

Lecture notes, Oct 27th, 2020

PS 3, Q 1

$$V = \max \left\{ \frac{w}{1-\beta}, c + \beta \mathbb{E} V \right\}$$

$$\frac{w^*}{1-\beta} = c + \beta \mathbb{E} V \Rightarrow w^* = (1-\beta)(c + \beta \mathbb{E} V)$$

CRRA

$$\frac{c^{1-\beta} - 1}{1-\beta} \quad \beta = 1 ?$$

$$\lim_{\beta \rightarrow 1} \frac{c^{1-\beta} - 1}{1-\beta} = \lim_{x \rightarrow \infty} \frac{c^x - 1}{x} \quad \begin{array}{l} \text{exp}(\lim_{x \rightarrow \infty} \frac{c^x}{x}) \\ \text{L'Hôpital} \end{array}$$

$$= \lim_{x \rightarrow \infty} \frac{\ln(c)}{1} = \ln(c)$$

Brute-force Value Function Iteration

$$V(z, k) = \max \{ u(c) + \beta \mathbb{E} V(z', k') \}$$

1 2 3 ... n-1 n grid of k $z \sim P_{\text{matrix}}$
m states

$$V_0(z, k) = 0 \quad \forall z, k$$

$$V_i(z_i, k_j) \quad \begin{array}{l} i=1 \dots m \\ j=1 \dots n \end{array}$$

$$\forall i, j \quad V_i(z_i, k_j) = \max_{k'_p} \left\{ u(c) + \beta \sum_{q=1}^m V_0(z_q, k'_p) \cdot P(z_i, z_q) \right\}$$

\uparrow
 $c(k_j, k'_p)$

enumerate $p = 1, \dots, n$ $y = k' - (1-\delta)k = c$

$$V_T \approx V_{T-1} \quad \forall i, j$$