Tauchen Method*

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We are going to discretize the continuous AR(1) process,

$$y_{t+1} = \rho y_t + \epsilon_{t+1}$$

where $\epsilon \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$.

The simplest way to discretize this process and to put into a discrete Markov chain form is using Tauchen (1986) method.

Let $\tilde{y}$ be the discrete-valued process that approximates y and let $\{y_1, y_2, \ldots, y_N\}$ be the finite set of possible realizations of $\tilde{y}$.

Choice of Points Tauchen suggests to select a maximum value y_N as a multiple m (e.g. $m = 3$) of the unconditional standard deviation, i.e.

$$y_N = m\left(\frac{\sigma_\epsilon^2}{1-\rho^2}\right)^{\frac{1}{2}}$$

and let $y_1 = -y_N$ (assuming G is symmetric), and $\{y_2, y_3, \ldots, y_{N-1}\}$ be located in an evenly-spaced manner over the interval $[y_1, y_N]$. Denote with d the distance between successive points in the state space.

Transition Probabilities Let

$$\begin{aligned}\pi_{jk} &= \Pr\left\{\tilde{y}_t = y_k | \tilde{y}_{t-1} = y_j\right\} = \Pr\left\{y_k - \frac{d}{2} < \rho y_j + \epsilon < y_k + \frac{d}{2}\right\} \\ &= \Pr\left\{y_k - \frac{d}{2} - \rho y_j < \epsilon < y_k + \frac{d}{2} - \rho y_j\right\}\end{aligned}$$

be the (generic) transition probability. Then, if $1 < k < N-1$, for each j choose

$$\pi_{jk} = F\left(\frac{y_k + d/2 - \rho y_j}{\sigma_\epsilon}\right) - F\left(\frac{y_k - d/2 - \rho y_j}{\sigma_\epsilon}\right)$$

where F is the cumulative density function of the standard normal distribution. And for boundaries of the interval $k = 1$ and $k = N$,

$$\begin{aligned}\pi_{j1} &= F\left(\frac{y_1 + d/2 - \rho y_j}{\sigma_\epsilon}\right) \\ \pi_{jN} &= 1 - F\left(\frac{y_N - d/2 - \rho y_j}{\sigma_\epsilon}\right)\end{aligned}$$

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