Lecture notes, Nov 19th, 2020

CORRECTION:

Budget constraint of hhld.

$$\boxed{c + a' \leq y + R \cdot a}$$

alternative: $c + \frac{a'}{R} \leq y + a$

Firm Dynamics model (Simplest version)

time is discrete, infinite horizon

Fixed production factor: capital k

production function

$$Y = s \cdot k^{\alpha} \ell^{1-\alpha}$$

ℓ: labor
hire from hhld

competitive labor market.
fixed wage w constant over time

$$\max_{\ell} \boxed{\overbrace{\boxed{Y}}^{\text{revenue}} - w \cdot \ell} \Leftarrow \text{profit (before tax)}$$

$$\frac{\partial(Y - w \cdot \ell)}{\partial \ell} = s \cdot (1-\alpha) \cdot k^{\alpha} \ell^{-\alpha} - w = 0 \quad \text{(FONC)}$$

$$\Downarrow$$

$$\frac{w}{s \cdot (1-\alpha) k^{\alpha}} = \ell^{-\alpha}$$

$$\Downarrow$$

$$\ell = \left(\frac{w}{s \cdot (1-\alpha) k^{\alpha}}\right)^{-\frac{1}{\alpha}}$$

$$= \left(\frac{s \cdot (1-\alpha)}{w}\right)^{\frac{1}{\alpha}} \cdot k$$

$$Y - w \cdot \ell = s \cdot k^{\alpha} \ell^{1-\alpha} - w \cdot \ell$$

$$= s \cdot k^{\alpha} \cdot \left(\frac{s \cdot (1-\alpha)}{w}\right)^{\frac{1-\alpha}{\alpha}} \cdot k^{1-\alpha} - w \cdot \left(\frac{s \cdot (1-\alpha)}{w}\right)^{\frac{1}{\alpha}} \cdot k$$

$$= \underbrace{\left[s \cdot \left(\frac{s \cdot (1-\alpha)}{w}\right)^{\frac{1-\alpha}{\alpha}} - w \cdot \left(\frac{s(1-\alpha)}{w}\right)^{\frac{1}{\alpha}}\right]}_{z(s, w)} \cdot k$$

w – constant
s – Markov, exogenous
$\Downarrow$
z – Markov, exogenous

$\pi = z \cdot k$ — CRS, C-D production function.
$\pi = z \cdot k^{\theta}$ — DRS, C-D, θ – span of control, $0 < \theta < 1$

TL;DR: z is exogenous, Markov

$$k' = (1-\delta)k + i$$

Therefore, Bellman Eq. of firm? π k, z

$$V(z, k) = \max_{k'} \left\{ \underbrace{\pi(z,k) - (k' - (1-\delta)k)}_{\text{dividend to shareholders}} + \beta \mathbb{E} V(z', k') \right\}$$

dividend to shareholders (doesn't have to be non-negative)

firms: risk-neutral in this case

(implicitly assuming Modigliani-Miller)

Balance sheet: $A = L + E$

A: capital, L: bondholders, E: hold by shareholders

Euler eq.?

$$\beta \cdot \mathbb{E}\left[\underbrace{(1-\delta) + \frac{\partial \pi'}{\partial k'}}_{R}\right] = 1$$

$\pi(z,k)$ if z is i.i.d. $\Rightarrow$ k' the same over (z,k)

if z is Markov $\Rightarrow$ k' is the same over k

CRS production? $\pi(z,k) = z \cdot k$ $\quad \frac{\partial \pi}{\partial k} = z$

DRS production $\pi(z,k) = z \cdot k^{\alpha}$ $\quad \underbrace{\frac{\partial \pi}{\partial k}}_{\text{marginal profit is decr. over } k} = \alpha \cdot z \cdot k^{\alpha - 1}$

Tobin's $q = \frac{V}{k}$ $\qquad$ Book-to-Market Ratio: $\frac{k}{V}$

(in data: market value: $p \cdot S$)

high q -> high value of investment

marginal Q $\Rightarrow$ $\frac{\partial V}{\partial k}$ vs. $\frac{V}{k}$ $\Leftarrow$ average Q

Hayashi (1982)

capital adjustment cost (quadratic)