

# Firms in International Trade

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This set of notes walks through concepts about how modern firms operate in an international context. The goal is to show how the behavior of firms can lead to new and different gains from trade than those that we explored in the Ricardian model.<sup>1</sup>

We want to think about a situation where firms have some market power but also face competition. Hence the term “monopolistic competition.” Now to deliver this we proceed in several steps. First, will review basic concepts about how a monopolist sets its price and quantity. Second, we will delve into the monopolistic competition model, that is how it encapsulates aspects of monopoly with competition. Third, we will think about how opening up to trade changes outcomes (i.e. prices, number of firms operating, etc.). At the end of these notes, we will the discuss how to think about the implications of this model with respect the world we live in.

## 0.1 Review of Monopoly

In economics, every thing boils down to how marginal benefit relates to marginal costs. The only issue for a monopolist is that she is able to set the price at which the product is sold.

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<sup>1</sup>These notes build on some of the material in Krugman, Obstfeld, and Melitz. If you want to know more about this subject, this book is a good place to start.

**Marginal Revenue and Price.** Now in this set of notes we assume that the demand curve a firm faces is linear (i.e. a straight line). Specifically, we will assume that:

$$Q = A - B \times P \quad (1)$$

where  $Q$  is the quantity demanded;  $A$  is a constant;  $B$  is a parameter representing the slope of the demand curve with respect to the price,  $P$ . In other words,  $B$  tells us something about the price elasticity of demand.

The revenue that the monopolist receives is simple:  $P \times Q$ . Now either through calculus or intuitive arguments, one can show that the marginal revenue (that is the incremental amount of revenue the monopolist receives from selling one extra unit) is

$$MR = P - \frac{Q}{B}. \quad (2)$$

First, notice that the price always lies above marginal revenue. Second, the more units sold (higher  $Q$ ) results in lower marginal revenue. The idea here is that the more a monopolist sells, he starts to “cannibalize” his own sales. The derivation of (2) is provided in the Appendix.

**Average and Marginal Costs.** Here we assume that there are economics of scale. The idea is that the more a firm sells, the firms average costs will decline. Recall that this differs relative the earlier presentation of the Ricardian model; there we assumed constant returns to scale. Constant returns to scale has the property that average costs are constant. In this set of notes we will assume declining average costs.

To be more specific about the technology of the firm we will describe the firms total costs as

$$C = F + c \times Q \quad (3)$$

where  $C$  is the firms total costs;  $F$  is the firms fixed costs; and  $c$  is a parameter determining a firm’s marginal costs.

So let’s define two concepts: (i) average cost and (ii) marginal cost. A firms average cost is total costs in (3) and divided by the quantity sold,  $Q$ . This gives

$$AC = \frac{F}{Q} + c, \quad (4)$$

which if you look carefully, it indicates that a firm’s average costs ( $AC$ ) decreases as it sells more ( $Q$  increases). You sell a lot, your average costs

are low; you sell a little, your average costs are high. This property comes from the presence of a fixed cost of production (given by  $F$ ).

A firm's marginal cost is the incremental costs to a firm from selling one extra unit of output. Either through calculus or intuitive arguments, you can calculate this as

$$MC = c, \quad (5)$$

which is a constant,  $c$ . One way to think about this is that the firm's marginal costs are the wages the firm pays for its production workers. So  $c$  would depend on the wage rate in the labor market. Another thing that goes into a firm's marginal cost of production is productivity. We typically think that more productive firms have lower marginal costs of production.

**Monopoly Equilibrium.** The profit maximizing monopolist sets marginal revenue equal to marginal cost and solves for prices and quantities. The key thing to understand is that the monopolist earns economic profits. It is straight forward to show mathematically or graphically that  $P > AC$  implying that the monopolist is making profits.

One important thing to note is how the firm's price relates to its marginal costs. Recall, that the firm sets marginal revenue to marginal cost. That is equate equation 2 with 5. This gives:

$$P = c + \frac{Q}{B}. \quad (6)$$

Notice that  $\frac{Q}{B} > 0$ , so this implies the firm is setting its price equal to its marginal cost *plus* a markup which is  $\frac{Q}{B}$ . Recall from micro, that in perfect competition, price reflects the firm's marginal costs. Here in the monopoly setting, the price now includes a markup. This markup is bad in the sense that it lowers consumer welfare. This observation is important to keep in mind in the monopolistic competition model—as we will see below, this markup changes as a country opens up to trade which is one source of the gains from trade in this model.

## 0.2 Monopolistic Competition

Monopolistic Competition relies on two key assumptions.

- First, each firm is assumed to produce a differentiated product. What we mean by this is that within an industry, we think of each firm producing a slightly different product from other firms in the industry. So each firm

produces a different product, but each product is a close substitute for other firms' products. As a real world example, you can think of smart phones: Apple produces a product (the Iphone) which is differentiated relative to Samsung (the Galaxy) and Microsoft/Nokia (Lumina), etc, but they are all close substitutes.

The purpose of this assumption is to give the firm some "local" monopoly power as each firm produces a unique product. This is why the model has the world "monopolistic" in it.

- Second, we assume that when the firm goes about making its pricing and production decisions, it takes other firms pricing and production decisions as given. Notice that this is very similar to what is assumed in models of perfect competition. In perfect competition, we assume that firms are small with respect to the market and thus their decisions have no effect on the market price.

In this situation, each firm sets its own price for its differentiated good, but this assumption means that the firm can abstract from thinking about how its pricing decisions will affect the decisions of other firms. This is the "competition" part of the word.

So the plan is to posit the firms demand curve for its product and solve for a market equilibrium in the two extreme cases: (i) autarky and (ii) frictionless trade. We will then compare these two cases to understand where the gains from trade come from.

**Demand.** What we are going to do is to describe the demand of a "typical" firm in a monopolistically competitive industry. The demand curve that this firm faces takes the following form

$$Q = S \times \left[ \frac{1}{n} - b \times (P - \bar{P}) \right]. \quad (7)$$

Where  $Q$  is the quantity demanded,  $S$  is the total output of the industry,  $n$  is the number of firms in the industry,  $b$  is a constant representing how elastic the a firm's sales are to its price,  $P$  is the price that this firm charges, and  $\bar{P}$  is the average price charged by its competitors. The intuition underlying this specification of the demand curve is simple. A firm's share of the industry output depends on the number of firms ( $n$ ) and the extent to which its price  $P$  differs from the the industry average.

Ok, so notice a couple of things about this demand curve. First, more overall demand (higher  $S$ ) or higher prices charged by the firms competitors (higher  $\bar{P}$ ) imply that the firm's quantity sold will be higher. In other words, if a lot of people demand these products and your competitors are charging high prices, you will sell more.

Second, if there are more firms ( $n$  is large) or your price is high ( $P$  is high), then you sell less. The last property is just a statement of the law of demand: if you raise your price, then the quantity demanded will decline. The first property is a bit less obvious, but all it is saying is that (holding  $S$  constant), if there are more firms, then the division of total demand amongst those firms is smaller. Thus this is a force at work to reduce the total quantity sold by an individual firm.

**Symmetry Assumption.** To make this model easy to solve, we assume that all  $n$  firms in the industry are symmetric (or identical). This means that...

- All firms have the same cost structure (outlined in equations 3, 4, 5).
- All firms (even though they produce differentiated products) face the same demand curve as shown in 7.

A couple of points about the assumption. To begin, it makes our life easy as it simplifies the computations below. However, the assumption of identical costs causes us to lose some important insights about which firms survive when a country opens up to trade. At the end of these notes we will revisit this issue and discuss the situation where firms differ in their cost structures.

## Autarky Market Equilibrium

We start with the autarky market equilibrium for a closed economy. Our task is to solve for: the prices (both  $P$  and  $\bar{P}$ ) and the number of firms ( $n$ ) that will operate in the industry.

How are we going to do this? The basic idea is to (i) utilize the assumption that all firms are symmetric and will make the same choices and (ii) rewrite the equations so it looks like the monopoly problem described above. We will then set marginal revenue equal to marginal cost and find  $P$  as a function of  $n$ . Finally, we will look for a “long-run” equilibrium where no profits are earned and price equals average costs (i.e.  $P = AC$ ). This will give us two equations in  $P$  and  $n$  which determines a market equilibrium.

Specifically, the assumption of symmetry has these implications:

- Since all firms are symmetric, they will have the same exact strategy, and thus charge the same price. This means that  $P = \bar{P}$  and thus now we only need to solve for  $P$ .

- Since all firms are charging the same price, it follows from 7, the quantity sold by each firm is  $Q = \frac{S}{n}$ . Again, because all firms are symmetric and following the same strategy, this is implying that the quantity that they sell, is simply the total industry demand  $S$  divided by the number of firms in the industry  $n$ . In other words, because all firms are symmetric, they have the same market share.

Given these observations, finding the equilibrium is simple. All we need to solve for is a price and the number of firms.

**Number of Firms and Average Costs.** Given the observations made above, we can relate an individual firm's average costs (see 4) to the number of firms and the size of the market by substituting in that  $Q = S/n$  to get

$$AC = \frac{F}{Q} + c, \quad (8)$$

$$= n \times \frac{F}{S} + c \quad (9)$$

Now here is an interesting feature of this model: The more firms there are (larger  $n$ ), average costs are higher. What this means then is that average costs are increasing the more firms there are in the industry. This is an implication of the fixed costs of production. If one firm services the industry, then that fixed cost is spread over the whole market  $S$ . If there are  $n$  firms operating in the industry, then each firm's market share is  $S$  divided by  $n$  firms. From one firm's perspective, its fixed costs are spread over a much smaller share of the market.

**Number of Firms and the Price.** Now here is the trick, we will rearrange the demand curve in 7, and show that it looks like 1 (the monopolist's demand curve). Then from there we can apply the same logic we discussed in the monopoly case.

So rearranging the demand curve 1 we have:

$$Q = \underbrace{\left[ \frac{S}{n} + S \times b \times \bar{P} \right]}_A - \underbrace{S \times b \times P}_B \quad (10)$$

which is of exactly the same form as the monopoly demand curve in 1 (that's why I have the underbraces showing how each term corresponds with the  $A$

and the  $B$  in the monopolists demand curve in 1).<sup>2</sup> This then implies that the firms marginal revenue is simply

$$MR = P - \frac{Q}{S \times b} \quad (11)$$

$$= P - \frac{1}{n \times b} \quad (12)$$

where 11 follows from substituting for  $B$  from 10 in 2. And the last step in 12 follows from the observation above that  $Q = \frac{S}{n}$ . What we have is the marginal revenue curve for  $a$  firm in this industry.

The final step is to take the marginal revenue of the firm and equate it with marginal costs in 5 and then solve for  $P$  in terms of the number of firms. Doing so gives

$$P = c + \underbrace{\frac{1}{n \times b}}_{\text{markup}}. \quad (13)$$

Now here is a second interesting feature of this model: The more firms there are (larger  $n$ ), prices are lower. Why are prices lower? Not because marginal costs are changing, those are fixed. Because markups  $\frac{1}{n \times b}$  are lower. The idea is super simple but powerful—more firms induce more competition and this shows up in lower markups over marginal cost and, in turn, lower prices for consumers. More competition leads to lower prices.

**Long-Run Market Equilibrium.** We are looking for an equilibrium where firms' prices are equal to their average costs, i.e.  $P = AC$ . This means that in equilibrium, economic profits are zero. This is sometimes called a long-run equilibrium.

Here is the idea: If the price is larger than average costs ( $P > AC$ ), then firms are earning positive economic profits. This would stimulate entry by new firms to try and capture some of these profits. As new firms enter the market two things occur: (i) average costs will increase and (ii) competition becomes fiercer and markups and prices will decrease. Both these forces are at work until there is no more incentive for firms to enter. That is  $P = AC$ .

Thinking through the opposite example is helpful. If ( $P < AC$ ), then firms are earning negative profits. Thus this situation would stimulate exit by incumbent firms. As firms exit two things occur: (i) average costs decrease

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<sup>2</sup>Recall that the general form of the monopolists demand curve is  $Q = A - B \times P$  and marginal revenues is then  $MR = P - Q/B$ .

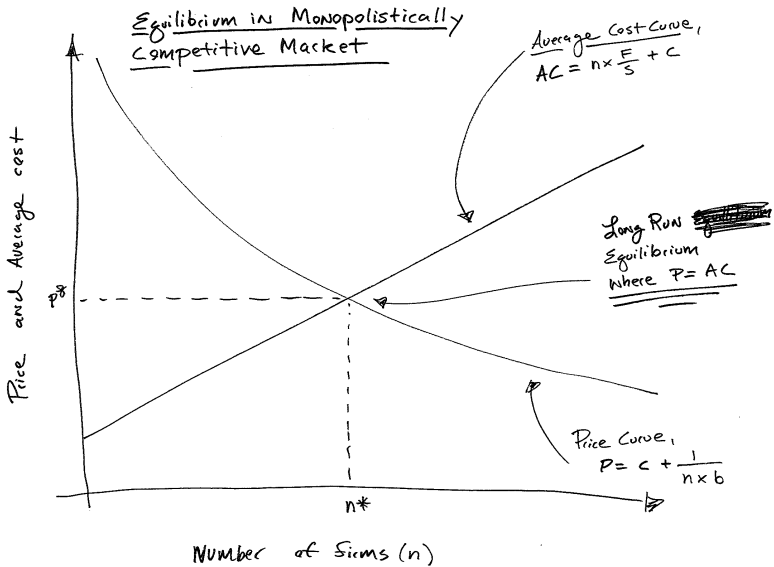


Figure 1: Equilibrium in a Monopolistically Competitive Market.

and (ii) reduced competition raises markups and prices. And all these forces push the economy back to the situation where  $P = AC$ .

Figure 1 illustrates this outcome. First, notice that the downward sloping line plots the price as it relates to the number of firms in the market (i.e. equation 13). Second, the upward sloping line plots average costs as it relates to the number of firms in the market (i.e. equation 9). Where these two curves intersect is where  $P = AC$  and the associated number of firms  $n^*$ . This is the market equilibrium.

### International Trade Market Equilibrium

Now let's consider an open economy where trade occurs. The way we will think about trade is the following: trade increases the industry demand  $S$  (loosely speaking it increases the "size of the market"). So mechanically, we will study an increase in market size from  $S$  to  $S'$  and call this the industry opening to trade.

How to think about this? Recall the smart phone example. If smart phones were only allowed to be sold in the US, total industry demand is limited by the approximately 300 million consumers in the US market. However,



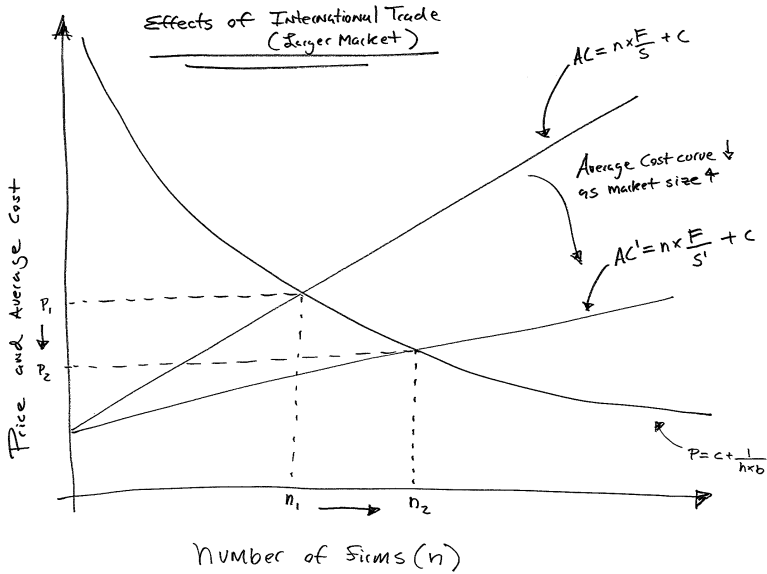


Figure 2: Effects of Trade (Larger Market Size)

if smart phones can be sold in the US and China, then the total industry demand would go up—a lot. So, in this model, we think about opening to trade as an increase in industry demand which was once limited by the border, but now combines the total industry demand of the world. The numerical example in the next section makes this idea even more concrete.

Industry demand increases from opening up to trade, then what happens? To answer this question, think of what would happen to the price and average cost curves in Figure 1. First, the price curve in Figure 1 (represented by equation 12) does not shift because the size of the market of  $S$  does not directly matter for the prices charged by firms

However, the average cost curve in Figure 1 (represented by equation 9) will rotate down. This is shown in 2. The idea here is simple. Holding fixed the number of firms, a larger market means there will be more sales per firm and this implies that the average costs of each firm declines. Again, this is about the fixed cost of production. Larger markets allow firms to “spread” the fixed costs over larger sales, which lowers their average costs.

Figure 2 illustrates how the long-run equilibrium changes. First, notice that in the new equilibrium, the industry supports a larger number of firms ( $n_1 < n_2$ ). Second, prices are lower. The later is interesting because marginal costs

did not change. This implies that the increase in the number of firms in the market results in lower markups and, hence, lower prices. In other words, a larger market allows more firms to operate and more firms results in fiercer competition resulting in lower prices.

What about the gains from trade? Let me summarize them here.

- **Trade leads to lower prices via increased competition.** This is obvious from Figure 2. Trade increases the purchasing power of consumers. This is one source of gains from trade.

The most important point is *why* lower prices are achieved. Recall that in the Ricardian model, opening to trade via comparative advantage allows countries to receive “lower prices” (more formally better terms of trade) to buy stuff. In this model, trade results in lower prices—but not through comparative advantage. In the monopolistic competition model, lower prices arise because of increases in competition because as more firms operate in the market. Some people call this effect the “pro-competitive” effect of trade.

- **Trade leads to more variety.** Recall that we think of a firm as a variety of the product. As illustrated in Figure 2, trade allows the industry to support a larger number of firms and, hence, more variety. The increase in variety is an additional gain from trade.

Why are increases in variety a gain from trade? One way to think about this is that when you show up to the store, there are now more options are available. And more options at least have to make you weakly better off relative to a situation with less options.

Think of cars. 40 years ago, options in the US were basically limited to the big three domestic auto makers (GM, Ford, Chrysler). Now, because of trade there are many more options available. So if you are diehard Ford consumer, no big deal. But if not, now you have the option to buy a Honda compared to the situation when the US auto market was relatively closed and this was not possible.

Again, this gain from trade is distinct from the gains in the Ricardian model. In the Ricardian model, the consumption bundle is fixed. Whether I’m open to trade or not, I’m always able to buy the same two goods. In the Ricardian model, trade just allows me to buy *more* of those same two goods. In the monopolistic competition model, trade provides more types of goods that were not available before.

Now there is an important, but subtle point to recognize: Some domestic firms may exit the industry, even though the total number of firms (both domestic and foreign) increased. This point has implications for the potential losers from trade and as an additional source of gains from trade.

Here is the way to think about this. Suppose we have two symmetric countries that are closed economies. In Figure 2, we can think of  $n_1$  as being the number of firms operating in each country when in isolation. Now when the countries integrate, the market size doubles from  $S$  to  $S'$ . However, the number of operating firms did not double. One can show that the total number of firms operating must be less than double. Before trade there were  $n_1$  firms in each country, now there are  $n_2 < 2 \times n_1$  firms.<sup>3</sup> This implies that some firms had to have exited the industry.

Similar to the Ricardian model, this implies that there are potential short run costs to international trade. Employees in firms that exit will lose their job. As we argued in the Ricardian model, these workers will eventually be reallocated to firms or industries that are expanding. But because labor markets do not operate frictionlessly in the real world, this implies there will be short run pain.

Second, certain consumers might loose as well. For example, if you loved a certain variety and that firm/variety exited (do you know what a Zune MP3 player is), then you are out of luck. Does this imply that trade is bad? No. My claim is that, on average, consumers gain from the expansion of variety.

### Numerical Example

This section walks through a numerical example to illustrate each of these points discussed above.

**Some Preliminaries.** To solve for an equilibrium, we set the price equal to average cost. This is shown below:

$$\underbrace{c + \frac{1}{n^* \times b}}_P = \underbrace{c + n^* \times \frac{F}{S}}_{AC} \quad (14)$$

$$\Rightarrow \frac{1}{n^* \times b} = n^* \times \frac{F}{S}$$

where the first line sets the price curve equal to the average cost curve. Then the second line illustrates that the equilibrium number of firms,  $n^*$ , must satisfy the bottom equation because the additive marginal cost terms cancel. Then from the bottom line we can solve for the equilibrium number of firms

$$n^* = \left( \frac{1}{b} \times \frac{S}{F} \right)^{\frac{1}{2}}. \quad (15)$$

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<sup>3</sup>To be clear, total variety did increase. From the perspective of a consumer, prior to opening to trade he had  $n_1$  varieties available to him. After trade he has  $n_2$  varieties available to him. So more options are available.

Equation 15 gives an expression relating characteristics of the industry to the equilibrium number of firms in the industry. Stare at equation 15 a bit and you can see (i) how larger markets result in more firms operating (ii) lower fixed costs  $F$  results in more firms operating (i.e. it is easier to enter). All this makes sense. Equation 15 is important because, given characteristics of the industry, we can compute the number of firms. And then from the price curve in equation 13, we can compute the price.

**A Numerical Example.** Table 1 reports some hypothetical industry characteristics in both the US and Mexico. First, we want to compute an equilibrium in each of these countries in autarky. That is when total industry demand in the US is only 16,000 units what is the US equilibrium price and number of firms. Then when total industry demand in Mexico is only 9,000 units, what is the Mexico equilibrium price and number of firms. Then we will think about what happens with trade, i.e. total industry demand combines the demand in the US and Mexico,  $16,000 + 9,000$ . Finally, in this example, we assume that firm characteristics and demand are the same in both countries.

Table 1: Industry Characteristics.

	USA	Mexico
Fixed Cost, $F$	75,000	75,000
Marginal Cost, $c$	5	5
Demand elasticity, $b$	$\frac{1}{300}$	$\frac{1}{300}$
Industry Demand in Autarky	16,000	9,000

- **Autarky in the United States.** First, use 15 to find the number of firms operating. Plugging in the numbers gives

$$\left( \frac{1}{1/300} \times \frac{16,000}{75,000} \right)^{\frac{1}{2}} = 8. \quad (16)$$

This tells us that the equilibrium number of firms in the US market is 8. Then to determine the price, we use the price curve in 13 to obtain

$$5 + \frac{1}{1/300} \times \frac{1}{8} = 42.5. \quad (17)$$

The equilibrium price in the US market is 42.5 dollars.

- **Autarky in Mexico.** Same thing! Use 15, which after plugging in the numbers

$$\left( \frac{1}{1/300} \times \frac{9,000}{75,000} \right)^{\frac{1}{2}} = 6, \quad (18)$$

which tells us that the equilibrium number of firms in Mexico is 6. Then to determine the price, we use the price curve in 13 to obtain

$$5 + \frac{1}{1/300} \times \frac{1}{6} = 55. \quad (19)$$

So the equilibrium price in Mexico is 55 dollars.

Before moving on, note the consequences of Mexico being a relatively small market: fewer firms are able to operate (i.e. there is less variety) and this results in higher prices because there is less competition.

Now we want to examine when both countries are open to trade. In the monopolistic competition model, what we mean by this is that the two industries are now integrated. Thus, total industry demand is now the sum of the demand coming from the US and the demand coming from Mexico or 16,000 + 9,000.

To compute the equilibrium number of firms in the integrated market we again use 15 which gives

$$\left( \frac{1}{1/300} \times \frac{16,000 + 9,000}{75,000} \right)^{\frac{1}{2}} = 10. \quad (20)$$

This tells us the equilibrium number of firms in the integrated market is 10. There are several things to notice here. First, total variety available to US and Mexican consumers increased. Prior to the markets being integrated, US consumers had access to only 8 varieties, now they have access to 10. Similarly, Mexican consumers had access to only 6 varieties and now they have access to 10. So consumers in both markets have more choices.

With that being said, notice that some firms had to have exited the industry. In autarky, the total number of firms operating in the US plus the total number of firms operating in Mexico is 14. In the integrated market, only 10 operate. So 4 firms must have exited the industry. Were these Mexican firms? US firms? We don't know because all firms are identical here. However, this observation does make the discussion above about the potential short-run costs of trade tangible. Workers who were employed by these firms may lose their jobs. Consumers who had a very strong attachment to these brands may be disappointed as these brands exited.

What about prices? Using the price curve given in 13

$$5 + \frac{1}{1/300} \times \frac{1}{10} = 35. \quad (21)$$

Table 2: Gains from Trade (Market Integration).

	USA	Mexico	Integrated
Number of Firms/Varieties	8	6	10
Price	42.5	55	35

This tells us that the equilibrium price declined as well. Consumers in both countries can now purchase these goods at lower prices. This is a good thing (who does not like lower prices) and illustrates the second source of gains from trade in this model. Table 2 summarizes the results.

It's worth emphasising what is going on here. Larger markets support a larger number of firms because average costs are lower. This leads to a gain in variety. More firms results in more competition, lower markups, lower prices. This leads to a gain in purchasing power. Furthermore, these gains are distinct relative to the Ricardian model where variety is fixed, and prices are lower but not because of pro-competitive effects.

## Heterogeneity in Productivity/Costs

Everything described up to this point assumed that firms were identical. Over the past 20-30 years, there has been a large amount of accumulated evidence showing firm performance (within very narrowly defined industries) can vary dramatically. Moreover, this evidence has shown that firm performance correlates strongly with a firms ability to export and succeed in international markets. In this section, I'm going to talk you through the implications of these facts in the the monopolistic competition model and how these facts suggest that there is another source of gains from trade.

In the model above, we assumed that all firms were identical and symmetric. Now we want to entertain a situation where firms differ in their marginal cost of production (i.e. some firms are more productive than others). Analytically this is hard to solve, be we can talk through this and get the flavor about what will occur.

Recall that when a country opens up to trade, some domestic firms will exit the industry.<sup>4</sup> Now ask yourself the question: If firms differ in their productivity (i.e. some are more productive than others), which domestic firms will exit? The high productivity firms? No, they are the firms most

<sup>4</sup>Not to confuse you, but it is important to emphasize that the total number of firms (domestic plus foreign) operating in the world goes up, just the number of operators in each country goes down.

likely to survive. The low productivity firm? Yes, these are the weakest firms and the competition from foreign firms will push them to exit the industry.

If the worst performing firms exit, what will then happen to average industry productivity in each country? Average productivity increase! Here opening to international trade “selects” only the most productive firms and it results in an increase in productivity. This is an additional source of gains from trade.

## **Executive summary**

1. The monopolistic competition focuses on the role of a firm in international trade and how industry responses lead to new and different sources (relative to the Ricardian Model) of the gains from trade.
2. Opening to trade leads to three sources of the gains from trade (i) lower markups and prices (ii) more variety and (iii) only the best/most productive firms survive foreign competition leading to increases in average industry productivity.

## Appendix

This section derives the marginal revenue curve in equation 2. There are several ways to do this. I will show you one intuitive way to do this. Then a second “brute force” approach.

**The “Finessed Approach.”** The monopolists revenue can be expressed as

$$R = P(Q) \times Q \quad (22)$$

where the price’s dependence on  $Q$  is noted. Now we want to think of how much will revenue change given a small change ( $\Delta$ ) in quantity. This is

$$\Delta R = P \times \Delta Q + Q \Delta P \quad (23)$$

which is just an application of the chain rule. Now let’s divide this expression through by  $\Delta Q$  and we have

$$\frac{\Delta R}{\Delta Q} = P + Q \frac{\Delta P}{\Delta Q} \quad (24)$$

What do we have here?  $\frac{\Delta R}{\Delta Q}$  is marginal revenue, that is how much revenue changes given a small change in the quantity. Then this says that marginal revenue equals the price plus  $Q$  times how the price responds to quantity.

To fill in this last bit, we can invert the demand curve to get

$$P = \frac{A}{B} - \frac{Q}{B} \quad (25)$$

which implies that  $\frac{\Delta P}{\Delta Q}$  is simply equal to  $\frac{-1}{B}$ . Then plug this into 24, we have marginal revenue (for the linear demand curve) expressed as

$$MR = \frac{\Delta R}{\Delta Q} = P - \frac{Q}{B}, \quad (26)$$

which is the same as in equation 2.

**The “Brute Force Approach.”** The plan here is simply to compute the derivative of the revenue function and then make some smart substitutions/rearrangements. So the revenue function is

$$R = P(Q) \times Q \quad (27)$$

$$R = \left( \frac{A}{B} - \frac{Q}{B} \right) Q \quad (28)$$



where  $R$  is total revenue and I substituted in that  $P = (\frac{A}{B} - \frac{Q}{B})$ . Then differentiating  $R$  with respect to  $Q$  tells us how much revenue changes with respect to a small change in quantity (i.e. marginal revenue (MR)),

$$MR = \frac{A}{B} - \frac{2Q}{B} \quad (29)$$

then I'm going to rearrange this in a very specific way

$$MR = \frac{A}{B} - \frac{Q}{B} - \frac{Q}{B} \quad (30)$$

$$MR = \left( \frac{A}{B} - \frac{Q}{B} \right) - \frac{Q}{B}, \quad (31)$$

and then notice the term in parentheses is the same as the demand curve in 25. Substituting this in gives

$$MR = P - \frac{Q}{B}. \quad (32)$$

which is the same as the marginal revenue curve shown in equation 2 of the text.

