① $[y^{new}, \theta, | y] \propto [y^{new}, \theta, y]$

$[y, y^{new}, \theta] =$

$[y | y_{new}, \theta] [y^{new} | \theta] [\theta]$

y is conditionally independent of y^{new}

$[y|\theta][y^{new}|\theta] [\theta]$

see that $[y|\theta][\theta] \propto [\theta|y]$
& substitute

$[y^{new}|\theta][\theta|y]$

from 1

$[y^{new}, \theta, | y] \propto [y^{new}|\theta][\theta|y]$

integrate out θ:

$$[y^{new}|y] = \int_{\theta} [y^{new} | \theta] [\theta | y] \, d\theta$$