# Futures, forwards and carry

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- Future < spot is *backwardation*

#### Price of futures 1

When the only cost is the cost of finance

$$F(t,T) = S(t) \times (1+r)^{(T-t)}$$
 (1)

or, in continuous time

$$F(t,T) = S(t)e^{r(T-t)}$$
 (2)

Where t is today, T is the exercise date, r is the interest rate, F is the future price and S is the spot price.

However, in many cases there are other costs and benefits that have to be included

Benefits

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$$\frac{F_{t,j}}{S_t} \times (1+i_{t,j}) = (1+i_{t,j}^*)$$

Where,  $F_{t,j}$  is the forward rate in terms of foreign currency for domestic at time t for j periods ahead,  $S_t$  is the spot rate,  $i_{t,j}$  is the nominal interest rate at time t for j periods ahead and  $i^*$  is the foreign rate.

#### Forward rate 1

$$F_{t,j} = S_t \times \frac{(1 + i_{t,j}^*)}{(1 + i_{t,j})} \tag{3}$$

Calculate the 1-year forward for USD-JPY with spot at JPY 150; the US 1-year rate at 1.0% and the Japanese 1-year rate at zero.

#### Forward rate 2

Action	USD	JPY
Borrow USD	1,000,000	
Owe	1,010,000	
Buy JPY		150,000,000
Receive		150,000,000
Sell	$\frac{150,000,000}{148.5149} = 1,010,000$	
	Table: Covered interest parity	

#### From Equation 3

$$\frac{F_{t,j}}{S_t} = \frac{(1+i_{t,j}^*)}{(1+i_{t,j})}$$

Taking 1 from each side and re-arranging.

$$\frac{F_{t,j} - S_t}{S_t} = \frac{(i_{t,j}^* - i_{t,j})}{(1 + i_{t,j})} \tag{4}$$

If the forward exposure is not *covered* there is a risk of a loss or a gain.

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- In this case, it would assume that investors assume that forward rate will give an *unbiased prediction* of the future rate
- If not, there is an arbitrage opportunity

# Expectations and forward rate 1

Action	USD	JPY
Borrow USD	1,000,000	
Owe	1,010,000	
Buy JPY		150,000,000
Receive		150,000,000
Sell	$\frac{150,000,000}{150.00} = 1,000,000$	

Table: Expected rate is above forward: Loss

# Expectations and forward rate 2

Action	USD	JPY
Borrow USD	1,000,000	
Owe	1,010,000	
Buy JPY		150,000,000
Receive		150,000,000
Sell	$\frac{150,000,000}{145.00} = 1,034,483$	

Table: Expected rate is below forward: gain

The expected future rate is equal to the forward rate so,

$$E[s_{t+j}] - s_t = \frac{(i_{t,j}^* - i_{t,j})}{(1 + i_{t,j})}$$
 (5)

or approximately,

$$E[\Delta s_{t+j}] = i_{t,j}^* - i_{t,j} \tag{6}$$

$$\Delta s_t = i_{t,j}^* - i_{t,j} \tag{7}$$

Where  $\Delta s_t$  is  $log(s_t) - log(s_{t-1})$ 



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A usual test of UIP is

$$\Delta s_{t+j} = \beta_0 + \beta_1 f_{t+j} + \varepsilon \tag{8}$$

Where  $f_{t+j}$  is the forward premium and UIP would mean that  $\beta_0=0$  and  $\beta_1=1$ 

Many tests find that  $\beta_1 < 0$ 

#### UIP does not usually seem to hold

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- UIP failure: are there market frictions that prevent arbitrage?
- Expectations failure: is there a risk premium that explains the divergence?

# Carry returns in calm and crisis

