An Introduction to Calculus

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Usain Bolt

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Speed

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- Instantaneous Speed $= \frac{\Delta y}{\Delta x}$

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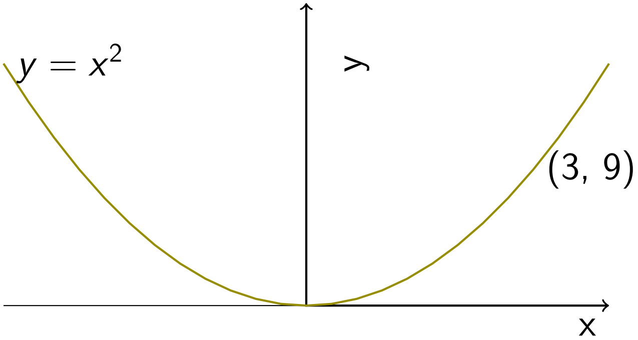
- h_0 is small
- Instantaneous speed $= \frac{f(x_0+h_0)-f(x_0)}{h_0}$

Example page 1

$f(x) = y = x^2$



Example page 2

Instantaneous change at (3, 9) when $f(x) = x^2$

$$\text{Instantaneous speed} = \frac{f(x_0+h_0)-f(x_0)}{h_0}$$

h	x + h	f(x + h)	$\frac{f(x+h)-f(x)}{h}$
0.1	3.1	9.61	6.1
0.01	3.01	9.0601	6.01
0.001	3.001	9.0060	6.001

Example page 3

Calculation

$$
\begin{aligned}
\text{Instantaneous speed} =& \frac{f(x_0 + h_0) - f(x_0)}{h_0} \\
=& \frac{(x_0 + h_0)^2 - x_0^2}{h_0} \\
=& \frac{x_0^2 + 2x_0 h_0 + h_0^2 - x_0^2}{h_0} \\
=& \frac{h_0(2x_0 + h_0)}{h_0} \\
=& 2x + h
\end{aligned}
$$

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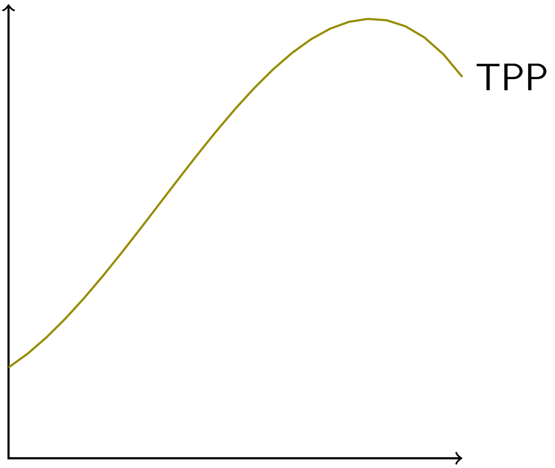
For any positive integer k, the derivative of $f(x) = x^k$ at x_0 is $f'(x) = kx^{k-1}$

Example 2 page 1

$TPP = 100 + 32Q + 10Q^2 - Q^3$



Example 2 page 2

Differentiating TPP

$$
\begin{aligned}
TPP =& 100 + 32Q + 10Q2 - Q^3 \\
TPP' =& 32 + 20Q - 3Q^2
\end{aligned}
$$

Gradient at maximum is zero, therefore

$$
\begin{aligned}
3Q^2 - 20Q - 32 =& 0 \\
(3Q + 4)(Q - 8) =& 0
\end{aligned}
$$

So $Q = 8$, or $Q = -1.33$

Quadratic Solution

For a quadratic equation of the form

$$ax^2 + bx + c$$

The *solution* or the roots can be found with

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$