Note on the Natural Debt Limit*

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In the model with income fluctuation, we impose an exogenous borrowing constraint $a_{t+1} \geqslant 0$. In fact, we can consider a more general problem by setting $a_{t+1} \geqslant -\phi$, where ϕ is a parameter. One may wonder if there is a "natural" borrowing limit that the household faces.

Suppose the income process $\{y_t\}_{t=0}^{\infty}$ is deterministic. Impose non-negativity of consumption throughout the life of the household, i.e., $c_t \geqslant 0$ for all t and iterate forward on the budget constraint

$$
\begin{aligned}
& c_t = a_t + y_t - \frac{a_{t+1}}{R} \geqslant 0 \\
& \quad \Downarrow \\
& a_t \geqslant -y_t + \frac{a_{t+1}}{R} \\
& \quad \Downarrow \\
& a_t \geqslant -y_t + \frac{1}{R}\left[-y_{t+1} + \frac{a_{t+2}}{R}\right] \\
& \quad \Downarrow \\
& a_t \geqslant -\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j y_{t+j}
\end{aligned}
$$

In other words, by imposing this constraint, the household is not allowed to accumulate more debt than what she will ever be able to repay by consuming just zero every period.

If the income process is stochastic, then how can we be sure that whatever the household borrows she will repay almost surely (i.e., with probability 1)? Then, we need to substitute y_t at each t with the lowest possible realization of the income shock, call it $y_{\min}$. Then we have the natural debt limit

$$
\begin{aligned}
a_t &\geqslant -\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j y_{\min} \\
&= -\frac{R}{R-1} y_{\min}
\end{aligned}
$$

No exogenous borrowing constraint can ever be looser than the natural debt limit. Note that if $y_{\min} = 0$, then the natural debt limit is 0. With Inada conditions on consumption preference, we can insure that the natural borrowing limit will never bind.

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