① - Discrete random variable:

what is it? Finite number of values $a < z < b$

not necessarily integer

② Notation

$$f(Z=z) = [z]$$

③ Probability mass function:

A function for which : 2 things

$$[z] \geq 0$$

$$\sum_{z \in S} z = 1$$

Support all values of z for which $[z] > 0$

Probability mass function

note → scale

$[z]$

Probability of z
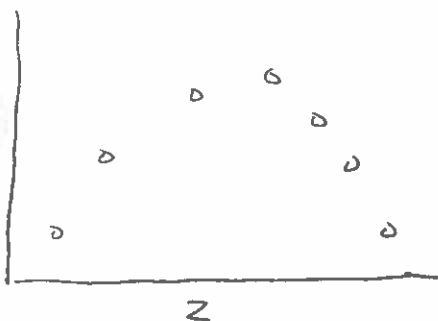
④ Example using Species Richness on many plots:

$$[z|\lambda] = \frac{\lambda^z e^{-\lambda}}{z!}$$

Moment generating function
$j = 1, 2, 3, 4$

$$M_j = \sum_{z \in S} (z-c)^j [z]$$

__Moments__

$C = 0$
$j = 1$

$$E(z) = \mu = \sum_{z \in S} z [z]$$

Expected value or mean
First moment

approximated by many random draws from the distribution z using

$$\frac{1}{n} \sum_{i=1}^{n} z_i$$

$C = \mu$
$j = 2$

$$E(z-\mu)^2 = \sigma^2 = \sum_{z \in S} (z-\mu)^2 [z]$$

Second central moment
__variance__

approximated as

$$\frac{1}{n} \sum_{i=1}^{n} (z_i - \mu)^2$$

Other moments

$j = 3$ skewness

$j = 4$ kurtosis "fatness of tails"

Probability mass function

1
$[u]$
0
μ

Cumulative distribution function

$$F(z) = \sum_{z \leq u} [u]$$

$F(z) = 1$
$Pr(z \leq u)$
0
z

who is it useful?

Quantile function

z
0
$F(z)$ 1
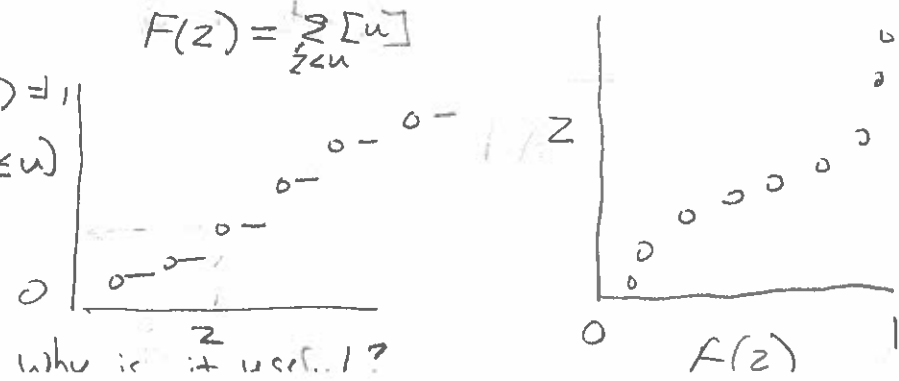
① Continuous random variable

$a < z < b$, infinite number of values

Requirements for probability density function

① $[z] \geq 0$

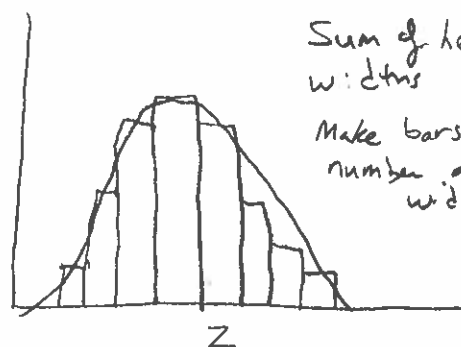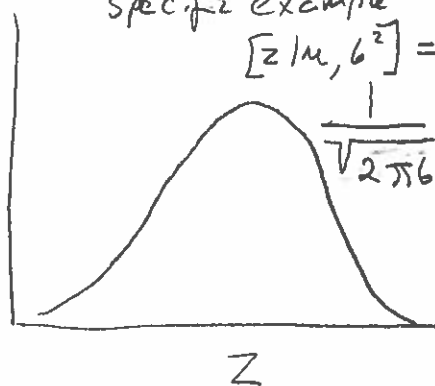② $\int_a^b [z]\, dz = Pr(a \leq z \leq b)$

③ $\int_{-\infty}^{\infty} [z]\, dz = 1$

$[z]$ probability density of z

support $[z] \geq 0$

specific example

$[z \mid \mu, 6^2] = \frac{1}{\sqrt{2\pi 6^2}} e^{-\frac{(z-\mu)^2}{26^2}}$

$[z]$

Sum of heights · widths = 1

Make bars infinite number and 0 width

Moment generating function

$m_j = \int_{-\infty}^{\infty} (z-c)^j [z]$

First moment:
Expected value or mean

$c=0$ $\quad M_1 = E(z) = \mu = \int_{-\infty}^{\infty} z\, [z]\, dz$
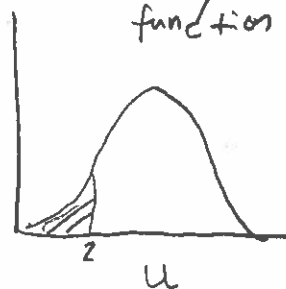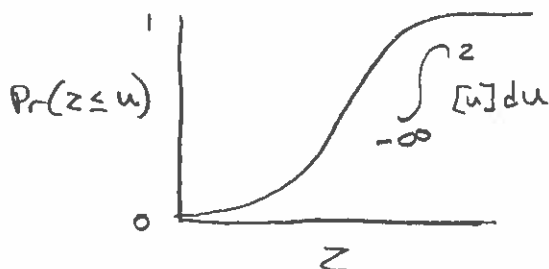$j=1$

approximated as $\frac{1}{n} \sum_{i=1}^{n} z_i$

Second central moment:

$c=\mu$ $\quad M_2 = E(z-\mu)^2 = 6^2 = \int_{-\infty}^{\infty} (z-\mu)^2 [z]$
$j=2$

approximated as $\frac{1}{n} \sum_{i=1}^{n} (z_i - \mu)^2$

Probability density are values on the continuous curve such that the area under the curve = 1

Scaling of y axis

Probability density function

$[u]$

$F(z) =$ Cummulative distribution function =

$Pr(z \leq u)$

$\int_{-\infty}^{z} [u]\, du$

$F'(z) =$ quantile function

z

$F(z)$