

Homework 1

DATA604 Simulation and Modeling

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1.1

Name several entities, attributes, activities, events, and state variables for the following systems.

(a) A cafeteria

Entities

- Serving Line
- Food Servers
- Tables

Attributes

- Number of Food Servers
- Number of seats per table
- Rate of serving for Food Servers
- Time range for eating the meal

Activities

- Waiting in line
- Being served by a Food Server
- Waiting for a table to eat
- Eating at a table

Events

- Arrival of new person in serving line to be served
- Person leaving serving line
- Person waiting for seat at table to eat
- Person finishing eating and leaving table

State Variables

- Number of people eating at tables
- Number of people waiting in line to be served

(b) A grocery store

Entities

- Checkout lanes

Attributes

- Max number of items allowed in checkout lane
- Rate of checkout for cashier

Activities

- Customer shopping in the grocery store
- Customer checking out (paying for goods)

Events

- Arrival of customer at grocery store
- Arrival of customer at checkout lane
- Customer completing checkout
- Customer departing store without purchasing anything

State Variables

- Number of customers in grocery store
- Number of customers in checkout lane lines

(c) A laundromat

Entities

- Washing machines
- Drying machines

Attributes

- Washing machine run time
- Drying machine run time
- Ratio of washing machine to drying machine capacity

Activities

- Washing clothes
- Drying clothes
- Loading washing machine
- Transferring from washing to drying machine
- Unloading from drying machine

Events

- Washing machine cycle starts
- Washing machine cycle stops
- Dryer cycle starts
- Dryer cycle stops

State Variables

- Number of busy washing machines
- Number of busy dryers

(d) A fast-food restaurant

Entities

- Cashiers
- Back-cooks (i.e. burger flippers)
- Fryers

Attributes

- Burgers per burger flipper
- Orders of fries per Fryer
- Cashier busy or not

Activities

- Cooking a burger
- Making french fries
- Cashier taking order, accepting payment

Events

- Order in
- Order ready for pickup
- French fries done cooking

State Variables

- Number of orders pending
- Number of burgers being cooked
- Orders of french fries cooked/ready for serving
- Number of burgers being ready for serving

(e) A hospital emergency room

Entities

- Doctors
- Beds
- Patients
- Admitting staff

Attributes

- Patients per Doctor

Activities

- Patient admitted
- Doctor take care of patient
- Patient discharged

Events

- Patient arrives
- Patient admitted
- Doctor discharges patient

State Variables

- Beds empty
- Patients awaiting admission
- Patients awaiting discharge

(f) A taxicab company with 10 taxis

Entities

- Taxis
- Dispatcher
- Customers

Attributes

- Taxi has customer
- Taxi enroute to customer
- Customer waiting for taxi

Activities

- Enroute to customer
- Transporting customer

Events

- Picking up customer
- Dropping off customer

State Variables

- Taxis with customers
- Customers waiting for available taxi

(g) An automobile assembly line

Entities

- Parts
- Assembly machines
- Workers

Attributes

- Parts inventory
- Assembly machine rate of production
- Worker rate of production

Activities

- Machine assembling car
- Worker assembling car
- Staging parts for use by Machine or Worker

Events

- Car assembly started
- Car assembly completed
- Parts depleted
- Car assembly by machine X completed
- Car assembly by worker Y completed

State Variables

- Cars on assembly line
- Parts inventory level
- Workers out sick/vacation
- Machines broken down

2.1

Consider the following continuously operating job shop. Interarrival times of jobs are distributed as follows:

Time Between Arrivals (hours)	Probability
0	0.23
1	0.37
2	0.28
3	0.12

Processing times for jobs are normally distributed, with mean 50 minutes, and standard deviation 8 minutes. Construct a simulation table and perform a simulation for 10 new customers. Assume that, when the simulation starts, there is one job being processed (scheduled to be completed in 25 minutes) and there is one job with a 50-minute processing time in the queue.

```
# Create a data frame of the pre-existing jobs
existingJobs <- data.frame(customer=c(-2, -1),
                          iaHrs=c(0,0),
                          iaMins=c(0,0),
                          arrivalMins=c(0,0),
                          svcTimeMins=c(25, 50),
                          timeSvcBegin=c(0, 25),
                          queueWaitMins=c(0,25),
                          timeSvcEnd=c(25, 75),
                          timeInSystem=c(25,75))

# Create a data frame of the new customers and their jobs
newJobs <- data.frame(customer=seq(1, 10),
                     iaHrs=c(0, sample(seq(0, 3),
                                       size=9,
                                       prob=c(.23, .37, .28, .12),
                                       replace=TRUE))),
                     iaMins=rep(NA, 10),
                     arrivalMins=rep(0, 10),
                     svcTimeMins=rnorm(10, mean=50, sd=8),
                     timeSvcBegin=rep(0, 10),
                     queueWaitMins=rep(0, 10),
                     timeSvcEnd=rep(0, 10),
                     timeInSystem=rep(0, 10))

# Convert from interarrival hours to minutes and
# determine overall arrival times
newJobs$iaMins <- newJobs$iaHrs * 60
newJobs$arrivalMins <- cumsum(newJobs$iaMins)
# Join the existing and new jobs into one table
simTable <- rbind(existingJobs, newJobs)
# Loop over the rows the compute the various activity and clock times
for(i in seq(3, nrow(simTable)))
{
  simTable[i,]$timeSvcBegin <- max(simTable[i,]$arrivalMins, simTable[i-1,]$timeSvcEnd)
  simTable[i,]$queueWaitMins <- simTable[i,]$timeSvcBegin - simTable[i,]$arrivalMins
}
```

```

simTable[i,]$timeSvcEnd <- simTable[i,]$timeSvcBegin + simTable[i,]$svcTimeMins
simTable[i,]$timeInSystem <- simTable[i,]$timeSvcEnd - simTable[i,]$arrivalMins
}
# Show the table
kable(simTable)

```

customer	iaHrs	iaMins	arrivalMins	svcTimeMins	timeSvcBegin	queueWaitMins	timeSvcEnd	timeInSystem
-2	0	0	0	25.00000	0.0000	0.00000	25.0000	25.00000
-1	0	0	0	50.00000	25.0000	25.00000	75.0000	75.00000
1	0	0	0	49.88872	75.0000	75.00000	124.8887	124.88872
2	3	180	180	44.90948	180.0000	0.00000	224.9095	44.90948
3	0	0	180	57.70883	224.9095	44.90948	282.6183	102.61831
4	1	60	240	49.44490	282.6183	42.61831	332.0632	92.06321
5	1	60	300	55.89174	332.0632	32.06321	387.9550	87.95496
6	2	120	420	46.70007	420.0000	0.00000	466.7001	46.70007
7	1	60	480	44.79027	480.0000	0.00000	524.7903	44.79027
8	1	60	540	64.67686	540.0000	0.00000	604.6769	64.67686
9	0	0	540	35.05879	604.6769	64.67686	639.7356	99.73565
10	2	120	660	50.96674	660.0000	0.00000	710.9667	50.96674

(a) What was the average time in the queue for the 10 new jobs? The average time in the queue for the 10 new jobs is computed below:

```
mean(simTable[seq(3, 12),]$queueWaitMins)
```

```
## [1] 25.92679
```

(b) What was the average processing time of the 10 new jobs? The average processing time is computed below:

```
mean(newJobs$svcTimeMins)
```

```
## [1] 50.00364
```

(c) What was the maximum time in the system for the 10 new jobs? The maximum time in the system for the 10 new jobs is computed below:

```
max(simTable[seq(3, 12),]$timeInSystem)
```

```
## [1] 124.8887
```


2.2

A baker is trying to figure out how many dozens of bagels to bake each day. The probability distribution of the number of bagel customers is as follows:

Customer/Day	8	10	12	14
Probability	0.35	0.30	0.25	0.10

Customers order 1,2,3 or 4 dozen bagels according to the following probability distribution:

Dozen Ordered/Customer	1	2	3	4
Probability	0.4	0.3	0.2	0.1

Bagels sell for \$8.40 per dozen. They cost \$5.80 per dozen to make. All bagels not sold at the end of the day are sold at half price to a local grocery store. Based on 5 days of simulation, how many dozen (to the nearest 5 dozen) bagels should be baked each day?

```
# Function to define a simulation at a specified
# level of dozens of bagels produced.
bakersProfit <- function(bagelsMade)
{
  simDays <- 5
  revPerDoz <- 8.40
  costPerDoz <- 5.80
  simTable <- data.frame(day=seq(1, simDays),
                        customers=sample(c(8,10,12,14),
                                         size=simDays,
                                         prob=c(0.35, 0.30, 0.25, 0.10),
                                         replace=TRUE),
                        dozenOrdered=rep(NA, simDays),
                        revenue=rep(NA, simDays),
                        lostProfit=rep(NA, simDays),
                        salvage=rep(NA, simDays),
                        dailyCost=rep(NA, simDays),
                        dailyProfit=rep(NA, simDays))

  for(i in seq(1, nrow(simTable)))
  {
    bagelsOrdered <- sample(c(1,2,3,4),
                          size=simTable[i,]$customers,
                          prob=c(0.4, 0.3, 0.2, 0.1),
                          replace=TRUE)

    simTable[i,]$dozenOrdered <- sum(bagelsOrdered)

    simTable[i,]$revenue <- min(simTable[i,]$dozenOrdered, bagelsMade) * revPerDoz
    simTable[i,]$lostProfit <- max(simTable[i,]$dozenOrdered - bagelsMade, 0) * (revPerDoz - costPerDoz)
    simTable[i,]$salvage <- max(bagelsMade - simTable[i,]$dozenOrdered, 0) * (revPerDoz / 2)
```

```

simTable[i,]$dailyCost <- bagelsMade * costPerDoz
simTable[i,]$dailyProfit <- simTable[i,]$revenue + simTable[i,]$salvage - simTable[i,]$dailyCost
}

return(simTable)
}
# Loop over a range of dozens of bagels (0, 5, 10, etc)
dozens <- seq(0, 30, by=5)
profitTable <- data.frame(dozPerDay=c(), fiveDayProfit=c())
for(d in dozens)
{
  # Run the simulation for the given level of production
  simTable <- bakersProfit(d)
  profitTable <- rbind(profitTable, cbind(dozPerDay=d, fiveDayProfit=sum(simTable$dailyProfit)))
  #print(paste(d, " dozen/day: 5 day profit is ", profitTable[profitTable$dozPerDay==d,]$fiveDayProfit, ".", sep=""))
}

```

The following table shows the profit associated with various levels of production:

dozPerDay	fiveDayProfit
0	0.0
5	65.0
10	130.0
15	186.6
20	251.6
25	199.0
30	222.0

In order to maximize profit, the baker should bake 20 dozen bagels/day as shown in the table above. The following table shows the details of the simulation related to the maximum profit shown above:

day	customers	dozenOrdered	revenue	lostProfit	salvage	dailyCost	dailyProfit
1	10	19	159.6	0.0	4.2	116	47.8
2	12	28	168.0	20.8	0.0	116	52.0
3	10	15	126.0	0.0	21.0	116	31.0
4	12	26	168.0	15.6	0.0	116	52.0
5	8	19	159.6	0.0	4.2	116	47.8

2.4

Smalltown Taxi operates one vehicle during the 9:00 A.M. to 5:00 P.M. period. Currently, consideration is being given to the addition of a second vehicle to the fleet. The demand for taxis follows the distribution shown:

Time Between Calls (minutes)	15	20	25	30	35
Probability	0.14	0.22	0.43	0.17	0.04

The distribution of time to complete a service is as follows:

Service Time (minutes)	5	15	25	35	45
Probability	0.12	0.35	0.43	0.06	0.04

Simulate 5 individual days of operation of the current system and of the system with an additional taxicab. Compare the two systems with respect to the waiting times of the customers and any other measures that might shed light on the situation.

One Taxi Simulation

```
# Function to wrap a single taxi cab on a single day
singleTaxiDailyCalls <- function(callsPerDay, maxDailyMinutes)
{
  # Create a data frame of the new customers and their jobs
  simTable <- data.frame(customer=seq(1, callsPerDay),
                        iaMins=sample(seq(15, 35, by=5),
                                     size=callsPerDay,
                                     prob=c(0.14, 0.22, 0.43, 0.17, 0.04),
                                     replace=TRUE),
                        arrivalMins=rep(0, callsPerDay),
                        svcTimeMins=sample(seq(5, 45, by=10),
                                           size=callsPerDay,
                                           prob=c(0.12, 0.35, 0.43, 0.06, 0.04),
                                           replace=TRUE),
                        timeSvcBegin=rep(0, callsPerDay),
                        queueWaitMins=rep(0, callsPerDay),
                        timeSvcEnd=rep(0, callsPerDay),
                        timeInSystem=rep(0, callsPerDay))

  # Determine overall arrival times
  simTable$arrivalMins <- cumsum(simTable$iaMins)
  # Loop over the rows the compute the various activity and clock times
```

```

for(i in seq(1, nrow(simTable)))
{
  if(i == 1)
  {
    simTable[i,]$timeSvcBegin <- simTable[i,]$arrivalMins

  }
  else
  {
    simTable[i,]$timeSvcBegin <- max(simTable[i,]$arrivalMins, simTable[i-1,]$timeSvcEnd)
  }
  simTable[i,]$queueWaitMins <- simTable[i,]$timeSvcBegin - simTable[i,]$arrivalMins
  simTable[i,]$timeSvcEnd <- simTable[i,]$timeSvcBegin + simTable[i,]$svcTimeMins
  simTable[i,]$timeInSystem <- simTable[i,]$timeSvcEnd - simTable[i,]$arrivalMins
}
# Convert queue wait of zero to NA so we can aggregate only those who actually waited.
simTable$queueWaitMins[simTable$queueWaitMins == 0] <- NA
# subset to the max daily minutes in the business day
simTable <- simTable[simTable$arrivalMins <= maxDailyMinutes,]

return(simTable)
}

```

The following code segment executes the **single** taxi simulation over 5 days and aggregates the results.

```

# Run the single taxi simulation over 5 days
daysToSimulate <- 5
multiDaySimTable <- data.frame()
for(i in seq(1, daysToSimulate))
{
  oneDayOneTaxi <- singleTaxiDailyCalls(32, 480)
  multiDaySimTableOneTaxi <- rbind(multiDaySimTable, cbind(day=i, oneDayOneTaxi))
}
kable(multiDaySimTableOneTaxi)

```

day	customer	iaMins	arrivalMins	svcTimeMins	timeSvcBegin	queueWaitMins	timeSvcEnd	timeInSystem
5	1	30	30	15	30	NA	45	15
5	2	25	55	25	55	NA	80	25
5	3	30	85	5	85	NA	90	5
5	4	30	115	25	115	NA	140	25
5	5	25	140	15	140	NA	155	15
5	6	25	165	5	165	NA	170	5
5	7	25	190	15	190	NA	205	15

day	customer	iaMins	arrivalMins	svcTimeMins	timeSvcBegin	queueWaitMins	timeSvcEnd	timeInSystem
5	8	25	215	15	215	NA	230	15
5	9	25	240	5	240	NA	245	5
5	10	20	260	25	260	NA	285	25
5	11	25	285	15	285	NA	300	15
5	12	25	310	25	310	NA	335	25
5	13	35	345	35	345	NA	380	35
5	14	25	370	15	380	10	395	25
5	15	30	400	25	400	NA	425	25
5	16	25	425	35	425	NA	460	35
5	17	30	455	25	460	5	485	30
5	18	25	480	5	485	5	490	10

```
# Show the table
kable(summary(multiDaySimTableOneTaxi[,c(2,7,9)]))
```

customer	queueWaitMins	timeInSystem
Min. : 1.00	Min. : 5.000	Min. : 5.00
1st Qu.: 5.25	1st Qu.: 5.000	1st Qu.:15.00
Median : 9.50	Median : 5.000	Median :20.00
Mean : 9.50	Mean : 6.667	Mean :19.44
3rd Qu.:13.75	3rd Qu.: 7.500	3rd Qu.:25.00
Max. :18.00	Max. :10.000	Max. :35.00
NA	NA's :15	NA

2 Taxi Simulation

Now lets develop the 2 taxi simulation.

```
# Function to wrap a duel taxi cab company on a single day
duelTaxiDailyCalls <- function(callsPerDay, maxDailyMinutes)
{
  # Create a data frame of the new customers and their jobs
  simTable <- data.frame(customer=seq(1, callsPerDay),
    iaMins=sample(seq(15, 35, by=5),
      size=callsPerDay,
      prob=c(0.14, 0.22, 0.43, 0.17, 0.04),
      replace=TRUE),
    arrivalMins=rep(0, callsPerDay),
    tc1Avail=rep(0, callsPerDay),
    tc2Avail=rep(0, callsPerDay),
```

```

    taxiChosen=rep(0, callsPerDay),
    svcTimeMins=sample(seq(5, 45, by=10),
                      size=callsPerDay,
                      prob=c(0.12, 0.35, 0.43, 0.06, 0.04),
                      replace=TRUE),
    timeSvcBegin=rep(0, callsPerDay),
    timeSvcEndTc1=rep(0, callsPerDay),
    timeSvcEndTc2=rep(0, callsPerDay),
    queueWaitMins=rep(0, callsPerDay),
    timeInSystem=rep(0, callsPerDay))

# Determine overall arrival times
simTable$arrivalMins <- cumsum(simTable$iaMins)
# Loop over the rows to compute the various activity and clock times
for(i in seq(1, nrow(simTable)))
{
  if(i == 1)
  {
    simTable[i,]$timeSvcBegin <- simTable[i,]$arrivalMins
    simTable[i,]$taxiChosen <- 1
  }
  else
  {
    # Determine availability
    simTable[i,]$tc1Avail <- max(simTable[seq(1, i),]$timeSvcEndTc1)
    simTable[i,]$tc2Avail <- max(simTable[seq(1, i),]$timeSvcEndTc2)
    # Select taxi
    simTable[i,]$taxiChosen <- if (simTable[i,]$tc1Avail <= simTable[i,]$arrivalMins ||
                                simTable[i,]$tc1Avail <= simTable[i,]$tc2Avail) 1 else 2
    # Determine service start based on selected taxi
    simTable[i,]$timeSvcBegin <- if(simTable[i,]$taxiChosen == 1) max(simTable[i,]$arrivalMins,
                                                                    simTable[i-1,]$tc1Avail) else max(simTable[i,]$arrivalMins,
                                                                    simTable[i-1,]$tc2Avail)
  }
  simTable[i,]$timeSvcEndTc1 <- if(simTable[i,]$taxiChosen == 1) simTable[i,]$timeSvcBegin + simTable[i,]$svcTimeMins else 0
  simTable[i,]$timeSvcEndTc2 <- if(simTable[i,]$taxiChosen == 2) simTable[i,]$timeSvcBegin + simTable[i,]$svcTimeMins else 0
  simTable[i,]$queueWaitMins <- simTable[i,]$timeSvcBegin - simTable[i,]$arrivalMins
  simTable[i,]$timeInSystem <- max(simTable[i,]$timeSvcEndTc1, simTable[i,]$timeSvcEndTc2) - simTable[i,]$arrivalMins
}
# Convert queue wait of zero to NA so we can aggregate only those who actually waited.
simTable$queueWaitMins[simTable$queueWaitMins == 0] <- NA
# subset to the max daily minutes in the business day
simTable <- simTable[simTable$arrivalMins <= maxDailyMinutes,]

```

```

return(simTable)
}

```

The following table shows the execution of a single day for the **two** taxi simulation.

customer	iaMins	arrivalMins	tc1Avail	tc2Avail	taxiChosen	svcTimeMins	timeSvcBegin	timeSvcEndTc1	timeSvcEndTc2	queueWaitMins	timeInSystem
1	25	25	0	0	1	15	25	40	0	NA	15
2	30	55	40	0	1	25	55	80	0	NA	25
3	20	75	80	0	2	5	75	0	80	NA	5
4	25	100	80	80	1	25	100	125	0	NA	25
5	25	125	125	80	1	5	125	130	0	NA	5
6	15	140	130	80	1	15	140	155	0	NA	15
7	20	160	155	80	1	25	160	185	0	NA	25
8	15	175	185	80	2	25	175	0	200	NA	25
9	25	200	185	200	1	25	200	225	0	NA	25
10	25	225	225	200	1	15	225	240	0	NA	15
11	20	245	240	200	1	15	245	260	0	NA	15
12	15	260	260	200	1	15	260	275	0	NA	15
13	25	285	275	200	1	5	285	290	0	NA	5
14	20	305	290	200	1	25	305	330	0	NA	25
15	20	325	330	200	2	45	325	0	370	NA	45
16	35	360	330	370	1	25	360	385	0	NA	25
17	20	380	385	370	2	45	380	0	425	NA	45
18	25	405	385	425	1	15	405	420	0	NA	15
19	25	430	420	425	1	25	430	455	0	NA	25
20	25	455	455	425	1	15	455	470	0	NA	15
21	15	470	470	425	1	25	470	495	0	NA	25

The following code segment executes the two taxi simulation over 5 days and aggregates the results.

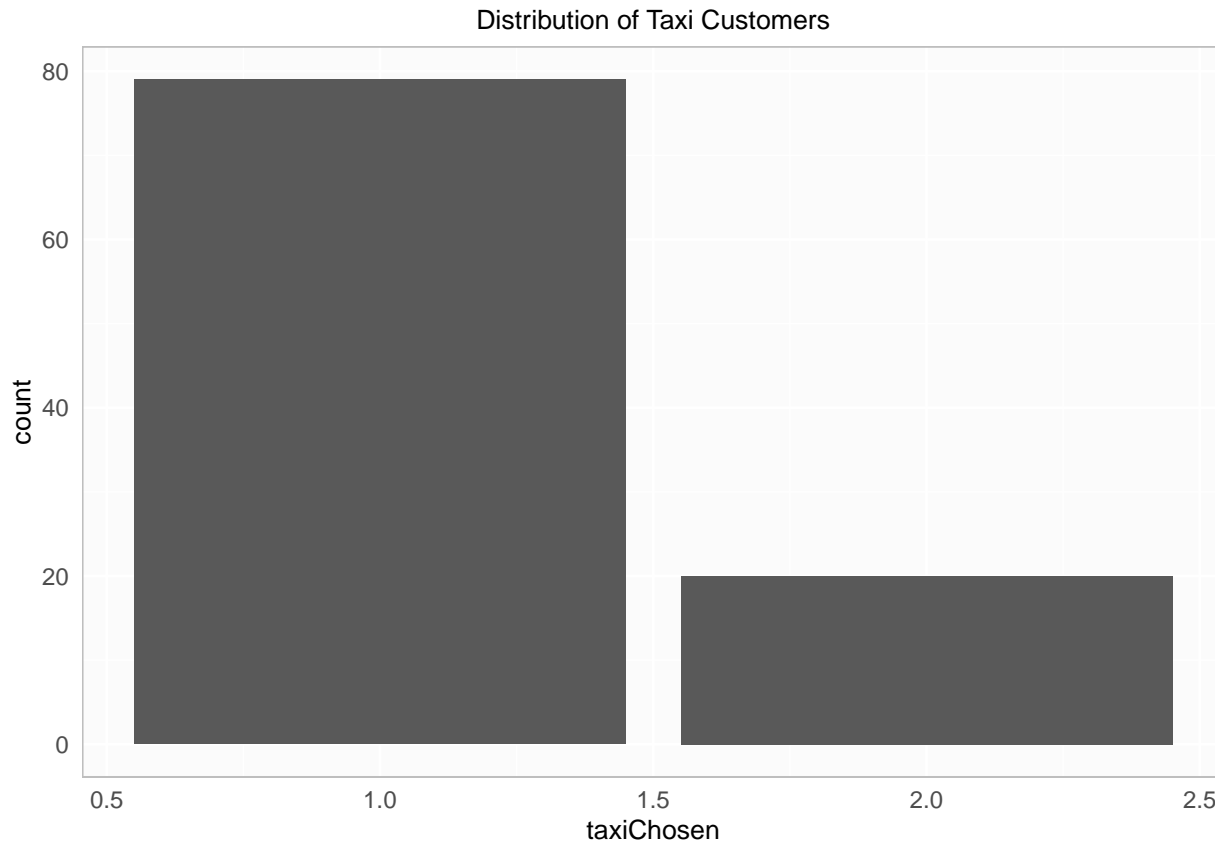
```

# Run the two taxi simulation over 5 days
daysToSimulate <- 5
multiDaySimTable <- data.frame()
for(i in seq(1, daysToSimulate))
{
  oneDayTwoTaxi <- duelTaxiDailyCalls(32, 480)
  multiDaySimTable <- rbind(multiDaySimTable, cbind(day=i, oneDayTwoTaxi))
}
#kable(multiDaySimTable)
# Show the table
kable(summary(multiDaySimTable[,c(2,12,13)]))

```


customer	queueWaitMins	timeInSystem
Min. : 1.00	Min. : NA	Min. : 5.00
1st Qu.: 5.50	1st Qu.: NA	1st Qu.:15.00
Median :10.00	Median : NA	Median :25.00
Mean :10.41	Mean :NaN	Mean :18.84
3rd Qu.:15.00	3rd Qu.: NA	3rd Qu.:25.00
Max. :21.00	Max. : NA	Max. :45.00
NA	NA's :99	NA

As can be seen in the column chart below, Taxi 1 has a much larger number of customers, but Taxi 2 does pick up approximately 20% of the customers.



2.5

The random variables X, Y, and Z are distributed as follows:

$$X \sim N(\mu = 100, \sigma^2 = 100)$$

$$Y \sim N(\mu = 300, \sigma^2 = 225)$$

$$Z \sim N(\mu = 40, \sigma^2 = 64)$$

Simulate 50 values of the random variable

$$W = \frac{X + Y}{Z}$$

Prepare a histogram of the resulting values, using class intervals of width equal to 3.

```
fxX <- function (n)
{
  return (rnorm(n, 100, 10))
}

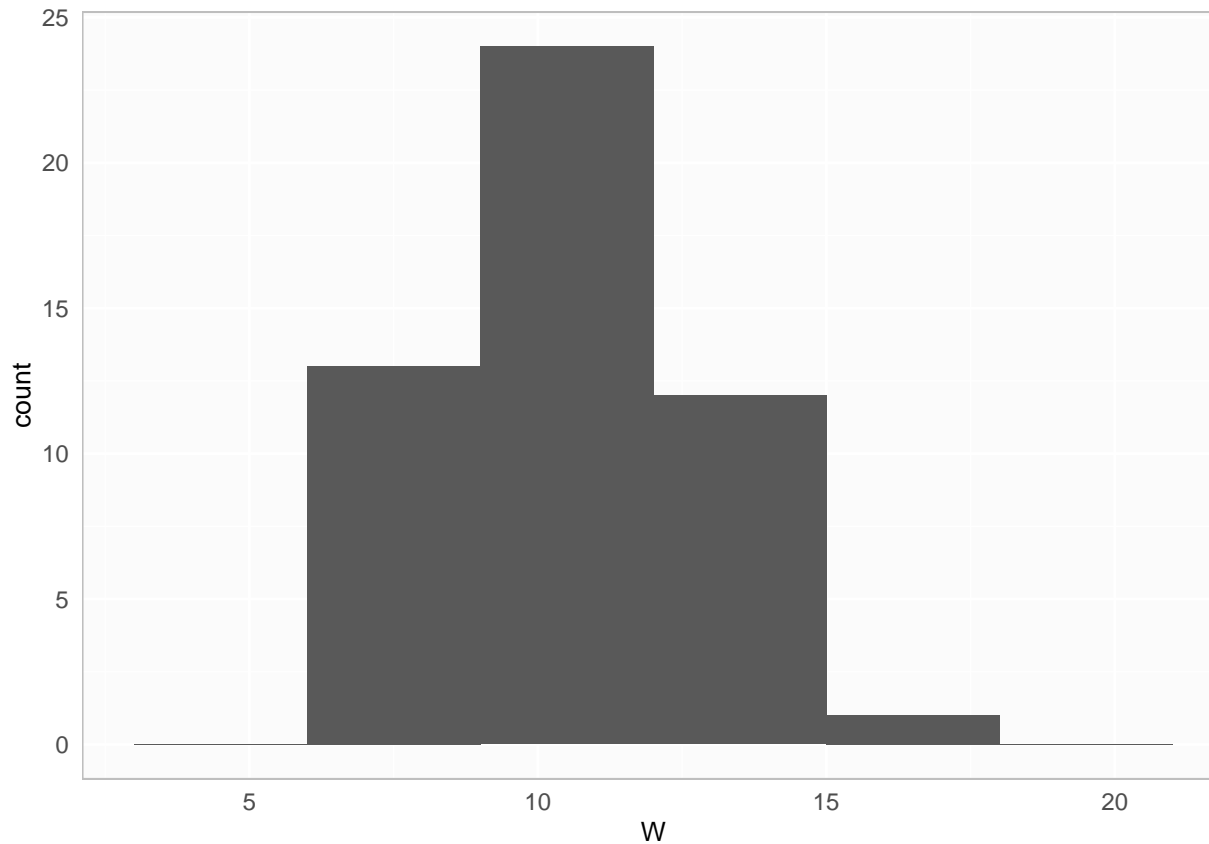
fxY <- function(n)
{
  return (rnorm(n, 300, 15))
}

fxZ <- function(n)
{
  return (rnorm(n, 40, 8))
}

n <- 50
dfVars <- data.frame(X=fxX(n), Y=fxY(n), Z= fxZ(n))
dfVars$W <- ( dfVars$X + dfVars$Y ) / dfVars$Z
```

The first 5 resulting random variables and the values of W are shown in the table below.

X	Y	Z	W
91.33994	309.2501	37.98667	10.545542
91.78661	300.2354	42.11673	9.307989
97.82386	272.7642	38.84069	9.541234
102.98633	305.7091	33.43615	12.223159
97.20652	294.7348	47.86196	8.188995
96.88976	292.6918	39.76631	9.796774



2.7

Estimate, by simulation, the average number of lost sales per week for an inventory system that functions as follows:

- Whenever the inventory level falls to or below 10 units, an order is placed. Only one order can be outstanding at a time.
- The size of each order is equal to $20 - I$, where I is the inventory level when the order is placed.
- If a demand occurs during a period when the inventory level is zero, the sale is lost.
- Daily demand is normally distributed, with a mean of 5 units and a standard deviation of 1.5 units. (Round off decimals to the closest integer during the simulation and, if a negative value results, give it a demand of zero.)
- Lead time is distributed uniformly between zero and 5 days - integers only.
- The simulation will start with 18 units in inventory.

- (g) For simplicity, assume that orders are placed at the close of the business day and received after the lead time has occurred. Thus, if lead time is one day, the order is available for distribution on the morning of the second day of business following the placement of the order.
- (h) Let the simulation run for 5 weeks.

```
inventorySim <- function(daysToSimulate, daysPerWeek)
{
  simTable <- data.frame(day=seq(0, daysToSimulate),
    #dayInCycle=c(daysPerWeek, rep(seq(1, daysPerWeek), daysToSimulate / daysPerWeek)),
    beginInv=c(NA, rep(NA, daysToSimulate)),
    demand=c(NA, round(rnorm(daysToSimulate, mean=5, sd=1.5))),
    endInv=c(18, rep(NA, daysToSimulate)),
    lostSales=rep(NA, daysToSimulate + 1),
    #shortage=c(0, rep(NA, daysToSimulate)),
    pendingOrder=c(0, rep(0, daysToSimulate)),
    leadTime=c(NA, rep(NA, daysToSimulate)),
    orderArriveDays=c(0, rep(0, daysToSimulate)))

  # Loop over the rows the compute the various activity and clock times
  for(i in seq(1, nrow(simTable)))
  {
    if(i == 1)
    {
      # Do nothing on first row (zeroth)
    }
    else
    {
      # Are any orders arriving today?
      pending0 <- 0
      if(simTable[i-1,]$orderArriveDays <= 1)
      {
        pending0 <- simTable[i-1,]$pendingOrder
        simTable[i,]$pendingOrder <- 0
      }
      # Adjust beginning inventory based on prior ending plus arriving orders
      simTable[i,]$beginInv <- simTable[i-1,]$endInv + pending0
      if(simTable[i,]$demand > 0 && simTable[i,]$beginInv == 0)
      {
        simTable[i,]$lostSales <- simTable[i,]$demand
      }
      # Adjust ending inventory
      endI <- simTable[i,]$beginInv - simTable[i,]$demand #- simTable[i-1,]$shortage
      simTable[i,]$endInv <- max(endI, 0)

      # Ordering
    }
  }
}
```

```

if(simTable[i,]$endInv <= 10 && simTable[i-1,]$orderArriveDays <= 1)
{
  # New Order
  simTable[i,]$pendingOrder <- 20 - simTable[i,]$endInv
  simTable[i,]$leadTime <- round(runif(1, 0, 5))
  simTable[i,]$orderArriveDays <- simTable[i,]$leadTime
}
else
{
  # Adjust arrival days for pending orders
  simTable[i,]$orderArriveDays <- if(simTable[i-1,]$orderArriveDays > 0) simTable[i-1,]$orderArriveDays - 1 else 0
  if(simTable[i,]$orderArriveDays > 0)
  {
    simTable[i,]$pendingOrder <- simTable[i-1,]$pendingOrder
  }
}
}
}

return(simTable)
}

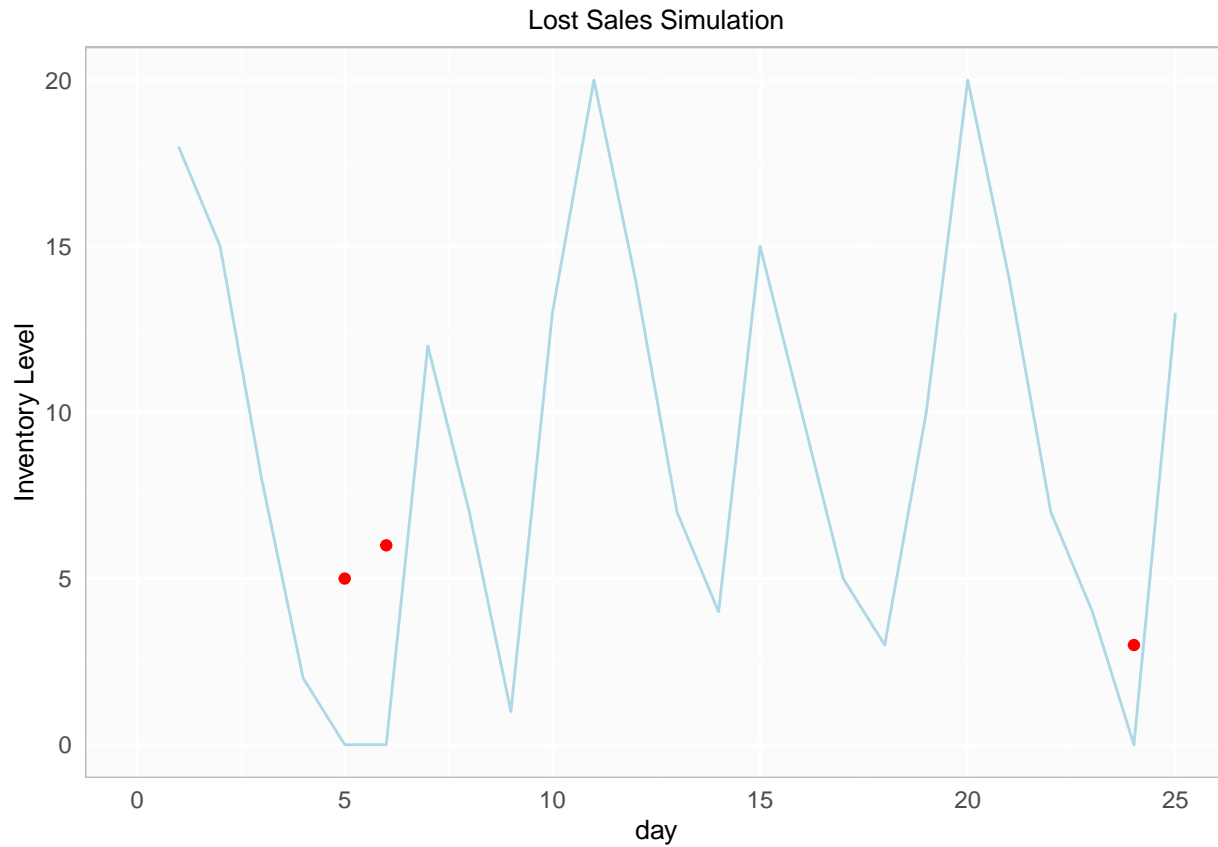
# Days based on 5 biz days/wk for n weeks
weeksToSim <- 5
daysToSim <- 5 * weeksToSim
inventorySimTable <- inventorySim(daysToSim)

```

day	beginInv	demand	endInv	lostSales	pendingOrder	leadTime	orderArriveDays
0	NA	NA	18	NA	0	NA	0
1	18	3	15	NA	0	NA	0
2	15	7	8	NA	12	5	5
3	8	6	2	NA	12	NA	4
4	2	5	0	NA	12	NA	3
5	0	5	0	5	12	NA	2
6	0	6	0	6	12	NA	1
7	12	5	7	NA	13	3	3
8	7	6	1	NA	13	NA	2
9	1	4	0	NA	13	NA	1
10	13	6	7	NA	13	0	0
11	20	6	14	NA	0	NA	0
12	14	7	7	NA	13	3	3
13	7	3	4	NA	13	NA	2
14	4	2	2	NA	13	NA	1
15	15	5	10	NA	10	4	4

day	beginInv	demand	endInv	lostSales	pendingOrder	leadTime	orderArriveDays
16	10	5	5	NA	10	NA	3
17	5	2	3	NA	10	NA	2
18	3	3	0	NA	10	NA	1
19	10	6	4	NA	16	1	1
20	20	6	14	NA	0	NA	0
21	14	7	7	NA	13	4	4
22	7	3	4	NA	13	NA	3
23	4	5	0	NA	13	NA	2
24	0	3	0	3	13	NA	1
25	13	4	9	NA	11	1	1

Based on the previously shown simulation data, the average lost sales per week are 2.8. The following chart shows lost sales as red points and the blue line represents the inventory levels during the simulation.



2.8

An elevator in a manufacturing plant carries exactly 400 kilograms of material. There are three kinds of material packaged in boxes that arrive for a ride on the elevator. These materials and their distributions of time between arrivals are as follows:

Material	Weight (kg)	Interarrival Time (minutes)
A	200	5 +- 2 (uniform)
B	100	6 (constant)
C	50	$P(2) = 0.33, P(3) = 0.67$

It takes the elevator 1 minute to go up to the second floor, 2 minutes to unload, and 1 minute to return to the first floor. The elevator does not leave the first floor unless it has a full load. Simulate 1 hour of operation of the system. What is the average transit time for a box of material A (time from its arrival until it is unloaded)? What is the average waiting time for a box of material B? How many boxes of material C made the trip in 1 hour?

```
# Function to wrap a single taxi cab on a single day
materialElevatorSim <- function(n, simMinutes, elevatorCapacity, travelUpMin, unloadMin, travelDownMin)
{
  # Material A interarrivals
  matAIA <- data.frame(mat=rep("A", n),
                      weight=rep(200, n),
                      matIA=sample(c(3, 7),
                                   size=n,
                                   prob=c(0.50, 0.50),
                                   replace=TRUE))
  matAIA$arrivalTime <- cumsum(matAIA$matIA)
  # Material B interarrivals
  matBIA <- data.frame(mat=rep("B", n),
                      weight=rep(100, n),
                      matIA=rep(6, n))
  matBIA$arrivalTime <- cumsum(matBIA$matIA)
  # Material C interarrivals
  matCIA <- data.frame(mat=rep("C", n),
                      weight=rep(50, n),
                      matIA=sample(c(2, 3),
                                   size=n,
                                   prob=c(0.33, 0.67),
                                   replace=TRUE))
  matCIA$arrivalTime <- cumsum(matCIA$matIA)

  materialArrivals <- rbind(matAIA, matBIA, matCIA)
  materialArrivals <- materialArrivals[with(materialArrivals, order(arrivalTime)),]

  simTable <- materialArrivals
  simTable$cumWeight <- rep(0, nrow(simTable))
```

```

simTable$waiting <- rep(1, nrow(simTable))
simTable$svcBeginTime <- rep(0, nrow(simTable))
simTable$svcEndTime <- rep(0, nrow(simTable))
simTable$waitTimeMins <- rep(0, nrow(simTable))
simTable$transitTime <- rep(0, nrow(simTable))

# Loop over the rows the compute the various activity and clock times
for(i in seq(1, nrow(simTable)))
{
  # Compute cumulative waiting weight
  if(i == 1)
  {
    simTable[i,]$cumWeight <- simTable[i,]$weight
  }
  else
  {
    cumWaitingWeight <- 0
    for(j in seq(1,i))
    {
      if(simTable[j,]$waiting == 1)
      {
        cumWaitingWeight <- cumWaitingWeight + simTable[j,]$weight
      }
    }
    simTable[i,]$cumWeight <- cumWaitingWeight
  }

  # Determine riders this iteration
  if(simTable[i,]$cumWeight >= elevatorCapacity)
  {
    # Determine which items move on to elevator and which don't
    riders <- c()
    if(simTable[i,]$cumWeight == elevatorCapacity)
    {
      # Mark all waiters as riders
      for(j in seq(1, i))
      {
        if(simTable[j,]$waiting == 1)
        {
          riders <- c(riders, j)
        }
      }
    }
  }
  else

```



```

{
  # Find waiters that fit
  diff <- simTable[i,]$cumWeight - elevatorCapacity
  excluder <- 0
  for(j in seq(i, 1, by=-1))
  {
    if(simTable[j,]$waiting == 1 && simTable[j,]$weight == diff)
    {
      excluder <- j
      break;
    }
  }

  for(j in seq(1, i))
  {
    if(simTable[j,]$waiting == 1 && j != excluder)
    {
      riders <- c(riders, j)
    }
  }
}

for(k in riders)
{
  simTable[k,]$waiting <- 0
  simTable[k,]$svcBeginTime <- max(simTable[i,]$arrivalTime, simTable[i,]$svcEndTime + travelDownMin)
  simTable[k,]$svcEndTime <- simTable[k,]$svcBeginTime + travelUpMin + unloadMin
}
}

#simTable[i,]$waitTimeMins <-

# simTable[i,]$queueWaitMins <- simTable[i,]$timeSvcBegin - simTable[i,]$arrivalMins
# simTable[i,]$timeSvcEnd <- simTable[i,]$timeSvcBegin + simTable[i,]$svcTimeMins
# simTable[i,]$timeInSystem <- simTable[i,]$timeSvcEnd - simTable[i,]$arrivalMins
}

# Convert queue wait of zero to NA so we can aggregate only those who actually waited.
#simTable$queueWaitMins[simTable$queueWaitMins == 0] <- NA

simTable$transitTime <- simTable$svcEndTime - simTable$arrivalTime
simTable[simTable$transitTime < 0,]$transitTime <- NA

simTable$waitTimeMins <- simTable$svcBeginTime - simTable$arrivalTime

```

```

simTable[simTable$waitTimeMins < 0,]$waitTimeMins <- NA

# Subset to the max simulation minutes
simTable <- simTable[simTable$arrivalTime <= simMinutes,]

return(simTable)
}

materialSimTable <- materialElevatorSim(60, 60, 400, 1, 2, 1)
rownames(materialSimTable) <- seq(1, nrow(materialSimTable))

```

mat	weight	matIA	arrivalTime	cumWeight	waiting	svcBeginTime	svcEndTime	waitTimeMins	transitTime
C	50	2	2	50	0	7	10	5	8
C	50	2	4	100	0	7	10	3	6
B	100	6	6	200	0	7	10	1	4
C	50	2	6	250	0	14	17	8	11
A	200	7	7	450	0	7	10	0	3
C	50	2	8	100	0	14	17	6	9
C	50	3	11	150	0	18	21	7	10
B	100	6	12	250	0	14	17	2	5
A	200	7	14	450	0	14	17	0	3
C	50	3	14	100	0	18	21	4	7
A	200	3	17	300	0	18	21	1	4
C	50	3	17	350	0	24	27	7	10
B	100	6	18	450	0	18	21	0	3
A	200	3	20	250	0	24	27	4	7
C	50	3	20	300	0	24	27	4	7
C	50	2	22	350	0	29	32	7	10
B	100	6	24	450	0	24	27	0	3
C	50	2	24	100	0	29	32	5	8
A	200	7	27	300	0	29	32	2	5
C	50	3	27	350	0	29	32	2	5
C	50	2	29	400	0	29	32	0	3
B	100	6	30	100	0	35	38	5	8
C	50	3	32	150	0	35	38	3	6
A	200	7	34	350	0	35	38	1	4
C	50	3	35	400	0	35	38	0	3
B	100	6	36	100	0	41	44	5	8
C	50	3	38	150	0	41	44	3	6
A	200	7	41	350	0	41	44	0	3
C	50	3	41	400	0	41	44	0	3
B	100	6	42	100	0	47	50	5	8
A	200	3	44	300	0	47	50	3	6
C	50	3	44	350	0	47	50	3	6

mat	weight	matIA	arrivalTime	cumWeight	waiting	svcBeginTime	svcEndTime	waitTimeMins	transitTime
C	50	3	47	400	0	47	50	0	3
B	100	6	48	100	0	53	56	5	8
C	50	3	50	150	0	53	56	3	6
A	200	7	51	350	0	53	56	2	5
C	50	3	53	400	0	53	56	0	3
A	200	3	54	200	0	57	60	3	6
B	100	6	54	300	0	57	60	3	6
C	50	3	56	350	0	57	60	1	4
A	200	3	57	550	0	57	60	0	3
C	50	2	58	50	0	61	64	3	6
A	200	3	60	250	0	61	64	1	4
B	100	6	60	350	0	61	64	1	4

Average transit time for material A is computed below:

```
transitTimeA <- mean(materialSimTable[materialSimTable$mat == "A",]$transitTime, na.rm=TRUE)
transitTimeA
```

```
## [1] 4.416667
```

Average waiting time for material B is computed below:

```
waitTimeMinsB <- mean(materialSimTable[materialSimTable$mat == "B",]$waitTimeMins, na.rm=TRUE)
waitTimeMinsB
```

```
## [1] 2.7
```

Number of material C which received service is computed below:

```
countCserviced <- nrow(materialSimTable[materialSimTable$mat == "C" &
                                          materialSimTable$waiting == 0,])
countCserviced
```

```
## [1] 22
```