Balanced Growth

Capital is NOT Sufficient for Sustained Growth in GDP

▶ Growth in output. . .

$$\frac{\Delta Y}{Y} = \underbrace{\frac{\Delta A}{A}}_{\text{Change in Tehcnology}} + \underbrace{\alpha \frac{\Delta K}{K}}_{\text{Change in Capital}} + \underbrace{(1-\alpha)\frac{\Delta L}{L}}_{\text{Change in Labor}} .$$

▶ What if all growth in GDP was coming from the change in capital? What would happen?

Capital is NOT Sufficient for Sustained Growth in GDP

► Krugman on the Cold War...

   ► "Soviet economy grew strongly after World War 2 and many in the West
   believed they would become more prosperous than capitalist economies.
   However, some economists that had examined the Soviet economy were
   less impressed."

   ► "What they [economists] found was that Soviet growth was based on rapid
   growth in inputs—end of story"

   ► In contrast, "capitalist growth had been based on growth in both inputs
   and efficiency, with efficiency the main source of rising per capita income."

Kaldor Facts of Economic Growth

1. Output per worker grows at a roughly constant rate that does not diminish over time.

2. Capital per worker grows over time and at a similar rate as output per worker.

3. The capital/output ratio is roughly constant. (1+2)

4. The share of payments to capital and labor in income is nearly constant.

   ▶ We have shown this already! Chapter 3, Problem Set # 1.

5. The rate of return to capital is constant.

6. Real wage grows over time.

Outline: Balanced Growth

- A **balanced growth path**: where Y and K grow at the same rate.

- Use Kaldor Facts # 1 and # 2, to derive. . .

    - How ALL growth in GDP is ultimately about growth in TFP and population growth.

    - How the real return on capital and real wage depend on growth in TFP and population growth, etc.

- Notation. . .

    - $n = \frac{\Delta L}{L}$ or annual growth rate of labor force.

    - $(1 - \alpha)g = \frac{\Delta A}{A}$ or annual growth rate of TFP.

    - Note. . . All I did is make a definition about how g relates to growth in A. The $(1 - \alpha)$ on the outside will make the algebra a bit simpler.

Balanced Growth I

▶ Kaldor Fact # 1 and # 2 imply. . .

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K}$$

▶ Then starting from our growth equation. . .

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L},$$

$$\frac{\Delta Y}{Y} - \frac{\Delta K}{K} = \frac{\Delta A}{A} - (1-\alpha)\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L},$$

$$(1-\alpha)\frac{\Delta K}{K} = (1-\alpha)\frac{\Delta L}{L} + \frac{\Delta A}{A},$$

$$\frac{\Delta K}{K} = \frac{\Delta L}{L} + \frac{1}{(1-\alpha)}\frac{\Delta A}{A}.$$

Balanced Growth II

▶ So we have shown Kaldor Fact # 1 and #2 imply that capital must grow according to. . .

$$\frac{\Delta K}{K} = \frac{\Delta L}{L} + \frac{1}{(1-\alpha)}\frac{\Delta A}{A}$$

▶ This implies that capital grows at rate

$$\frac{\Delta K}{K} = n + g$$

▶ In words, on a balance growth path, **capital must grow at the rate of population growth + technological progress!**

▶ Given this result, at what rate does output grow?

Kaldor Fact # 5, Returns to Capital are Constant

▶ Recall, that the real rental rate is

$$\frac{R}{P} = \text{MPK} = \alpha\frac{Y}{K}$$

▶ So if capital is growing at the same rate of output, then the marginal product of capital and the real rental rate must be constant.

▶ More interesting question: How does the rental rate depend on technological progress, population growth, etc.?

▶ Next slide. . .

Capital Accumulation

- Capital accumulation equation

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- δ is the rate of depreciation of capital.

- Savings equals investment (loanable funds equilibrium) (end of Chapter 3)!

$$I_t = S_t = s \times Y_t$$

- s is the savings rate in the economy.
  - Note that this is one minus the Marginal Propensity to Consume (MPC) in an economy with no Government discussed at the end of Chapter 3.

Output-to-Capital Ratio on the Balanced Growth Path

- Capital accumulation equation

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$$(1 + n + g)K_t = (1 - \delta)K_t + sY_t$$

$$(n + g + \delta)K_t = sY_t$$

- Then the output-to-capital ratio on the balanced growth path is

$$\frac{(n + g + \delta)}{s} = \frac{Y}{K}$$

- Important result!!!

  - Because the rental rate on capital $= \text{MPK} = \alpha \frac{Y}{K}$, this determines the **level** of returns on capital on a balanced growth path.

  - How do returns on capital depend on savings rate, technology growth, population growth. . . In-class practice problem.

Kaldor Fact # 6: Real Wages on the Balanced Growth Path

▶ Recall the real wage $= \text{MPL} = (1 - \alpha)\frac{Y}{L}$. So we just need to determine how output per worker is growing. . .

$$\underbrace{\frac{\Delta Y}{Y} - \frac{\Delta L}{L}}_{\text{Growth in Output per Worker}} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} - \alpha \frac{\Delta L}{L},$$

$$\underbrace{\frac{\Delta Y}{Y} - \frac{\Delta L}{L}}_{\text{Growth in Output per Worker}} = (1 - \alpha)g + \alpha(n + g) - \alpha n,$$

$$= g$$

▶ **Real wage growth depends only on the rate of technological progress, g.**

▶ Blog post: Gordon's argument that the rate of technological progress is slowing and its implications.

Food for thought. . .

- ▶ What role does the savings rate (little s) play in determining if a country has a high standard of living or not. . . if at all?

    - ▶ Key to answering this question is to distinguish between **level** versus **growth** effects.

    - ▶ Previous result: the **growth** rate of real wages (or output per worker) only depends on growth rate of technological progress.

        This is only about the rate of growth.

    - ▶ What about the **level** of real wages? Does this depend on the savings rate?