Bayesian Regression

ESS 575 Models for Ecological Data

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Normal data, continuous and real valued

$$
\begin{aligned}
[\beta_0, \beta_1, \sigma \mid \mathbf{y}] &\propto \prod_{i=1}^{n} \text{normal}\big(y_i \mid g(\beta_0, \beta_1, x_i), \sigma^2\big) \times \\
&\quad \text{normal}\big(\beta_0 \mid 0, 1000\big)\, \text{normal}\big(\beta_1 \mid 0, 1000\big) \times \\
&\quad \text{uniform}\big(\sigma \mid 0, 100\big) \\
g(\beta_0, \beta_1, x_i) &= \beta_0 + \beta_1 x_i
\end{aligned}
$$

```
b0 ~ dnorm(0, .001)
b1 ~ dnorm(0, .001)
sigma ~ dunif(0, 100)
tau <- 1/sigma^2
for (i in 1:length(y)){
  mu[i] <- b0 + b1 * x[i]
  y[i] ~ dnorm(mu[i], tau)
}
```

Poisson, discrete and positive

$$
\begin{aligned}
\left[\beta_0, \beta_1 \mid \mathbf{y}\right] &\propto \prod_{i=1}^{n} \mathrm{Poisson}\left(y_i \mid g\left(\beta_0, \beta_1, x_i\right)\right) \times \\
&\quad \mathrm{normal}\left(\beta_0 \mid 0, 1000\right) \mathrm{normal}\left(\beta_1 \mid 0, 1000\right) \\
g\left(\beta_0, \beta_1, x_i\right) &= e^{\beta_0 + \beta_1 x_i}
\end{aligned}
$$

```
b0 ~ dnorm(0, .001)
b1 ~ dnorm(0, .001)
for(i in 1:length(y)){
  log(mu[i]) <- b0 + b1 * x[i]
  y[i] ~ dpois(mu[i])
}
```

or

```
mu[i] <- exp(b0 + b1 * x[i])
y[i] ~ dpois(mu[i])
```

Poisson with offset

$\log(u_i) =$ *offset* for observation i

$$
\begin{aligned}
[\beta_0, \beta_1 \mid \mathbf{y}] &\propto \prod_{i=1}^{n} \mathrm{Poisson}\big(y_i \mid g\left(\beta_0, \beta_1, x_i, u_i\right)\big) \times \\
&\quad \mathrm{normal}\big(\beta_0 \mid 0, 1000\big)\,\mathrm{normal}\big(\beta_1 \mid 0, 1000\big) \\
g\left(\beta_0, \beta_1, x_i, u_i\right) &= u_i e^{\beta_0 + \beta_1 x_i}
\end{aligned}
$$

```
b0 ~ dnorm(0, .001)
b1 ~ dnorm(0, .001)
for(i in 1:length(y)){
  log(mu[i]) <- log(u[i]) + b0 + b1 * x[i]
  y[i] ~ dpois(mu[i])
}
```

Bernoulli, data 0 or 1 (aka logistic)

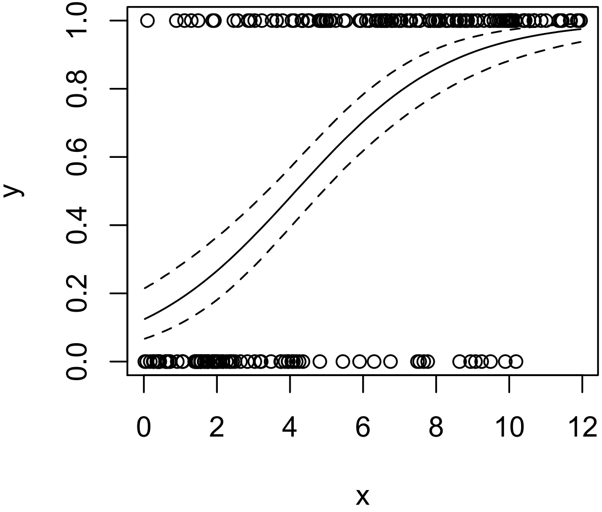
$$
\begin{aligned}
\left[\beta_0, \beta_1 \mid \mathbf{y}\right] &\propto \prod_{i=1}^{n} \mathrm{Bernoulli}\big(y_i \mid g\left(\beta_0, \beta_1, x_i\right)\big) \times \\
& \quad \mathrm{normal}\big(\beta_0 \mid 0, 2\big)\,\mathrm{normal}\big(\beta_1 \mid 0, 2\big) \\
g\left(\beta_0, \beta_1, x_i\right) &= \frac{e^{\beta_0 + \beta_1 x_i}}{e^{\beta_0 + \beta_1 x_i} + 1}
\end{aligned}
$$

```
b0 ~ dnorm(0, .5)
b1 ~ dnorm(0, .5)
for(i in 1:length(y)){
  logit(p[i]) <- b0 + b1 * x[i]
    y[i] ~ dbern(p[i])
 }
```

or

```
p[i] <- inv.logit(b0 + b1 * x[i])
y[i] ~ dberb(p[i])
```

Bernoulli, data 0 or 1 (aka logistic)



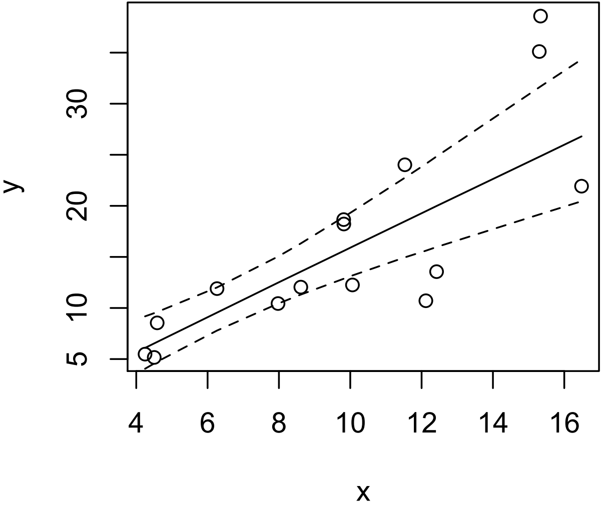
lognormal, data continuous and > 0

$$\begin{aligned}
\left[\beta_0, \beta_1, \sigma \mid \mathbf{y}\right] &\propto \prod_{i=1}^{n} \text{lognormal}\left(y_i \mid \log\left(g\left(\beta_0, \beta_1, x_i\right)\right), \sigma^2\right) \times \\
&\quad \text{normal}\left(\beta_0 \mid 0, 1000\right) \text{normal}\left(\beta_1 \mid 0, 1000\right) \times \\
&\quad \text{uniform}\left(\sigma \mid 0, 5\right) \\
g\left(\beta_0, \beta_1, x_i\right) &= e^{\beta_0 + \beta_1 x_i}
\end{aligned}$$

Talk about the interpretation of σ.

```
b0 ~ dnorm(0, .001)
b1 ~ dnorm(0, .001)
sigma ~ dunif(0, 5)
tau <- 1/sigma^2
for(i in 1:length(y)){
  mu[i] <- exp(b0 + b1 * x[i])
  y[i] ~ dlnorm(log(mu[i]), tau)
}
```

lognormal, data continuous and > 0



lognormal, data continuous and > 0

$$\begin{aligned}
\left[\beta_0, \beta_1, \sigma \mid \mathbf{y}\right] &\propto \prod_{i=2}^{n} \text{lognormal}\Big(y_i \mid \log\big(g\left(\beta_0, \beta_1, y_{i-1}, H_i\right)\big), \sigma^2\Big) \times \\
&\quad \text{normal}\big(\beta_0 \mid 0, 1000\big)\,\text{normal}\big(\beta_1 \mid 0, 1000\big) \times \\
&\quad \text{uniform}\big(\sigma \mid 0, 5\big) \\
g\left(\beta_0, \beta_1, y_{i-1}, H_i\right) &= y_{i-1} e^{\beta_0 + \beta_1 y_{i-1}} - H_i
\end{aligned}$$

Talk about the bounding trick.

```
b0 ~ dnorm(0, .001)
b1 ~ dnorm(0, .001)
sigma ~ dunif(0, 5)
tau <- 1/sigma^2
for(i in 2:length(y)){
  mu[i] <- y[i-1] * exp(b0 + b1 * y[i-1]) - H[i]
  y[i] ~ dlnorm(log(max(.000001, mu[i])), tau)
}
```