# Lecture notes. Nov 5th, 2020

AR(1) process

$$x_{t+1} = \rho \cdot x_t + \varepsilon_{t+1} \qquad |\rho| < 1$$

$$x_{t+1}^2 = \rho^2 \cdot x_t^2 + 2\rho x_t \varepsilon_{t+1} + \varepsilon_{t+1}^2$$

$$\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

$$Var(X) = \rho^2 \cdot Var(X) + \sigma_\varepsilon^2$$

$$\{\varepsilon_1, \cancel{\varepsilon_2} \dots \varepsilon_N\} \quad \lim_{N \to \infty} Var(\varepsilon) \to \sigma_\varepsilon^2$$

$$Var(x) = \frac{\sigma_\varepsilon^2}{1-\rho^2}$$

Tauchen method $[x_1, \dots x_m]$ $P = [\pi_{ij} \ \ ]$

$\{x_t\}_{t=1,\dots T}$ $corr\{[x_1, \dots x_{T-1}], [x_2, \dots x_T]\}$ (when Y, X are vectors)

$$\underset{[x_2 \dots x_T]}{Y} = \beta \underset{[x_1 \dots x_{T-1}]}{X} + \varepsilon \quad \Rightarrow \quad \beta = \frac{Cov(Y,X)}{Var(X)}$$

$$= corr(X, Y)$$

Auto-cor of X

$Var\{X_t\}_{t=1,\dots T}$

---

An example of some risky asset

$$Y = [1.5,\ 1,\ 0.5] \qquad \Pi = \begin{pmatrix} 0.5 & 0.25 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$$

$$m = \beta \cdot \frac{u'(c')}{u'(c)} \qquad u(c) = \ln(c) \qquad u'(c) = \frac{1}{c}$$

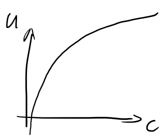
states next period

$$M = \beta \cdot \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1.5 & 3 \\ 2 & \frac{1}{1.5} & 1 & 2 \\ 3 & \frac{1}{3} & \frac{1}{2} & 1 \end{array}$$

states today

$u'(c) > 0$

$u''(c) < 0$




$\underline{cov(m, R) < 0}$

price of this asset?

$$P = \beta \mathbb{E}(m \cdot (p' + y')) \qquad P_1,\ P_2,\ P_3$$

$$P_1 = \beta \cdot [0.5 \cdot 1 \cdot (P_1 + 1.5) + 0.25 \cdot 1.5 \cdot (P_2 + 1) + 0.25 \cdot 3 \cdot (P_3 + 0.5)]$$

$$= \beta \cdot [0.5 P_1 + 0.75 + 0.375 P_2 + 0.375 + 0.75 P_3 + 0.375]$$

$$= \beta \cdot [0.5 P_1 + 0.375 P_2 + 0.75 P_3 + 1.5]$$

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

$$m = \beta \cdot \left(\frac{c'}{c}\right)^{-\gamma} \qquad \mathbb{E}(m) \approx \frac{1}{R^f}$$

$$\beta \cdot \frac{c'}{c} \qquad R^f$$

$\gamma$: empirically $\approx 2, 3$ theory $\gamma \approx 100 \sim 500$