Lecture 16

1 Parametric Bootstrap

Suppose that we have a statistical model $f(x;\theta)$ for data X. Suppose that we have an estimator $\widehat{\theta}(x)$ for the parameter θ (for example, by doing OLS or ML).

The estimator $\widehat{\theta}(x)$ is a random variable, as it depends on the data. If θ were known, we could approximate the distribution of $\widehat{\theta}(x)$ by Monte-Carlo methods: generate I independent draws from $x \sim f(x;\theta)$, and evaluate $\widehat{\theta}(x)$ over the I new data sets that we have generated.

The true parameter that generated the data is unknown; hence it is not possible to generate draws from the model $f(x;\theta)$. However, since there is an estimator for θ, one could perform the Monte-Carlo approximation exercise described above by using draws from the model:

$$f(x;\widehat{\theta}(x)).$$

If we do this, we will effectively end with I estimators (one for each new data set). If the sample size is large and the parametric model $f(x;\theta)$ varies smoothly with θ, the c.d.f. based on the I estimators can be shown to be a reasonable approximate for the distribution of $\widehat{\theta}$.

The Monte-Carlo type exercise describe above (which you will implement in the homework) is known as the <u>parametric bootstrap</u> and it is a general statistical technique used to estimate the distribution of $\widehat{\theta}(x)$.