# An Introduction to Calculus

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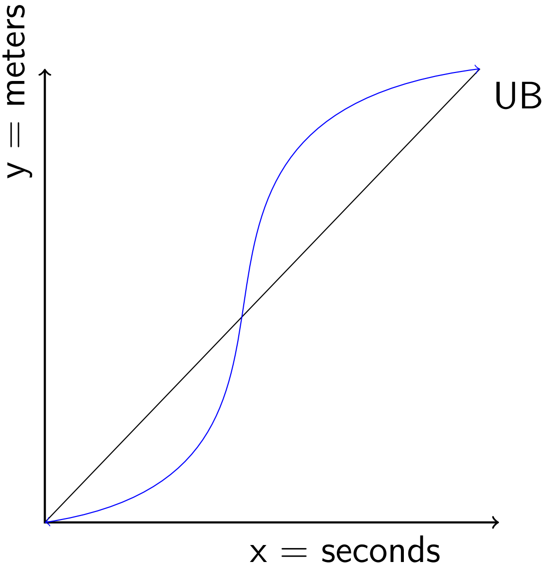
# Usain Bolt



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- Instantaneous Speed $= \frac{\Delta y}{\Delta x}$

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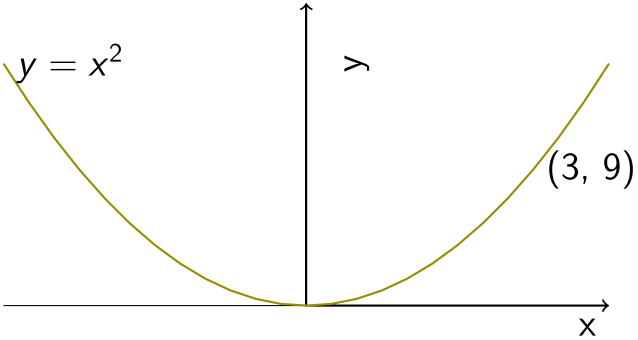
- Instantaneous speed $= \frac{f(x_0+h_0)-f(x_0)}{h_0}$

# Example page 1

$f(x) = y = x^2$



# Example page 2

## Instantaneous change at (3, 9) when $f(x) = x^2$

Instantaneous speed $= \frac{f(x_0+h_0)-f(x_0)}{h_0}$

| h | x + h | f(x + h) | $\frac{f(x+h)-f(x)}{h}$ |
|---|---|---|---|
| 0.1 | 3.1 | 9.61 | 6.1 |
| 0.01 | 3.01 | 9.0601 | 6.01 |
| 0.001 | 3.001 | 9.0060 | 6.001 |

# Example page 3

## Calculation

$$
\begin{aligned}
\text{Instantaneous speed} =& \frac{f(x_0+h_0)-f(x_0)}{h_0} \\
=& \frac{(x_0+h_0)^2 - x_0^2}{h_0} \\
=& \frac{x_0^2 + 2x_0h_0 + h_0^2 - x_0^2}{h_0} \\
=& \frac{h_0(2x_0+h_0)}{h_0} \\
=& 2x + h
\end{aligned}
$$

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For any positive integer k, the derivative of $f(x) = x^k$ at $x_0$ is $f'(x) = kx^{k-1}$

# Quadratic Solution

For a quadratic equation of the form

$$ax^2 + bx + c$$

The *solution* or the roots can be found with

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$