

# Small Assignment 1 Solutions

## Part 1.

We are going to explore the nuts and bolts of regressions by doing one by hand. *Whether you are using R or excel, please do not use canned functions to calculate sample variances and covariances and to estimate regression coefficients.*

Start by reading in the `energy.csv` file.

```
In [1]: #Read in the data
df <- read.csv('energy.csv', header=T)
```

(a) Create two new variables, GDP (\$) per capita and energy use (KT oil equivalent) per capita. *Hint: use the mutate function.*

```
In [2]: #Load tidyverse
library(tidyverse)
#Generate per capita variables
df <- mutate(df, gdp_pc= gdp/pop, energy_pc=energy_use/pop)
#head(df)
```

Warning message:

"As of rlang 0.4.0, dplyr must be at least version 0.8.0.

\* dplyr 0.7.8 is too old for rlang 0.4.5.

\* Please update dplyr to the latest version.

\* Updating packages on Windows requires precautions:

<<https://github.com/jennybc/what-they-forgot/issues/62>>"

```
-- Attaching packages ----- tidyverse
1.2.1 --
```

```
v ggplot2 3.1.0      v purrr 0.2.5
```

```
v tibble 2.0.1       v dplyr 0.7.8
```

```
v tidyr 0.8.2        v stringr 1.3.1
```

```
v readr 1.3.1        v forcats 0.3.0
```

```
-- Conflicts ----- tidyverse_conflicts() --
```

```
x dplyr::filter() masks stats::filter()
```

```
x dplyr::lag()     masks stats::lag()
```

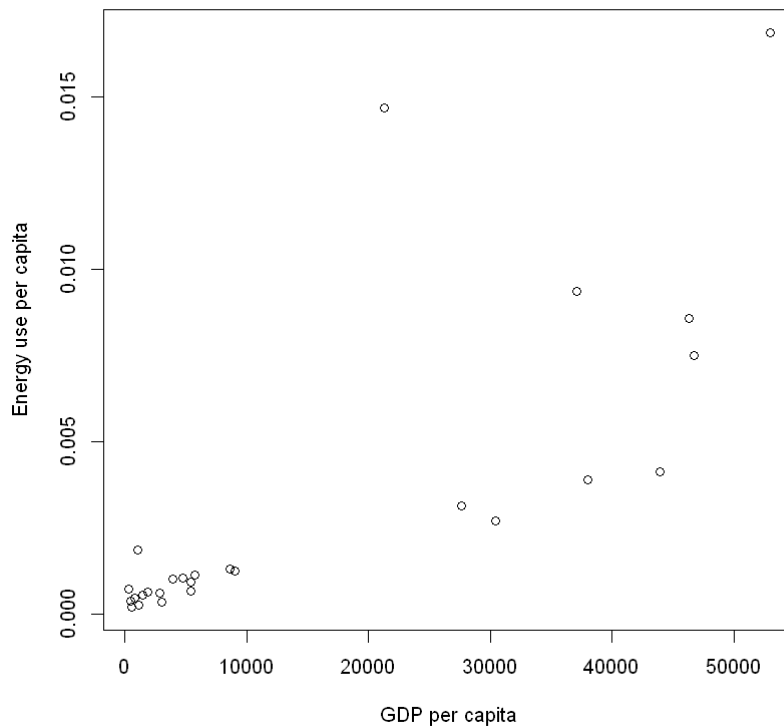
Warning message:

"The `printer` argument is deprecated as of rlang 0.3.0.

This warning is displayed once per session."

**(b)** Generate a scatter plot comparing energy use per capita (on the y-axis) to GDP per capita (on the x-axis) for the countries in your sample. *Hint: Use the `plot` function.*

```
In [3]: #Simple scatterplot
plot(df$gdp_pc, df$energy_pc, xlab='GDP per capita', ylab='Energy use pe
r capita')
```



**(c)** Calculate the sample mean of GDP per capita, sample mean of energy per capita, covariance of GDP per capita and energy use per capita, and variance of GDP per capita.

```

In [4]: #Get means
xbar <-mean(df$gdp_pc)
ybar <-mean(df$energy_pc)

#Create new columns with residuals (x_i-\bar{x}) and (y_i-\bar{y}) as part of df
df <- mutate(df, xres= gdp_pc-xbar, yres= energy_pc-ybar)
#Create new columns with _i*y_i and x_i^2
df <- mutate(df, xy= xres*yres, xx=xres^2)

#Get covariance and variance by summing up xy and xx and dividing by n-1.
covxy<- sum(df$xy)/(nrow(df)-1)
varx<- sum(df$xx)/(nrow(df)-1)

#Display results
c(xbar, ybar, covxy, varx) #note the use of c to create a vector!

```

15419.2698158678 · 0.0032316582767966 · 62.272034896757 · 326578025.706693

**(d)** Suppose  $y_i$  is energy use per capita in country  $i$  and  $x_i$  is GDP per capita in country  $i$ . Suppose your statistical model is

$$y_i = \beta_0 + \beta_1 x_i + u_i.$$

Estimate  $\hat{\beta}_0$  and  $\hat{\beta}_1$  using your results in part **(c)**.

```

In [5]: #Solve for \hat{\beta}_1 and \hat{\beta}_0
b1hat<- covxy/varx
b0hat<- ybar-xbar*b1hat
#Display
c(b0hat,b1hat)

```

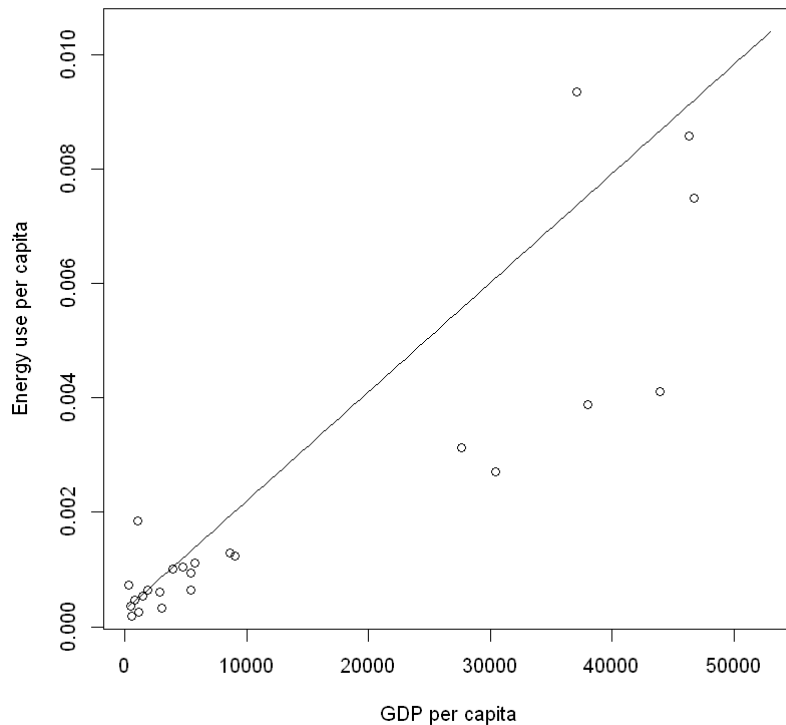
0.000291505442023208 · 1.90680419363809e-07

**(e)** Plot a line which indicates the predictions from your estimated model,

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i.$$

Hint: use the `curve` function.

```
In [6]: #plot the line
curve(b0hat+b1hat*x, min(df$gdp_pc), max(df$gdp_pc), xlab='GDP per capita', ylab = 'Energy use per capita')
#overlay with data points
points(df$gdp_pc,df$energy_pc)
```



(f) Interpret your estimates  $\hat{\beta}_0$  and  $\hat{\beta}_1$ .

$\hat{\beta}_0 = 0.00003$  means that the model predicts that a country with 0 GDP would have energy use equivalent to .0003 KT of oil.  $\hat{\beta}_1 = 1.9e-07$  means that model predicts that each additional dollar of GDP leads to 1.9e-07 addition KT of oil equivalent energy use.

## Part 2.

Suppose you estimate a weekly wage equation which suggests

$$\log(wage_i) = 5.45 + .09 * Ed_i + u_i$$

where  $Ed_i$  is the number of years of education completed by person  $i$ .

(a) Predict the wage for a person with 12 years of education (that is a person who completes high school).

```
In [7]: exp(5.45+.09*12)
```

685.398211491809

**(b)** Interpret your estimated  $\beta_1 = 0.09$ .

An additional year of education is associated with 9% higher weekly wages.