Model Building for Multilevel Models

Data and Packages for these Notes

Load the following packages

- AICcmodavg
- dplyr
- foreign
- ggplot2
- lme4

We will use the NBA data.

- Read in the level-1 data (player-level data)
- Read in the level-2 data (team-level data)
- Join the two datasets together

```
> head(nba)

  Team_ID Shots_on_five Life_Satisfaction Coach_Experience
1      01             3            18.804                2
2      01             3            18.000                2
3      01             4            21.000                2
4      01             4            20.500                2
5      01             3            19.000                2
6      01             2            12.100                2

> tail(nba)

    Team_ID Shots_on_five Life_Satisfaction Coach_Experience
295      30             3             19.90                3
296      30             1             13.90                3
297      30             2             14.01                3
298      30             2             12.99                3
299      30             3             13.01                3
300      30             3             14.78                3
```

Unconditional Random Intercepts Model (Baseline)

The Unconditional Random Intercepts Model

Partitioning Total Outcome Variation Between and Within Teams

Level-1 Model

$$Y_i = \beta_0^* + \epsilon_i \quad \text{where} \quad \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Level-2 Model

$$\beta_0^* = \beta_{00} + \eta_{0j} \quad \text{where} \quad \eta_{0j} \sim \mathcal{N}(0, \sigma_0^2)$$

Composite Model

$$Y_i = \underbrace{\boxed{\beta_{00}}}_{\text{Mean life satisfaction across players}} + \underbrace{\boxed{\eta_{0j}}}_{\text{Team-specific deviation}} + \underbrace{\boxed{\epsilon_i}}_{\text{Player-specific deviation}}$$

j is the team subscript

i is the player subscript

```
# Unconditional random intercepts model
> lmer.a = lmer(Life_Satisfaction ~ 1 + (1 | Team_ID), data = nba)

> summary(lmer.a)



Random effects:
 Groups   Name        Variance Std.Dev.
 Team_ID  (Intercept) 14.96    3.868
 Residual             14.61    3.822
Number of obs: 300, groups:  Team_ID, 30



Fixed effects:
            Estimate Std. Error t value
(Intercept)  14.8067     0.7398   20.01
```

Interpreting the Fixed-Effects

Predicted Level-1 Model

$$\widehat{\text{Life Satisfaction}} = 14.81$$

Interpretation of Intercept

The estimated mean life satisfaction score for all players is 14.81.

Interpretation of the variance component for the random-effect of intercept

The estimated variance for the random-effect of intercept provides a measure of the between-team (team-to-team) variation of life satisfaction scores.

In our example...

There seems to be between-team variation in life satisfaction.

$$\hat{\sigma}_0^2 = 14.96$$

Interpretation of the residual variance component

The estimated residual variance provides a measure of the within-team (player-to-player) variation of life satisfaction scores.

In our example...

There seems to be within-team variation in life satisfaction.

$$\hat{\sigma}_\epsilon^2 = 14.61$$

An estimated 50.6% of the total variation in life satisfaction is attributable to differences between teams.

$$\hat{\rho} = \frac{14.96}{14.96 + 14.61} = 0.506$$

...which means that an estimated 49.4% of the total variation in life satisfaction is attributable to differences between players.

Interpreting the Random-Effects

Composite Model

$$\hat{Y}_i = \hat{\beta}_0 + b_{0j}$$

Interpretation of RE of intercept

The b_{0j} estimate for each team is the difference in predicted life satisfaction between the team average and the sample average (grand mean).

To obtain the random-effects we will use the `ranef()` function.

```
# Get estimates of the random-effects
> ranef(lmer.a)

$Team_ID
   (Intercept)
01   1.6870159
02   5.5804726
03   4.3624008
04   1.8123761
05  -3.8844101
06   1.0334296
```

Team 1: $\hat{Y}_i = 14.81 + 1.69 = 16.5$

The estimated life satisfaction for a player on team 1 is 1.69 points higher than the grand mean.

The estimated life satisfaction for a player on team 1 is 16.5.

Team 2: $\hat{Y}_i = 14.81 + 5.58 = 20.39$

The estimated life satisfaction for a player on team 2 is 5.58 points higher than the grand mean.

The estimated life satisfaction for a player on team 1 is 20.39.

Team 5: $\hat{Y}_i = 14.81 - 3.88 = 10.93$

The estimated life satisfaction for a player on team 5 is 3.88 points lower than the grand mean.

The estimated life satisfaction for a player on team 1 is 10.93.

Maximum Likelihood (ML) vs. Restricted Maximum Likelihood (REML)

Likelihood in Fixed Effects Regression (Intercept-Only Model)

$$\mathcal{L}(\boldsymbol{\beta}; \mathbf{y}) = \left(\frac{1}{\sqrt{2\pi\sigma_\epsilon^2}}\right)^N \times \exp\left[-\frac{\sum(y_i - \beta_0)^2}{2\sigma_\epsilon^2}\right]$$

Likelihood function

$$\ell(\boldsymbol{\beta}; \mathbf{y}) = -\frac{N}{2} \times \ln\left(2\pi\sigma_\epsilon^2\right) - \frac{1}{2\sigma_\epsilon^2} \times \sum(y_i - \beta_0)^2$$

Log-likelihood function

We can use calculus to show that the estimate for β_o that maximizes the log-likelihood is

$$\hat{\beta}_0 = \frac{\sum y_i}{n}$$

Similarly the estimate for $\sigma^2{}_e$ that maximizes the log-likelihood is

$$\hat{\sigma}_\epsilon^2 = \frac{\sum \hat{\epsilon}_i^2}{n}$$

The maximum likelihood estimate for the variance,

$$\hat{\sigma}^2_\epsilon = \frac{\sum \hat{\epsilon}^2_i}{n}$$

is a **biased estimator** of the population variance. It *underestimates* the population value in repeated sampling.

$$E\left(\hat{\sigma}^2_\epsilon\right) < \sigma^2_\epsilon$$

To correct for the bias, a different denominator is used,

$$\hat{\sigma}^2_\epsilon = \frac{\sum \hat{\epsilon}^2_i}{n - k}$$

where k is the number of regression coefficients being estimated. This is referred to as the **restricted maximum likelihood (REML)** estimator

The `var()` function and the model summary output give REML estimates.

```
# REML Estimates (Default)

Random effects:
 Groups   Name        Variance Std.Dev.
 Team_ID  (Intercept) 14.96    3.868
 Residual             14.61    3.822
Number of obs: 300, groups:  Team_ID, 30


Fixed effects:
            Estimate Std. Error t value
(Intercept)  14.8067     0.7398   20.01
```

```
# ML Estimates

> lmer.a_ml = lmer(Life_Satisfaction ~ 1 + (1 | Team_ID), data = nba, REML = FALSE)
> summary(lmer.a_ml)


Random effects:
 Groups   Name        Variance Std.Dev.
 Team_ID  (Intercept) 14.41    3.796
 Residual             14.61    3.822
Number of obs: 300, groups:  Team_ID, 30


Fixed effects:
            Estimate Std. Error t value
(Intercept)  14.8067     0.7274   20.36
```

Note that REML only affects the variance estimates and SEs…the fixed-effects coefficients are exactly the same.

ML

$$\hat{\sigma}_{\epsilon}^{2} = \frac{\sum \hat{\epsilon}_i^2}{n}$$

REML

$$\hat{\sigma}_{\epsilon}^{2} = \frac{\sum \hat{\epsilon}_i^2}{n-k}$$

- When n is very large the ML and REML estimates for variance will be pretty much the same.
- The magnitude of the difference in the ML and REML estimates for variance are dependent on the value of k (the number of fixed-effects in the model)

When you are comparing models to select fixed effects, then ML estimation should be used. When you are comparing models to select random effects, then REML estimation should be used. For interpretation of any models, use REML estimation.

Selecting Fixed Effects

Next we will include any player-level (level-1) predictors to explain within-team variation. When we do this, we will keep the random effects the same as for the unconditional intercept model.

Level-1 Model:

$$Y_i = \beta_0^* + \beta_1^*(X_{1i}) + \epsilon_i \quad \text{where} \quad \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Level-2 Model:

$$\beta_0^* = \beta_{00} + \eta_{0j}$$
$$\beta_1^* = \beta_{10} \quad \text{where} \quad \eta_{0j} \sim \mathcal{N}(0, \sigma_0^2)$$

Composite Model:

Examine the Shots_on_five Predictor

```
# Fit the unconditional intercepts model using ML estimation (to compare fixed effects)
> lmer.a_ml = lmer(Life_Satisfaction ~ 1 + (1 | Team_ID), data = nba, REML = FALSE)


# Fit model with fixed and random effects of intercept AND fixed effect of slope using
# ML estimation
> lmer.b_ml = lmer(Life_Satisfaction ~ 1 + Shots_on_five +
    (1 | Team_ID), data = nba, REML = FALSE)

> aictab(
   cand.set =  list(lmer.a_ml, lmer.b_ml),
   modnames = c("Intercept", "Intercepts + Slopes")
   )

Model selection based on AICc:

                    K    AICc Delta_AICc AICcWt Cum.Wt      LL
Intercepts + Slopes 4 1385.98       0.00      1      1 -688.92
Intercept           3 1733.44     347.46      0      1 -863.68
```

Selecting Random Effects

Now we can also examine additional random effects (in this case a potential RE of slopes as well as intercepts.

Level-1 Model:

$$Y_i = \beta_0^* + \beta_1^*(X_{1i}) + \epsilon_i \qquad \text{where} \qquad \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Level-2 Model:

$$\begin{aligned} \beta_0^* &= \beta_{00} + \eta_{0j} \\ \beta_1^* &= \beta_{10} + \eta_{1j} \end{aligned} \qquad \text{where} \qquad \begin{bmatrix} \eta_{0j} \\ \eta_{1j} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

Composite Model:

$$Y_i = \beta_{00} + \eta_{0j} + (\beta_{10} + \eta_{1j})X_{1i} + \epsilon_i$$

Team-specific deviation in slope

Player-specific deviation

Mean life satisfaction for all players with $X_i = 0$

Team-specific deviation in intercept

Average difference in life satisfaction for a one-unit difference in X_i

Typically before one would do any interpretation of this model, we would compare it to the model that only includes the RE of intercept. However, here we introduce the interpretations as a pedagogical exercise.

Composite Model:

$$Y_i = \beta_{00} + \eta_{0j} + (\beta_{10} + \eta_{1j})X_{1i} + \epsilon_i$$

$$Y_i = \beta_{00} + \beta_{10}(X_{1i}) + [\eta_{0j} + \eta_{1j}(X_{1i}) + \epsilon_i]$$

```
# Fixed and random effects of intercept AND fixed and random effects of slope

> lmer.b = lmer(Life_Satisfaction ~ 1 + Shots_on_five +
    (1 + Shots_on_five | Team_ID), data = nba)

> summary(lmer.b)

Random effects:
 Groups   Name          Variance Std.Dev. Corr
 Team_ID  (Intercept)   0.09279  0.3046
          Shots_on_five 0.09913  0.3148   1.00
 Residual               5.10616  2.2597
Number of obs: 300, groups:  Team_ID, 30


Fixed effects:
              Estimate Std. Error t value
(Intercept)     6.4296     0.3169   20.29
Shots_on_five   3.2887     0.1340   24.55
```

Interpreting the Fixed-Effects

Predicted Level-1 Model

$$\text{Life Sat}\hat{\text{i}}\text{sfaction} = 6.43 + 3.2(\text{SO5})$$

Interpretation of Intercept

The mean life satisfaction for players who have a shooting success (`Shots_on_five`) of 0 is 6.43.

Interpretation of the slope

Each one-unit difference in shooting success (`Shots_on_five`) is associated with a 3.29 unit change in life satisfaction, on average.

Interpretation of the residual variance component

The estimated residual variance provides a measure of the within-team (player-to-player) variation of life satisfaction scores after accounting for shooting success.

In our example...

There seems to be within-team variation in life satisfaction scores after accounting for shooting success.

$$\hat{\sigma}_\epsilon^2 = 5.11$$

The residual variation (level-1) decreased from 14.61 to 5.11.

$$r^2 = \frac{14.61 - 5.11}{14.61} = 0.650$$

The change in level-1 residual variation should **always** be compared to the *unconditional random intercepts* model.

This is a **Pseudo R^2**. Similar to the R^2 in OLS models, it measures the reduction in the level-1 residual variance.

Interpreting the Random-Effects

Fitted Composite Model

$$\hat{Y}_{ij} = \hat{\beta}_{00} + b_{0j} + \hat{\beta}_{10}(X_i) + b_{1j}(X_i)$$

Interpretation of RE of intercept

The b_{0j} estimate for each team is the difference in predicted life satisfaction between the team average and the sample average, for a shooting success of 0.

Interpretation of RE of slope

The b_{1j} estimate for each team is the difference in the effect of shooting success on life satisfaction between the team and the overall sample.

Interpretation of the variance component for the REs of intercept

The variance in the b_{0j} estimates indicates the variation in average life satisfaction across teams for a shooting success of 0.

Interpretation of the variance component for the REs of slope

The variance in the b_{1j} estimates indicates the variation in the effect of shooting success on life satisfaction across teams.

```
# Get estimates of the random-effects
> ranef(lmer.b)

$Team_ID
   (Intercept) Shots_on_five
01  0.07792009    0.08053477
02  0.36112562    0.37324350
03  0.38440629    0.39730537
04 -0.07373235   -0.07620650
05 -0.23807533   -0.24606415
06  0.17342369    0.17924307
```

Team 1: $$\hat{\text{Life Satisfaction}} = 6.43 + 0.08 + (3.2 + 0.08)\text{SO5}$$

$$= 6.51 + 3.37(\text{SO5})$$

The estimated life satisfaction for a player whose shooting success is 0 (`Shots_on_five` = 0) on team 1 is, on average, 6.51.

On team 1, each one-unit difference in shooting success (`Shots_on_five`) is associated with a 3.37 unit change in life satisfaction.

```
# Get estimates of the random-effects
> ranef(lmer.b)

$Team_ID
   (Intercept) Shots_on_five
01  0.07792009    0.08053477
02  0.36112562    0.37324350
03  0.38440629    0.39730537
04 -0.07373235   -0.07620650
05 -0.23807533   -0.24606415
06  0.17342369    0.17924307
```

There seems to be between-team variation in intercepts.

$$\hat{\sigma}_0^2 = 0.09$$

There seems to be between-team variation in slopes.

$$\hat{\sigma}_1^2 = 0.10$$

```
# Estimates of the variance-covariance matrix of the random effects
> varCorr(lmer.b)$Team_ID

              (Intercept) Shots_on_five
(Intercept)    0.09279463    0.09590843
Shots_on_five  0.09590843    0.09912672
attr(,"stddev")
  (Intercept) Shots_on_five
    0.3046221     0.3148440
attr(,"correlation")
              (Intercept) Shots_on_five
(Intercept)             1             1
Shots_on_five           1             1
```

Square roots of the variance estimates

$$\mathbf{G} = \begin{bmatrix} 0.093 & 0.096 \\ 0.096 & 0.099 \end{bmatrix}$$

The b_{0j} estimates and b_{1j} estimates are positively related. Teams that have a higher intercept also tend to have higher slopes.

Now we will actually perform the comparison of this model (RE of intercept and slope) with the model that only includes the RE of intercept. Remember, this should have been done prior to any interpretations.

```r
# Fixed and random effects of intercept AND fixed effect of slope
# Fit using REML estimation
> lmer.b0 = lmer(Life_Satisfaction ~ 1 + Shots_on_five +
    (1 | Team_ID), data = nba)



# Fixed and random effects of intercept AND fixed and random effects of slope
# Fit using REML estimation
> lmer.b = lmer(Life_Satisfaction ~ 1 + Shots_on_five +
    (1 + Shots_on_five | Team_ID), data = nba)



> aictab(
   cand.set =  list(lmer.b0, lmer.b),
   modnames = c("Varying Intercepts", "Varying Intercepts and Slopes")
   )
```

```
Model selection based on AICc:

                              K    AICc Delta_AICc AICcWt Cum.Wt  Res.LL
Varying Intercepts            4 1389.63       0.00    0.7    0.7 -690.75
Varying Intercepts and Slopes 6 1391.30       1.67    0.3    1.0 -689.51


Warning message:
Model selection for fixed effects is only appropriate with ML estimation:
REML (default) should only be used to select random effects for a constant set of fixed
effects
```

Note the model without the random effect of slope reduces the number of parameters being estimated by 2 (no longer need to estimate the variance estimate of the slopes, nor the covariance estimate between intercepts and slopes).

Conditional Models: Adding Level-2 Predictors

Now we will include any team-level (level-2) predictors to explain between-team variation. First, we will examine the model if we a**dd a level-2 predictor to only the level-2 intercept model**.

Level-1 Model:

$$Y_i = \beta_0^* + \beta_1^*(X_{1i}) + \epsilon_i \qquad \text{where} \qquad \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Level-2 Model:

$$\beta_0^* = \beta_{00} + \beta_{01}(G_j) + \eta_{0j}$$

$$\beta_1^* = \beta_{10} + \eta_{0j}$$

$$\text{where} \qquad \begin{bmatrix} \eta_{0j} \\ \eta_{1j} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

Composite Model

$$Y_i = \beta_{00} + \beta_{01}(G_j) + \eta_{0j} + (\beta_{10} + \eta_{1j})\, X_{1i} + \epsilon_i$$

$$Y_i = \beta_{00} + \beta_{01}(G_j) + \beta_{10}(X_{1i}) + [\eta_{0j} + \eta_{1j}(X_{1i}) + \epsilon_i]$$

Composite Model

$$Y_i = \beta_{00} + \boxed{\beta_{01}(G_j)} + \beta_{10}(X_{1i}) + [\eta_{0j} + \eta_{1j}(X_{1i}) + \epsilon_i]$$

Adding a level-2 predictor to only the level-2 intercept model adds a main-effect of the level-2 predictor into the composite model.

What happens when we include a level-2 predictor **in both the level-2 models for intercept and slope**?

Level-1 Model:

$$Y_i = \beta_0^* + \beta_1^*(X_{1i}) + \epsilon_i \qquad \text{where} \qquad \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Level-2 Model:

$$\beta_0^* = \beta_{00} + \beta_{01}(G_j) + \eta_{0j}$$

$$\beta_1^* = \beta_{10} + \beta_{11}(G_j) + \eta_{1j}$$

$$\text{where} \qquad \begin{bmatrix} \eta_{0j} \\ \eta_{1j} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

Composite Model $Y_i = \beta_{00} + \boxed{\beta_{01}(G_j)} + \beta_{10}(X_{1i}) + \boxed{\beta_{11}(G_j)(X_{1i})} + [\eta_{0j} + \eta_{1j}(X_{1i}) + \epsilon_i]$

Adding a level-2 predictor to the level-2 intercept model **adds a main-effect of the level-2 predictor** into the composite model. While, adding a level-2 predictor to the level-2 slope model **adds an interaction-effect of the level-2 predictor with the level-1 predictor** (sometimes referred to as a cross-level predictor) into the composite model.

Note since adding a a level-2 predictor to the level-2 slope model adds an interaction-effect, all constituent main-effects also need to be included. This means that to include a level-2 predictor in the slope model, the same predictor has to be included in the intercept model as well.

Typically a model with the level-2 predictor in the intercept model only is compared to a model that includes the level-2 predictor in both the intercept and the slope models.

Now we will include any team-level (level-2) predictors to explain between-team variation.

Level-1 Model:

$$Y_i = \beta_0^* + \beta_1^*(X_{1i}) + \epsilon_i \qquad \text{where} \qquad \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Level-2 Model:

$$\beta_0^* = \beta_{00} + \beta_{01}(G_j) + \eta_{0j}$$

$$\beta_1^* = \beta_{10} + \beta_{11}(G_j) + \eta_{1j}$$

$$\text{where} \qquad \begin{bmatrix} \eta_{0j} \\ \eta_{1j} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

Composite Model

$$Y_i = \beta_{00} + \beta_{01}(G_j) + \eta_{0j} + (\beta_{10} + \beta_{11}(G_j) + \eta_{1j})\, X_{1i} + \epsilon_i$$

$$Y_i = \beta_{00} + \beta_{01}(G_j) + \beta_{10}(X_{1i}) + \beta_{11}(G_j)(X_{1i}) + [\eta_{0j} + \eta_{1j}(X_{1i}) + \epsilon_i]$$

In our example thus far, we had adopted the fixed effects of intercept and shots_on_five,and the random effects for intercept only. We will first fit the model that includes coaching experience as a predictor in only the level-2 model for intercepts.

Level-1 Model: $\text{LifeSatisfaction}_i = \beta_0^* + \beta_1^*(\text{SO5}) + \epsilon_i$

Level-2 Model: $\beta_0^* = \beta_{00} + \beta_{01}(\text{CoachExperience}) + \eta_{0j}$

$\beta_1^* = \beta_{10}$

Composite Model

$$\text{LifeSatisfaction}_i = \beta_{00} + \beta_{01}(\text{CoachExperience}) + \beta_{10}(\text{SO5}) + \eta_{0j} + \epsilon_i$$

We will also fit the model that includes coaching experience as a predictor in the level-2 model for both intercepts and slopes.

Level-1 Model: $$\text{LifeSatisfaction}_i = \beta_0^* + \beta_1^*(\text{SO5}) + \epsilon_i$$

Level-2 Model: $$\beta_0^* = \beta_{00} + \beta_{01}(\text{CoachExperience}) + \eta_{0j}$$
$$\beta_1^* = \beta_{10} + \beta_{11}(\text{CoachExperience})$$

Composite Model

$$\text{LifeSatisfaction}_i = \beta_{00} + \beta_{01}(\text{CoachExperience}) + \beta_{10}(\text{SO5}) + \beta_{11}(\text{CoachExperience})(\text{SO5}) + \eta_{0j} + \epsilon_i$$

Here are the composite models for three cases: (1) No level-2 predictors for either intercept or slope; (2) Level-2 predictor for intercept only; and (3) Level-2 predictor for both intercept and slope.

Which estimation method should we use to fit and compare these models? Why?

Model 1

$$\text{LifeSatisfaction}_i = \beta_{00} + \beta_{10}(\text{SO5}) + \eta_{0j} + \epsilon_i$$

Model 2

$$\text{LifeSatisfaction}_i = \beta_{00} + \beta_{01}(\text{CoachExperience}) + \beta_{10}(\text{SO5}) + \eta_{0j} + \epsilon_i$$

Model 3

$$\begin{aligned}\text{LifeSatisfaction}_i = \beta_{00} + \beta_{01}(\text{CoachExperience}) + \beta_{10}(\text{SO5}) + \\ \beta_{11}(\text{CoachExperience})(\text{SO5}) + \eta_{0j} + \epsilon_i\end{aligned}$$

```
# No level-2 predictors
> lmer.c1 = lmer(Life_Satisfaction ~ 1 + Shots_on_five +
   (1 | Team_ID), data = nba, REML = FALSE)


# Level-2 predictor for intercept
> lmer.c2 = lmer(Life_Satisfaction ~ 1 + Shots_on_five + Coach_Experience +
   (1 | Team_ID), data = nba, REML = FALSE)


# Level-2 predictor for intercept and slope
> lmer.c3 = lmer(Life_Satisfaction ~ 1 + Shots_on_five + Coach_Experience +
    Shots_on_five:Coach_Experience + (1 | Team_ID), data = nba, REML = FALSE)


# AICc evidence
> aictab(
   cand.set =  list(lmer.c1, lmer.c2, lmer.c3),
   modnames = c("None", "Intercept", "Intercept and Slopes")
   )


Model selection based on AICc:

                     K    AICc Delta_AICc AICcWt Cum.Wt      LL
Intercept            5 1347.21       0.00   0.66   0.66 -668.50
Intercept and Slopes 6 1348.54       1.33   0.34   1.00 -668.13
None                 4 1385.98      38.77   0.00   1.00 -688.92
```

```
# Re-fit the adopted model using REML estimation
> lmer.c2 = lmer(Life_Satisfaction ~ 1 + Shots_on_five + Coach_Experience +
   (1 | Team_ID), data = nba)

> summary(lmer.c2)


Random effects:
 Groups   Name        Variance Std.Dev.
 Team_ID  (Intercept) 0.000    0.000
 Residual             5.098    2.258
Number of obs: 300, groups:  Team_ID, 30



Fixed effects:
                 Estimate Std. Error t value
(Intercept)        3.9474     0.3694  10.686
Shots_on_five      3.1343     0.1231  25.471
Coach_Experience   1.5533     0.2148   7.231
```

Interpret the model output

- Variance in the Random Effects
  - ★ How much has the error variance decreased from the baseline model?
  - ★ How much of the error variance in the current model is within-team variation? How much is between-team variation?

- Fixed Effects
  - ★ Interpret each of the fixed effects

- Random Effects
  - ★ Compute the predicted life satisfaction for a player from Team 01 and for a player from Team 02.