Lecture notes, Oct 1st, 2020

s – states

V – value function

$$V(s) = \max_{s' \in \mathbb{S}} \{ f(s, s') + V(s') \}$$

Goal: find the shortest path from $A \to G$

$$V(s) = \min_{s' \in \mathbb{S}} \{ c(s, s') + V(s') \}$$

$\mathbb{S} =$ {(B, D, C) for A; (D, E) for B; … ; ϕ null set for G}

$V(s)$: shortest distance from s to G

$c(s, s')$: number on the arrow from s to s'

$c(A, D) = 3$ $\quad c(D, G) = 8$

A: $V(A) = \min_{B,C,D} \{ 5 + V(C),\ 3 + V(D),\ 1 + V(B) \} = 8$

B: $V(B) = \min_{D/E} \{ 9 + V(D),\ 6 + V(E) \} = 10$

C: $V(C) = 2 + V(F) = 3$

D: $V(D) = \min_{F/G} \{ 4 + V(F),\ 8 + V(G) \} = 5$

E: $V(E) = 4 + V(G) = 4$

F: $V(F) = 1 + V(G) = 1$

G: $V(G) = 0$

McCall model

1. time periods – discrete & infinite, i.e.

   $t = 0, 1, 2, 3 \ldots \infty$

2. S_t $\quad W_t = W(S_t)$ $\quad S_t \in \mathbb{S} = (1, 2, 3, \ldots, n)$

Discount factor β

	0	1	2	3	…	N …
		β	β^2	β^3	…	β^N
Y_t	Y_0	Y_1	Y_2	Y_3	…	Y_N

βY_1

$\beta^2 Y_2$

$\beta^3 \cdot Y_3$

…

$$U = \sum_{t=0}^{\infty} \beta^t \cdot Y_t$$

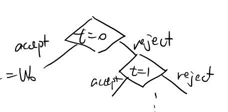
Accept W_t at t: $W_t = Y_t = Y_{t+1} = Y_{t+2} \ldots$

reject W_t at t: $Y_t = c$

Y_{t+1} … ? we don't know at period t

At period 0 $\quad \max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \cdot Y_t$

$W_t = W(S_t)$ $\quad Y_t = W_0$



$$V(S_t) = \max_{accept/reject} \begin{cases} W_t + \beta \cdot W_t + \beta^2 \cdot W_t \cdots = \dfrac{W_t}{1-\beta}, & \text{if accept} \\ c + \mathbb{E}_t \beta \cdot V(S_{t+1}), & \text{if reject} \end{cases}$$

$$\max \sum_{t=0}^{\infty} \beta^t Y_t = Y_0 + \sum_{t=1}^{\infty} \beta^t Y_t$$

$$V_0 \;"\; = Y_0 + \beta \cdot \sum_{s=0}^{\infty} \beta^s Y_{1+s}$$

$$= Y_0 + \beta V_1$$