Lecture notes, Oct 22nd, 2020

Envelope theorem (unconstrained case)

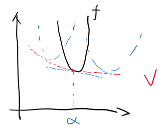
$f(x, \alpha)$ $\quad V(\alpha) = \max/\min_{x} f(x, \alpha) = f(x^*(\alpha), \alpha)$

$x^*(\alpha) = \operatorname{argmax/min}_{x} f(x, \alpha)$

$$\underbrace{\frac{\partial V(\alpha)}{\partial \alpha} = \left.\frac{\partial f(x,\alpha)}{\partial \alpha}\right|_{x=x^*}}$$

proof? $\quad \frac{\partial V(\alpha)}{\partial \alpha} = \frac{\partial f(x^*, \alpha)}{\partial \alpha} + \boxed{\frac{\partial f(x^*, \alpha)}{\partial x^*}} \cdot \frac{\partial x^*}{\partial \alpha}$ $\quad \square$

$\overset{\shortparallel}{0}$ when x^* is extreme point



RBC

$\mathbb{E} \sum_{\{c_t\}} \beta^t u(c_t) \qquad c_t + (k_{t+1} - (1-\delta)k_t) = Y_t$

$Y_t = f(z_t, k_t)$

$V(z, k) = \max_{c} \{ u(c) + \beta \mathbb{E} V(z', k') \}$

s.t. $\quad c + (k' - (1-\delta)k) = f(z, k) \quad \cdots \quad \lambda$

$\mathcal{L} = u(c) + \beta \mathbb{E} V(z', k') - \lambda [c + (k' - (1-\delta)k) - f(z, k)]$

$\underline{\frac{\partial \mathcal{L}}{\partial c} = u'(c) - \lambda = 0} \qquad \underline{\frac{\partial \mathcal{L}}{\partial k'} = \beta \mathbb{E} \frac{\partial V(z', k')}{\partial k'} - \lambda}$

$\underline{\boxed{\frac{\partial V(z,k)}{\partial k} = \frac{\partial \mathcal{L}}{\partial k}} = \lambda [(1-\delta) + f_k(z, k)]}$

$\Uparrow$ Envelope theorem

$\lambda = u'(c)$

$\beta \mathbb{E} \frac{\partial V(z', k')}{\partial k'} = u'(c)$

$\beta \mathbb{E} \frac{u'(c')}{u'(c)} \cdot [(1-\delta) + f_k(z', k')] = 1 \quad \Leftarrow$ Euler Equ.

$-\frac{u''(c)}{u'(c)}$ Absolute Risk Aversion

$-\frac{u''(c)}{c \cdot u'(c)}$ Relative Risk Aversion

$\times$ Increasing / Constant / Decreasing

CRRA $\subseteq$ DARA

$\frac{c^{1-\gamma}}{1-\gamma} \qquad \gamma \cdots$ risk-aversion

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C-D $\quad$ constant returns to scale

$f = k^{\alpha} n^{1-\alpha}$

capital $\quad$ labor

$\underset{\shortparallel}{\underbrace{\text{labor share}}}$ + capital share = 1

$\frac{W = w \cdot n}{R}$ (competitive environment)

$p = 1 \Rightarrow R = f(k, n)$

$\max_{n} k^{\alpha} n^{1-\alpha} - w \cdot n$

$1 - \alpha \approx 0.63 - 0.66$

$\Downarrow$

$\alpha = \frac{1}{3}$

$\beta \qquad \boxed{\frac{1}{1 + r_f} = \beta}$

risk-free rate

$r_f \approx 1\%$ per quarter

$\Downarrow$

4% per year

K & P = $\beta = 0.96$ annual, 0.99 quarterly

r_f after 2000 ... 1% per year