Question 4 - by iterating on the first-order conditions

The problem can be written as:

$$W(k) = \max_{s} V(k,s) + \gamma W(Ask^{\alpha})$$

where, A is an known constant which equals $\frac{1-\alpha}{1+\lambda}$. The first-order condition reads:

$$0 = \frac{\partial V}{\partial s} + \gamma W'(k')Ak^{\alpha}$$

meaning

$$W'(k') = -\frac{1}{\gamma Ak^{\alpha}}\frac{\partial V}{\partial s} \tag{1}$$

The envelop implies:

$$W'(k) = \frac{\partial V}{\partial k} + \gamma W'(k')A\alpha sk^{\alpha-1}$$

Forwarding this expression one period ahead:

$$W'(k') = \frac{\partial V}{\partial k'} + \gamma W'(k'')A\alpha s'k'^{(\alpha-1)} \tag{2}$$

Now we can use (1) to eliminate $W'(k')$ and $W'(k'')$ in (2), and reach the Euler equation.

$$-\frac{1}{\gamma Ak^{\alpha}}\frac{\partial V}{\partial s} = \frac{\partial V}{\partial k'} - \alpha\frac{s'}{k'}\frac{\partial V}{\partial s'} \tag{3}$$

We know that s is a function of k': $s = \frac{1}{A}\frac{k'}{k^{\alpha}}$ Therefore, by using the chain rule $\frac{\partial V}{\partial s}$ can be written as

$$\frac{\partial V}{\partial s}\frac{\partial s}{\partial k'}$$

As $\frac{\partial s}{\partial k'} = \frac{1}{A}\frac{1}{k^{\alpha}}$ Then equation (3) can be rearranged as:

$$(\frac{1}{\gamma}+1)\frac{\partial V}{\partial s} = \frac{\partial V}{\partial s'}$$

And this is the Euler equation. From the expression of V(k,s), we can obtain the $\frac{\partial V}{\partial s}$ and $\frac{\partial V}{\partial s'}$

Now we define the euler residual as:

$$euler\_resi = (\frac{1}{\gamma}+1)\frac{\partial V}{\partial s} - \frac{\partial V}{\partial s'}$$

The MatLab codes for this method can be found in the appendix.