Problem 1

The distribution of x is unspecified in the question, and here we assume that $x \sim N(0,1)$.

With Gaussian-Hermite integration, when # nodes = $[2, 3, 4, 5, 7]$, the numerical results are:

$$E[X^4] = [1.000, 3.000, 3.000, 3.000, 3.000]$$
$$E[X^6] = [1.000, 9.000, 15.000, 15.000, 15.000]$$
$$E[1/(1+X^2)] = [0.500, 0.750, 0.600, 0.692, 0.672]$$

With Monte Carlo integration, when $n = [10^2, 10^3, 10^4, 5 * 10^5]$, the numerical results are:

$$E[X^4] = [2.802, 2.963, 2.946, 3.012]$$
$$E[X^6] = [11.098, 15.096, 14.270, 14.705]$$
$$E[1/(1+X^2)] = [0.649, 0.669, 0.653, 0.655]$$

Matlab codes

Problem 1

```
clc; clear all; close all;

%% Initialization
syms x;
y = [x^4, x^6, 1/(1+x^2)]';
my = matlabFunction(y);

%% Guassian-Hermite Quadrature Integration
n= [2, 3, 4, 5, 7];
for i = 1:length(n)
    [x, w] = qnwnorm(n(i), 0, 1); % by Miranda and Fackler (2002)
    int = sum(repmat(w',3,1).*my(x'),2);
    disp(sprintf('When n = %d, GH integrations are %4.3f, %4.3f,
       %4.3f', n(i), int));
end

%% Monte Carlo Integration
n = [1e2, 1e3, 1e4, 5*1e4];
for i =1:length(n)
    r = randn(1,n(i));
    int = sum(my(r),2)./n(i);
    disp(sprintf('When n = %d, MC integrations are %4.3f, %4.3f,
       %4.3f', n(i), int));
end
```