



# Complexity

**Outline of the NWO strategic theme**  
***Dynamics of complex systems***

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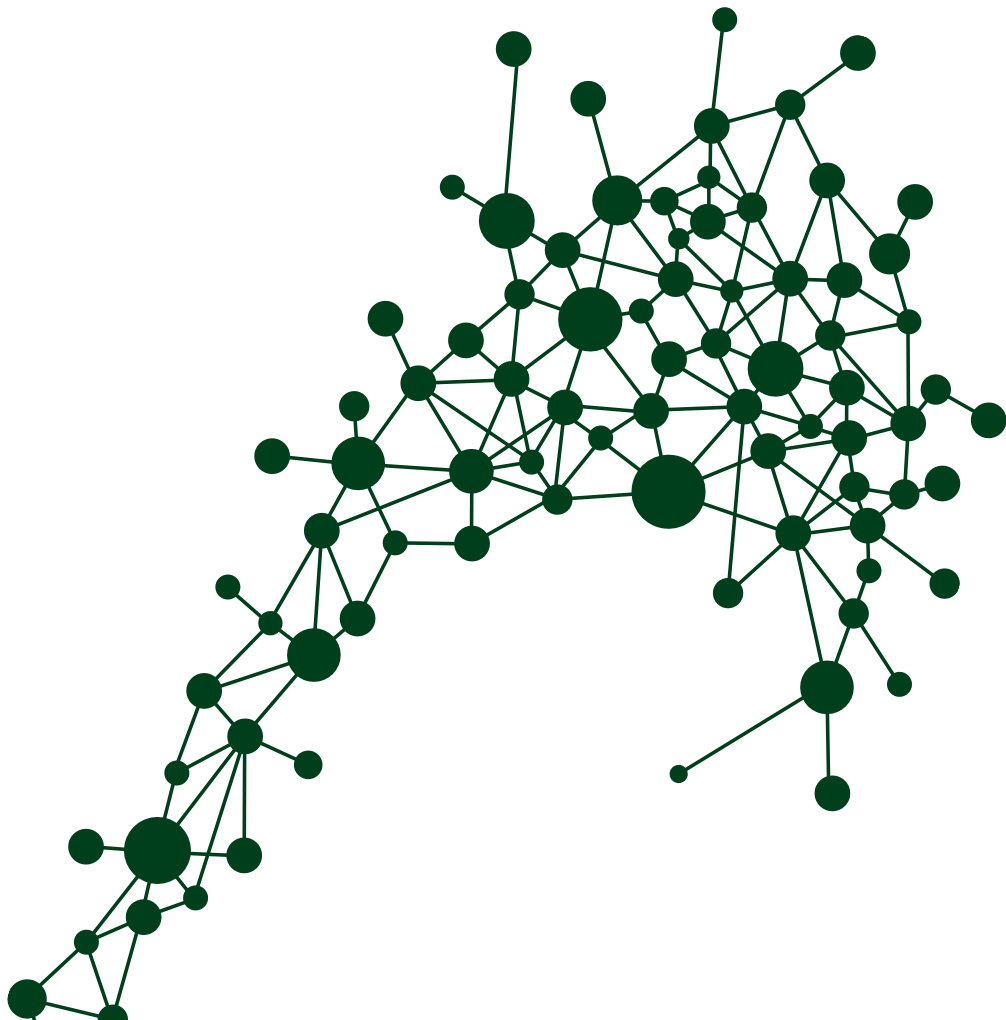
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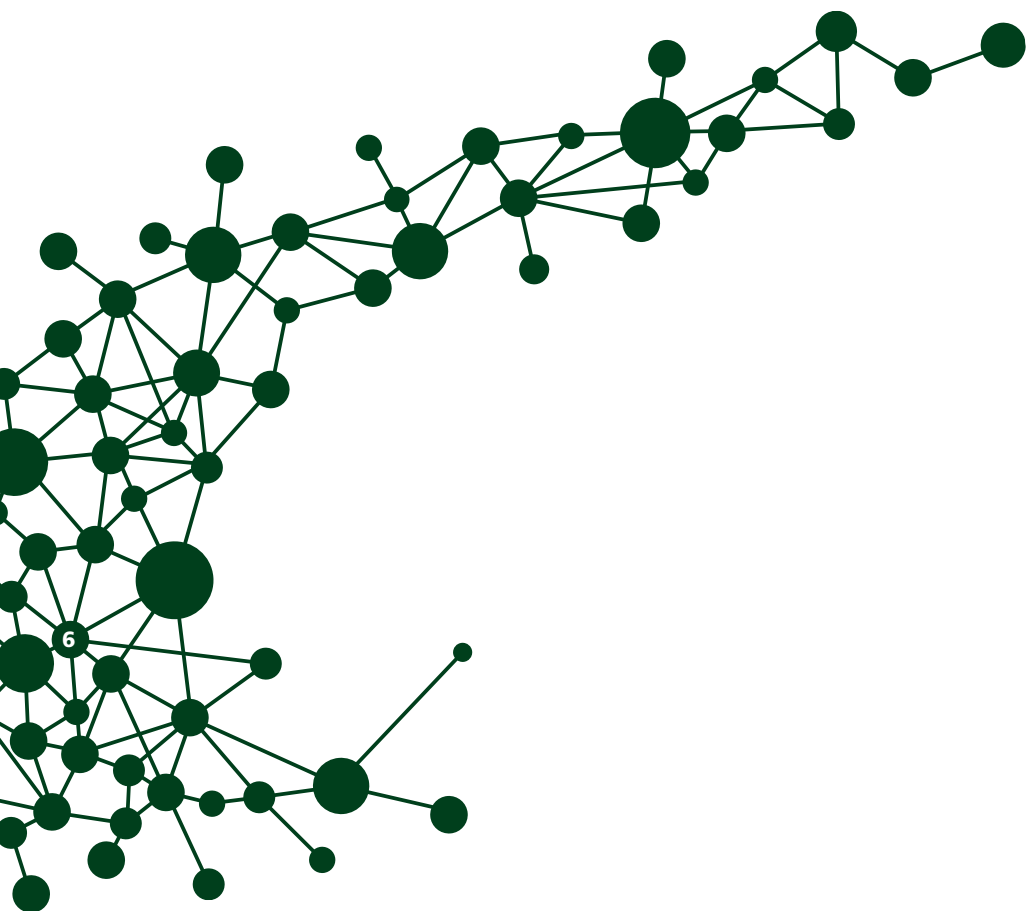
# Preface

Dynamics of complex systems is one of the program themes in the NWO (Netherlands Organisation for Scientific Research) strategy for the years 2007-2011. The ambition of the current proposal is to initiate integrated activities in the field of complex systems within the Netherlands, to provide opportunities for the Dutch scientific community to start up a research program on '*Complexity*', and to join in, and give direction to, European activities in this field.

At many levels of activity, Dutch researchers take prominent positions and are internationally recognized for their research in complexity science – a field that by its very nature requires a multi-disciplinary collaboration. However, this has not yet led to an established national community that crosses existing disciplinary boundaries.

In this proposal, we call for an effort to bring together ideas, insights, and knowledge from various disciplines and to initiate inspiration and cross-fertilization among traditionally separated fields.

We sketch the contours of a scientific '*Complexity*' program by focusing on three trans-disciplinary research themes. The choice for these themes is determined by the opportunities offered and challenges posed by the field of '*Complexity*', in combination with the existing strengths and highlights of the Dutch scientific community.





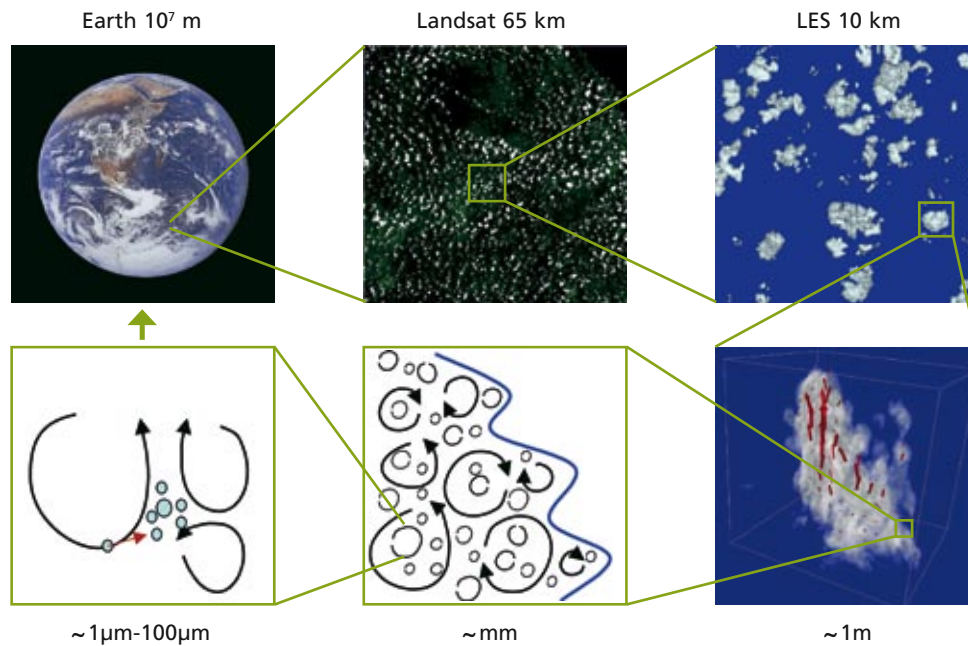
# Introduction

## What is 'Complexity'? What is a complex system?

In science, the notion of '*Complexity*' is associated with a vast number of phenomena observed in nature, in society, in laboratory experiments, and in mathematical models. Increasingly, scientists dealing with complex systems in different fields of enquiry realise that a proper understanding of such systems requires an approach that transcends the boundaries between the classical disciplines. Therefore, defining and developing the cross-disciplinary field of complexity research is a timely challenge.

There are a number of characteristic features that are shared by almost all complex systems. A complex system can often be seen as a large collection of small elements that interact with each other at a micro-level. Such elements may be atoms in physics, molecules or cells in biology, or consumers in socio-economics. However, '*more is different*' (Anderson, 1972) in complex systems. Phenomena observed at a global, *macro*-level, typically cannot be reduced to the properties of the constituent elements: these are *emergent* properties that arise through '*self-organizing*' local interactions. This is in sharp contrast to the classical reductionistic idea that nature can only be understood by reducing or decomposing its processes into elementary building blocks that can be studied independently. The concept of '*Complexity*' has been introduced as the associated paradigm shift in the study of natural phenomena. »





**8** Complex systems are, without exception *nonlinear* and have a strong *multiple-scales* or even *scale-free* character, in time as well as in space. The formation, the structure and the dynamics of clouds is a prototypical example of a – largely not understood – complex process. The spatial scales involved range from micrometers to thousands of kilometres (*Figure 1*). In fact, cloud formation is in itself only one of the components of the complex system formed by our climate, a slow time process at the global scale that is driven by the ‘fast’ dynamics of day-to-day weather.

Another feature shared by many complex systems, is the fact that they can be modelled as networks or graphs. The notion of a network, or its more abstract representation as a graph, allows for dealing with the often inhomogeneous patterns of interactions within complex systems. The brain, with its billions of interconnected neurons in which bio-chemical processes at the micro-level give rise to consciousness and emotions at the macro-level, is perhaps the most challenging complex network of all. More often than not, these networks evolve – new connections may form, others may break: complex systems develop and adapt dynamically. Contrary to the fundamental processes that form the foundation of the reductionistic approach, such as Newton’s laws, complex systems often have a memory. Complex cause and effect relations characterize many psychological disorders, the emotional responses of an individual (and thus her/his brain) are for a large part driven by past events. As stated by Ilya Prigogine, ‘*complex systems carry their history on their back*’, in the sense that not only the dynamics but even the nature of a complex system is determined by its past evolution.

## The added value of complexity research

Complexity research is an intrinsically transdisciplinary enterprise. The phenomena studied in complexity science originate from disciplines ranging from statistical physics to economics, from mathematics to sociology, from chemistry to neuroscience, from computer science to genomics, etc. Moreover, many challenges faced by our present-day society, such as the global spread of infectious diseases, climate change and traffic control translate to scientific questions that are at the core of complexity research.

Once again, ‘*more is different*’: complexity research extends beyond the straightforward combination of two or more disciplines, it introduces new dimensions, challenges and opportunities. Its core consists of studying the underlying concepts, or mechanisms, that can be distilled from the complex phenomena exhibited by nature, society or a computer simulation, and that are characteristic for complex systems *in general*. For instance, emergent behaviour at the macro-level, such as the outbreak of panic in a crowd, the catastrophe in which a vegetated area collapses into a desert, or the sudden ‘electric storm’ of an epileptic seizure, shares essential characteristics with well-studied concepts such as phase transitions in physics and bifurcations in mathematics. Likewise, questions about the prediction and possible control of congestions in traffic flow or about the growth and dynamics of sandbanks in coastal areas, correspond directly to theories on pattern formation that have been developed in mathematics and physics. There is a world to be gained in science by crossing boundaries.



Figure 1 Clouds, Climate and Complexity. Clouds exhibit complex behaviour on scales that vary from global patterns, to the size of thunderstorm cells, to micro-physics. While large-scale conditions impact small-scale organization, there also is a direct link from micro scale reflectivity properties to earth albedo. (courtesy Harm Jonker, TU Delft)

However, it is obvious to all scientists working in fields related to complexity research that existing fundamental insights so far do not go beyond scratching the surface of most complex phenomena. In other words, novel ideas that penetrate the subtle interplay between many interconnected elements on various, often entangled, scales, need to be developed. These novel insights can be reached only by a direct cross-fertilization between different disciplines. A question formulated by an observation in cancer research may open up a new area of mathematical research; a thermodynamical concept may inspire and guide a breakthrough psychological experiment. In fact, complexity theory and its predecessors have already established that this is (much) more than wishful thinking. Scientific computing, i.e. the combination of modelling and computer simulations, is revolutionizing various areas in science and society; the impact of the work by the meteorologist Edward Lorenz on the (mathematical) field of dynamical systems can hardly be overestimated.

By their nature and origin, insights obtained in the dynamics, prediction or control of complex systems have a potentially decisive impact on human society, as for instance has been demonstrated by the worldwide effects of the ideas by Black, Merton, and Scholes – that are based on concepts stemming from mathematics and physics – on financial markets.

Complexity research has another, extremely valuable aspect: it may bridge the gap between the three main scientific communities in the Netherlands, known as ‘alpha’, ‘beta’, and ‘gamma’ sciences. The ‘alpha’ sciences concentrate on studies

such as literature, history, and philosophy, whereas the ‘beta’ sciences cover studies like mathematics, physics, and chemistry. More or less in between these two communities, the ‘gamma’ sciences concerns studies of man and society, for example, sociology, psychology, law, and economics. Complexity research has its roots in each of these three communities. Especially in the study of complex systems, the ‘alpha’, ‘beta’, and ‘gamma’ sciences use to a high extent the same language, tools, and methodology, as illustrated by the numerous examples mentioned in this document. There is much to gain by bringing these different focal points into a single framework for joint research, with already the clear advantage of having a common ground and understanding of complexity.

## Complexity research in the Netherlands

At the international scale, ‘Complexity Science’ has been evolving as an independent and highly relevant multi-disciplinary scientific field for more than a decade now. For instance in the U.S.A., the National Science Foundation (NSF) has initiated many activities in this area. Examples within Europe include the EU-funded new and emerging science and technology (NEST) specific targeted research projects (STREPs) ‘Complexity Pathfinder’ in which multi-disciplinary complexity research has been implemented in various disciplines, such as physics, chemistry, biology, psychology, sociology, political science and economics. Another recent European initiative is the FP6 NEST project on ‘Tackling Complexity in Science’. »

A large number of researchers in the Netherlands, working in various fields, have been actively involved in complexity research at an individual or at a local level. For instance, several Dutch universities and institutes participate in the European '*Tackling Complexity in Science*' projects '*Starflight*' (Starlings in flight), '*Complex Markets*', '*CREEN*' (Critical Events in Evolving Networks) and '*EMBIO*' (Emergent Organisation of Complex Biomolecular Systems).

In contrast to developments abroad, these activities have not yet led to an established national community that crosses the existing disciplinary boundaries. A national program on '*Complexity*' will have a significant impact on the foundation, success and vigour of such a community, and thus on the evolution and embedding of '*Complexity Science*' within the Netherlands. Such a program will bring together researchers from traditionally separated disciplines, and will create opportunities for joint research projects that are completely novel within the Dutch scientific landscape.

Moreover, the formation of a program on '*Complexity*' is especially timely, as can be concluded from the following initiatives, activities and recent developments:

- The NWO-GBE/GBN program '*Dynamics of Patterns*' has been running since 2005. It has a multi-disciplinary scope and emphasis that is similar to that of the intended '*Complexity*' program. In fact, a '*Complexity*' program may be seen as the natural next step that extends the scope of the '*Dynamics of Patterns*' program into the realm of economics, biology, psychology, sociology and

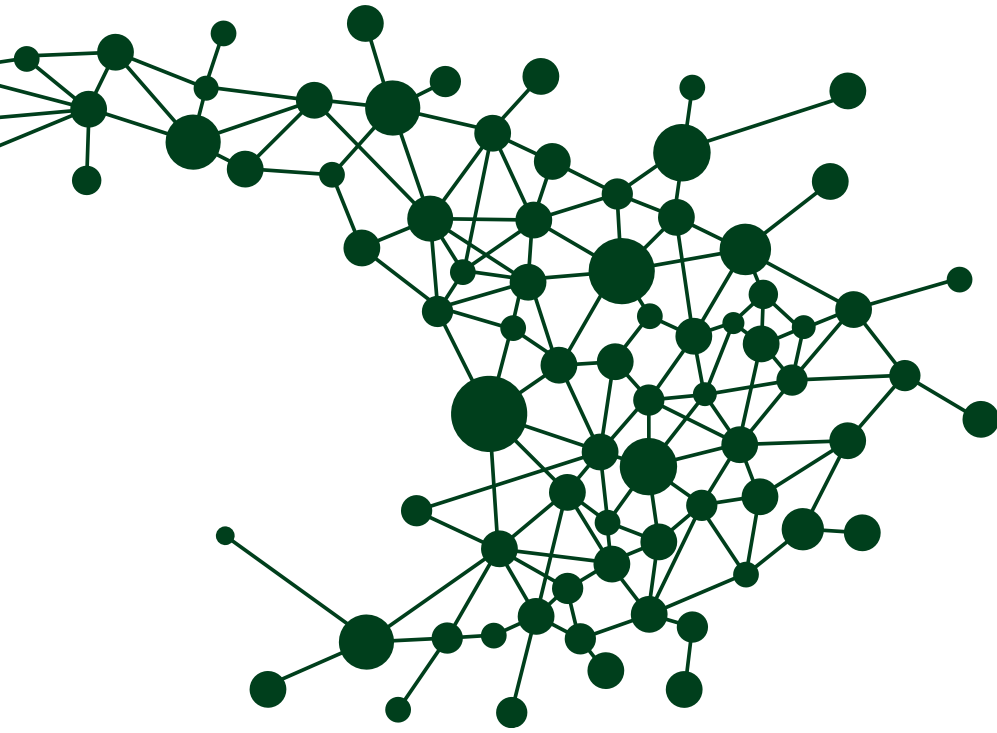
the earth sciences. There is a mutual understanding within the (active) '*Dynamics of Patterns*'-community that it is now an appropriate moment to reach out to adjoining disciplines.

- Last year, a new centre for research on complex systems, *Para Limes*, has been established (in Doesburg) as a private initiative by leading figures from science and industry. This centre may be seen as a European counterpart of the world-renowned '*Santa Fe Institute for Complex Systems*'. *Para Limes* is expected to attract international complexity researchers of the highest level. Their presence will stimulate complexity research in the Netherlands. On the other hand, a well-coordinated national '*Complexity*' program will increase the impact and attractiveness of this centre.
- In 2006 the ERANET on complexity, called the Complexity-NET, was initiated by 11 European Research Councils and Ministries, including NWO. This ERANET resulted from an initiative of CREST, the EU Scientific and Technical Research Committee, which identified complexity as one of the top five priority areas. Through an analysis of national research funding activities and funding procedures, it has been possible to define and specify a Coordinated Action on complexity, which sets the scene for a strategic funding of complexity research and research training on the European level. A joint action plan includes the opening of national programs and the possibility of joint research or research training programs.

- Within the Dutch scientific community, there is a number of activities that indicate that now indeed is the moment to build robust and permanent bridges between the alpha, beta and gamma sciences. The foundation in 2005 of mathematics ('wiskunde')-cluster '*Nonlinear Dynamics of Natural Systems*', that has a research focus on the (interactions of mathematics with) the life and earth sciences, is one example. Another one is the recent initiative by the *Netherlands Institute for Advanced Study* in the Humanities and Social Sciences (NIAS) and the Lorentz Center, that has its roots in the beta sciences, to jointly organize interdisciplinary  $\alpha\beta\gamma$ -workshops – a cooperation that is also quite unique from an international point of view.
- At present, there is a number of promising initiatives at the national and more local scales (e.g. Science Park Amsterdam) in the field of '*Computational Science*' and/or '*e-Science*'. Although there certainly is a difference between their scientific approaches, methods and central questions, the fields of '*Complexity*' and '*Computational e-Science*' are compatible and share common interests. In fact, '*Complexity*' research profits from the '*e-Science*' initiatives, and vice versa. Moreover, the associated communities are well-mixed, and strongly support each others initiatives.
- In the social, behavioural and life sciences, the number of complexity research institutes is rapidly increasing. 'CeNDEF' (Center for Nonlinear Dynamics in Economics and Finance) at the Faculty of Economics and

Econometrics of the University of Amsterdam studies the economy and financial markets as nonlinear, complex, evolving systems. '*DRIFT*' (Dutch Research Institute For Transitions) at the Faculty of Social Sciences of the Erasmus University Rotterdam employs theories and concepts from a wide range of scientific disciplines, such as complex systems science, governance, sociology, culture sciences and policy analysis, to study major transitions in society and technology. 'CSCA' (Cognitive Science Center Amsterdam) is related to the Faculties of Science, Social and Behavioral Sciences, and Humanities of the University of Amsterdam, this consortium studies human cognition and for instance applies nonlinear mathematical models to study complex behaviour of the neural system of the brain. '*RIKS*' (Research Institute for Knowledge Systems), located in Maastricht and connected to the Faculty of General Science at Maastricht University, applies complex cellular automata models to predict future land-use patterns.

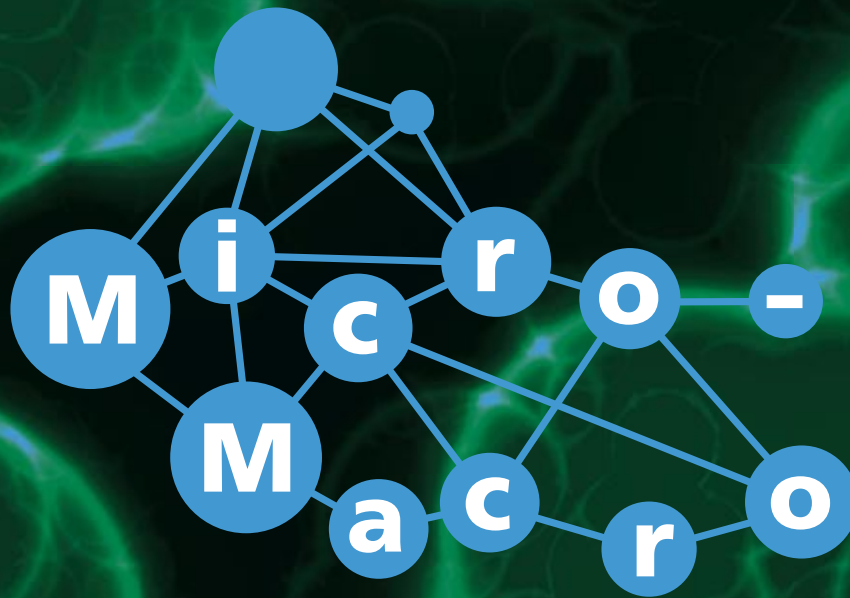
The Dutch scientific community is especially well-equipped, also from an international perspective, to initiate activities in the field of complexity, and has the opportunity to create a starting point from which it may join in, and give direction to, European initiatives. In order to do so, the community would strongly benefit from a common research platform that allows for maximal cross-disciplinary interactions. We feel that a national program on complexity science could provide such a platform, as well as provide opportunities for young researchers to be trained within this rapidly developing field. ■



# Research themes

In the second part of this proposal, we sketch the contours of a '*Complexity*' program by focusing on three research themes. Each of these has been chosen to highlight a broad area within the field of '*Complexity*'. Based on its existing strengths and potential, the Dutch scientific community may be expected to provide major contributions to the opportunities offered and challenges posed by each of these themes. The themes are by no means independent, in fact they are often strongly overlapping and integrated. However, they represent complementary approaches to the understanding and the study of complex systems.

The theme Micro-Macro focuses on the relationship between the micro-components and the macro-behaviour of complex systems. The Networks theme highlights the role of the topology and strengths of the connections between the components of a complex system. The Predictability theme, finally, deals with the behaviour of complex systems and the extent to which this behaviour can be predicted or controlled on the basis of (partial) knowledge of its workings.



## Micro-Macro

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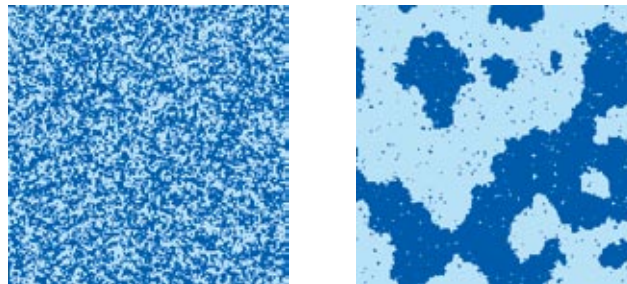
An important characteristic of complex systems is that properties observed at a global, *macro* level typically cannot be reduced to the properties of the constituent elements. In a complex system, macro-level phenomena are often *emergent properties* that arise through, often nonlinear, local micro-level interactions.

The first rigorous theory explaining macro behaviour through micro interactions originated in physics, following the work of Ludwig Boltzmann on statistical thermodynamics. The simplest, but simultaneously the most influential, model in statistical mechanics is the Ising model, originally proposed by Lenz and Ising as a model for explaining ferromagnetism. It consists of a regular array of units that can be in either of two states ('up' or 'down'). These units interact only with their immediate neighbours, in a manner that tends to favour having a common state. At the same time all units are exposed to an external randomizing influence (temperature). The key insight from the Ising model is that at a certain critical 'temperature', a phase transition occurs in which the collective, macroscopic

state of the system changes to one in which large, coherent domains with a common choice for the micro-state appear. Generalizations of the Ising model have been applied in many fields. Applications include the modelling of flocking birds, or beating heart cells and social interactions and herding behaviour in financial markets. In psychology, the model has been used to describe neurons in the brain, which can be either active or inactive. Hopfield (1982), for example, suggested that a dynamical Ising model provides a first order approximation of a neural network, which is capable of learning.



Figure 2 Snapshots from an Ising model simulation. Left: a disordered high temperature state. Right: an ordered low temperature state.



## Examples

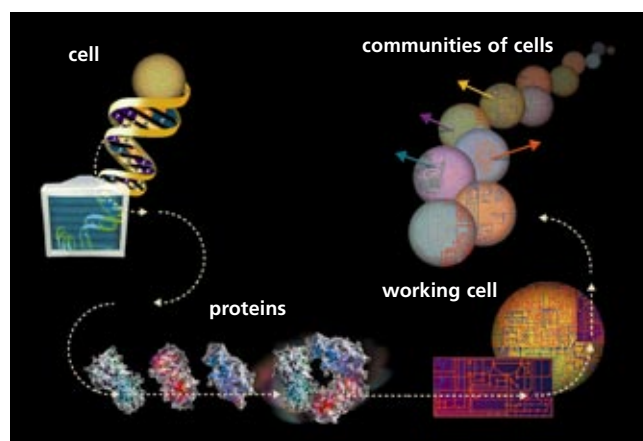
Complex systems often have *multiple scales* in spatial structure (e.g., quarks-atoms-molecules-material-planet-galaxies) and in time (from nano-seconds to galactic time, from tic-by-tic trading to lifetime investment). These distinct scales reflect a hierarchical organization of nature. Both individual and collective emergent properties play a role as one moves from one scale to the next. We discuss a number of examples of interaction at the micro-level and emergent properties at the macro-level.

**Molecule to Material** The best known example of an emergent property is the *phase transition* in physics, which underlies the transition from gas into liquid or from liquid into a solid state. For example, when water freezes to ice when the temperature falls below a critical level, small changes in local interactions of water molecules suddenly lead to a large change, a phase transition, at the macro-level. Nowadays physics has a fairly solid understanding of emergent behaviour in *equilibrium* systems through the statistical mechanical theory of phase transitions. However, the field of *self-organization* phenomena in driven, *non-equilibrium* systems is still in its infancy. This holds a fortiori for systems, in which the micro-components are dynamical systems in their own right, like polymers, which consist of many individual segments coupled to form a larger unit. Modern cell biology has provided physics with novel examples of molecular systems, which display emergent properties. One example is that of the cytoskeleton in which energy-consuming interacting entities like micro-tubules and motor-proteins show striking self-organizing behaviour. The appropriate analytic tools to describe such systems, which are characterized by disparate length scales (nm for motor proteins, microns for micro-tubules), time scales (ms for motors, minutes for micro-tubules), and novel effects like

the random switching between growth and shrinking of the micro-tubules, have yet to be developed. Such systems are also characterized by the large number of physical parameters that exert various degrees of control. Many of these elude simple experiment-driven measurement or assessment. This poses the additional challenge of identifying the relevant parameters, by searching through high-dimensional spaces, a tough problem in its own right.

**Gene to organism** In evolutionary biology, a great challenge is to understand the scaling-up, over many orders of magnitude (both in time and space), of genomic change to changes in the organism as a whole, and how this is ultimately shaped by ongoing evolutionary processes. The current genomic data explosion (and the insights obtained from the data through bio-informatics studies) gives us raw material to work from. We need to develop methodologies to study the interlocking of processes at multiple space and time scales. We need to learn to recognize which part of (relative) macro-level behaviour is due to general self-organizing processes and which parts reflect rare, but evolvable, cases in a high-dimensional specification space. We need to further our understanding of how Darwinian mutation selection processes interface with self-organizing processes at multiple levels. Not only do we need to understand the macro-level behaviour in terms of given micro-level behaviour, but we also need to understand how the (relative) micro-level behaviour emerged as a consequence of the macro-level behaviour that it generates (as is the case in multi-level evolutionary processes). A case in point is the question of how new species emerge in specific contexts, depending on the competition (or collaboration) with other species and environmental conditions. »

Figure 3 The multi-level path from gene to organism  
(from: <http://genomics.energy.gov> )



**16 Neuron to mind** Love, humour, solving a puzzle, remembering the name of one's first teacher, the idea for a novel, etc., this is all in the brain. All psychological processes find their origin in the interactions between billions of neurons: the brain appears to be the ultimate complex system. This version of the micro-macro problem, or specifically brain-mind problem, has generated an enormous body of research, and is the subject of one of the most central and long-lasting debates in psychology and philosophy. It now appears that the complex system account is an important and viable alternative for the traditional reductionist and dualist accounts for this mind-body problem. For instance, using the approach of synergetics, Kelso and others have been able to demonstrate empirically, and to model mathematically, stimulus-induced phase transitions in the brain, relating macroscopic levels of synchronization to micro levels of interacting neural modules. Both from practical and theoretical points of view, the methods and ideas in complex systems theory will play an important role in the future research in psychology.

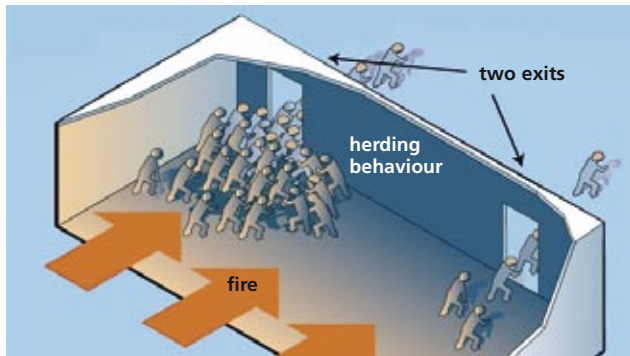
**Individual to population** In biological and socio-economic systems, properties at the population level are ultimately the result of interactions among individual elements within these systems. Biological systems are characterized by selection among individuals and competition among species. Ecosystems are shaped by evolutionary selection and mutation. Socio-economic systems consist of many individuals in different roles (consumers, producers, investors), who interact and compete in local (e.g., regional, national) and global market institutions. Remarkably, traditional neoclassical economics has completely ignored interaction and evolution. Rather it is characterized by

an extreme reductionist approach, in which aggregate macro behaviour is reduced to the study of an average representative consumer and firm, behaving rationally and optimally. However, economics is witnessing a paradigm shift from the representative rational agent framework to an interacting agents hypothesis and complexity view. Here, markets are viewed as conglomerates of many heterogeneous consumers. Firms and investors are represented as having bounded rationality, while employing simple, behavioural decision rules (Hommes, 2006). Evolutionary selection of behavioural heuristics and social interactions can infuse discipline into the new research program, by taming the 'wilderness of bounded rationality'. Interaction among consumers, firms, and investors at the micro level may explain emergent properties at the macro level, such as power-law distributions of firm size, wealth, and financial asset returns (Axtell, 2001). A classical example of micro-macro interaction, and arguably the first application of complexity in economics, is the model of racial segregation of Schelling (1971). He discovered that the emergent property of racial segregation in densely populated urban cities, may arise from a tiny change in initial conditions, namely slight preferences of individuals for neighbours from their own ethnic group.

## Challenges

At present there is a rapidly growing literature on applications of complexity models of micro-macro interaction in fields such physics, biology, psychology, and economics. These models share many features, but also differ importantly in many ways. At present, a multi-disciplinary approach would be very fruitful to develop fundamental methods in complexity modelling, to combine the specific

Figure 4 Models of interacting agents are used to study the herding behaviour of panicking crowds. (from: Low, 2000)



methodological strengths of each discipline, and to determine generality and specificity in methodology, to be able to efficiently tailor modelling effort to the substantive problem at hand. Some challenges include the following.

**Theoretical issues** Complex systems evolve over time, and a systematic study of the key features of the dynamics of complex systems is essential. Challenging questions about the dynamics of micro-macro transitions include: (i) how are emergent properties related to micro interactions?; (ii) How does self-organization in complex systems arise? (iii) what are the 'simplest complex systems' or reduced form models, which still explain the most important emergent properties and stylized facts observed in data?; (iv) how can we reverse-engineer the mechanics of complex systems from their behaviour under a controlled set of external stimuli?; (v) which features can be explained by a deterministic model, and which need a stochastic explanation?; (vi) how does feedback from macro behaviour to micro behaviour affect the aggregate outcome and emergent properties of complex systems? (vii) how can we go beyond the paradigm of 'simple rules give rise to complex behaviour' to 'complex rules leading to complex behaviour'?; (viii) how do we deal with searching high-dimensional parameter spaces for identifying relevant behaviour in complex systems (the needle in the haystack problem).

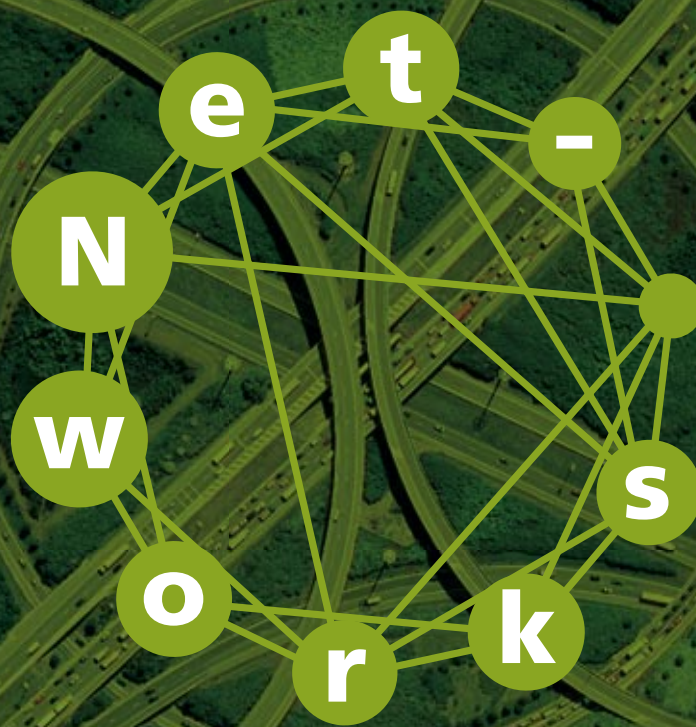
**Adaptive micro-entities** A key difference between biological and social systems relative to physical systems is that the micro entities considered are often not invariant, but may change their properties over various time scales. An important challenge therefore is how to adapt physics

models of interacting particle systems and apply them to biological or socio-economic systems. A complexity theory of socio-economics should contain a theory of 'smart atoms'. In particular, the tradition rational expectations paradigm has to be replaced by a universal theory of heterogeneous expectations, bounded rationality and learning (e.g. Brock and Hommes, 1997). A complexity theory of biological systems should incorporate the interrelation between adaptive processes of micro entities at regulatory and evolutionary time scales and the emergent macro-scale properties (and vice versa).

**Brain and mind** Relevant research questions in the context of brain research are: What are the emergent properties of different kind of neural network structures? Can we understand the properties and limitations of macro processes, such as working memory, from the complex emergent properties of neural activity? How do emergent properties of neural activation such as awareness, influence this neural activity? But other questions also arise. One cause (say, child abuse) has often many different effects, and one effect (say depression) can have many different origins. Complex system theory may provide insight in this type of relationships and points to new types of interventions that are not based on simple cause-effect models. Similar considerations hold for understanding neurological and psychiatric disease from a complex systems point of view.

**Prebiotic evolution** The transition of non-living to living systems is a major transition in levels of complexity. How did self-organizing processes and genetic information accumulation codetermine this major transition is an ultimate question for complexity theory. ■



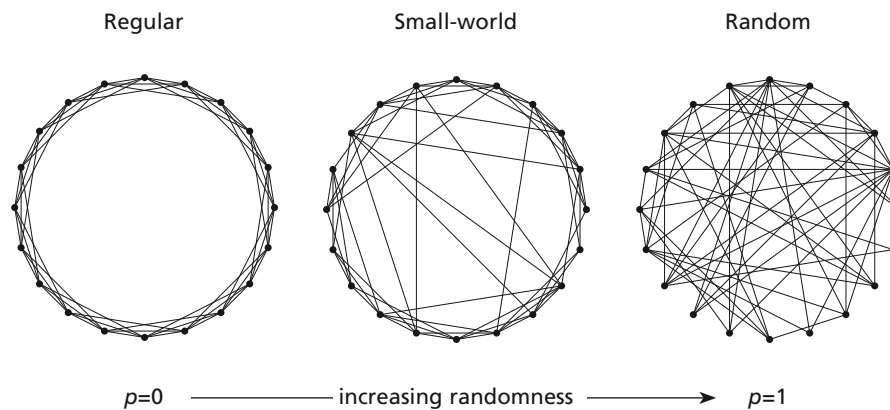


## Networks

**18** Complex systems often consist of large numbers of elements or modules that interact. Such systems can be modelled as networks or graphs. Since the discovery of small-world networks and scale-free networks in the late 1990s, we have witnessed a rapidly increasing research activity throughout disciplines including natural sciences, social science and even the humanities. This was initiated by the introduction of random graphs in which nodes were connected by links in a random manner (Solomonov and Rapoport, 1951; Erdős and Rényi, 1960). However, truly random graphs appeared to be unsuitable for the study of networks encountered in nature and society, as it was found that these networks are highly structured. Indeed, most natural networks display features like clustering, skewed degree distributions and degree correlations that cannot be explained by comparably simple random networks.

A major breakthrough in modern network theory occurred with the work Watts and Strogatz (1998). They proposed a simple model of a network on a ring in which each node is connected only to its  $k$  nearest neighbours. By rewiring the edges with a probability  $p$ , the whole range between ordered and random networks could be covered. Networks with intermediate  $p$  values were shown to have both high clustering and short path lengths. Such networks, designated as 'small-world' networks, proved to be excellent models on many real complex systems in nature, including the nervous system of *c. Elegans*, a social network of movie actors and the network of power plants in the US. Watts and Strogatz (1998) also showed that a small-world architecture may facilitate the spread of infection or information in networks.

Figure 5 Network topologies for different values of  $p$   
(after: Watts and Strogatz, 1998)



## Examples

A second profound discovery was published a year later by Barabasi and Albert (1999). They introduced the concept of 'preferential attachment' and proposed a model for the *growth* of a network where the likelihood that a newly added node will connect to an existing node depends upon the connectivity of this node: new nodes prefer to attach themselves to a well-connected node as to minimise the distance to other nodes. The obtained degree distributions can be described by a power law. Such a power law implies that few nodes have high connectivity ('hubs'), while most nodes have low connectivity.

It has been shown that many real networks, such as for instance the World Wide Web, collaboration networks of scientists, networks of airports and possibly brain networks are probably scale-free, at least to some extent (Boccaletti et al., 2006). Scale-free networks have many interesting properties including an extremely short path length as in small-world networks, and resilience to 'random attack' but vulnerability for targeted attacks on hubs. Compared to small-worlds, where all nodes have the same degree, the degree of nodes is very different in scale-free networks. Yet, if we assume that, apart from preferential attachment, geographical proximity affects the probability of linking (as a cost constraint), small-world properties can be obtained from a preferential attachment algorithm as well.

One important success factor of the modern theory of complex networks, in particular of the Barabasi-Albert model, is that the basic logic of preferential attachment applies to virtually all networks, while the logic can be extended with additional variables and constraints to understand the properties that are specific to the network at hand. This versatility explains its widespread use and further development in natural and social sciences. Four examples – out of many – readily illustrate the adaptability of complex networks to specific domains.

**Spatial systems** Networks that have a spatial dimension – like train or airline networks, electricity networks, neural networks, corporate networks or social networks – are affected by transportation costs: the cost of a link increases with geographical distance. This implies that the probability of a new node linking to an existing node is not only dependent on preferential attachment logic but also inversely on distance. As a result, one typically obtains a hub-and-spoke structure with large distances between hubs each connected with a subset of nodes at small distance. Complex networks models can be used to simulate the future spatial evolution of such networks depending on possible scenarios (changing the transportation costs, changing the scale economies at the hub, etc.). Of interest, similar considerations may play a role in biological networks. »

Figure 6 The official map of the network formed by the London underground



**20 Brain research** There is now increasing evidence that anatomical and functional connectivity networks in the brain display the small-world phenomenon and have degree distributions with 'heavy tails' (Stam and Reijneveld, 2007). However, most studies show that brain networks deviate in significant ways from the classical scale-free model of Barabasi and Albert. In particular, fMRI studies have suggested a degree distribution with power law and exponential components. Such a degree distribution can be explained by growth models of brain networks, which take into account biological constraints. Furthermore, there is increasing evidence that the complex structure of brain networks breaks down in various neurological and psychiatric disorders such as Alzheimer's disease, brain tumours, epilepsy and schizophrenia. Modern network theory provides a general framework for understanding how different types of network damage ('random error' versus 'targeted attack') could bring about these pathological network changes.

**Text** With the advent and success of digital text archives, there is an increasing interest in the application of network theory on inter-textual reference structures as pioneered by De Solla Price (1965) in the context of scientific citation networks. Other examples of such reference structures are citations between patents, references between legal sentences, hyperlinks between Web pages, etc. Generally, a reference can be interpreted as a recommendation, thus guiding search processes through complex networks. The unchallenged success of Google's search engine, which is primarily based on hyperlink structures, is a prime example of the relevance of complex network theory in this domain.

**Food webs** The structure of food webs has been shown to exhibit small-world properties. An important implication of food-web research holds that one can assess the resilience of ecosystems regarding the extinction of one or more species. More specific co-evolutionary models have been proposed that model the interaction structure of species as a complex graph using a variety of techniques. In such studies, the similarities between extinction events of species in ecosystems and extinction events of technologies in economies, are often highlighted, and provide fertile ground for cross-disciplinary research between biologists and economists (Frenken, 2006).

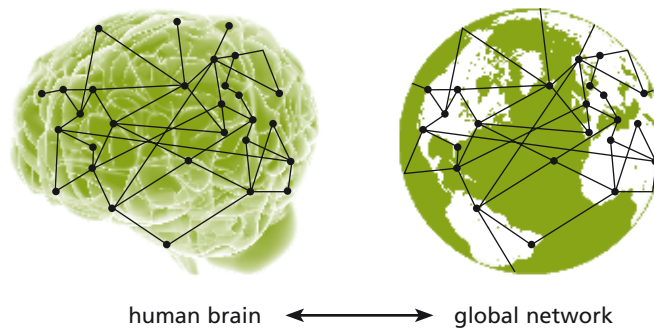
## Challenges

The discovery of small-world networks in 1998 and of scale-free networks in 1999 was noted by scientists in many different fields, and sparked a large body of theoretical and experimental research that is growing to this day. Some excellent reviews are provided of the current state of network theory and its empirical basis (Boccaletti et al., 2006), which reflects the rapid progress as well as the newly emerging topics, which we summarise under four challenges:

**Dynamics of networks** First, theoretical advances reside in the generalisation of the Barabasi-Albert model of evolving networks, or dynamics of networks, for (i) directed graphs in which links are not necessarily mutual, (ii) to weighted networks in which links can be more or less strong and (iii) to hierarchical networks in which a hierarchy exists of modules-within-modules-within-modules, where a module is characterised by a high density of links. Though some of its



**Figure 7** Anatomical and functional connectivity networks in the brain display the small-world characteristics which become disrupted in the case of disease



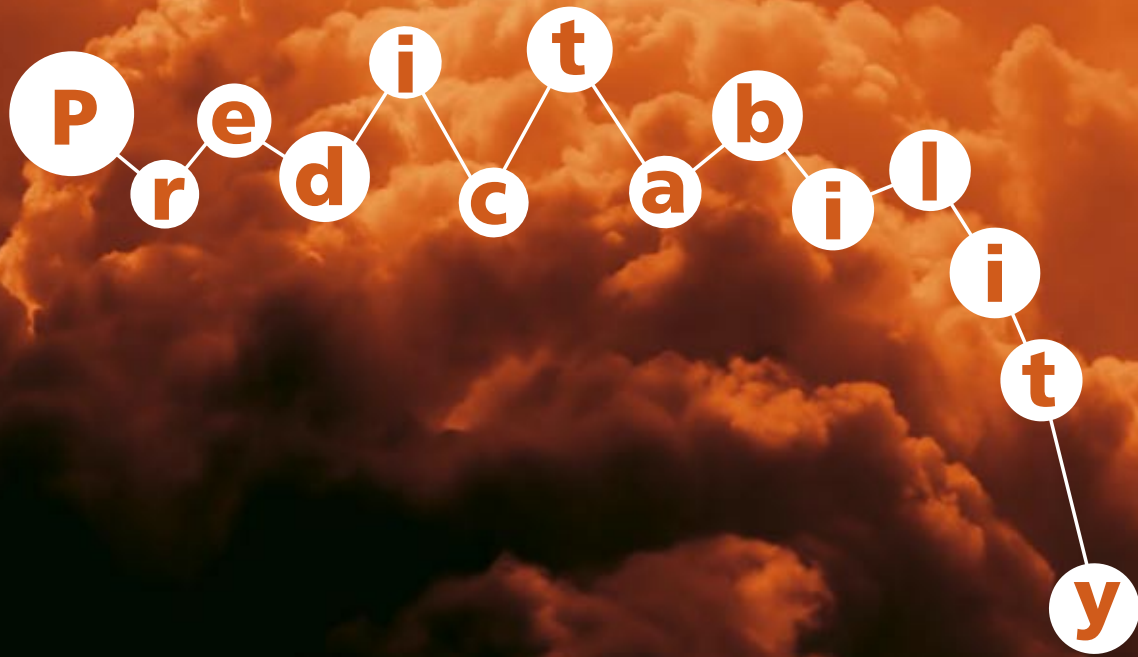
properties are being understood, explanatory models are still to be developed that can generate such network structures dynamically under realistic assumptions.

**Dynamics on networks** Second, complex networks research is used to study the diffusion processes otherwise indicated as dynamics on networks as distinct from dynamics of networks themselves. Examples include the diffusion of information and knowledge, the diffusion of diseases as in epidemics, and synchronization processes in model and real neural networks. For random networks, such dynamics are now being understood well. The dynamics on small-world and scale-free networks, let alone directed, weighted and hierarchical networks are much less well understood as they rely more heavily on computational methods. In particular, a relation has been suggested between network topology and the threshold for transitions between asynchronous and synchronous states on such networks, which could be relevant for understanding phenomena such as epileptic seizures.

**Development constraints** Though complex-network models replicate global properties of empirical networks, they often fail to explain more specific properties stemming from development constraints. To advance empirical application, both retrospectively and prospectively, models need to be elaborated to include congestion effects, entry barriers effects, aging of nodes and geographical constraints. Such an approach also explains why there can exist critical time windows during development. Related to this, little is known about the adaptive capability of networks in the face of environmental influences, including random and targeted

attack removing nodes and links, where adaptive capability (resilience) can be understood as the probability of recovery. Typical fields of application here are critical infrastructures breakdowns, loss of biodiversity, group conflicts, and recovery from disease.

**Design of networks** Complex network models are recently being used in the design of networks for distributed information systems. In contrast to classical top-down approaches, a complex-network approach allows decentralised nodes to use local information for managing connections to other nodes. Decentralised approaches often exploit what are known as epidemic protocols, and have been followed for the construction of overlays, semantic clustering of nodes, information dissemination, and data aggregation (see also Kermarrec and van Steen, 2007). ■



## Predictability

**22** Due to its immediate relevance in science, technology and society, the predictability of the behaviour of a complex system is one of the central themes within complexity research. It is related to the dim, and not well-defined, transition from almost linear short-time dynamics to fully developed, long-term, spatio-temporal chaos. In fact, the 'butterfly effect', associated with one of the first examples of a chaotic system (Lorenz, 1963), has become a well-known metaphor for the inherent limitations of the predictability of nonlinear processes. It dates back to the 1960s, originated at the intersection of meteorology and mathematics, and - in retrospect - can be seen as an early example of complexity research.

Problems with predictability are not restricted to only chaotic phenomena. For instance, predicting whether a vegetated area may collapse into a desert, or whether dense traffic will congest, involves issues like stability with respect to stochastic fluctuations and the dependence of the process on the variation of parameters.

Predictions of the behaviour of complex real-world processes are increasingly based on simulations of extended multi-scale nonlinear model systems. For instance, meteorologists wish to issue warnings for severe weather events on the basis of cloud-resolving simulations. Central questions are how to assess the predictability of these multi-scale systems and how to determine the reliability of a prediction. These questions go beyond the explicit context of the multi-scale model and thus lie at the core of complex-systems research.

**Figure 8** The basis of ensemble prediction illustrated by the prototypical Lorenz (1963) model showing that predictability is flow dependent. (a) A forecast with high probability (b) forecast with moderate predictability (c) forecast with low predictability. (From: Palmer, 2007)



## Examples

**Earth system modelling** Leith and Kraichnan (1972) estimated that the predictability of large-scale weather extends well beyond a week. At that time actual forecasts had a scale which was an order of magnitude smaller. This encouraged research in improving the quality of numerical weather forecasts. One of the priorities was to improve methods for feeding corrections from observations into the model simulation. An important development was the pioneering of so-called ensemble forecasting techniques (see Figure 8).

Using weather forecasts models for climate forecasts seems like a trivial extension, but it was an extension right into the domain of complexity theory. To start with, one does not look at the daily weather, but at the climate. This also means that the details of the initial state of the atmosphere are not important anymore. Secondly, other components do enter the system, such as the ocean, the biosphere and cryosphere, each with its own time scale or time scales. The complex systems that result from combining these components are called Earth System Models. Although Earth System Models form the basis for much of the IPCC Climate Assessment, the predictability of climate change and the estimation of the uncertainty in our estimates have remained central issues.

Desertification is another example of a complex earth-system process in which predictability plays a central role. In desertification, well-vegetated areas exhibit vegetation patterns before undergoing a sudden catastrophic and

irreversible transition to a desert state. The work by Kefi et al. (2007) indicates that this process is governed by a power-law behaviour in the spread and magnitude of the vegetation patches. This phenomenon may, in principle, be used as a predictive tool for the occurrence of desertification, even if the insight in the mechanism that drives the power-law behaviour is still limited. Although desertification and weather systems have quite different dynamics, in both cases there is an intimate relationship between complex behaviour and predictability.

**Financial markets** The stock market presents yet another example of a highly unpredictable system. According to the traditional view financial investors are fully rational and stock markets perfectly efficient. In such a world movements in stock prices are only driven by random news about the economy (e.g. interest rates, economic growth, etc.). But the standard view is at odds with extreme movements observed frequently in financial markets worldwide, for example the 20% drop of the Dow Jones index on black Monday, October 19, 1987 or, more recently, the large movements due to the credit crisis.

In the last decade an alternative view based upon complexity theory has been proposed. The most prominent example has been the Santa Fe artificial stock market (Arthur et al., 1997), where the interaction of a large population of fundamental traders and technical analysts leads to large swings in stock prices triggered by news but reinforced by herding »

Figure 9 The global spread of the H5N1 influenza, or bird flu (2007). In yellow, countries with poultry or wild birds killed by the H5N1 virus. In orange, countries with human cases of the H5N1 influenza. (from Wikipedia)



**24** behaviour. Using a simpler, stylized version of this model, Brock and Hommes have shown that evolutionary selection among simple investment strategies may lead to instability and chaotic stock price fluctuations, with temporary bubbles and unpredictable crashes, very similar to real markets (see Hommes 2006). In fact, agent-based financial market models can reproduce important 'stylized facts' or emergent properties observed in real financial data, such as unpredictable asset returns, clustered volatility (i.e. irregular switching between quiet and turbulent phases) and fat tails.

**Epidemiology of diseases** Infectious diseases have had a decisive influence on the history of mankind, and also the future asks for predictions and rational control decisions. The spatio-temporal dynamics of an infectious disease, such as HIV or the bird flu is a complex process, driven by the spread of micro-organisms and resistance genes, and often fuelled by the very use of antibiotics. It has a dominant multi-scale character, for instance since the transmission is determined by the nature of individual contacts - compare HIV to the bird flu - while the disease occurs on a global scale. It is natural to model the evolution of a disease by a dynamically evolving network. Contacts between individuals change and are predominantly short range and centred, but the relatively few long range contacts are essential for the global dynamics of an epidemic.

The predictability of the possible occurrence of an outbreak and the evolution of an epidemic are central themes in the study of epidemiology of diseases, see for instance Day and Proulx (2004) in which a theory for predicting both the short term and long-term evolution of virulence is developed.

**Traffic management** Transport is another discipline witnessing a shift to multi-scale modelling. The interest of transport modellers involves different time horizons. First, traffic-flow prediction is focused on traffic flows during the day or even segments of the day. Second, transport demand models are focused on longer term prediction of transport demand as a function of demographic and economic change, jointly with the impact of infrastructure, institutional, land use and economic policies.

Traditionally, traffic-flow forecasting has been based on operations-research allocation algorithms applied to aggregates of travellers. Lately, however, scientists have started applying agent-based micro-level simulation models, with emerging, nonlinear aggregate traffic-flow patterns (see e.g., Balmer et al, 2005; Rosetti et al, 2005). An interesting aspect is that travellers can actively decide what to do if traffic-flow predictions are available. Different situations may emerge – multiple user equilibrium, bifurcation or oscillatory behaviour.

For transport demand-modelling activity-based models are the state-of-the-art. They simulate which activities are conducted where, when, for how long, and with whom. These models depend on data collected for typical days. Only very recently, dynamic models have been developed, these models are based on notions of multi-scale interactions, nonlinear dynamics, agent-based technology, and emerging aggregate behaviour. They incorporate both processes of gradual change, but also sudden bifurcation and phase transitions.

## Challenges

Many of the problems that face society involve inherently complex processes which are studied by extended models that include many different scales in time and in space. However, it is a priori far from clear how the outcome of such a model simulation relates to the behaviour of the original real-world system. Therefore, it is essential to assess the uncertainty in the model predictions, rather than just making predictions. Moreover, it is also of interest to estimate the predictability of the process itself, i.e. the reliability and the resolution of a prediction with the best possible model. Of course, all this applies especially if one tries to extend the frontier of predictions.

We see a number of basic research challenges of a nature that combines theoretical and empirical aspects, which are relevant for predicting the dynamics of complex systems.

**The predictability of multi-scale systems** When does including more details induce better predictability? When is it possible to average small scales into large-scale effects without affecting the long-term predictability of a model? When does including new, large-scale, components affect the stability and the regimes of the system?

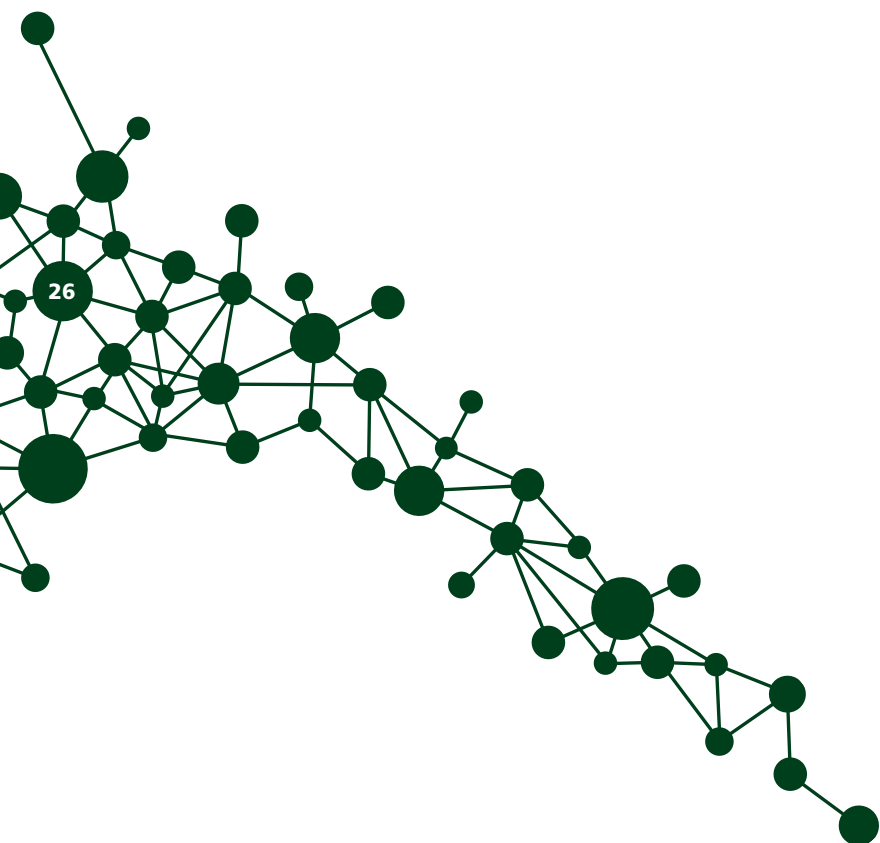
### **What determines the predictive power of a model?**

This is a fundamental issue that is especially relevant in the context of long term predictions. A model may be accurate on a short time scale, but may exhibit unrealistic behaviour in the long run. What determines whether a weather model is suitable for modelling climate?

**Surprising consequences of changes in control parameters** Is it possible to quantify the vulnerability of a system to a catastrophic event? For example, can it be recognized whether a vegetated area is on the verge of desertification?

**Managing uncertainty in complex systems** How is the uncertainty in predictions related to uncertainties in the input data and model formulation? When should we measure more, and when should we model better?

Each of these questions has the promise of paradigm-shifting advances in understanding complex systems, and is relevant for more than one field of science. For complex-systems research, the challenge is in developing new tools and strategies that have value beyond one specific context and that can be connected to real-world situations. ■







# Opportunities

'Complexity' has evolved into a unifying concept in the study of processes and phenomena that appear in nature and society. Increasing computer power has enabled explicit simulations of elaborate, extended, multiple-scale, and thus highly complex, models. This has opened up completely new avenues in science and technology.

In this proposal, three transdisciplinary research themes – Micro-Macro, Networks, and Predictability – have been identified, that transcend the boundaries of traditional disciplines and that arise in many complex systems. It is argued by way of examples from a large variety of disciplines – ranging from physics and meteorology to neurophysiology and sociology – that as the systems under investigation grow in complexity, the associated scientific questions share more and more common features. The challenges formulated in the context of the three research themes show that the most promising opportunities for complexity research lie in combining approaches and ideas from different disciplines. This way, novel and potentially breakthrough insights can be obtained that go beyond the setting of specific systems. Moreover, there is an intimate interplay between developing these transdisciplinary insights and a specific disciplinary complex system: novel insights can both be inspired by, and be applied to a given complex system in psychology, genetics, economics, etc.

For this reason, there is a need for instruments that stimulate the study of transdisciplinary questions in complexity research and that promote the exchange of ideas within the scientific community. Such a community does not yet exist in the Netherlands. However, in combination with the existing strengths and highlights of the Dutch scientific community, recent developments within the Dutch scientific landscape show that there is a huge potential for a scientifically excellent, transdisciplinary, 'Complexity' community in the Netherlands.

Therefore, we – the authors of this proposal – firmly believe that it is now time to seize the opportunity and to build such a community by initiating a program on 'Complexity' research. ■

# Who was involved in the preparation of this document?

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**Paulien Hogeweg** is head of the research group 'Theoretical Biology and Bioinformatics', within the focus area Biocomplexity and Bioinformatics of the Science Faculty of Utrecht University. The aim of her research is to understand biotic systems as dynamic information processing systems at many interconnected space and time scales. To this end she developed novel modelling formalisms and data analysis methods. The overall research question is how complex organisms evolved and are evolving, with a focus on the interface between self-organization and evolution.

**Cars Hommes** obtained his MSc in mathematics and Ph.D. in economics at the University of Goningen, and is now professor of economic dynamics at the University of Amsterdam. In 1998 he received a NWO Pionier grant to set up the Center for Nonlinear Dynamics in Economics and Finance (CeNDEF), a multi-disciplinary research group pursuing a complexity approach to economics.

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**Bela Mulder** was trained as a theoretical physicist. Although, originally active in the field of statistical physics of soft matter systems, his current activities focus on understanding molecular self-organization processes in living cells. He heads the Theory of Biomolecular Matter group at the FOM Institute for Atomic and Molecular Physics in Amsterdam, and is Adjunct Professor of Theoretical Cell Physics at the Laboratory for Plant Cell Biology of Wageningen University.

**Kees Stam** was trained as a neurologist and clinical neurophysiologist. Since 2000 he is professor of clinical neurophysiology in the VU University Medical Center. Following his PhD work on cognitive dysfunction and event-related potentials in Parkinson's disease, he became interested in nonlinear dynamics and more recently graph theory as tools to explore the dynamics of brain networks in various neurological disorders such as Alzheimer's disease, Parkinson's disease, brain tumours and epilepsy. He developed various new techniques for nonlinear time series analysis such as the synchronization likelihood and the phase lag index.

**Maarten van Steen** is full professor in Computer Science at VU University Amsterdam. His research concentrates on large-scale distributed systems in which (up to hundreds of) thousands of computers collaborate in realizing a coherent view of a system. This research focuses on two application areas. The first is on designing and developing collaborative Web-based content distribution networks in which data,

processes, and control are distributed to achieve scalability. Second, similar scalability problems are addressed in large-scale wireless (sensor) networks, with the additional constraint that resources are often extremely scarce. In both areas, concepts of self-organization and management play a crucial role.

**Lex Zandee** was an undergraduate student in physics and mathematics at the University of Amsterdam. He obtained his PhD at the Radboud University Nijmegen. After a couple of postdoc positions in the United States, he took up a position as geophysicist at Royal Dutch Shell, where he was responsible for signal analysis in seismic research. At present, he coordinates the activities for the support of mathematical research in the Netherlands at NWO (Netherlands Organisation for Scientific Research). He also is the Dutch coordinator of the Complexity-NET, the EU ERANET on complexity.

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