Multi-Level Modeling
Testing Fixed and Random Effects

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Read in and Prepare Data for these Notes

```
# Load foreign package to be able to read in SPSS data
> library(foreign)

# Read in the level-1 (player-level) data
> nbaL1 = read.spss(file = "http://www.tc.umn.edu/~zief0002/data/nbaLevel1.sav",
    to.data.frame = TRUE)

# Read in the level-2 (team-level) data
> nbaL2 = read.spss(file = "http://www.tc.umn.edu/~zief0002/data/nbaLevel2.sav",
    to.data.frame = TRUE)

# Merge nbaL2 into nbaL1 using the Team_ID variable
> nba = merge(nbaL1, nbaL2, by = "Team_ID")

# Load libraries
> library(ggplot2)
> library(lmerTest)
> library(dplyr)
```

Group Mean Center the Level-1 Predictor

```
# Compute the mean for each team
> teams = nba %.%
    group_by(Team_ID) %.%
    summarise(teamMean = mean(Shots_on_five))

> head(teams)

  Team_ID meanShots
1      01       3.0
2      02       3.7
3      03       3.3
4      04       3.3
5      05       1.5
6      06       2.7

# Merge the team means with the nba data frame
> nba = merge(nba, teams, by = "Team_ID")

# Compute the group mean deviations
> nba$SO5 = nba$Shots_on_five - nba$teamMean
```

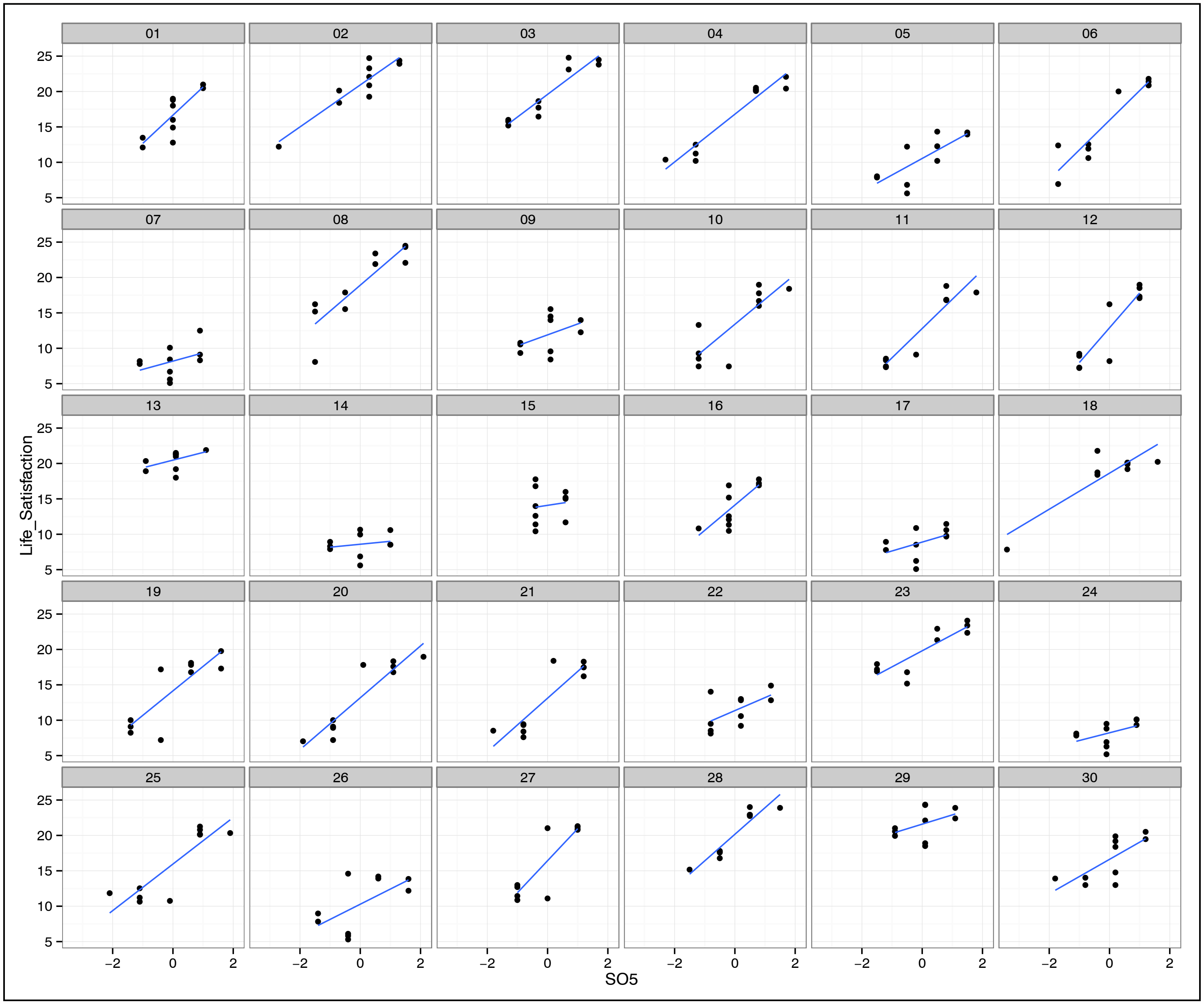
Functional Form of the Level-1 Relationship

What is an Appropriate Functional Form for the Level-1 Model?

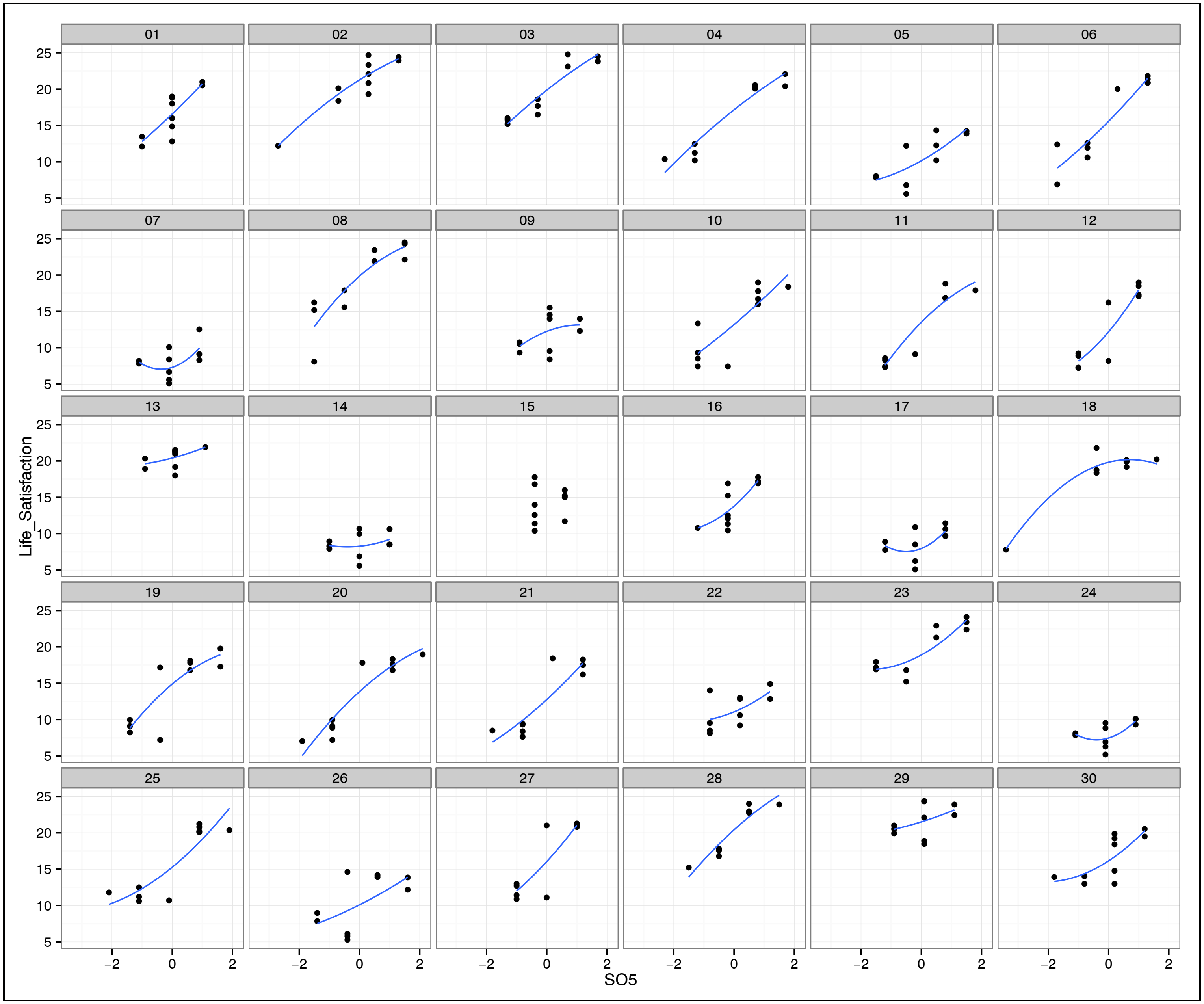
(Examining Empirical Plots with Superimposed OLS Trajectories)

Previously, we fitted a linear relationship between the level-1 predictor and the outcome.

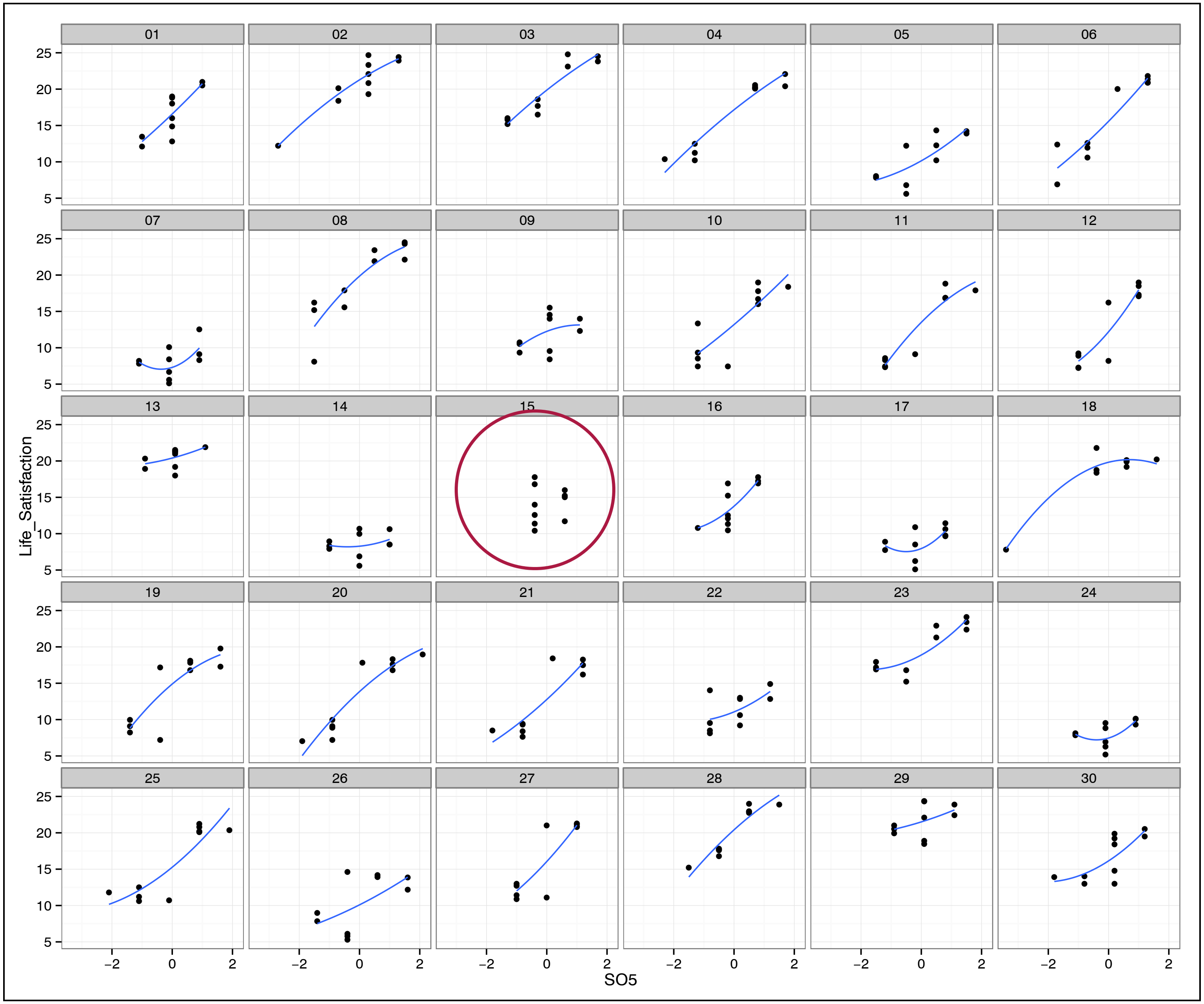
Does a linear model make sense here?



Does a quadratic or log model fit the data better?



Does a quadratic model fit the data better?



Linear Model

Level-1: $Y_{ij} = \beta_0^* + \beta_1^*(X_i) + \epsilon_{ij}$ where $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$

Level-2:

$$\beta_0^* = \beta_{00} + b_{0j}$$
$$\beta_1^* = \beta_{10}$$

Composite: $Y_{ij} = \beta_{00} + \beta_{10}(X_i) + [b_{0j} + \epsilon_{ij}]$ where $b_{0j} \sim N(0, \sigma_0^2)$

Quadratic Model

Level-1: $Y_{ij} = \beta_0^* + \beta_1^*(X_i) + \beta_2^*(X_i^2) + \epsilon_{ij}$ where $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$

Level-2:

$$\beta_0^* = \beta_{00} + b_{0j}$$
$$\beta_1^* = \beta_{10}$$
$$\beta_2^* = \beta_{20}$$

Composite: $Y_{ij} = \beta_{00} + \beta_{10}(X_i) + \beta_{20}(X_i^2) + [b_{0j} + \epsilon_{ij}]$ where $b_{0j} \sim N(0, \sigma_0^2)$

Fit the Linear and Quadratic Models

In order to make a legitimate comparison between two multi-level models (e.g., compare "apples" to "apples") we need to have models that either differ in the random-effects…or in the fixed-effects, but not both.

We still need to account for the within-team correlation among the outcomes (non-independence), so we will include a random intercept in each model.

In order to compare the models, we will fit them both using ML rather than REML.

```
# Linear model with random intercepts
> lmer.l = lmer(Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID), data = nba, REML = FALSE)

# Quadratic model with random intercepts
> lmer.q = lmer(Life_Satisfaction ~ 1 + SO5 + I(SO5 ^ 2) + (1 | Team_ID), REML = FALSE)
```

These two models have the exact same random-effects (i.e., random intercepts), but differ in their fixed-effects structure.

Also note that the linear model can be nested in the quadratic model.

When comparing models that differ in the number of fixed effects, **we must use maximum likelihood (ML)** to estimate the models.

Recall the variance estimate using REML is

$$\hat{\sigma}_{\epsilon}^{2} = \frac{\sum \hat{\epsilon}_i^2}{N - p - 1}$$

This estimate depends on the number of fixed effects in the model and thus is not comparable for models that differ in the number of fixed effects.

Using REML is inappropriate for comparing models that differ in their fixed-effects.

	Parameter	Model L	Model Q
Fixed effects			
Intercept		14.81 (0.72)	14.81 (0.73)
SO5 (Linear)		3.02 (0.14)	3.02 (0.13)
SO5 (Quadratic)		✔	–0.05 (0.12)
Variance components			
Level-1	Within-persons	5.14	5.14
Level-2	Intercepts	15.36	15.36
Goodness-of-fit			
	Deviance	1445.6	1445.6
	AIC	1453.6	1455.6
	BIC	1468.4	1474.1

You can use deviance statistics to compare two models if *two criteria* are satisfied:

1. Both **models are fit to the same exact data**—beware missing data
2. **One model is nested within the other**—we can specify the less complex model (e.g., Model A) by imposing constraints on one or more parameters in the more complex model (e.g., Model B), usually, but not always, setting them to 0)

If these conditions hold, then:

- Difference in the two deviance statistics is asymptotically distributed as $\chi 2$
- df = # of independent constraints

We can obtain Model A from Model B by invoking 1 constraint:

✓ $\beta_{20} = 0$

H_0: $\beta_{20} = 0$

Compute **difference in Deviance statistics** and compare to appropriate $\chi 2$ distribution

$\Delta Deviance = 0.0022$, $df = 1$ ($p = 0.9626$)
⇛ Fail to reject H_0

The BIC measures also suggest that the linear model should be adopted.

```
# Test of the difference in deviance between two nested models
> anova(lmer.l, lmer.q)

Data: nba
Models:
object: Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID)
..1: Life_Satisfaction ~ 1 + SO5 + I(SO5^2) + (1 | Team_ID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object  4 1453.6 1468.4 -722.78   1445.6
..1     5 1455.5 1474.1 -722.78   1445.5 0.0022      1     0.9626
```

So the evidence points toward adopting the linear model rather than the quadratic model.

Random-Effects

Linear Model w/RE for intercept

Level-1: $Y_{ij} = \beta_0^* + \beta_1^*(X_i) + \epsilon_{ij}$ where $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$

Level-2:
$$\begin{aligned} \beta_0^* &= \beta_{00} + b_{0j} \\ \beta_1^* &= \beta_{10} \end{aligned}$$
where $b_{0j} \sim N(0, \sigma_0^2)$

Composite: $Y_{ij} = \beta_{00} + \beta_{10}(X_i) + [b_{0j} + \epsilon_{ij}]$

Linear Model w/RE for intercept and slope

Level-1: $Y_{ij} = \beta_0^* + \beta_1^*(X_i) + \epsilon_{ij}$ where $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$

Level-2:
$$\begin{aligned} \beta_0^* &= \beta_{00} + b_{0j} \\ \beta_1^* &= \beta_{10} + b_{1j} \end{aligned}$$
where $\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix}\right)$

Composite: $Y_{ij} = \beta_{00} + \beta_{10}(X_i) + [b_{0j} + b_{1j}(X_i) + \epsilon_{ij}]$

```
# Linear model with random effects for intercepts and slopes
> lmer.l3 = lmer(Life_Satisfaction ~ 1 + SO5 + (1 + SO5 | Team_ID),
    data = nba, REML = FALSE)
```

Since `lmer.l` is nested in `lmer.l3`, we can test the difference in deviance to see if the random effect of slope is necessary

```
# Test for difference in deviance
> anova(lmer.l, lmer.l3)

Data: nba
Models:
object: Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID)
..1: Life_Satisfaction ~ 1 + SO5 + (1 + SO5 | Team_ID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object  4 1453.6 1468.4 -722.78   1445.6
..1     6 1448.3 1470.5 -718.13   1436.3 9.2884      2   0.009617 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

It appears that we may need the random effect of slope....HANG ON!

```
# Test for difference in deviance
> anova(lmer.l, lmer.l3)

Data: nba
Models:
object: Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID)
..1: Life_Satisfaction ~ 1 + SO5 + (1 + SO5 | Team_ID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object  4 1453.6 1468.4 -722.78   1445.6
..1     6 1448.3 1470.5 -718.13   1436.3 9.2884      2   0.009617 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Why is there a TWO parameter difference.
Didn't we only add a RE for slope?

Actually, we added two parameters to the random intercepts only model...(1) the variance for the slope REs, and (2) the covariance between the intercept and slope REs

The chi-squared test here is actually testing the hypothesis

$$H_0 : \sigma_{01} = \sigma_1^2 = 0$$

Zoinks! Either one (or both) of the parameters may be zero...hmmm...

Linear Model w/RE for intercept

$$b_{0j} \sim N(0, \sigma_0^2)$$

Linear Model w/RE for slope and intercept

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

Linear Model w/independent RE for slope and intercept

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix} \right)$$

This model assumes there is **no** correlation between the intercept and slope REs...that they are independent.

This model has one additional parameter than the random intercepts only model (the covariances are constrained, or fixed, to 0 so the only additional parameter to be estimated is the variance for the RE of slope).

```
# Linear model with independent random effects for intercepts and slopes
> lmer.l4 = lmer(Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID) + (0 + SO5 | Team_ID),
    data = nba, REML = FALSE)
```

To fit the independent RE model, we include two RE terms in the lmer() function. The first is the RE for intercept, and the second is the RE for slope and also constrains the covariance to 0.

```
> summary(lmer.l4)

     AIC      BIC   logLik deviance df.resid
  1449.1   1467.7   -719.6   1439.1      295

Random effects:
 Groups    Name        Variance Std.Dev.
 Team_ID   (Intercept) 15.4099  3.9255
 Team_ID.1 SO5          0.5812  0.7623
 Residual               4.6256  2.1507
Number of obs: 300, groups: Team_ID, 30

Fixed effects:
            Estimate Std. Error      df t value Pr(>|t|)
(Intercept)  14.8067     0.7274 29.9980   20.36  < 2e-16 ***
SO5           2.9038     0.1956 21.3750   14.85 9.75e-13 ***
```

Note the lack of the correlation b/w the REs.

Is the Random Effect for Slope = 0?

The chi-squared test here is testing the hypothesis

$$H_0 : \sigma_1^2 = 0$$

```
# Test for difference in deviance
> anova(lmer.l, lmer.l4)

Data: nba
Models:
object: Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID)
..1: Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID) + (0 + SO5 | Team_ID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object  4 1453.6 1468.4 -722.78   1445.6
..1     5 1449.1 1467.7 -719.57   1439.1 6.4138      1    0.01132 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

There is evidence that there are between-team differences in the effect of shooting success on life satisfaction scores, p = .011.

Is the Covariance between the Random Effects = 0?

The chi-squared test here is actually testing the hypothesis

$$H_0 : \sigma_{01} = 0$$

```
# Test for difference in deviance
> anova(lmer.l4, lmer.l3)

Data: nba
Models:
object: Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID) + (0 + SO5 | Team_ID)
..1: Life_Satisfaction ~ 1 + SO5 + (1 + SO5 | Team_ID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object  5 1449.1 1467.7 -719.57   1439.1
..1     6 1448.3 1470.5 -718.13   1436.3 2.8746      1    0.08999 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

There is not evidence that the RE for intercepts and slope are related, $p = .089$.

Should we Trust these Tests?

Keep in mind that the results of these tests are only asymptotically correct.

There is good methodological evidence to suggest that the *p*-values we get are conservative....too big.

Are there better options for examining the need for different random effects?

Yes. Options include making a decision based on a graphical examination of the level-2 residuals (did); use of a bootstrap simulation; and also examination of different information criteria (e.g., AICC)

These sound super-interesting. Where can I learn about such methodologies?

You can learn about these methodologies by (1) Googling; (2) reading a book exclusively about about mixed-effects modeling; or (3) taking an advanced course such as HLM (Epsy 8268) or Longitudinal Data Analysis (Epsy 8282).

Specifying the Level-2 Model

Level-2 Predictor for Intercept Only

Level-1: $Y_{ij} = \beta_0^* + \beta_1^*(X_i) + \epsilon_{ij}$ where $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$

Level-2:

$$\beta_0^* = \beta_{00} + \beta_{01}(P_j) + b_{0j}$$
$$\beta_1^* = \beta_{10} + b_{1j}$$

where $\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix}\right)$

Composite: $Y_{ij} = \beta_{00} + \boxed{\beta_{01}(P_j)} + \beta_{10}(X_i) + [b_{0j} + b_{1j}(X_i) + \epsilon_{ij}]$

The level-2 predictor in the intercept equation are main effects in the composite model.

Remember...when comparing models that differ in their fixed effects structure, we must use **maximum likelihood** (ML) to estimate the models.

Centering the Level-2 Predictor?

Centering level-2 predictors is much less complicated than centering level-1 predictors.

- Use the raw metric (i.e., do not center)
- Center using the grand mean

Group-mean centering isn't an option with level-2 predictors since every case in the group will have the same value of the level-2 predictor.

Essentially then, it boils down to whether the value of 0 is interpretable in the raw metric (if so, don't bother centering) and if you care (do you need to interpret the intercept).

In our data `Coach_Experience` does not take on the value of 0 (i.e., it has a valid interpretation to interpret `Coach_Experience` = 0), but, since interpreting the intercept is not of interest for the analysis, we won't bother to center the level-2 predictor.

Fit the linear model with independent random effects for intercept and slope, and a level-2 predictor for intercepts.

```
> lmer.i = lmer(Life_Satisfaction ~ 1 + SO5 + Coach_Experience +
    (1 | Team_ID) + (0 | SO5), data = nba, REML = FALSE)
```

We can test the difference in deviance to see if the additional main effect of `Coach_Experience` is statistically important.

```
> anova(lmer.l4, lmer.i)

Data: nba
Models:
object: Life_Satisfaction ~ 1 + SO5 + (1 | Team_ID) + (0 + SO5 | Team_ID)
..1: Life_Satisfaction ~ 1 + SO5 + Coach_Experience + (1 | Team_ID) +
..1:     (0 + SO5 | Team_ID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object  5 1449.1 1467.7 -719.57   1439.1
..1     6 1400.4 1422.7 -694.21   1388.4 50.717      1  1.067e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

There is evidence that coaching experience is predicting variation in the random intercepts, $p < .001$.

Level-2 Predictor for Intercept and Slope

Level-1: $Y_{ij} = \beta_0^* + \beta_1^*(X_i) + \epsilon_{ij}$ where $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$

Level-2:

$$\begin{aligned} \beta_0^* &= \beta_{00} + \beta_{01}(P_j) + b_{0j} \\ \beta_1^* &= \beta_{10} + \beta_{11}(P_j) + b_{1j} \end{aligned}$$

where

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix} \right)$$

Composite: $Y_{ij} = \beta_{00} + \beta_{01}(P_j) + \beta_{10}(X_i) + \boxed{\beta_{11}(P_j)(X_i)} + [b_{0j} + b_{1j}(X_i) + \epsilon_{ij}]$

The level-2 predictor in the slope equation are interaction effects in the composite model.

Fit the linear model with independent random effects for intercept and slope, and a level-2 predictor for intercepts and slopes.

```
> lmer.i.s = lmer(Life_Satisfaction ~ 1 + SO5 + Coach_Experience +
    SO5: Coach_Experience + (1 | Team_ID) + (0 | SO5), data = nba, REML = FALSE)
```

We can test the difference in deviance to see if the additional interaction effect between `Coach_Experience` and `SO5` is statistically important.

```
> anova(lmer.i, lmer.i.s)

Data: nba
Models:
object: Life_Satisfaction ~ 1 + SO5 + Coach_Experience + (1 | Team_ID) +
object:     (0 + SO5 | Team_ID)
..1: Life_Satisfaction ~ 1 + SO5 + Coach_Experience + SO5:Coach_Experience +
..1:     (1 | Team_ID) + (0 + SO5 | Team_ID)
       Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
object  6 1400.4 1422.7 -694.21   1388.4
..1     7 1400.5 1426.4 -693.25   1386.5 1.9167      1     0.1662
```

There is **no** evidence that coaching experience is predicting variation in the random slopes, $p = .166$.

adopting a "final" model

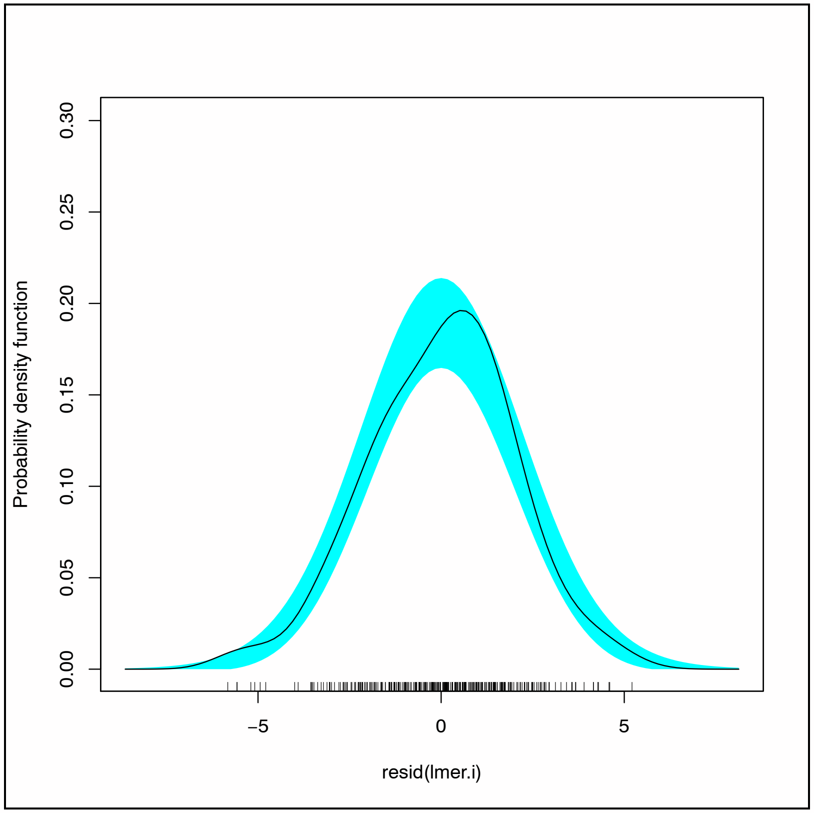
So....it seems that we have a final model that is:

- linear;
- Includes **independent** random effects for intercept and slope; and
- Includes a level-2 predictor for intercepts

```
# Re-fit the model using REML for better variance estimates
> lmer.i = lmer(Life_Satisfaction ~ 1 + SO5 + Coach_Experience +
    (1 | Team_ID) + (0 | SO5), data = nba)
```

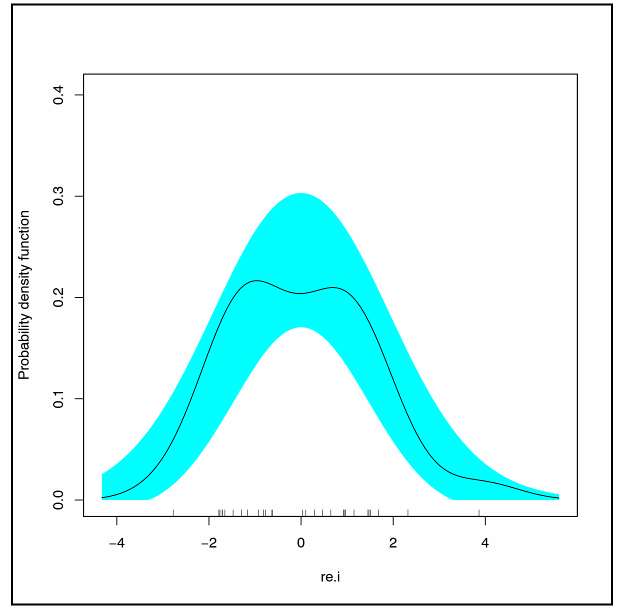
Before interpreting any coefficients from the model, we should examine the residuals at level-1 and at level-2.

```
> library(sm)

# Examine level-1 residuals
> sm.density(resid(lmer.i), model = "normal")


# Examine level-2 residuals (intercept)
> re.i = ranef(lmer.i)$Team_ID[ , 1]
> sm.density(re.i, model = "normal")

# Examine level-1 residuals
> re.s = ranef(lmer.i)$Team_ID[ , 2]
> sm.density(re.s, model = "normal")
```

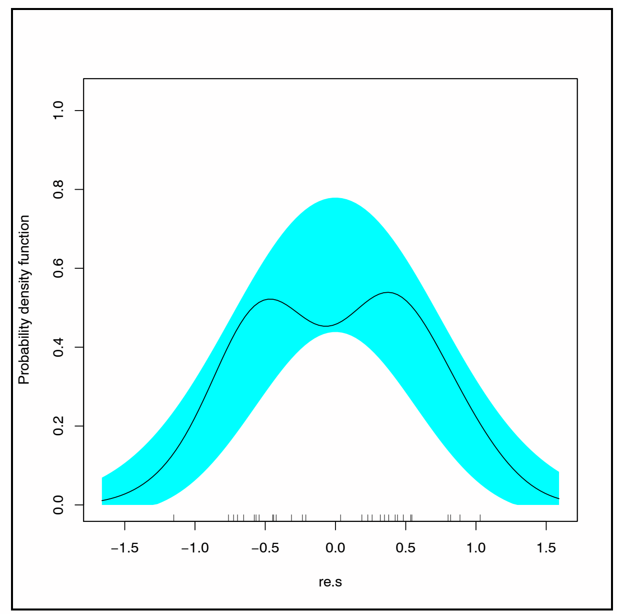


Within-team
(level-1) residuals

Between-team
(level-2) residuals
for the intercepts



Between-team
(level-2) residuals
for the slopes



```
> summary(lmer.i)

REML criterion at convergence: 1390.2

Random effects:
 Groups    Name        Variance Std.Dev.
 Team_ID   (Intercept) 2.6742   1.6353
 Team_ID.1 SO5         0.6338   0.7961
 Residual              4.6198   2.1494
Number of obs: 300, groups: Team_ID, 30

Fixed effects:
                 Estimate Std. Error      df t value Pr(>|t|)
(Intercept)        5.3975     0.9053 27.9960   5.962 2.03e-06 ***
SO5                2.8978     0.2002 21.0620  14.474 2.04e-12 ***
Coach_Experience   4.7843     0.4299 27.9960  11.128 8.66e-12 ***
---
```

Interpreting the Fixed Effects

Fitted Model

$$\hat{\text{Life Satisfaction}} = 5.40 + 2.90(\text{Shooting success}) + 4.78(\text{Coaching Experience})$$

Intercept

The estimated average life satisfaction score for players in the NBA who have an average amount of shooting success on their team (`S05` = 0) and a coach with an experience level of 0 is 5.40. (remember that experience of 0 is extrapolation)

Effect of Shooting Success on Life Satisfaction

Players with more shooting success have higher life satisfaction scores, on average, controlling for their coaches level of experience.

The estimated difference in life satisfaction scores for players with a one-unit difference in shooting success is 2.90, controlling for their coaches level of experience.

Effect of Coaching Experience on Life Satisfaction

Players with coaches who have more experience have higher life satisfaction scores, on average, controlling for their shooting success.

The estimated difference in life satisfaction scores for players whose coaches have a one-unit difference in coaching experience is 2.90, controlling for shooting success.

The residual variation (level-1) decreased from 14.61 to 4.62.

$$R^2_\epsilon = \frac{14.61 - 4.62}{14.61} = 0.684$$

The predictors of shooting success and coaching experience reduced the within-team variation by 68%.

The residual variation in the intercept (level-2) decreased from 15.96 to 2.67.

$$R^2_0 = \frac{15.96 - 2.67}{15.96} = 0.833$$

The predictor of coaching experience reduced the between-team variation in intercepts by 83%.

The residual variation in the slopes (level-2) decreased from 0.6338 to 0.6338.

$$R^2_1 = \frac{0.6338 - 0.6338}{0.6338} = 0$$

The predictor of coaching experience did not reduce the between-team variation in slopes at all.

This was expected since we didn't include coaching experience as a predictor for slopes in our "final" model.

PRESENTING RESULTS FROM MULTI-LEVEL MODELS

Table 1.

Parameter Estimates for a Taxonomy of Fitted Multi-Level Models Predicting Life Satisfaction Scores for 300 NBA Players

Parameter		Model A	Model B	Model C
Fixed effects				
Intercept		14.81 (0.74)	14.81 (0.74)	5.40 (0.91)
Shooting success[†]			2.90 (0.20)	2.90 (0.20)
Coaching experience				4.78 (0.43)
Variance components				
Level-1	Within-persons	14.61	4.62	4.62
Level-2	Intercepts	14.96	15.96	2.67
	Slopes		0.63	0.63
Pseudo R^2 statistics and Goodness-of-fit				
	R^2_ϵ		0.684	0.684
	R^2_0			0.833
	R^2_1			0.000
	$R^2_{Y,\hat{Y}}$	0.547	0.866	0.864
	Deviance	1726.1	1439.3	1390.2
	AIC	1732.1	1449.3	1402.2
	BIC	1743.3	1467.8	1424.4

[†]Group-mean centered

Displaying Analytic Results

Constructing Prototypical Fitted Plots

Key idea: Pick one predictor to display on the *x*-axis. Leave this predictor as a variable in the fitted model.

Key idea: Substitute prototypical values for the predictors into the fitted models to yield prototypical fitted growth trajectories

$$\text{Life } \widehat{\text{Satisfaction}} = 5.40 + 2.90(\text{Shooting success}) + 4.78(\text{Coaching Experience})$$

1. Pick the predictor to display on the *x*-axis...`Shooting Success`...leave it as a variable in the fitted model.
2. Pick prototypical values for the other predictors. Coaching Experience takes on the values of 1, 2, and 3.
3. Substitute each prototypical value into the fitted model, one at a time to get a partial regression model.

Coaching Experience = 1

$$\text{Life } \widehat{\text{Satisfaction}} = 5.40 + 2.90(\text{Shooting success}) + 4.78(1)$$

$$\text{Life } \widehat{\text{Satisfaction}} = 10.18 + 2.90(\text{Shooting success})$$

Coaching Experience = 2

$$\text{Life } \widehat{\text{Satisfaction}} = 10.18 + 2.90(\text{Shooting success}) + 4.78(2)$$

$$\text{Life } \widehat{\text{Satisfaction}} = 19.74 + 2.90(\text{Shooting success})$$

Coaching Experience = 3

$$\text{Life } \widehat{\text{Satisfaction}} = 10.18 + 2.90(\text{Shooting success}) + 4.78(3)$$

$$\text{Life } \widehat{\text{Satisfaction}} = 24.52 + 2.90(\text{Shooting success})$$

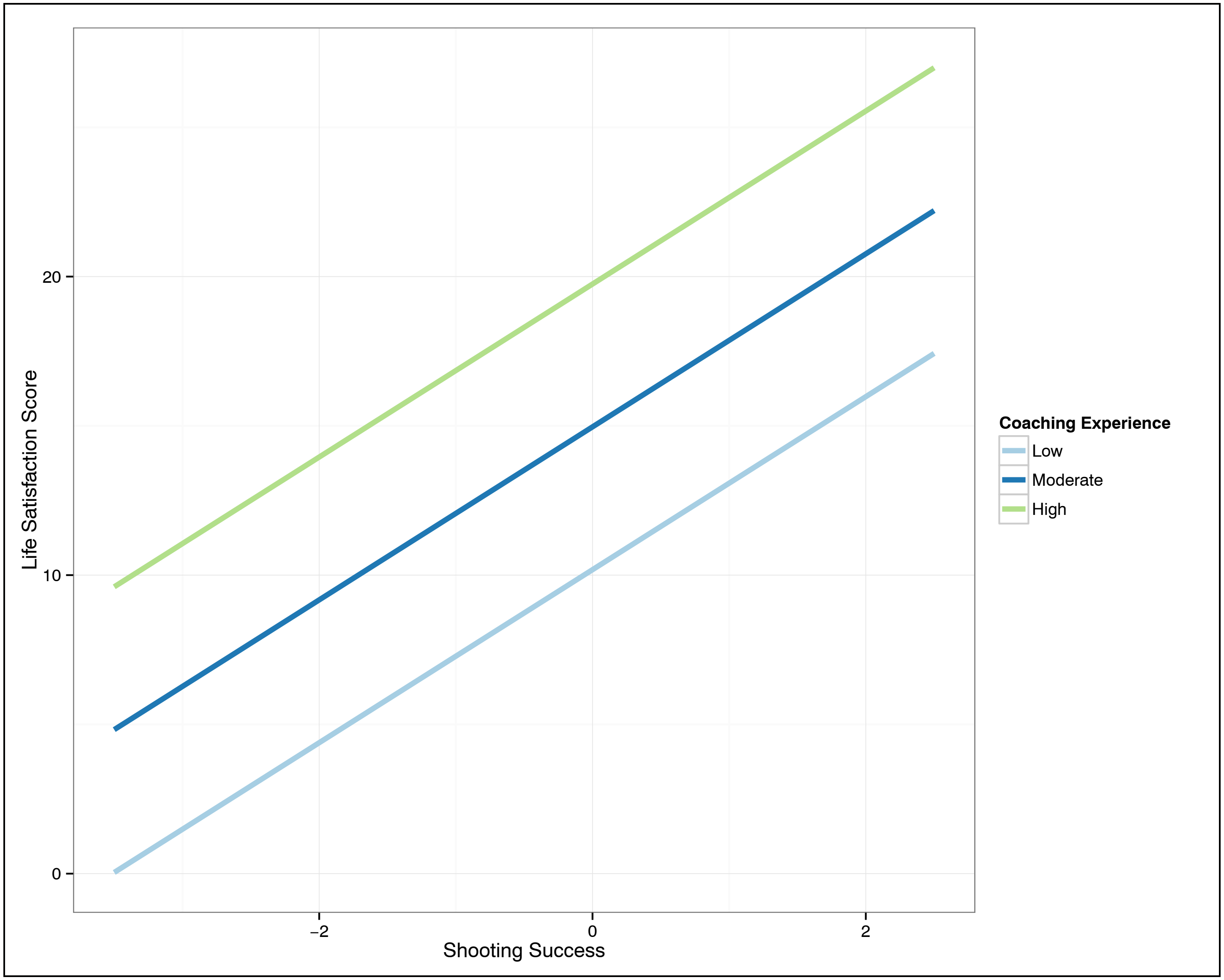


Figure 1. Fitted lines showing the fixed effects of shooting success and coaching experience for Model C.

Picking Prototypical Values when there are Many to Choose From

Key idea

Select "interesting" values of continuous predictors and plot prototypical trajectories by selecting:

1. **Substantively interesting values.** This is easiest when the predictor has inherently appealing values (e.g., 8, 12, and 16 years of education in the U.S.)

2. **A range of percentiles.** When there are no well-known values, consider using a range of percentiles (either the 25th, 50th and 75th or the 10th, 50th, and 90th)

3. **The sample mean ± .5 (or 1) standard deviation.** Best used with predictors with a symmetric distribution

4. **The sample mean (on its own).** If you don't want to display a predictor's effect but just control for it, use just its sample mean

Tip

Remember that exposition can be easier if you select whole number values (if the scale permits) or easily communicated fractions (eg.,¼, ½, ¾, ⅛)

using R to plot REGRESSION lines

Create a clean data set to get the fitted values from.

1. Use `expand.grid()`
2. Use `seq()` to set a range of X values for the predictor you want on the x-axis
3. Use `c()` to set the prototypical values for **all** other predictors

```
> myData = expand.grid(
    SO5 = seq(from = -3.5, to = 2.5, by = 0.1),
   Coach_Experience = c(1, 2, 3)
    )
```

Use the `fixef()` function to get the fixed effects from the LMER model and compute the fitted values.

```
> fixef(lmer.i)

    (Intercept)               SO5 Coach_Experience
       5.397541          2.897842         4.784303

> myData$fitted = fixef(lmer.i)[[1]] +
    fixef(lmer.i)[[2]] * myData$SO5 +
    fixef(lmer.i)[[3]] * myData$Coach_Experience
```

Once the fitted values have been computed, coerce any predictors that have multiple prototypical values (except the predictor on the x-axis) into factors. This will be beneficial when we plot.

```
> head(myData)

   SO5 Coach_Experience     fitted
1 -3.5                1 0.03939566
2 -3.4                1 0.32917990
3 -3.3                1 0.61896414
4 -3.2                1 0.90874838
5 -3.1                1 1.19853262
6 -3.0                1 1.48831686

> myData$Coach_Experience = factor(
    myData$Coach_Experience,
    levels = c(1, 2, 3),
    labels = c("Low", "Moderate", "High")
    )

> head(myData)

   SO5 Coach_Experience     fitted
1 -3.5              Low 0.03939566
2 -3.4              Low 0.32917990
3 -3.3              Low 0.61896414
4 -3.2              Low 0.90874838
5 -3.1              Low 1.19853262
6 -3.0              Low 1.48831686
```

Plot the fitted values vs. the predictor on the *x*-axis.

```
> ggplot(data = myData, aes(x = SO5, y = fitted, color = Coach_Experience)) +
    geom_line(lwd = 1.5) +
    scale_color_brewer(palette = "Paired", name = "Coaching Experience") +
    theme_bw() +
    xlab("Shooting Success") +
    ylab("Life Satisfaction Score")
```