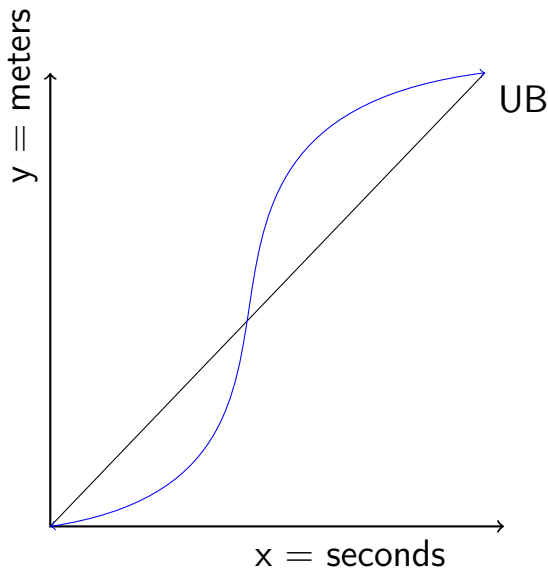


An Introduction to Calculus

Rob Hayward

Friday, 20th September 2013

Usain Bolt



Speed

Average and Instantaneous Speed

Slope is constant with average speed but varies over time.

Speed

Average and Instantaneous Speed

Slope is constant with average speed but varies over time.

$$\blacksquare \text{ Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

Speed

Average and Instantaneous Speed

Slope is constant with average speed but varies over time.

- Average Speed = $\frac{\text{Distance}}{\text{Time}}$

- Average Speed = $\frac{100}{9.58} = 10.43 \frac{m}{s}$

Speed

Average and Instantaneous Speed

Slope is constant with average speed but varies over time.

- Average Speed = $\frac{\text{Distance}}{\text{Time}}$
- Average Speed = $\frac{100}{9.58} = 10.43 \frac{m}{s}$
- Need to find the tangent to the line for *Instantaneous Speed*

Speed

Average and Instantaneous Speed

Slope is constant with average speed but varies over time.

- Average Speed = $\frac{\text{Distance}}{\text{Time}}$
- Average Speed = $\frac{100}{9.58} = 10.43 \frac{m}{s}$
- Need to find the tangent to the line for *Instantaneous Speed*
- Instantaneous Speed = $\frac{\Delta y}{\Delta x}$

Instantaneous Speed

Small change (h)

Instantaneous speed is the tangent to the curve. For $f(x)$

Instantaneous Speed

Small change (h)

Instantaneous speed is the tangent to the curve. For $f(x)$

■ Instantaneous speed $= \frac{f(x_0+h_0)-f(x_0)}{(x_0+h_0)-x_0}$

Instantaneous Speed

Small change (h)

Instantaneous speed is the tangent to the curve. For $f(x)$

■ Instantaneous speed $= \frac{f(x_0+h_0)-f(x_0)}{(x_0+h_0)-x_0}$

■ h_0 is small

Instantaneous Speed

Small change (h)

Instantaneous speed is the tangent to the curve. For $f(x)$

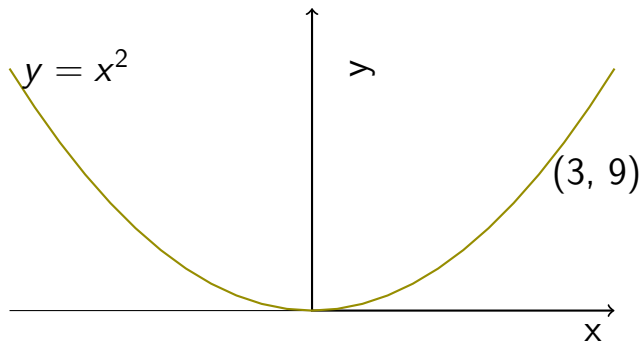
- Instantaneous speed $= \frac{f(x_0+h_0)-f(x_0)}{(x_0+h_0)-x_0}$

- h_0 is small

- Instantaneous speed $= \frac{f(x_0+h_0)-f(x_0)}{h_0}$

Example page 1

$$f(x) = y = x^2$$



Example page 2

Instantaneous change at $(3, 9)$ when $f(x) = x^2$

$$\text{Instantaneous speed} = \frac{f(x_0 + h_0) - f(x_0)}{h_0}$$

h	$x + h$	$f(x + h)$	$\frac{f(x+h) - f(x)}{h}$
0.1	3.1	9.61	6.1
0.01	3.01	9.0601	6.01
0.001	3.001	9.0060	6.001

Example page 3

Calculation

$$\begin{aligned}\text{Instantaneous speed} &= \frac{f(x_0 + h_0) - f(x_0)}{h_0} \\ &= \frac{(x_0 + h_0)^2 - x_0^2}{h_0} \\ &= \frac{x_0^2 + 2x_0h_0 + h_0^2 - x_0^2}{h_0} \\ &= \frac{h_0(2x_0 + h_0)}{h_0} \\ &= 2x + h\end{aligned}$$

The derivative

Instantaneous rate of change

The instantaneous rate of change

The derivative

Instantaneous rate of change

The instantaneous rate of change

$$\blacksquare f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$$

The derivative

Instantaneous rate of change

The instantaneous rate of change

■ $f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$

■ $f'(x)$ or

The derivative

Instantaneous rate of change

The instantaneous rate of change

- $f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$
- $f'(x)$ or
- $\frac{dy}{dx}(x)$

The derivative

Instantaneous rate of change

The instantaneous rate of change

- $f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$
- $f'(x)$ or
- $\frac{dy}{dx}(x)$

The derivative

Instantaneous rate of change

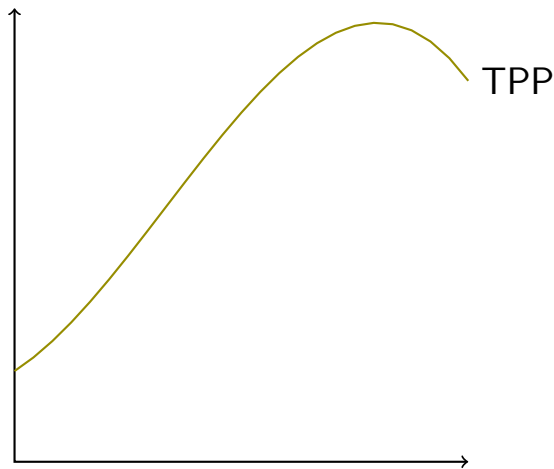
The instantaneous rate of change

- $f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$
- $f'(x)$ or
- $\frac{dy}{dx}(x)$

For any positive integer k , the derivative of $f(x) = x^k$ at x_0 is $f'(x) = kx^{k-1}$

Example 2 page 1

$$TPP = 100 + 32Q + 10Q^2 - Q^3$$



Example 2 page 2

Differentiating TPP

$$TPP = 100 + 32Q + 10Q^2 - Q^3$$

$$TPP' = 32 + 20Q - 3Q^2$$

Gradient at maximum is zero, therefore

$$3Q^2 - 20Q - 32 = 0$$

$$(3Q + 4)(Q - 8) = 0$$