Lecture notes, Oct 8^{th}, 2020

$$E_0 \sum_{t=0}^{\infty} \beta^t Y_t$$

t $\quad S_t \quad \overset{\text{wage}}{W_t = W(S_t)}$ $\qquad S_t$: i.i.d.

decision? accept / reject

accept: $Y_t = W_t$, $Y_{t+1} = W_t$, $Y_{\vdots} = W_t$

reject: $Y_t = c$

$t+1 \cdots$ S_{t+1}? accept / reject

$$V(S_t) = \max_{\text{accept/reject}} \left\{ W_t + \beta \cdot W_t + \beta^2 \cdot W_t \cdots = \frac{W_t}{1-\beta}, \; c + \beta E V(S_{t+1}) \right\} \quad \text{– Bellman Eq.}$$

Solve the McCall model

$$S_t \in S = \left\{ \begin{array}{ccc} 1 & W_{s1} & F(1) \\ 2 & W_{s2} & F(2) \\ 3 & \vdots & \vdots \\ 4 & & \\ \vdots & & \\ n & W_{sn} & F(n) \end{array} \right.$$

S_t. i.i.d.

$W(S_1) < W(S_2) \cdots < W(S_n)$ $\quad \sum_{i=1}^{n} F(i) = 1$

$0 < W_{s1} < W_{s2} \cdots < W_{sn}$

$0 < c$

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if, instead of living forever, the agent lives for T periods

$$V_T(s) = \begin{cases} \text{accept} & \text{if } W(s) > c \\ \text{reject} & \text{if } W(s) < c \end{cases}$$

then, V_{T-1}

$V_{T-1}(s) =$ accept if $W(s) + \beta \cdot W(s) > c + \beta \cdot E V_T(s')$

$$\| \\ c + \beta \cdot \sum F(s') \left(W(s') \cdot \mathbb{1}(W(s') > c) + c \cdot \mathbb{1}(W(s') < c) \right)$$

$$\| \\ c + \beta \cdot \sum_{s' \in S} F(s') \cdot V_T(s')$$

$$\Downarrow$$

$$V_{T-1}(s) = \max \left\{ W(s) \cdot (1+\beta), \; c + \beta \cdot \sum_{s' \in S} F(s') \cdot V_T(s') \right\}$$

$$V_{T-2}(s) = \max \left\{ W(s) \cdot (1+\beta+\beta^2), \; c + \beta \cdot \sum_{s' \in S} F(s') \cdot V_{T-1}(s') \right\}$$

$$\vdots$$

$$\underbrace{V_0(s)}_{T \to \infty} = \max \left\{ W(s) \cdot \frac{1-\beta^{T+1}}{1-\beta}, \; c + \beta \cdot \sum_{s' \in S} F(s') \cdot V_1(s') \right\}$$