Lecture notes, Nov 10^{th}, 2020

Numerical Example of CCAPM

$u(c) = \ln(c)$ $\quad y = (1.5, 1, 0.5]$ $\quad \Pi = [\quad]$

$\beta = 0.96$

SDF ? $\quad M_{t,t+1} = \beta \cdot \frac{u'(c')}{u'(c)} = \beta \cdot \frac{c}{c'}$

$$M_{ij} = \beta \frac{c_i}{c_j}$$

$$P_i = \beta \cdot \mathbb{E}\left[\frac{u'(c')}{u'(c)} (P' + y')\right] \qquad n \text{ states}$$

$$= \beta \cdot \sum_{j=1}^{n} \frac{u'(c_j)}{u'(c_i)} \cdot (P_j + y_j) \cdot \Pi_{ij}$$

put all P_i into a vector ?

$$\begin{pmatrix} P_1 \\ \vdots \\ P_N \end{pmatrix} = \Pi \odot M \cdot \left[\begin{pmatrix} P_1 \\ \vdots \\ P_N \end{pmatrix} + \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}\right]$$

Denote $\begin{pmatrix} P_1 \\ \vdots \\ P_N \end{pmatrix} = \tilde{P}$ $\quad \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} = \tilde{y}$

where $M_{ij} = \frac{u'(c_j)}{u'(c_i)} \cdot \beta$

$$\tilde{P} = \Pi \odot M \cdot [\tilde{P} + \tilde{y}] \Rightarrow (I - \Pi \odot M) \cdot \tilde{P} = \Pi \odot M \cdot \tilde{y}$$

$$\Downarrow$$

$$\tilde{P} = (I - \Pi \odot M)^{-1} (\Pi \odot M \, \tilde{y})$$

CRRA preference $\quad \frac{c^{1-\nu}}{1-\nu}$ when $\nu > 1$

$\ln c \qquad \nu = 1$

$$u'(c) = c^{-\nu}$$

Linear Price when $\nu = 1$???

$\nu = 1 \Rightarrow u'(c) = \frac{1}{c}$

$$P = \beta \mathbb{E}\left[\frac{y}{y'} \cdot (p' + y')\right] \qquad \text{in eqm. } c = y$$

$$\Downarrow$$

$$\frac{P}{y} = \beta \cdot \mathbb{E}\left[\frac{p'}{y'} + 1\right] \Leftarrow \frac{P}{y} \text{ is linear}$$

Constant if stationary

Household Problem

Endowment Economy

$$\max_{\{c_t\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

endowment y_t : stochastic exogenous

asset position a_t

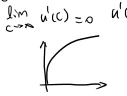
(Gross) return R

$$c_t + a_{t+1} = y_t + R \cdot a_t$$

(Technical assumption $u \in$ Inada condition. $\lim_{c \to 0} u'(c) = \infty$ $\quad u''(c) < 0$

$\lim_{c \to \infty} u'(c) = 0$ $\quad u'(c) > 0$)

$\boxed{\beta R < 1}$



Euler ? $\quad \beta \mathbb{E} R \cdot \frac{u'(c')}{u'(c)} = 1$

Incomplete market ?

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \mathbb{E} \, \beta^t u(c_t)$$

s.t. $\quad c_t + a_{t+1} \leq y_t + R \cdot a_t$

$a_t \geq 0 \Leftarrow$ additional constraint comparing to the model above