Introduction to Ramsey Problem*

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Ramsey problem is to think about the design and implementation of optimal (fiscal) policy, under a dynamic equilibrium framework.

1 Model

Time period is discrete and infinite.

The representative household maximizes

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right)$$

subject to a budget constraint

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = \left(1 - \tau_t^l\right) w_t l_t + \left[1 + \left(1 - \tau_t^k\right)\left(r_t - \delta\right)\right] k_t + b_t$$

where k is for capital, b is for bonds issued by the government.

The representative firm produces final goods with production function $f\left(k_t, l_t\right)$. Therefore suppose the output is y_t then firms will minimize their cost,

$$\min f\left(k_t, l_t\right) - w_t l_t - r_t k_t$$

subject to

$$f\left(k_t, l_t\right) = y_t$$

The resource constraint of the overall economy is therefore

$$c_t + g_t + k_{t+1} = f\left(k_t, l_t\right) + (1 - \delta) k_t$$

The government raises taxes from the household and also issues short-term bond to fund government expenditure g_t,

$$g_t = \tau_t^k \left(r_t - \delta\right) k_t + \tau_t^l w_t l_t + \frac{b_{t+1}}{R_t} - b_t$$

In equilibrium,

- Households solve their maximization problem given prices and the government policy

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- Firms minimize their costs
- Government satisfies its budget constraint (this one, as well as the budget constraint of households, would imply market clearing under Walrasian equilibrium)

Ramsey Equilibrium

- Fix a sequence of exogenously given government purchases $\{g_t\}_{t=0}^{\infty}$.
- The "best" competitive equilibrium given $\{g_t\}_{t=0}^{\infty}$, k_0, b_0, and bounds on τ_t^k.

2 Solution and Analysis

We write down the first-order necessary conditions for the household,

$$\begin{aligned}\mathrm{SDF}(t|0) &= \beta^t \mathbb{E}\frac{u_c(t)}{u_c(0)} \\ \frac{u_l(t)}{u_c(t)} &= \left(1-\tau_t^l\right) w_t\end{aligned}$$

and from firm's problem,

$$\begin{aligned}r_t &= f_k(t) \\ w_t &= f_l(t)\end{aligned}$$

and then we substitute the conditions in the budget constraint of household

$$\sum_{t=0}^{\infty} \beta^t \left(u_c(t) c_t - u_l(t) l_t\right) - u_c(0)\left\{\left(1+\left(1-\tau_0^k\right)(r_0-\delta)\right)k_0 + b_0\right\} = 0$$

and we define the second term as $A\left(c_0, l_0, \tau_0^k, b_0\right)$. This can be treated as an "implementability constraint". Denote the Lagrangian multiplier to this constraint as Φ.

Then we write down the social planner's problem. Define

$$W(c_t, l_t, \Phi) = \left(u(c_t, l_t) + \Phi\left(u_c(t) c_t - u_l(t) l_t\right)\right)$$

and the objective function becomes

$$\sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \Phi) + \theta_t\left(f(k_t, l_t) + (1-\delta) k_t - c_t - g_t - k_{t+1}\right) - \Phi A\left(c_0, l_0, \tau_0^k, b_0\right)$$

where θ_t is the Lagrange multiplier of the resource constraint. With the assumption that the solutions are interior,

$$\begin{aligned}W_c(t) &= \beta W_c(t+1)\left(f_k(t+1) + 1 - \delta\right) \\ W_l(t) &= -W_c(t) f_l(t) \\ W_l(0) &= \left[\Phi A_c - W_c(0)\right] f_l(t) + \Phi A_l\end{aligned}$$

Implications At the "steady state" of social planner's problem,

$$1 = \beta \left(f_k + 1 - \delta \right)$$

and the "steady state" of the competitive equilibrium,

$$1 = \beta \left(1 + \left(1 - \tau_{t+1}^{k} \right) \left(r_{ss} - \delta \right) \right)$$

That means the optimal tax rate on capital should be always 0.