Notes on the Permanent Income Hypothesis*

November 12, 2020

The strict version of the Permanent Income Hypothesis (PIH) is a special case of the endowment economy with two key assumptions: 1) household have quadratic utility

$$u(c) = b_1 c - \frac{1}{2} b_2 c^2$$

and 2) $\beta R = 1$. Then the Euler equation reads

$$b_1 - b_2 c = \beta R \mathbb{E}\left(b_1 - b_2 c'\right)$$

which means

$$\mathbb{E}\left(c'\right) = c$$

From the law of iterated expectations,

$$\mathbb{E}_t c_{t+j} = c_t, \forall j \geqslant 0$$

Then we iterate the one-period budget constraint forward,

$$\begin{aligned}
c_t &= y_t + a_t - \frac{1}{R} a_{t+1} \\
c_t + \frac{1}{R} c_{t+1} &= a_t + y_t + \frac{1}{R} y_{t+1} - \left(\frac{1}{R}\right)^2 a_{t+2} \\
&\vdots \\
\sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i c_{t+i} &= a_t + \sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i y_{t+i}
\end{aligned}$$

and then we take expectations on both sides,

$$\begin{gathered}
\sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i \mathbb{E}_t c_{t+i} = a_t + \sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i \mathbb{E}_t y_{t+i} \\
\Downarrow \\
c_t \frac{R}{R-1} = a_t + \sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i \mathbb{E}_t y_{t+i}
\end{gathered}$$

and we can define the RHS of the above equation as wealth.

*Notes for UBC ECON 407, fall 2020

Moreover, we repeat the equation above, but now denote with $t-1$, and we take the difference between these two equations,

$$c_t - c_{t-1} = \frac{R-1}{R} \sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i \left(\mathbb{E}_t - \mathbb{E}_{t-1}\right) y_{t+i}$$

therefore, under the PIH, the change in consumption between $t-1$ and t is proportional to the revision in expected earnings due to the new information accruing in that time.