

Game Theory

EC 201: Principles of Microeconomics

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Winter 2020

Prologue

Agenda

Game Theory

- How can we model strategic interactions?
- Last topic for Midterm 2.

Midterm 2 Overview

- One week from today!

Game Theory

Game Theory

What is it?

A framework of analysis in which two or more players compete for payoffs.

A player's payoff depends on

1. Her decisions.
2. All other players' decisions.

Useful for understanding a variety of "games" people play.

- **Competitive scenarios:** Politics, collective bargaining, war, sports, *etc.*
- **Cooperative scenarios:** Resource management, public goods, *etc.*

What defines a game?

Who are the **players**?

- Pete Buttigieg and Bernie Sanders.
- Saudi Arabia, Venezuela, and other OPEC countries.
- USA and USSR.

What are the **strategies** available to each player?

- Focus on early primary states or Super Tuesday states.
- Collude (restrict oil output) or compete (increase oil output).
- Develop nuclear arsenal or not.

What are the **payoffs** associated with each combination of strategies?

- Democratic nominee or just another rank-and-file progressive.
- Split higher revenues or lower revenues.
- Continue to exist or total annihilation.

The Prisoner's Dilemma

		Prisoner 2	
		Deny	Accuse
Prisoner 1	Deny	Short Jail Sentence Short Jail Sentence	No Jail Sentence Longest Jail Sentence
	Accuse	Longest Jail Sentence No Jail Sentence	Long Jail Sentence Long Jail Sentence

Story

An interrogator keeps **Prisoner 1** and **Prisoner 2** in separate cells and interviews them separately.

Each prisoner can either deny the charges or accuse the other prisoner.

The Prisoner's Dilemma

		Prisoner 2	
		Deny	Accuse
Prisoner 1	Deny	-3, -3	0, -12
	Accuse	-12, 0	-8, -8

Analysis

Step 1: Quantify the payoffs.

Step 2: Find each prisoner's *best response* for each strategy the other could play.

- Both players have a *dominant strategy* to accuse.

Step 3: Find the *Nash equilibrium*.

- Equilibrium = (Accuse, Accuse)

Nash Equilibrium

Definition

A set of strategies such that no player has the incentive to deviate unilaterally from her chosen strategy.

Q: What logic supports the (Accuse, Accuse) equilibrium?

- Given that Prisoner 1 decided to Accuse, Prisoner 2 is better off having played Accuse.
- Given that Prisoner 2 decided to Accuse, Prisoner 1 is better off having played Accuse.

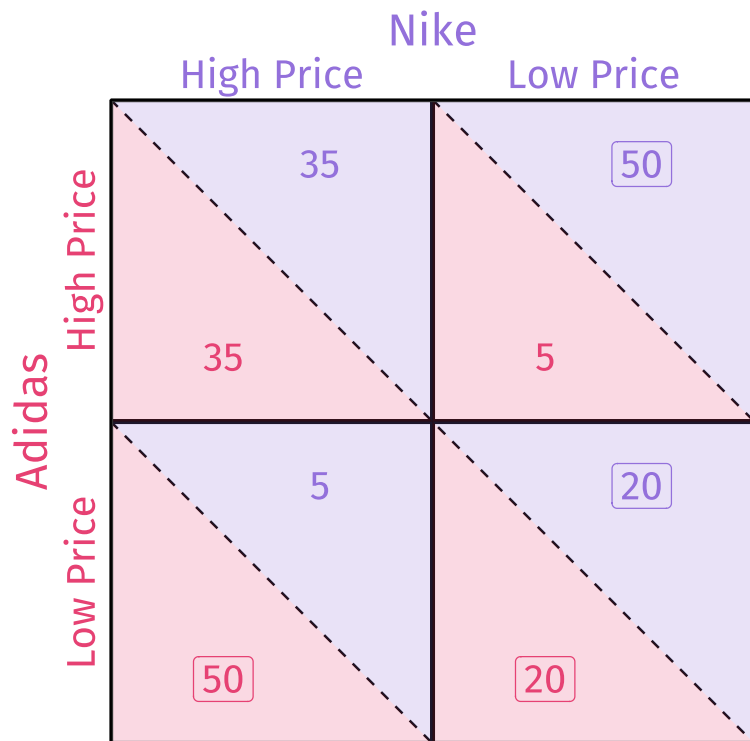
Q: Cooperate or defect?

- If everyone chooses to cooperate, then everyone will receive 2 bonus participation points.
- If everyone chooses to cooperate, but one person defects, then the person who defects will receive 6 bonus participation points and everyone else will receive 0 bonus participation points.
- If more than one person chooses to defect, then everyone will receive 0 bonus participation points.

A. Cooperate.

B. Defect.

Cournot Game



Q: What are the equilibrium strategies?

A: Equilibrium = (Low Price, Low Price), even though a bilateral deviation would be mutually beneficial!

- Another dilemma!

Prisoner's Dilemma 2.0

		Prisoner 2	
		Deny	Accuse
Prisoner 1	Deny	<div><div>-3</div><div>-3</div></div>	<div><div>-4</div><div>-12</div></div>
	Accuse	<div><div>-12</div><div>-4</div></div>	<div><div>-8</div><div>-8</div></div>

Story

An interrogator keeps **Prisoner 1** and **Prisoner 2** in separate cells and interviews them separately.

Each prisoner can either deny the charges or accuse the other prisoner.

Twist: Pre-heist contract.

Prisoner's Dilemma 2.0

		Prisoner 2	
		Deny	Accuse
Prisoner 1	Deny	<div><div>-3</div><div>-3</div></div>	<div><div>-4</div><div>-12</div></div>
	Accuse	<div><div>-12</div><div>-4</div></div>	<div><div>-8</div><div>-8</div></div>

Q: What are the equilibrium strategies?

A: There are two equilibria!

- (Accuse, Accuse)
- (Deny, Deny)

No longer a prisoner's dilemma.

- Now a coordination game.

Golden Balls - £100,000 Split Or Steal? 14/03/08



Split or Steal?

		Stephen	
		Split	Steal
Sarah	Split	50k 50k	100k 0
	Steal	0 100k	0 0

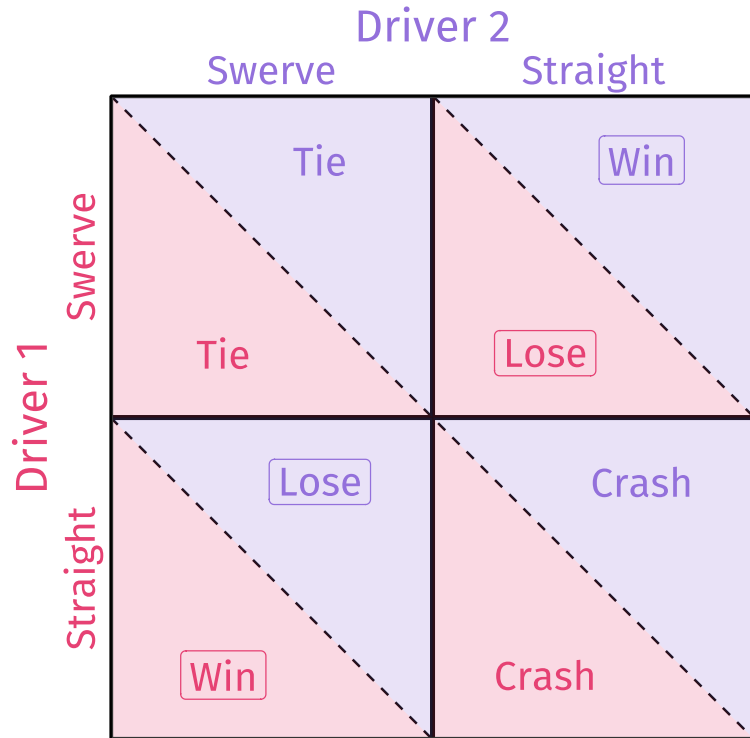
Observed outcome: (Steal, Split)

Q: What are the equilibrium strategies?

A: Three equilibria.

- (Steal, Split)
- (Split, Steal)
- (Steal, Steal)

Chicken



Q: What are the equilibrium strategies

A: Two equilibria.

- (Swerve, Straight)
- (Straight, Swerve)

Matching Pennies

		Roommate 2	
		Heads	Tails
Roommate 1	Heads	-1	$+1$
	Tails	$+1$	-1

Q: What are the equilibrium strategies

A: No mutual best responses!

- The only equilibrium strategy is to randomize.

Pascal's Wager

		Nature	
		God	No God
Pascal	Live as if God	Heaven	A
	Live as if No God	Hell	B

Analysis

$\Pr[G]$ = Probability that God exists.

Expected payoff from belief
 $= \Pr[G] \times \text{Heaven} + (1 - \Pr[G]) \times A$

Expected payoff from non-belief
 $= \Pr[G] \times \text{Hell} + (1 - \Pr[G]) \times B$

Pascal's Conclusion: If $\Pr[G] > 0$,
Heaven > Hell, and A is at least as
good as B, **belief dominates non-**
belief.

Brinkmanship

Brinkmanship

A **threat** provides a way to make a strategic move.

- But, you often only have one large threat available.

Some threats are so big that your opponent doesn't believe you'll carry it out.

- *e.g.*, "give me \$50 or I will kill myself" is unlikely to provide a credible threat.

Brinkmanship

A **credible threat** is costly enough, but not too costly, for opponents to take you seriously.

- *e.g.*, "give me \$50 or I will run across the road with my eyes closed" is more likely to induce other player to fork over money.
 - Probability of death is positive, but less than one.

Brinkmanship

Brinkmanship requires that process is at least partially out of control.

- *e.g.*, if the threatener hears traffic, then the threat wouldn't work.
 - All parties would know the threatener wouldn't run if there were a vehicle coming.
- The probability of the threat actually being carried out must be outside the threatener's control.

Brinkmanship

Cuban Missile Crisis

A striking example of successful brinkmanship?

- JFK took the world to the brink of nuclear war, and by doing so persuaded Khrushchev to remove missiles from Cuba.

But it seems as if game theory cannot explain this.

- Why didn't Khrushchev apply backward induction, figure out the final outcome, and decide not to start the whole process?

Brinkmanship

George Bush vs. Saddam Hussein

Bush played a similar strategy against Hussein: he gambled.

- **Preferred outcome:** Compliance or removal of Hussein as a response to Bush's threat.
- So...Bush lost.

Bush and JFK played essentially the same brinkmanship strategy.

- Why didn't Hussein employ backward induction and quit before the war?

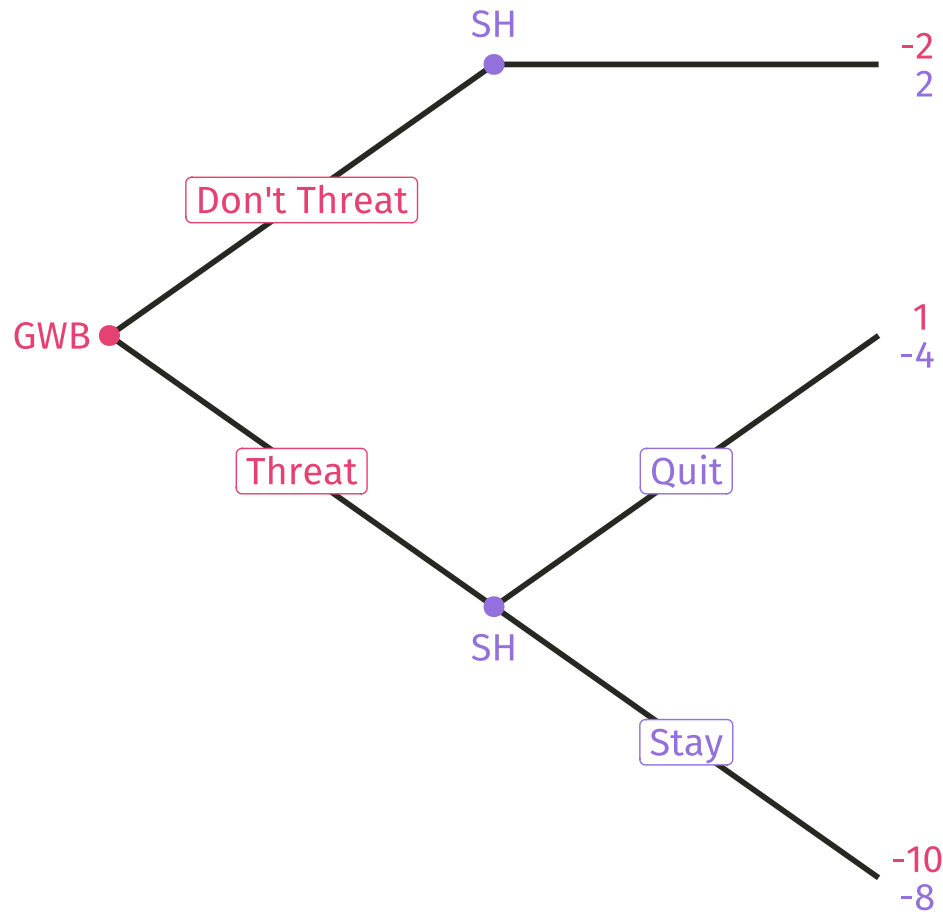
Brinkmanship

We can't say Bush played the game better or worse than JFK.

- Brinkmanship necessarily involves taking risks. There is a chance of success and failure.
- Something went wrong in the Persian gulf that didn't in the Cuban missile crisis, but what was it?

Brinkmanship

Simple Threat Model

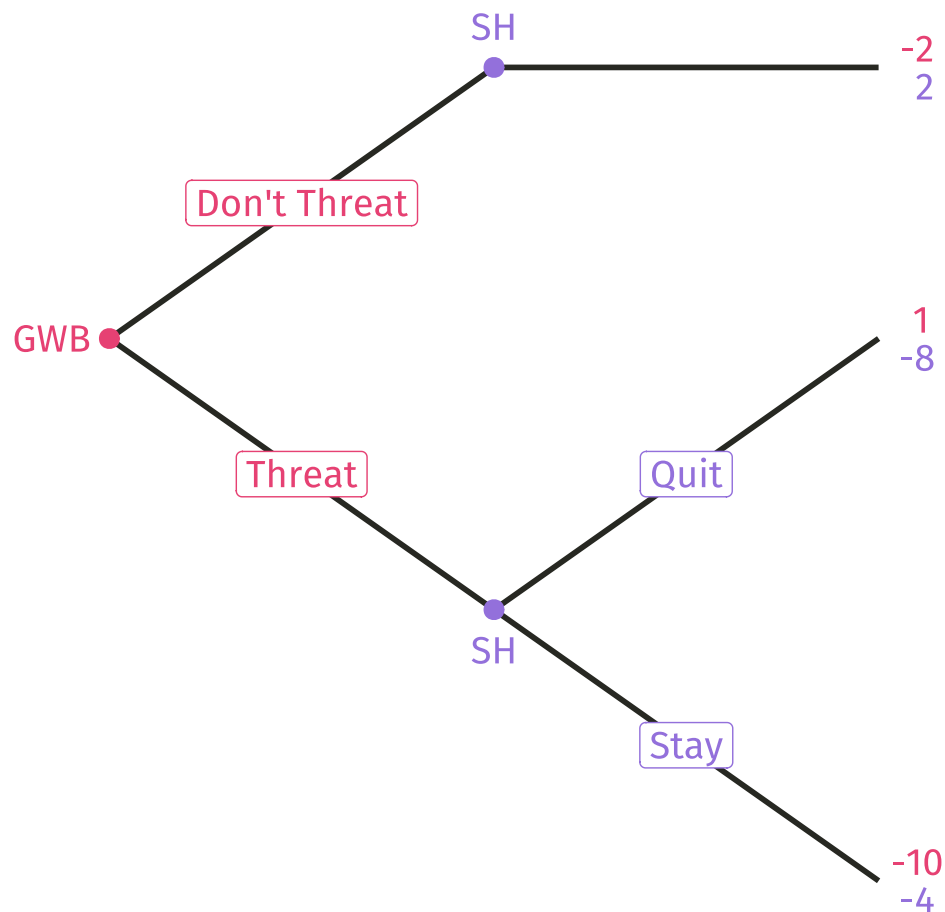


Problem: Backward induction tells that GWB always **threats** and SH always **quits**.

- But SH didn't quit!
- Maybe we have the payoffs wrong?
- Maybe SH values defying the US and simply prefers not to quit?

Brinkmanship

Take 2



Problem: Backward induction tells that SH always stays and GWB doesn't threat.

- But GWB didn't quit!

Brinkmanship

At least two elements are missing in our analysis:

1. GWB didn't know SH's payoffs: "Am I in game 1 or game 2?"
 - Expensive intel to know how SH valued defying the US.
2. Neither SH nor GWB was sure GWB would invade.
 - Many unknowns (*e.g.*, opposition to GWB's plans in the UN by France, Germany, and Russia).

Suppose that SH is tough with probability $p \in (0, 1)$ and GB is committed to his threat with probability $q \in (0, 1)$

- New game, similar to Cuban missile crisis.

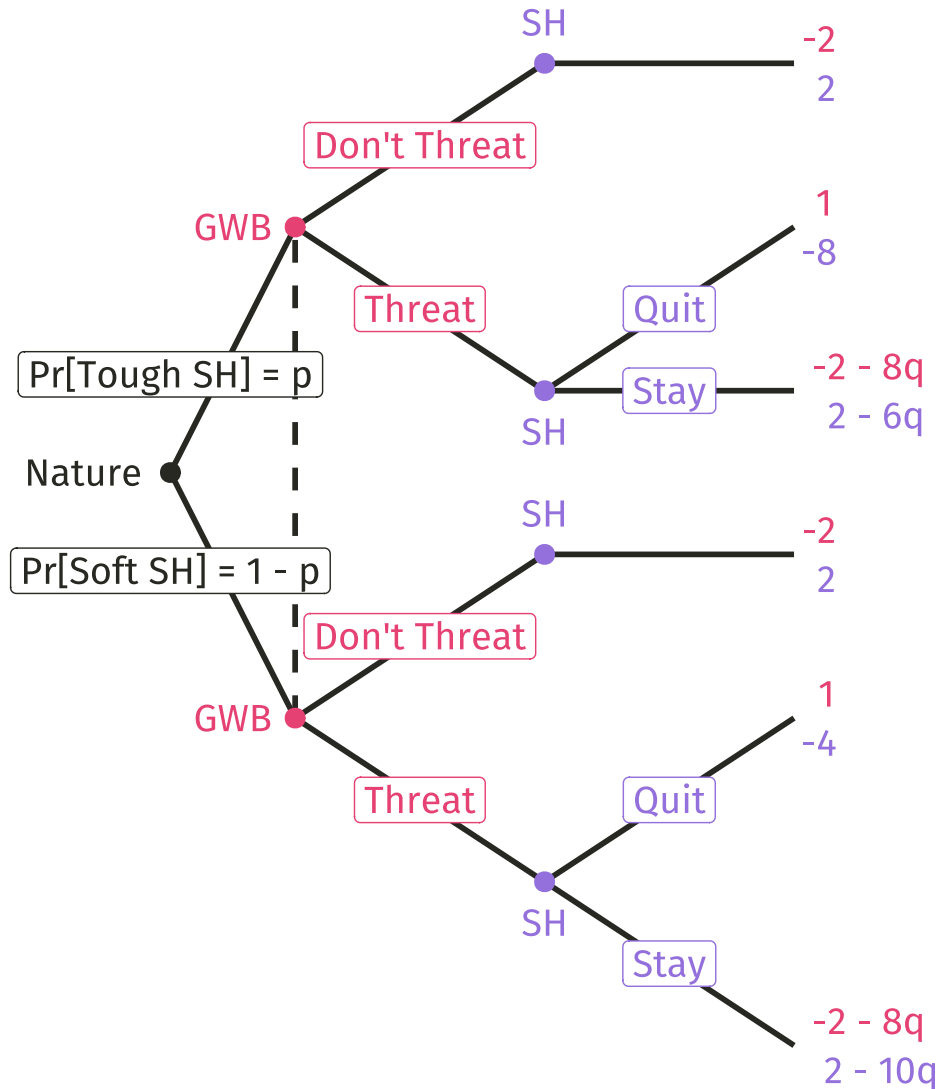
Backward induction

Tough SH won't quit for any q .

- Since $-8 > 2 - 6q$.

Soft SH quits if $q > 0.6$

- *i.e.*, $2 - 10q < -4$.



GWB's options

Threaten and get

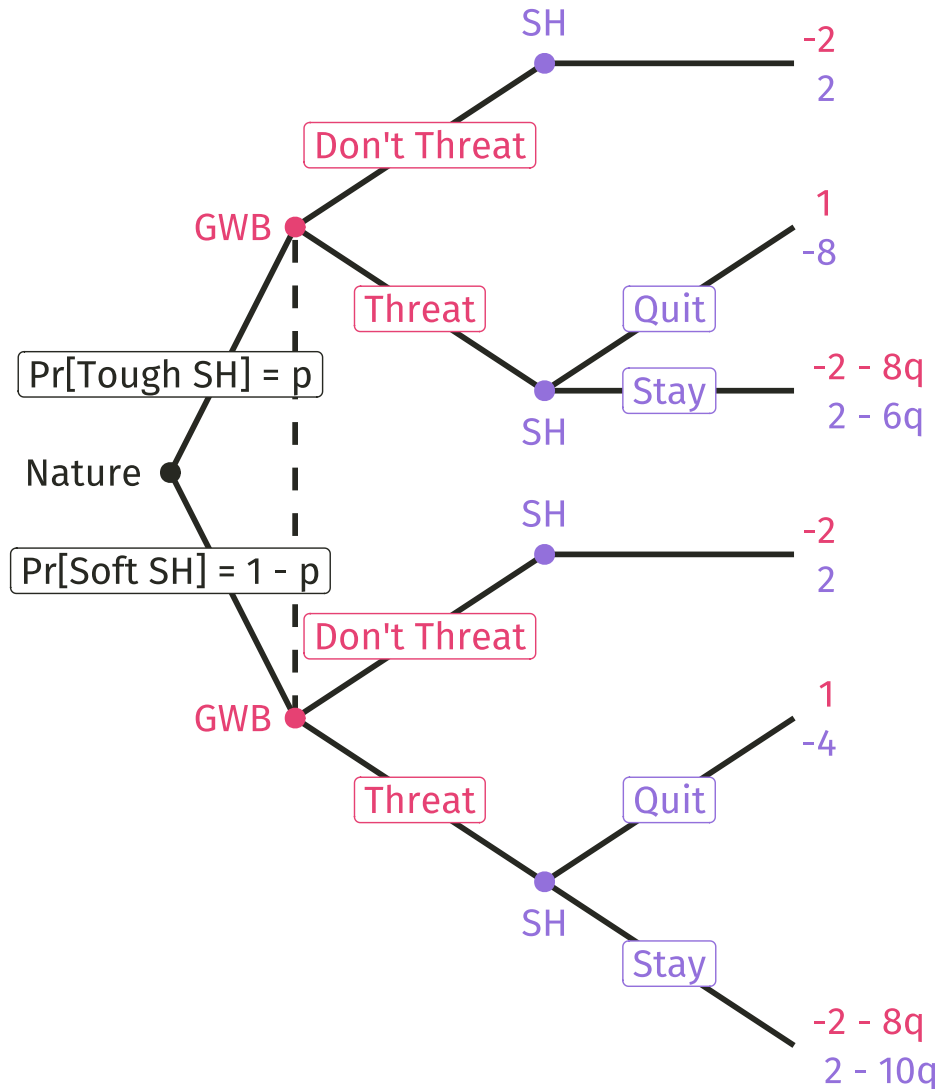
$$p(-2 - 8q) + (1 - p)(1)$$

Don't threaten and get

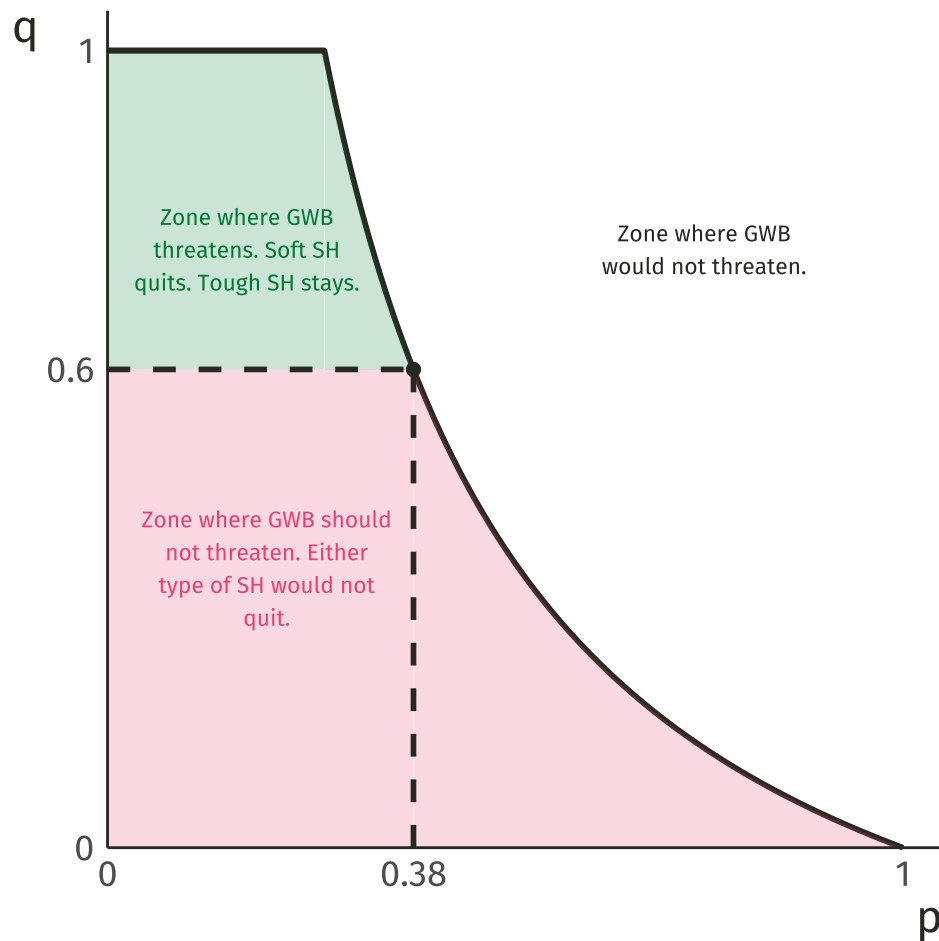
$$p(-2) + (1 - p)(-2) = -2$$

Will threaten if

$$p < 3/[3 + 8q]$$



Brinkmanship



GWB tries a $q < 0.6$. If this doesn't work and doesn't trigger conflict, he tries a slightly higher q .

Either GWB reaches $q = 0.6$ and a soft SH quits or we're going to war.

No different from JFK and Khrushchev, except for the outcome.

Midterm 2

Midterm Topics

Anything from lectures or discussions (weeks 5 through 7 only) **is fair game!**

1. Policy Levers: Taxes & Subsidies
2. Policy Levers: Price Controls
3. How Economists Learn from Data I
4. How Economists Learn from Data II
5. Market Failure: Externalities
6. Game Theory

Midterm Structure

Multiple Choice

- 50 questions
- 1 point per question
- Multiple groups of sequential questions (*e.g.*, about graphs or tables)

Midterm Protocol

Materials

- Writing utensil
- 3-inch-by-5-inch note card
- Basic or scientific calculator (no graphing or programming capabilities)
- UO ID card
- **Nothing else**

Procedure

- **Randomized** seating chart
- 80 minutes from *"you may begin"* to *"pencils down"*
- First 30 minutes: **quiet period** (no questions, no getting up)
- Last 50 minutes: ask lots of questions
- Show your UO ID card as you turn-in your exam

Midterm Preparation

1. Lecture slides
2. Discussion worksheets
3. Practice midterm questions
 - No solutions posted, but you can ask about the questions in office hours.
4. Extended office hours
 - M and T 14:00-16:00 in **412 PLC**
5. Prepare your note card