Questions on Calculus

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1. What are the drivatives of

   - $y = 2 + 6x$

     $\frac{d(y)}{d(x)} = 6$

   - $y = 5 - 4x + 2x^3$

     $\frac{d(y)}{d(x)} = 4 + 6x$

   - $y = 25 + 6x^2 - 3x^3 + 25x^4$

     $\frac{d(y)}{d(x)} = 12x - 9x^2 + 100x^3$

   - $y - 3 = 2x$

     $\frac{d(y)}{d(x)} = 2$

   - $TPP = 24 + 5L + 2L^2 - L^3$

     $\frac{d(TPP)}{d(L)} = 5 + 4L - 3L^2$

   - What does your answer to the prevous question tell you about the shape of the Total Physical Product Curve?

     It goes up initially but starts rise at a slower pace. There are diminishing returns.

     Find the second derivative of the following

- $TU = 25 + X_1 - X_1^2$

  $\frac{d^2(U)}{d(X_1)} = -2$

- $TU = 25 + 25X_1 - 2X_1^2$

  $\frac{d^2(U)}{d(X_1)} = -4$

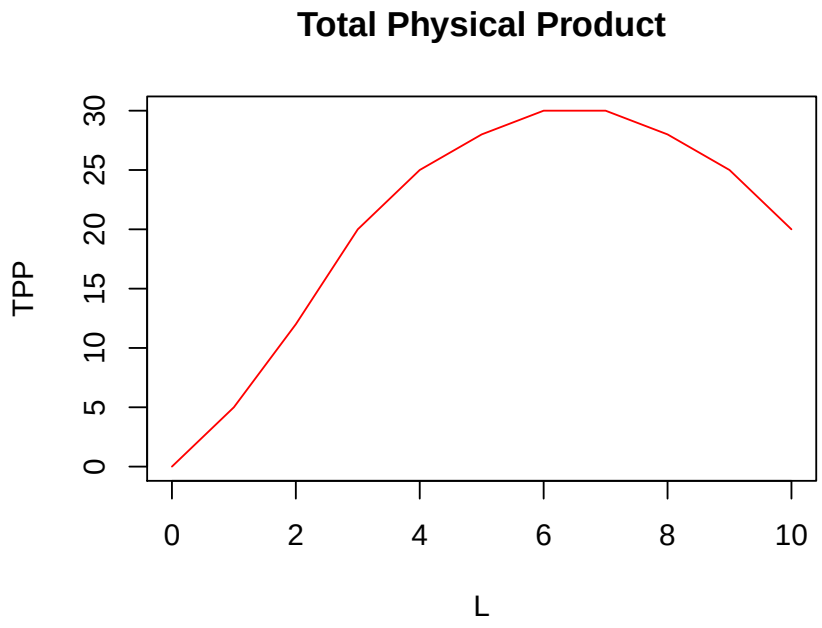
- $TPP = 15 + 15Q + Q^2 - Q^3$

  $\frac{d^2(TPP)}{d(L)} = 2 - 6Q$

- What does your answer to the previous question tell you about the shape of the Total Physical Product Curve?

  It shows that the rate of change is falling.

- Here is the total physical product curve. Show the *average physical product*? for a particular point on the graph.



- How would you calculate the *marginal physical product*?

2. What is the gradient of the TPP at its peak?

3. What is the value of the MPP when TPP is at its peak?

4. Given the $TPP = 100 + 32Q + 23Q^2 - Q^3$,

   - What is the TPP' or MPP?

     $32 + 46Q - 3Q^2$

   - How would you find the maximum TPP?

$$3Q^2 - 46Q - 32 = 0$$

$$(3Q + 2)(Q - 16) = 0$$

Therefore $3Q = -2 \quad \text{or} \quad Q = 16$

5. Given the $TPP = 500 + 180Q + 15Q^2 - 2Q^3$,

   - What is the TPP' or MPP?

     $MPP = 180 + 30Q - 6Q^2$

   - How would you find the maximum TPP?

$$6Q^2 - 30Q - 180 = 0$$

$$(6Q + 90)(Q - 20) = 0$$

So $Q = -15 \quad \text{or} \quad Q = 20.$

6. Given the $TU = 25X - 0.5X^2$

   - What is the drivative of TU?

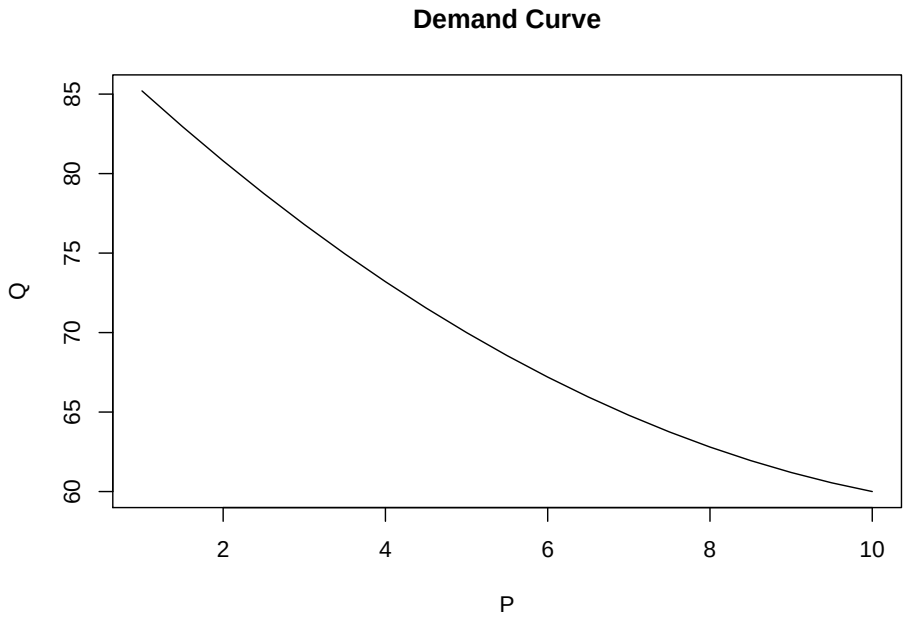
     $TU = 25 - X$

   - What can we say about the utility of X?

     There are diminishing returns.

   Given a demand curve

$$Q_d = 90 - 5P + 0.2P^2$$



   What is the elasticity of demand at the point $P = 5, Q = 70$?

The equation for elasticity is

$$\begin{aligned}\varepsilon_d =& \frac{d(Q)}{d(P)} \times \frac{P}{Q} \\ =& \frac{-5 - 0.4 \times 5}{\times} \frac{5}{70} \\ =& -5 - 2 \times \frac{5}{70}\end{aligned}$$

$$= -0.21428$$

Therefore the demand is inelastic