Lecture notes, Oct 20^{th}, 2020

RBC solution

$$\max \ \mathbb{E}\sum_{t=0}^{\infty}\beta^t u(C_t)$$

s.t. $C_t + (k_{t+1} - (1-\delta)k_t) = f(z_t, k_t) \quad \cdots \quad \lambda_t$

$$\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\left\{\beta^t u(C_t) - \lambda_t\left[C_t + k_{t+1} - (1-\delta)k_t - f(z_t, k_t)\right]\right\}$$

w.r.t.

$C_t: \quad \beta^t \cdot u'(C_t) = \lambda_t$

$k_{t+1}: \quad -\lambda_t + \lambda_{t+1}\left((1-\delta) + f_k(z_{t+1}, k_{t+1})\right) = 0$

$$\beta^t \cdot u'(C_t) = \beta^{t+1} \cdot u'(C_{t+1}) \cdot \left((1-\delta) + f_k(z_{t+1}, k_{t+1})\right)$$

$$\mathbb{E}\ \beta\frac{u'(C_{t+1})}{u'(C_t)} \cdot \left[(1-\delta) + \frac{\partial f(z_{t+1}, k_{t+1})}{\partial k_{t+1}}\right] = 1 \quad \cdots \ \forall t$$

Euler condition

Bellman Equation of simple RBC

k_t -- state variable

$C_t, k_t, Y_t \rightarrow c, k, y$

$C_{t+1}, k_{t+1}, Y_{t+1} \rightarrow c', k', y'$

$$V(z, k) = \max_{C}\left\{u(C) + \mathbb{E}\left(\beta V(z', k')\right)\right\}$$

s.t. $C + (k' - (1-\delta)k) = f(z, k) \quad \cdots \quad \lambda$

$$\mathcal{L} = u(c) + \mathbb{E}\left(\beta V(z', k')\right) - \lambda\left[c + k' - (1-\delta)k - f(z, k)\right]$$

$$\frac{\partial \mathcal{L}}{\partial c} = u'(c) - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial k'} = \frac{\partial \mathbb{E}\left(\beta V(z', k')\right)}{\partial k'} - \lambda = 0$$

$$\Rightarrow u'(c) = \frac{\partial \mathbb{E}\left(\beta V(z', k')\right)}{\partial k'}$$

MU of consumption — Marginal Benefit of investment

$$\frac{\partial V(z,k)}{\partial k} = (1-\delta) + \frac{\partial f(z,k)}{\partial k} + \underbrace{\frac{\partial V}{\partial k'} \cdot \frac{\partial k'}{\partial k}}_{=0}$$

Envelope Theorem