Lecture notes, Nov 3rd, 2020

AR(n) => VAR(1) ?

$$X_t = \rho_1 X_{t-1} + \rho_2 X_{t-2} \cdots + \rho_n X_{t-n} + \varepsilon_t \leftarrow AR(n)$$

AR(2) $\quad x_t = \rho_1 x_{t-1} + \rho_2 x_{t-2} + \varepsilon_t$

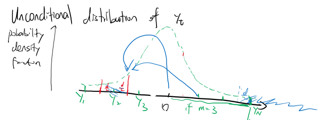
$\downarrow$

VAR(1) ? $\quad X_t = \begin{pmatrix} Y_t \\ Z_t \end{pmatrix} \qquad Y_t \equiv X_t, \quad Z_t \equiv X_{t-1}$

$$\begin{aligned} Y_t &= \rho_1 Y_{t-1} + \rho_2 Z_{t-1} + \varepsilon_t \\ Z_t &= Y_{t-1} \end{aligned} \implies X_t = \begin{pmatrix} \rho_1 & \rho_2 \\ 0 & 1 \end{pmatrix} X_{t-1} + \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix}$$

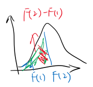
For AR(1) process $\{Y_t\}$ $\quad Y_{t+1} = \rho Y_t + \varepsilon_{t+1} \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

Unconditional distribution of Y_t



$N = 7 \qquad d = \dfrac{2 \cdot m \cdot \sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}}{N-1} \qquad 3 \cdot \sqrt{\dfrac{\sigma_\varepsilon^2}{1-\rho^2}}$

$Pr(Y_{t+1} = y_j \mid Y_t = y_i)$?



if $x \sim N(0, \sigma^2)$

$\Downarrow$

then $\frac{x}{\sigma} \sim N(0,1)$

Bellman eq. of the endowment economy

$\max \mathbb{E} \sum \beta^t u(c_t)$

s.t.

$c_t + P_t z_{t+1} \le z_t \cdot Y_t + z_t \cdot P_t$

$z_t \le 1$

$$\Rightarrow \quad V(z) = \max_{z'} \{ u(c) + \beta \mathbb{E} V(z') \}$$

s.t. $\quad c + p \cdot z' \le z \cdot y + z \cdot p \quad \cdots \lambda$

$$\mathcal{L} = \max_{z'} \{u(c) + \beta \mathbb{E} V(z')\} - \lambda (c + p \cdot z' - z \cdot y - z \cdot p)$$

$$\beta \mathbb{E} \frac{\partial V(z')}{\partial z'} - \lambda \cdot p = 0$$

$$u'(c) = \lambda$$

$$\beta \mathbb{E} \frac{\partial V(z')}{\partial z'} = u'(c) \cdot p$$

Envelope thm. to compute $\frac{\partial V(z)}{\partial z} = u'(c) \cdot (y + p)$

$$\boxed{p \cdot u'(c) = \beta \mathbb{E}[u'(c') \cdot (y' + p')]}$$

Euler eq.

$$E(mR) = 1$$

BOTH ARE RANDOM VARIABLES!!!

An example (could be trivial) of Lucas tree

$y = [0.9, \ 1.1]$ we assume it is i.i.d.

$Pr(y) = [0.4, \ 0.6]$

$$p \cdot u'(c) = \beta \mathbb{E}[u'(c') \cdot (y' + p')]$$

if $p' = \begin{cases} 1 & \text{-- for state 1} \\ 2 & \text{-- for state 2} \end{cases}$ $\qquad \Downarrow$ in equilibrium, $c = y$

then p ? $\quad p = \beta \cdot \mathbb{E}\left[\frac{u'(y')}{u'(y)} \cdot (y' + p')\right]$

$$= \beta \cdot \Big[0.4 \cdot \frac{u'(0.9)}{u'(y)} \cdot (0.9 + 1) \quad \text{--- state 1}$$

$$+ \ 0.6 \cdot \frac{u'(1.1)}{u'(y)} \cdot (1.1 + 2)\Big] \quad \text{--- state 2}$$

$\mathbb{E}[mR] = 1$ what's m & R here?

$\mathbb{E}[mR] = 1$

$\Downarrow$

$0.4 \cdot [m_1 \cdot R_1]$

$+$

$0.6 \cdot [m_2 \cdot R_2]$

$\shortparallel$

1

m: a random variable

state 1, prob 40% $\quad m_1 = \beta \frac{u'(0.9)}{u'(y)}$

state 2, prob 60% $\quad m_2 = \beta \cdot \frac{u'(1.1)}{u'(y)}$

R: a random variable

state 1 40% $\quad R_1 = \frac{y' + p'}{p} = \frac{0.9 + 1.0}{p}$

state 2 60% $\quad R_2 = \frac{y' + p'}{p} = \frac{1.1 + 2.0}{p}$

$$P_t = \beta \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \cdot (P_{t+1} + Y_{t+1})\right]$$

$$= \beta \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \cdot \left(\beta \mathbb{E}_{t+1}\left(\frac{u'(C_{t+2})}{u'(C_{t+1})} \cdot (P_{t+2} + Y_{t+2})\right] + Y_{t+1}\right)\right]$$

$$= \beta \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \cdot Y_{t+1}\right] + \beta^2 \mathbb{E}_t \left(\frac{u'(C_{t+2})}{u'(C_t)} \cdot Y_{t+2}\right) + \beta^2 \left(\frac{u'(C_{t+2})}{u'(C_t)} \cdot P_{t+2}\right)$$

$\vdots$

$$= \sum_{i=1}^{N} \beta^i \cdot \mathbb{E}_t \left(\frac{u'(C_{t+i})}{u'(C_t)} \cdot Y_{t+i}\right) + \underbrace{\beta^N \cdot \mathbb{E}_t \left(\frac{u'(C_{t+N})}{u'(C_t)} \cdot P_{t+N}\right)}_{\text{if this} \to 0 \text{ when } N \to \infty \text{ ?!}}$$

then

$$P_t = \sum_{i=1}^{\infty} \beta^i \mathbb{E}_t \left(\frac{u'(C_{t+i})}{u'(C_t)} \cdot Y_{t+i}\right)$$