EPI 809

(Biostatistics II)

Introduction:

$\Rightarrow$ Syllabus

$\Rightarrow$ GitHub (Scripts and other materials)

$\Rightarrow$ D2L (INCLASS, HW, Grades)

Types of Random Variables

⇒ **Quantitative (usually continuous).** Can take on an (conceptually) infinite number of values, and there is a notion of order and distance between values (e.g., body height)

⇒ **Categorical**; take on a finite number of values (e.g., presence/absence of disease).

⇒ Categorical RVs can be divided into various subtypes (binary, ordinal, multinomial)

Example

CPS5 data set (Goldberger 1998, adapted from Berndt 1991)
http://www.hup.harvard.edu/features/golint/CPS5.txt.

⇒ This data comprise information from 528 people surveyed in 1985.

⇒ The variables included in the data set are:

- years of education (integer, but can be analyzed as continuous)
- years of experience in the labor market (integer, but can be analyzed as continuous)
- wage USD/hour (continuous)
- Sex (Male/Female, binary)
- Region (South/non-South, multinomial)
- Marital status (married/ not married)

⇒ We illustrate regression analysis to quantify effects of education on wages, after accounting for differences due to sex, and region.

The first rows of the CPS5 data set…

education	south	ehtnicGroup	female	married	experience	unionized	hourlyWage
10	0	White	0	1	27	0	9
12	0	White	0	1	20	0	5.5
12	0	White	1	0	4	0	3.8
12	0	White	1	1	29	0	10.5
12	0	White	0	1	40	1	15
16	0	White	1	1	27	0	9
12	0	White	1	1	5	1	9.57
14	0	White	0	0	22	0	15
8	0	White	0	1	42	0	11

Quantitative

- Education
- Experience and
- Hourly-wage

Multinomial:

- Ethnicity

Binary:

- Region (south=1)
- Sex (female=1)
- Married (yes=1)
- Unionizes (yes=1)

I-Loading the data in R from an text-file

```
DATA=read.table(file='~/Dropbox/STATCOMP/2018/wage.txt',header=T)
```

II-Checking and inspecting the data set

```
dim(DATA)  # returns number of rows and columns
str(DATA)  # describe the data type and structure
head(DATA) # shows the first rows
tail(DATA) # same for last rows
fix(DATA)  # shows data in an spreadsheet format
summary(DATA) # summaries by variable
```

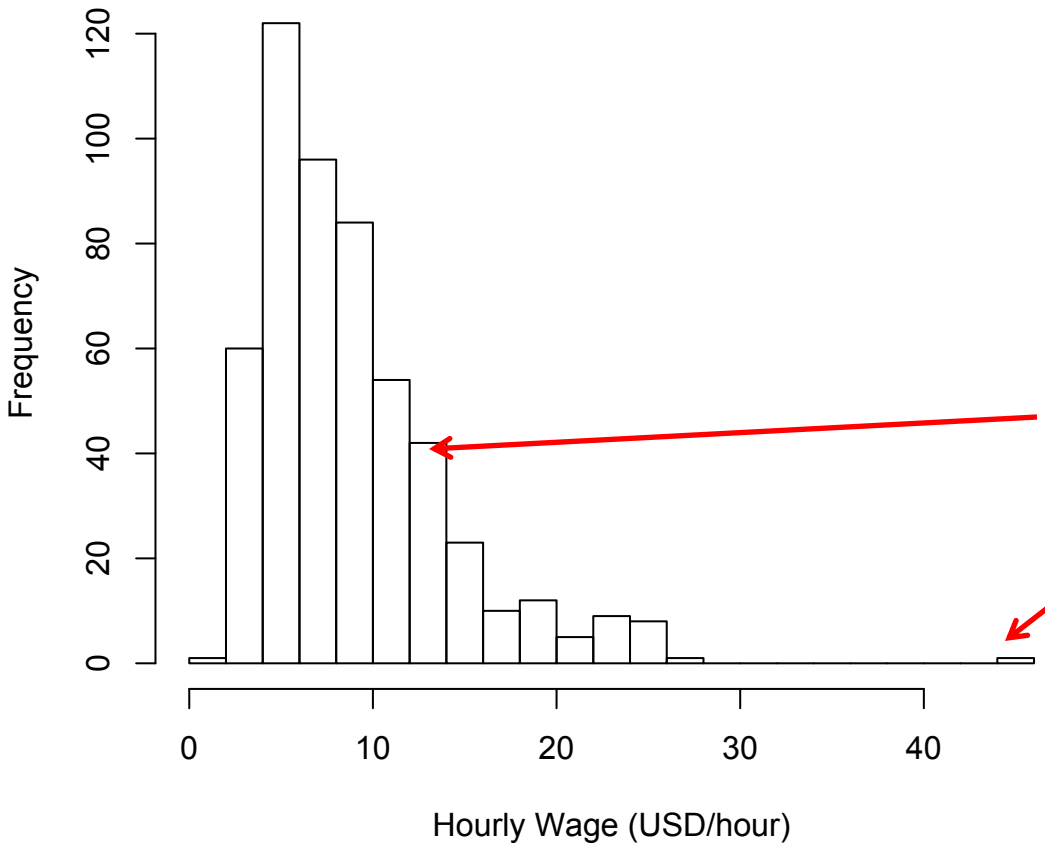
III-Univariate & Bi-variate descriptive analysis

Hourly Wages

```
hist(DATA$Wage,30)
summary(DATA$Wage)
```

```
> summary(DATA$Wage)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.750   5.250   7.790   9.048  11.250  44.500
```

Histogram



The variable on the x-axis is divided into bins of equal width.

Here, each bin has $2 increments.

The height of each bar reflects the number of subjects falling in each bin.

Here, 40 people make $12-14 USD/hr

Outlier?

The distribution appears to be skewed

Univariate Descriptive Statistics & Graphs: Continuous RVs

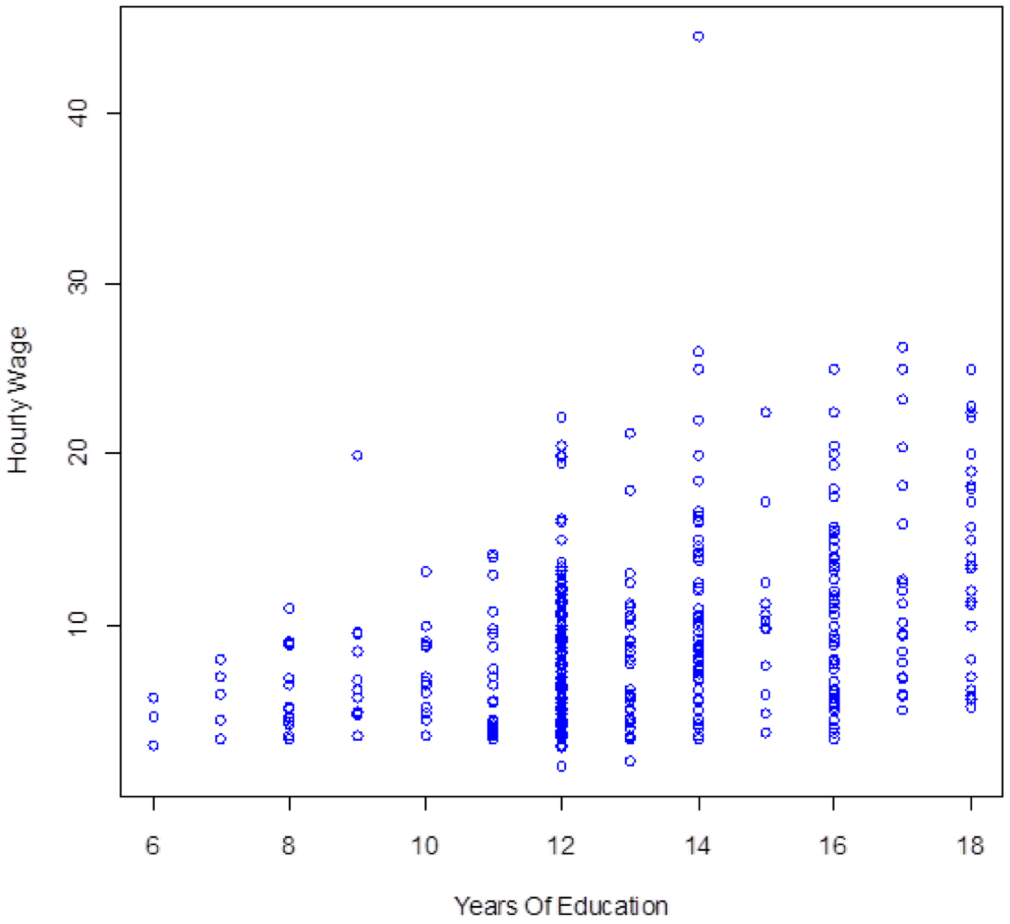
```
table(DATA$Education)
```

```
> table(Education)
Education
  6   7   8   9  10  11  12  13  14  15  16  17  18
  3   5  15  12  17  27 218  37  56  13  70  24  31
>
> summary(DATA$Education)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   6.00   12.00   12.00   13.09   15.00   18.00
```

Bi-variate analysis

Bi-variate Descriptive Statistics (Quantitative variables)

```
> plot(Wage~Education, col='blue',data=DATA)
```

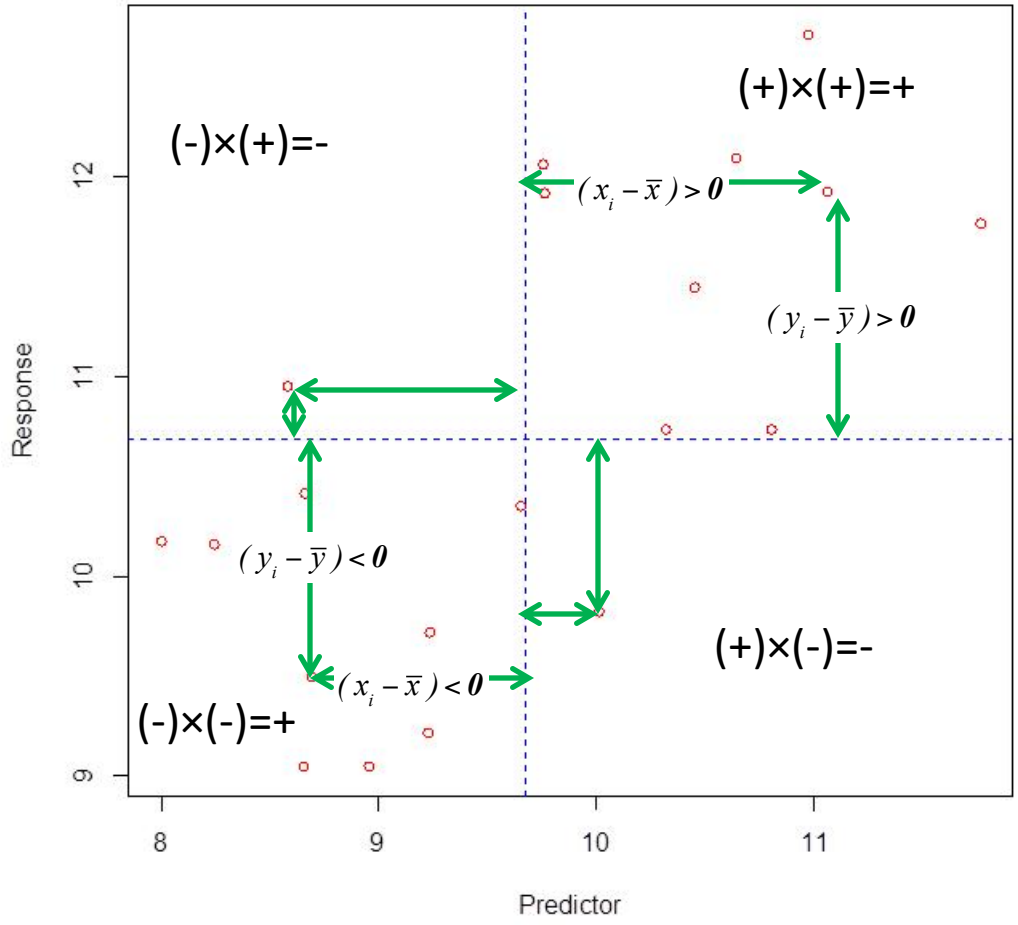


Key Concepts we will discuss

- Variance & Co-variance
- Correlation

Quantifying Linear Dependence Between Two Continuous Variables

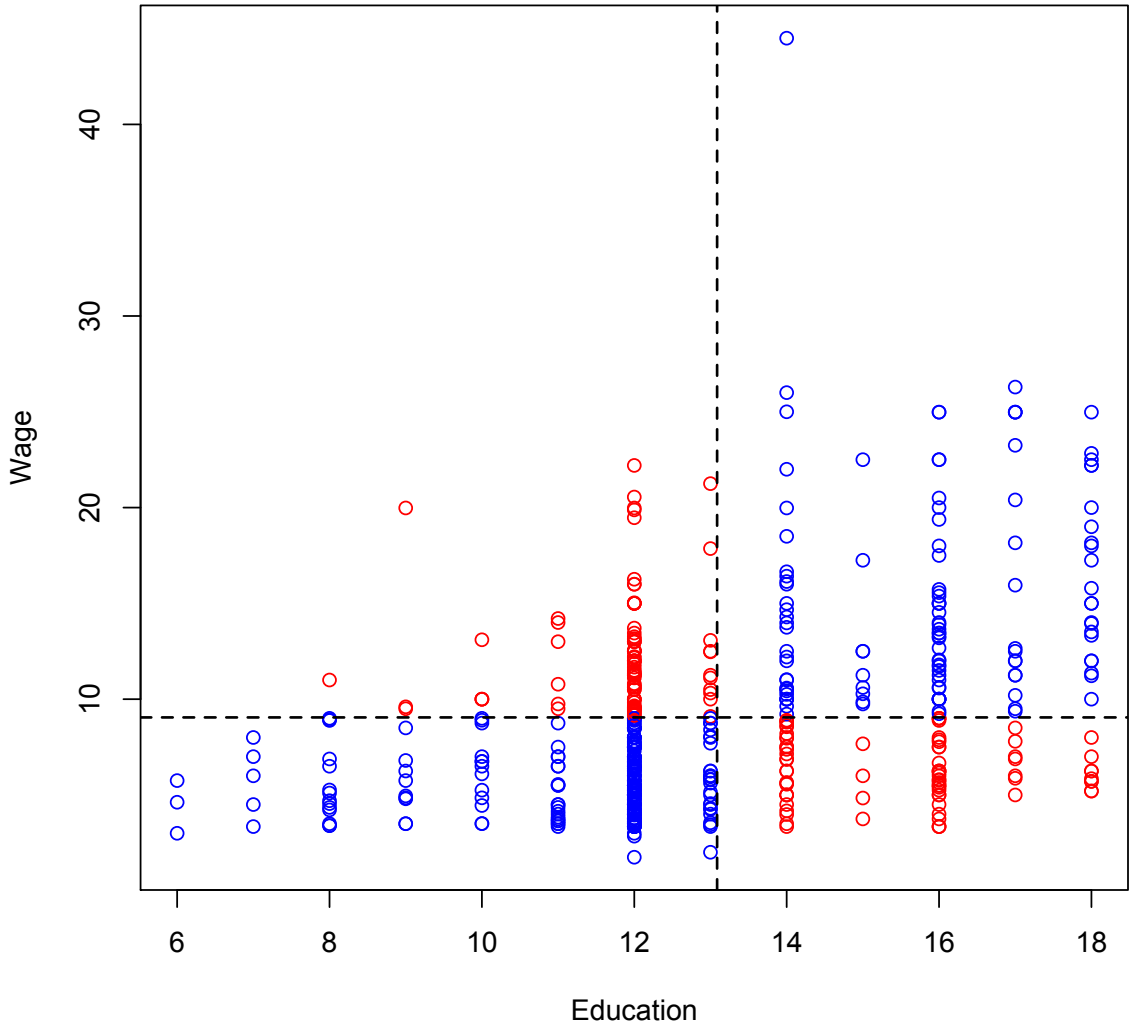
Quantifying Co-variability (co-variance)



Observation	x_i	y_i	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})(y_i - \bar{y})$
1	10.65	12.09	0.97	1.40	1.37
2	11.76	11.77	2.09	1.08	2.26
3	10.80	10.73	1.13	0.04	0.05
4	9.23	9.22	-0.44	-1.47	0.65
5	10.46	11.45	0.78	0.76	0.59
6	9.23	9.72	-0.44	-0.97	0.43
7	8.66	10.42	-1.01	-0.27	0.28
8	10.97	12.72	1.30	2.02	2.63
9	9.76	11.92	0.09	1.23	0.11
10	8.66	9.05	-1.02	-1.64	1.67
11	11.06	11.93	1.39	1.24	1.72
12	8.25	10.16	-1.43	-0.53	0.76
13	8.69	9.50	-0.98	-1.19	1.18
14	10.01	9.82	0.34	-0.87	-0.29
15	10.32	10.74	0.65	0.05	0.03
16	8.58	10.95	-1.09	0.26	-0.28
17	9.65	10.35	-0.02	-0.34	0.01
18	8.95	9.05	-0.72	-1.64	1.18
19	8.00	10.18	-1.67	-0.52	0.86
20	9.76	12.06	0.08	1.37	0.11
Sum	193.45	213.82	0.00	0.00	15.31
Average	9.67	10.69	0.00	0.00	0.765
Sample Covariance					0.806

$$S_{XY} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$$

Quantifying Co-variability (co-variance)




```
x=DATA$Education
y=DATA$Wage
meanX=mean(x)
meanY=mean(y)
dx=(x-meanX)
dy=(y-meanY)

signColor=ifelse(sign(dx)*sign(dy)==1,
                 'blue','red')

plot(Wage~Education,data=DATA,
             col=signColor);
abline(h=meanY,lwd=1.5,lty=2)
abline(v=meanX,lwd=1.5,lty=2)
```

Discuss In-class #1 (see https://github.com/gdlc/EPI809/)

Variance & Covariance

$$S_{XY} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$$

Sample co-variance: Measures Association between Y and X.

$$S_{XX} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(x_i - \bar{x}\right)$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

Sample variance: Measures Variability.

Sample variance is simply the sample covariance of a variable with itself.

Covariance is location invariant but scale dependent

$$S_{XY} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$$

Location Invariant: adding a constant to any of the variables does not change the sample co-variance (why? Because Sxy is based on deviations from the sample mean)

Scale dependent: multiplying any of the variables by a constant will alter the Sxy.

Proof: let Z=a+b×Y

$$S_{XZ} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(a + by_i - \frac{1}{n}\sum_{j=1}^{n} a + by_j\right)$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(a + by_i - \frac{n}{n}a - \frac{1}{n}\sum_{j=1}^{n} by_j\right)$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(by_i - b\bar{y}\right)$$

$$= b\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$$

$$= bS_{XY}$$

Co-variance is location invariant but scaled-dependent.

Summary: Sample Covariance

$\Rightarrow$ Measures patterns of (linear) association between two variables

$\Rightarrow$ Sxy is computed using cross-products of deviations from the sample mean.

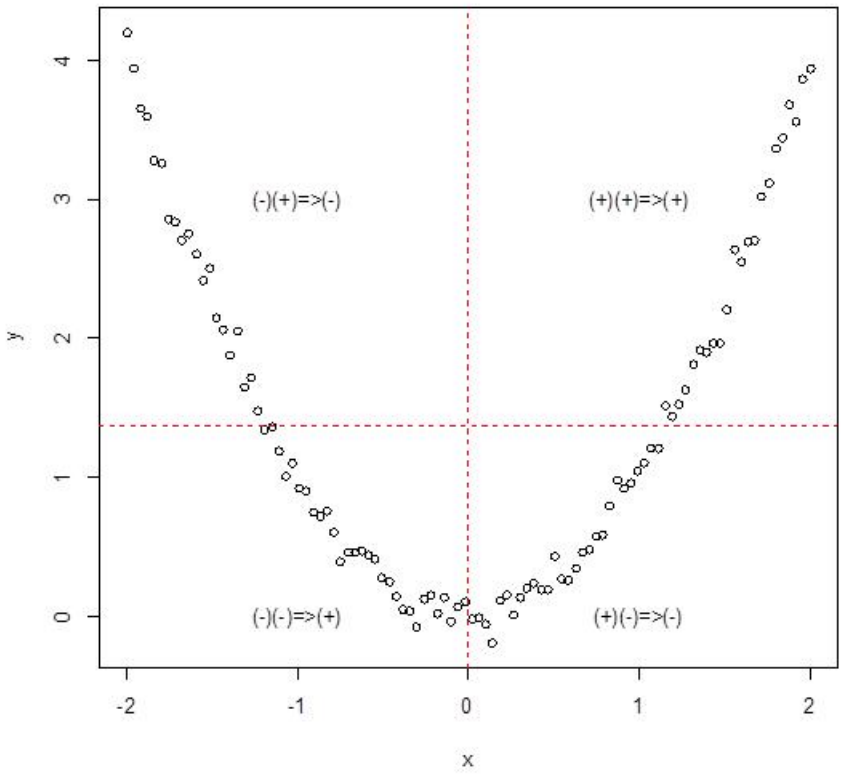
$\Rightarrow$ Therefore, it is location invariant (adding a constant to either x or y does not change covariance).

$\Rightarrow$ However, it is scale dependent.

$\Rightarrow$ Next we will discussed a standardized version of the co-variance that is scale invariant: Pearson's Product Moment Correlation Coefficient.

Covariance Versus Independence

Covariance Vs Independence



⇒ Absence of covariance does not imply independence.

⇒ Two random variables are independent if the joint distribution p(X,Y) is equal to the product of the marginal distributions, that is p(X,Y)=p(X)p(Y).

⇒ Independence implies that the means, variances (and all the moments) of one of the RV does not depend on the other.

⇒ Absence of covariance implies absence of dependence of the mean E[Y|X] only in a linear sense.

Pearson’s Correlation Coefficient

Correlation: a scale-invariant measure of linear association.

$$R_{yx} = \frac{S_{XY}}{\sqrt{S_{XX} S_{YY}}}$$

$$-1 \leq R_{yx} \leq 1$$

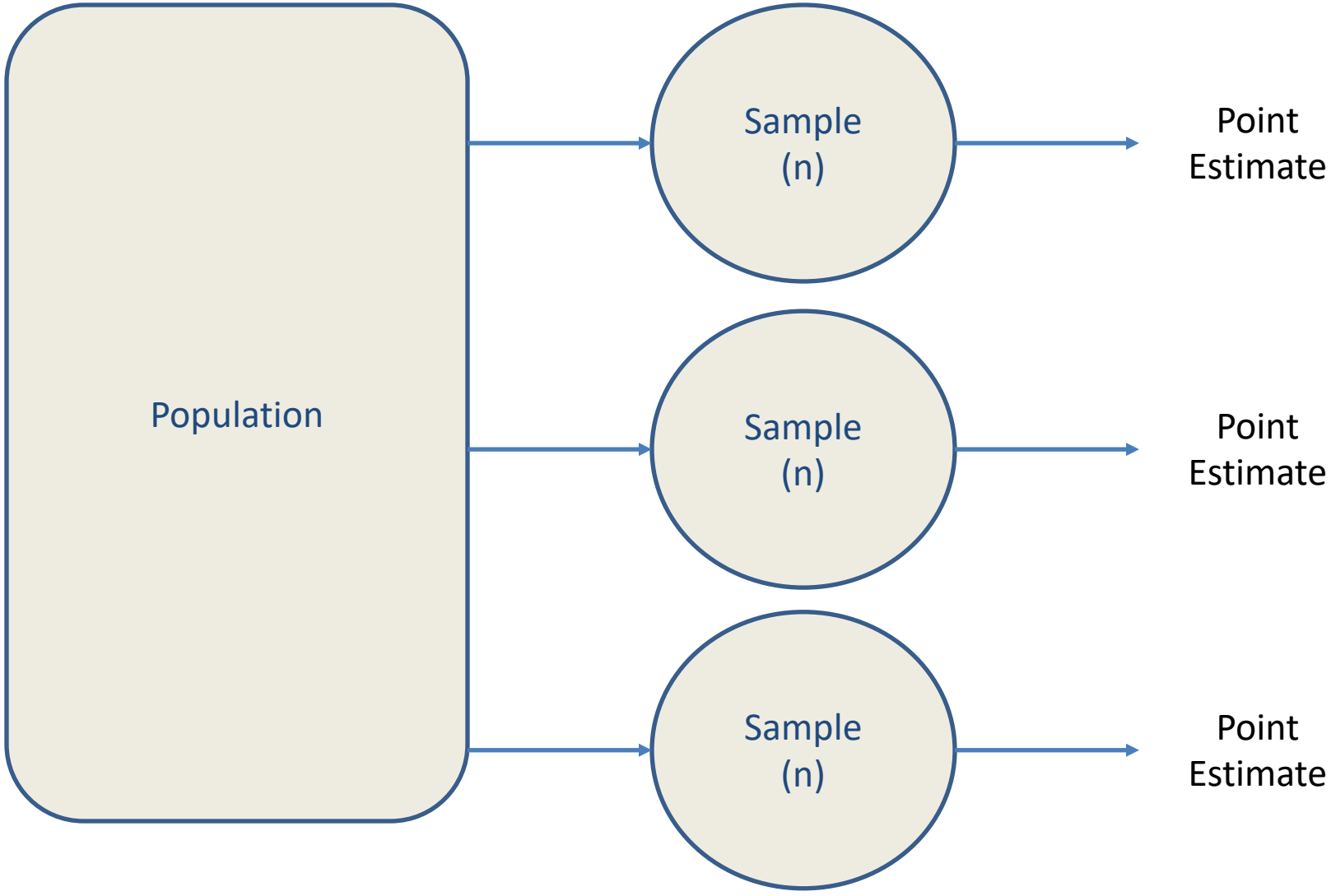
Pearson's product-moment correlation:

- Measures linear association
- Absence of correlation does not mean independence.
- It is scale and location invariant
- COR=0 indicates absence of linear association,
- Positive correlation means that as X increases so does Y,
- Negative correlation means that as X increases Y decreases
- COR = +/- 1 implies perfect linear dependence between X and Y.

Inference on the correlation coefficient

Statistical Inference

(& Conceptual Repeated Sampling from the Population)



Statistical Inference

(& Conceptual Repeated Sampling from the Population)

$\Rightarrow$ $\boldsymbol{\theta}$, a population parameter of unknown value (e.g., correlation coefficient)

$\Rightarrow$ $\widehat{\boldsymbol{\theta}}$, a point-estimate derived from a finite sample (e.g., the correlation of two variables in a sample)

$\Rightarrow$ $\widehat{\boldsymbol{\theta}}$ informs us about $\boldsymbol{\theta}$, but we have uncertainty about $\boldsymbol{\theta}$ because $\widehat{\boldsymbol{\theta}}$ was derived from a finite sample.

$\Rightarrow$ How do we quantify such uncertainty?

$\Rightarrow$ We often use Standard Errors (SE), which represent the standard deviation of the estimator over conceptual repeated sampling.

$\Rightarrow$ We can also build confidence intervals, intervals that over conceptual repeated sampling will contain the true population parameter x% of the time (x=confidence)

$\Rightarrow$ How do we estimate SEs and CIs?

Inference on the correlation coefficient

Standard Error: $$SE(r) = \sqrt{\frac{1-r^2}{n-3}}$$

t-stat: $$tstat = \frac{r}{\sqrt{\frac{1-r^2}{n-3}}}$$

For n reasonably large (e.g., >50) and intermediate coefficient, we can safely assume that *r* follows a normal distribution.

Under normality, a CI is given by

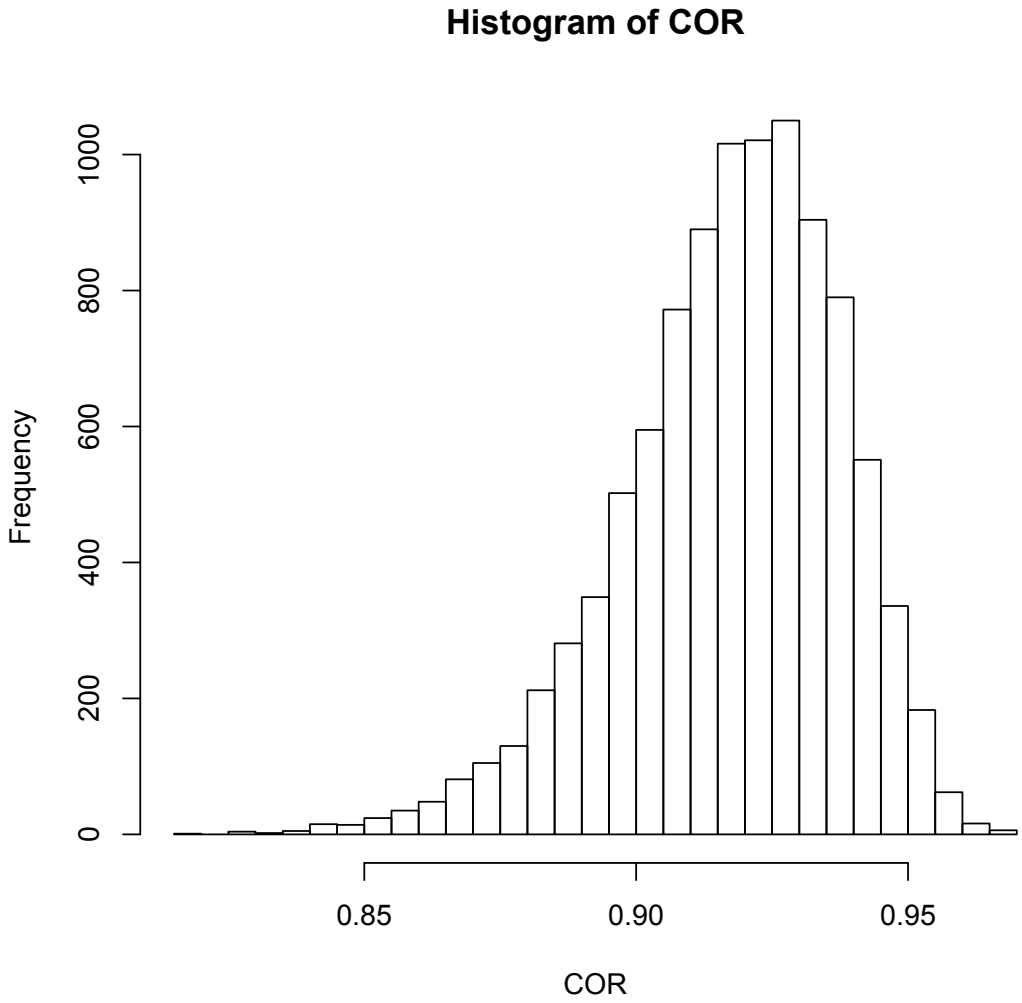
$$r \pm qnorm\left(mean = 0, sd = SE(r), p = 1 - \frac{x}{2}\right)$$

For 95% (99%) CI use x=0.05 (0.01)

However, when n is small and/or the correlation coefficient is close to either 1 or -1, the sampling distribution is not symmetric.

In this case we may want to use Fisher's z-transform

Distribution of the correlation coefficient

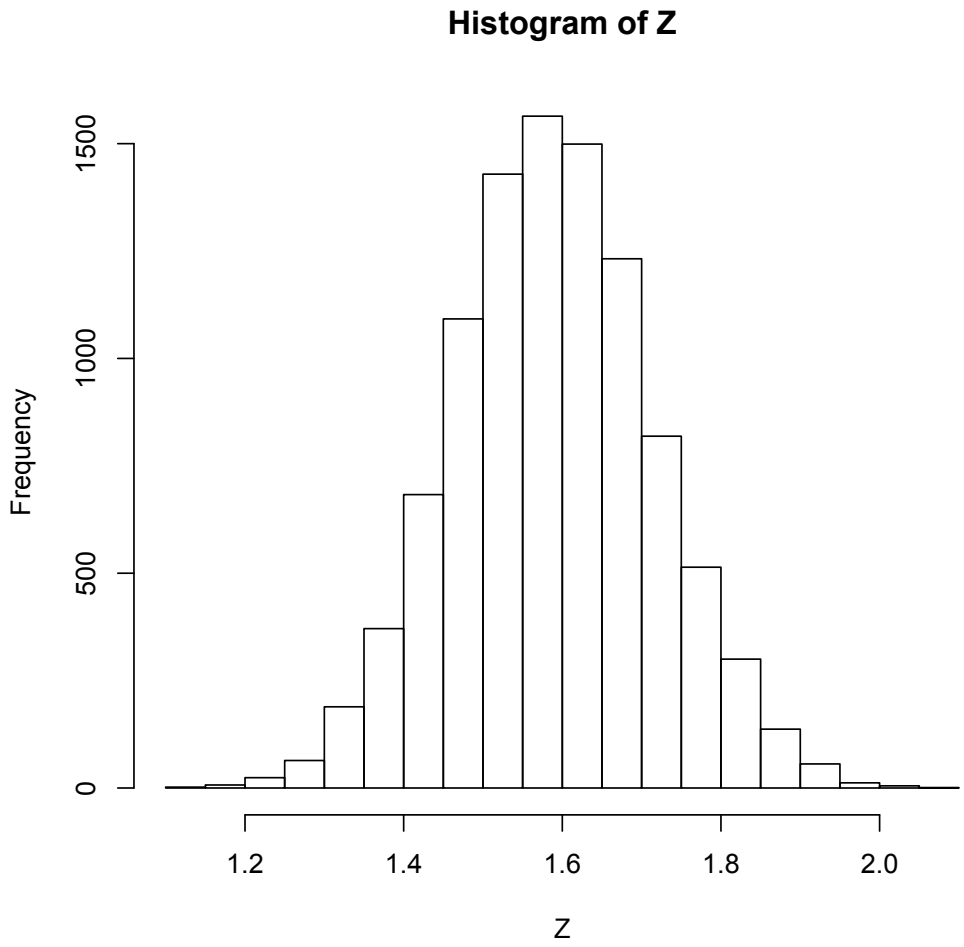



```
# Monte Carlo
 nRep=10000
 COR=rep(NA,nRep)
 for(i in 1:10000){
  x=runif(n)
  y=x+rnorm(n)/8
  COR[i]=cor(x,y)
 }

 hist(COR,30)
```

Fisher's Z-transform

$$Z = 0.5 \times \log\left(\frac{1+r}{1-r}\right)$$




```
# Monte Carlo
 nRep=10000
 COR=rep(NA,nRep)
 for(i in 1:10000){
  x=runif(n)
  y=x+rnorm(n)/8
  COR[i]=cor(x,y)
 }
 Z=0.5*log((1+COR)/(1-COR))
 hist(Z,30)
```

Fisher's Z-transform

$$Z = 0.5 \times \log\left(\frac{1+r}{1-r}\right) \sim N\left(0, \frac{1}{n-3}\right)$$

We can compute a CI for Z, then map-back the interval into *r*.

$$Z = 0.5 \times \log\left(\frac{1+r}{1-r}\right)$$

$$r = \frac{e^{2Z}-1}{1+e^{2Z}}$$

Github:

https://github.com/gdlc/EPI809/blob/master/CORRLEATION.md