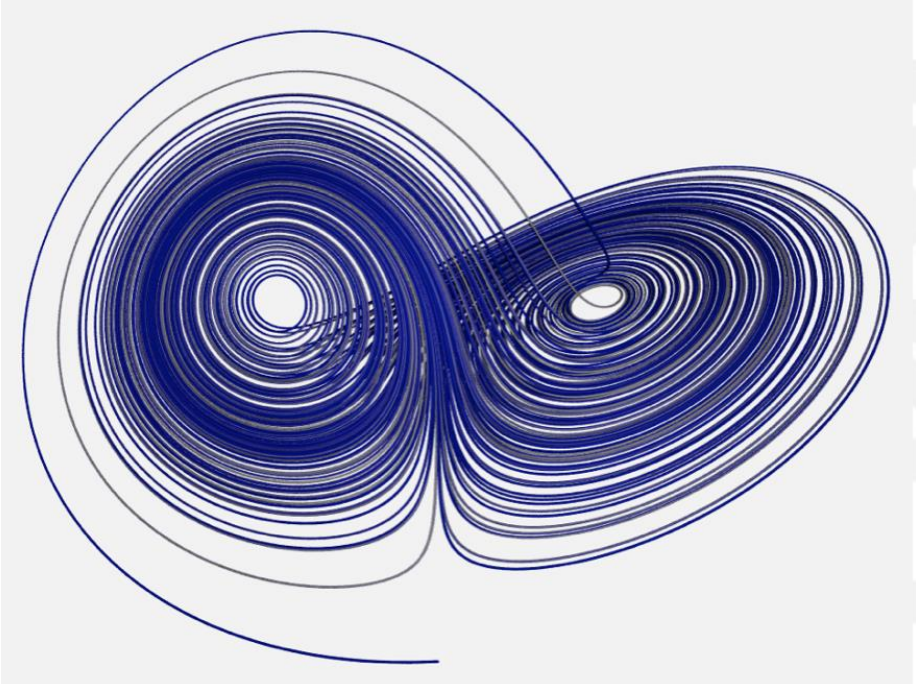
Interaction dominant dynamics
Multiple processes (3)
Multiple Scales (time)

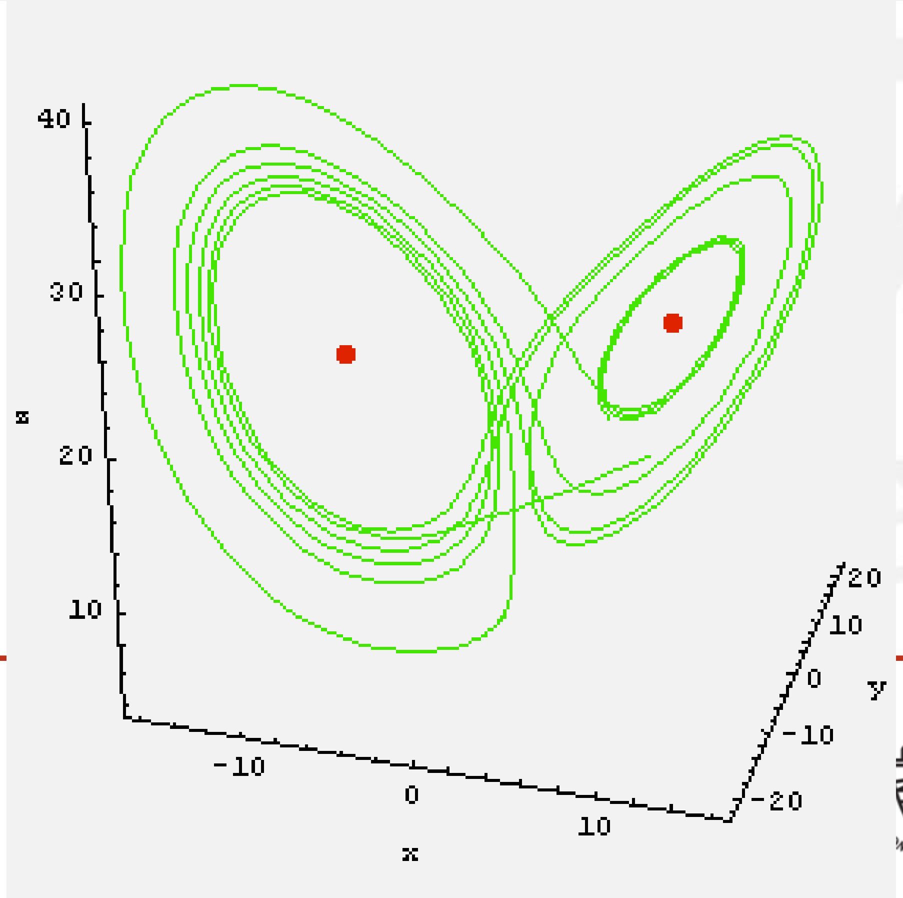
x depends on y and x
y depends on x, y and z
z depends on x, y and z

A 3-D state space
3 coupled flows ~

Lorenz System - 3D State Space







Anticipation

Component-dominant

Use rules learned in the past, map those representations to the future. Requires memory, attention, perception, motor etc. modules.

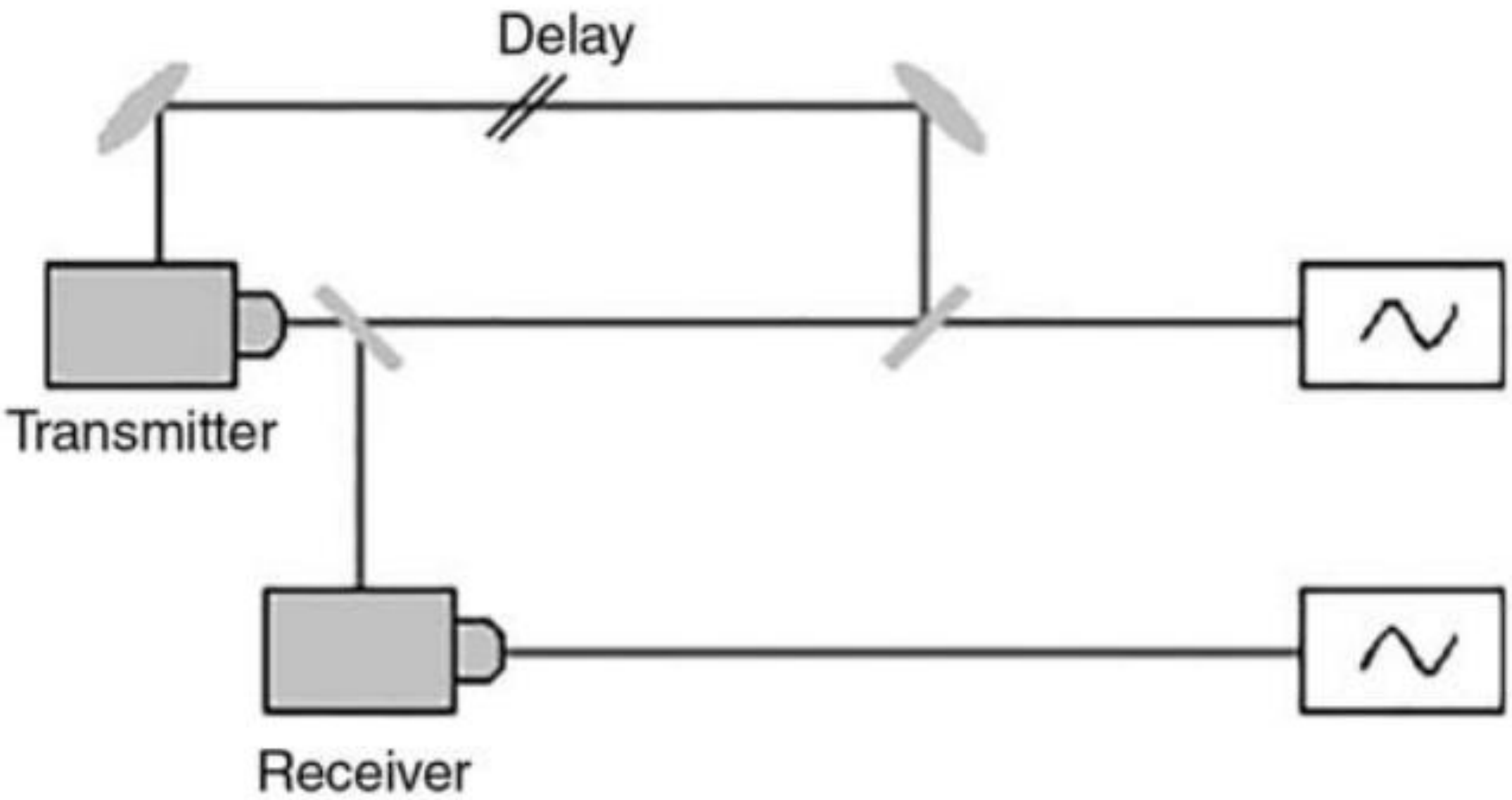
Interaction-dominant

State of y provides information about the state of x at a future time from the coupled dynamics of the system itself.

Not by any explicit predictive mechanism.

Anticipation without an internal model

Anticipating chaotic synchronization:
- Unidirectional coupling from the transmitter to receiver



Sivaprakasam, S., Shahverdiev, E. M., Spencer, P. S. & Shore, K. A. (2001). Experimental demonstration of anticipating synchronization in chaotic semiconductor lasers with optical feedback. Phys. Rev. Lett., 87, 154101.

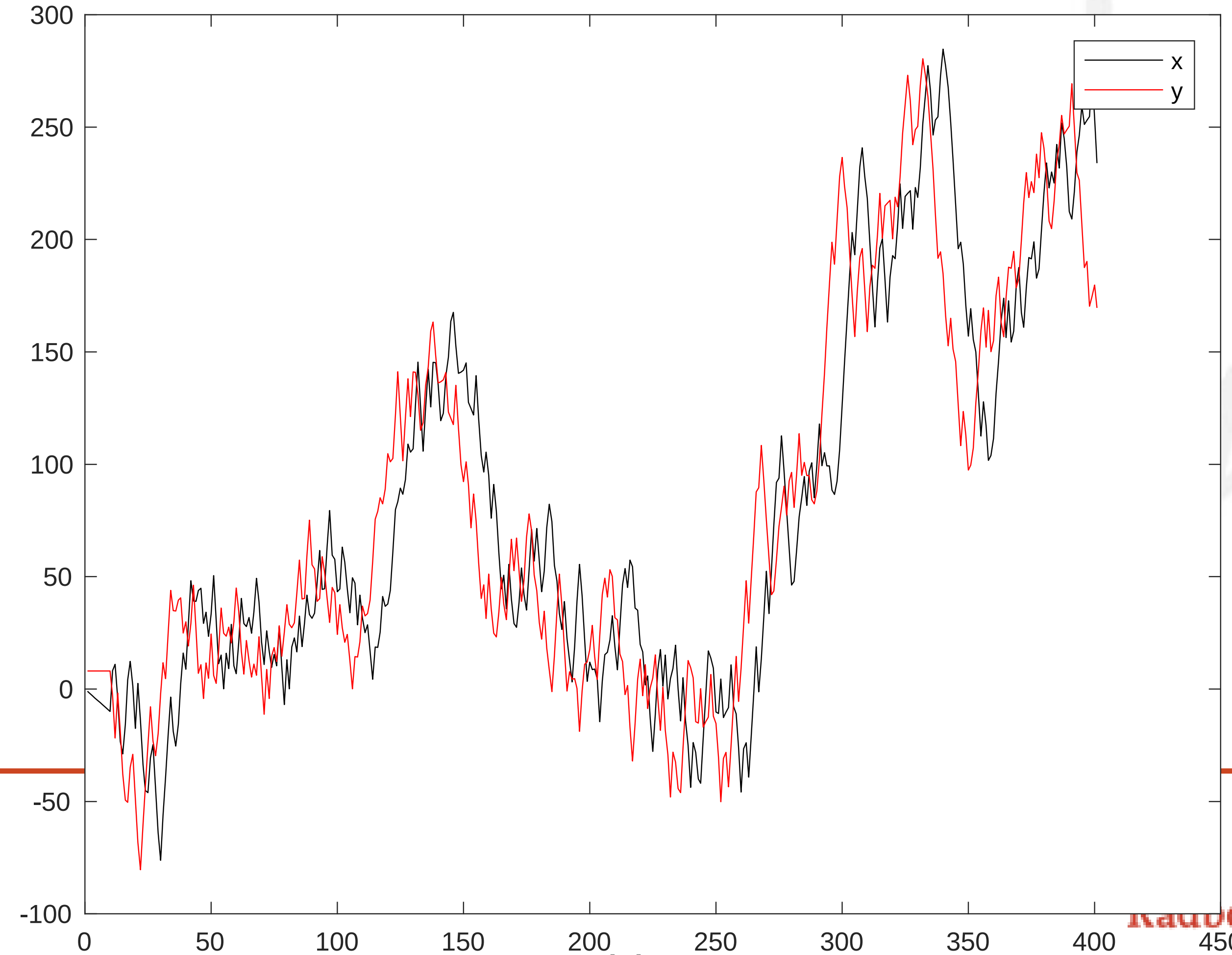
Anticipation without an internal model

Sender:

$$\frac{\partial x(t)}{\partial t} = -\alpha x(t) - \beta \sin x(t-\tau) \tag{1}$$

Receiver:

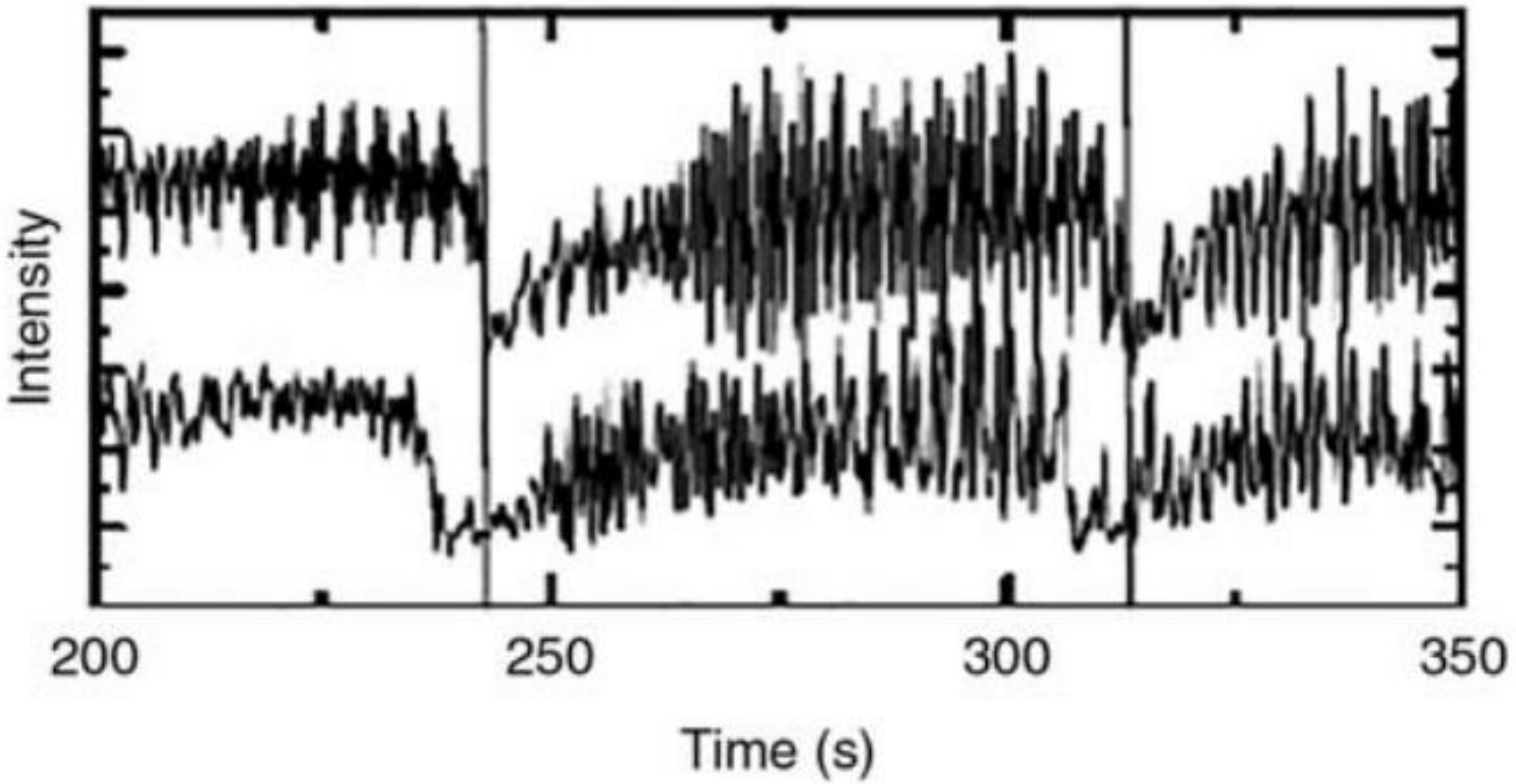
$$\frac{\partial y(t)}{\partial t} = -\alpha y(t) - \beta \sin x(t) \tag{2}$$



Anticipation without an internal model

Yt / Δ*t* does not contain within it a model, or representation, of *Xt* / Δ*t* .

This anticipation arises from within the lawful evolution of the system itself.



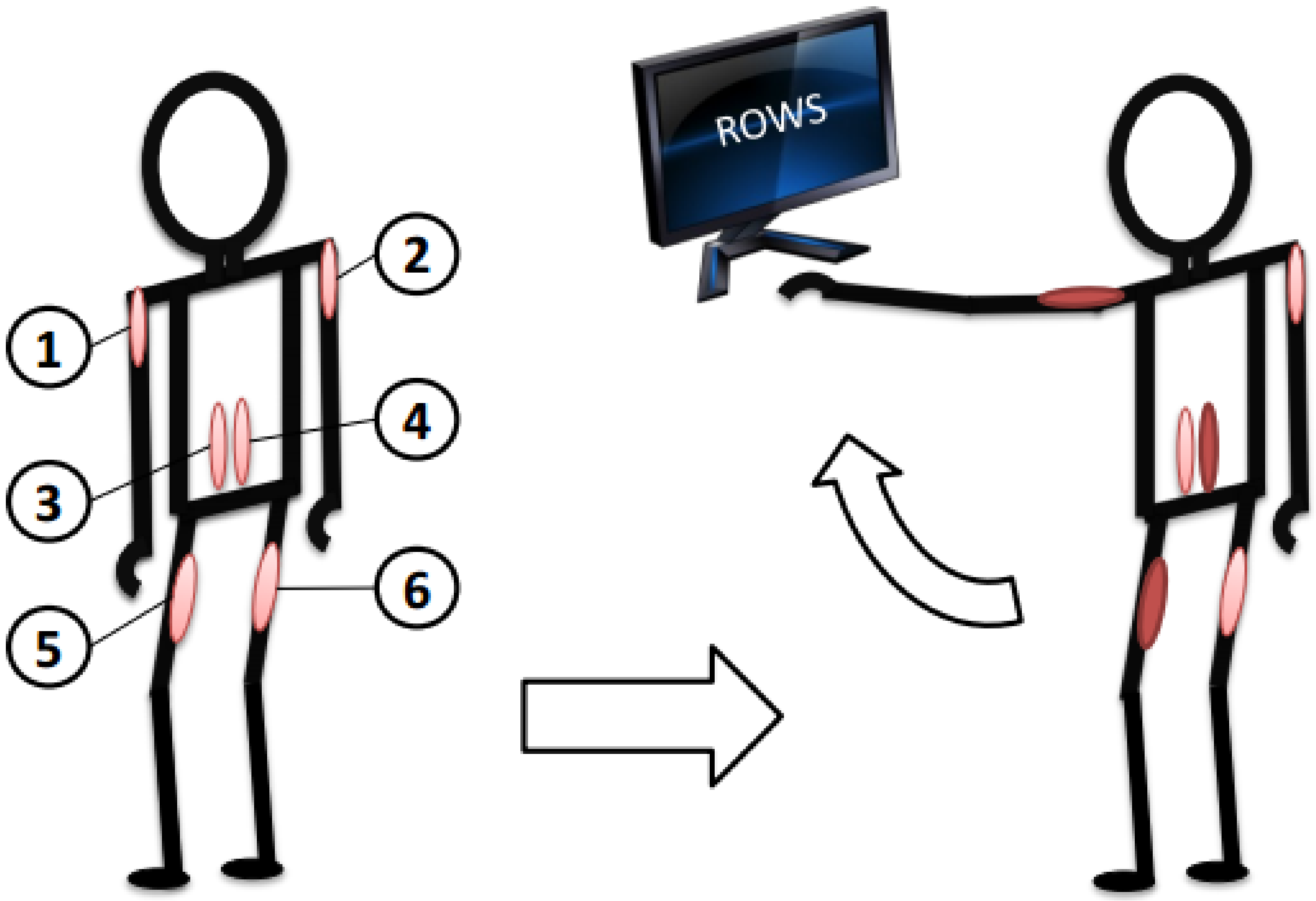
Anticipation without an internal model

Lexical decision

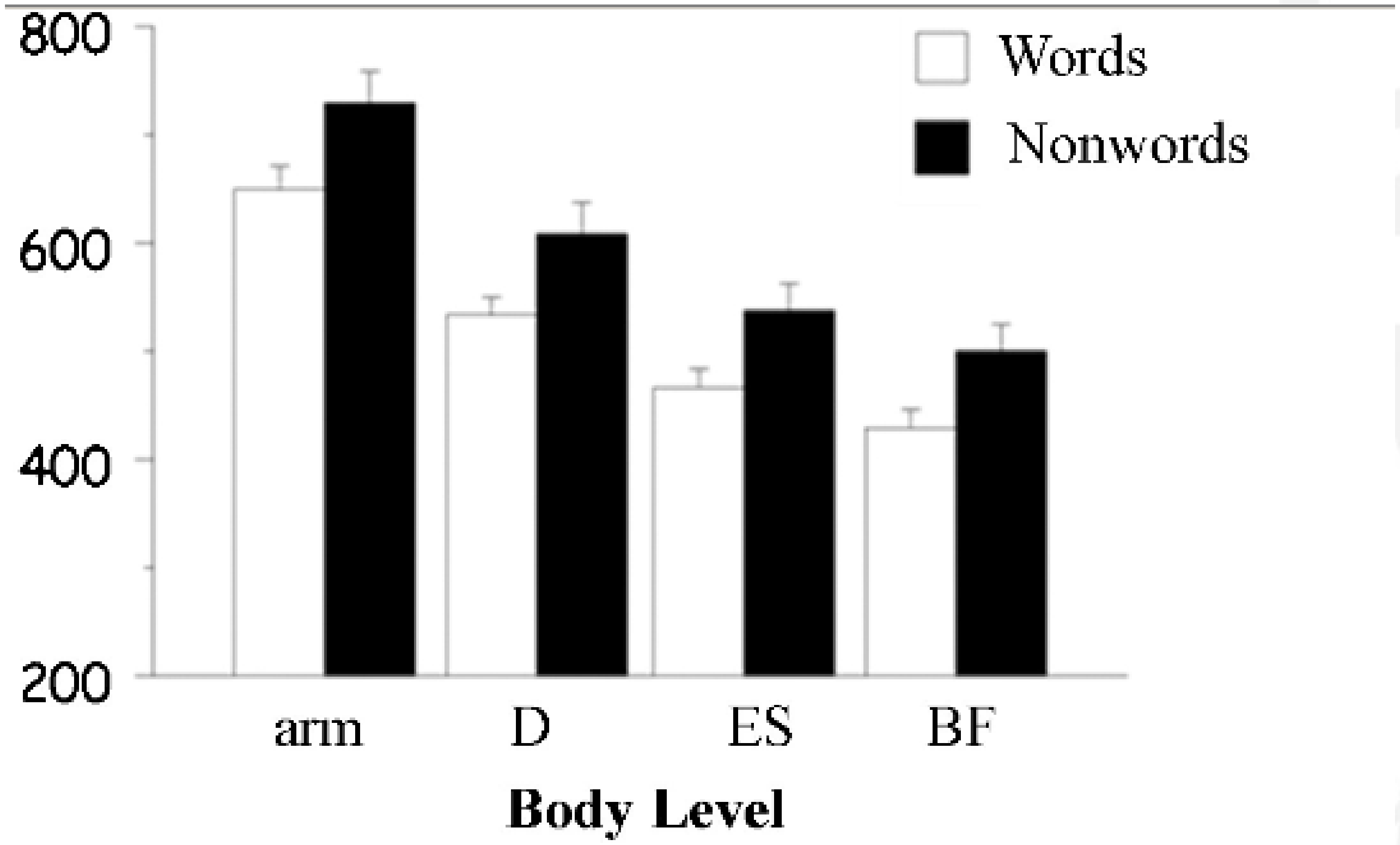
Anticipation without an internal model

RT arm
Muscle contraction in:

- The shoulder
- Lower back
- Thigh



Anticipation without an internal model



Moreno, M. A., Stepp, N., & Turvey, M. T. (2011). Whole body lexical decision. *Neuroscience Letters, 490*, 126-129.

A note on simulating differential equations: ~flows ~

Differential equations are **continuous**…

To find out how they behave when there is no solution we need to ‘discretise’ them and approximate the solution with a difference equation: *Numerical integration*

The easiest (but most error prone) method is Euler’s method (18th century):

$\mathbf{X_{n+1} = X_n + H * f(X_n)}$ where **H** = step length

Checking how well the approximation is, can easily be done if we know an analytic algebraic solution

Also see the notes in Chapter 2

A note on simulating differential equations: ~flows ~ Euler's method

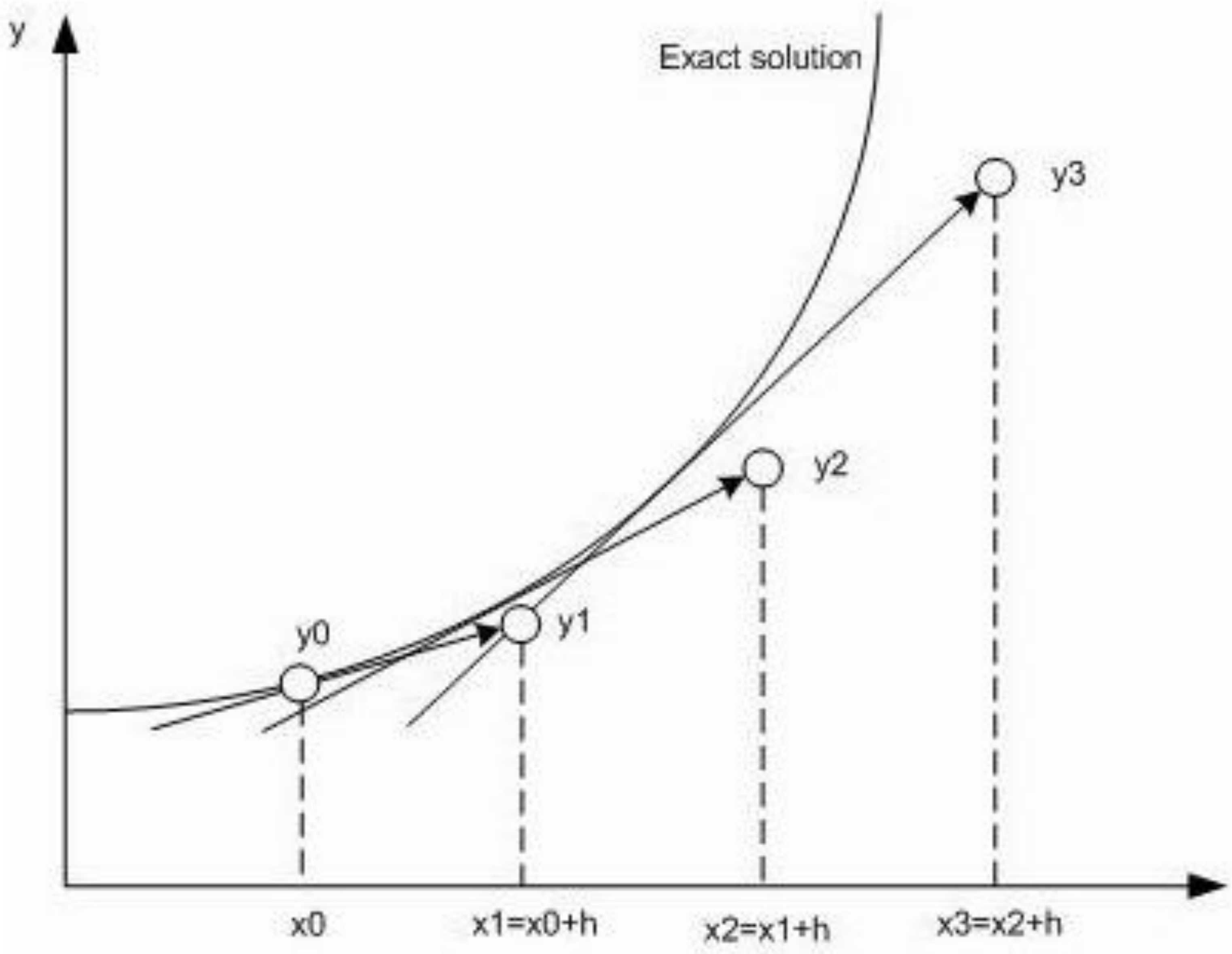
Basic Idea:

1. Calculate next time step

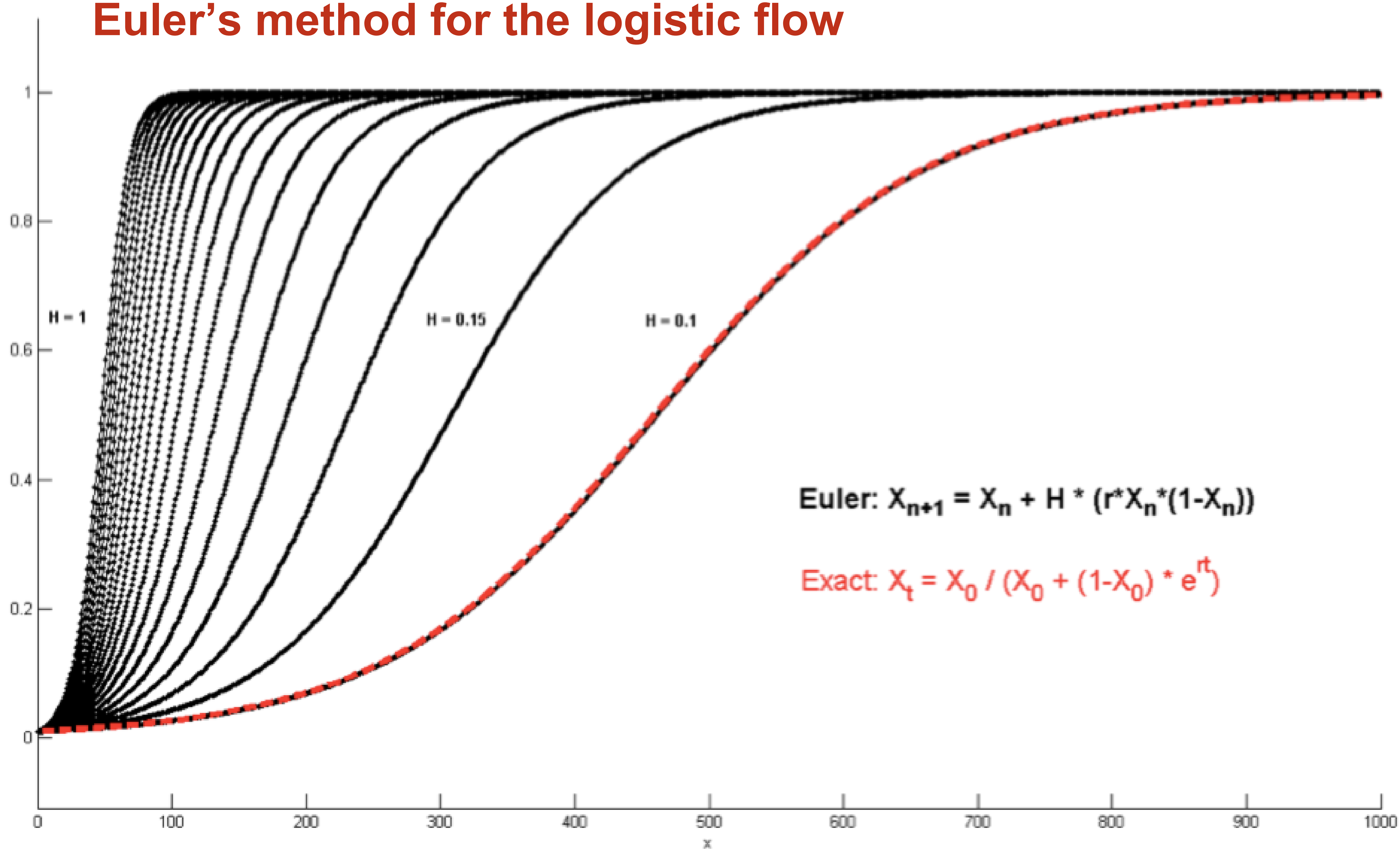
2. Don't enter that value into the iterative process as input!

3. Take some smaller proportion representing a smaller step in time

4. Effect is that large fluctuations are iteratively 'smoothed' and a continuous flow is approximated



Euler's method for the logistic flow



Runge-Kutta 4th Order Method (harmonic mean of 4 points)

$$\mathbf{k}_1 = h \cdot f(\mathbf{y}_n)$$

$$\mathbf{k}_2 = h \cdot f\left(\mathbf{y}_n + \frac{\mathbf{k}_1}{2}\right)$$

$$\mathbf{k}_3 = h \cdot f\left(\mathbf{y}_n + \frac{\mathbf{k}_2}{2}\right)$$

$$\mathbf{k}_4 = h \cdot f(\mathbf{y}_n + \mathbf{k}_3)$$

$$\Rightarrow \mathbf{y}_{n+1} = \mathbf{y}_n + \frac{\mathbf{k}_1}{6} + \frac{\mathbf{k}_2}{3} + \frac{\mathbf{k}_3}{3} + \frac{\mathbf{k}_4}{6}$$

Comparison of accuracy of methods is only possible for systems that have an analytic, exact solution