Homework 1 (Lecture 9-10)

1 The AR(1) model

Consider the linear process given by:

$$X_t = \sum_{j=0}^{\infty} \theta^j \varepsilon_{t-j}, \tag{1}$$

where $\{\epsilon_t\}$ is a sequence of white noise with variance σ^2. This process is usually called the autoregressive model of order 1.

Question 1: Compute the <u>autocorrelation</u> function of the linear process X_t.

Question 2: Show that the model above satisfies the <u>auto regression</u>:

$$X_t = \theta X_{t-1} + \epsilon_t \tag{2}$$

Question 3: A Gaussian AR(1) is given by (1) with $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ i.i.d. How could you generate random draws from a Gaussian AR(1) using Python?
Question 4 (Optional): Show that if $|\theta| < 1$, then the impulse-response coefficients satisfy the summability condition discussed in lecture.

2 The AR(2) model

Consider the causal linear process that satisfies the equation:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t, \tag{3}$$

where $\{\epsilon_t\}$ is a sequence of white noise with variance σ^2. This process is usually called the Autoregressive model of order 2.

Following what we did in Lecture 8-9, I would like you to answer the following questions:

Question 1: What are the IRF coefficients of the AR(2) model? In other words: what is the MA(∞) representation of the AR(2) model?

Question 2: What is the autocovariance function of the AR(2) model?

Question 3 (Optional, try this question once you are done with all the HW): What are the restrictions that ϕ_1 and ϕ_2 would need to satisfy in order for the IRF coefficients to be summable?

3 From ACF to IRFs

Suppose that I tell you that I used an MA(1) model with $\sigma^2 = 1$ to generate the following ACF:

$$\gamma(0) = 1, \quad \gamma(1) = \frac{1}{2}$$

Can you figure the combination of IRF coefficients that I used to generate this ACF? Are the IRF coefficients <u>identified</u> from the ACF (that is; is there a unique combination of θ_0 and θ_1 that gives the ACF above?)