Math Practice Solutions

1. **Find the derivative of the following functions:**

   a. $y = 3x^9$

   $dy / dx = 27x^8$

   b. $y = 5x^{-2}$

   $dy / dx = -10x^{-3}$

   c. $y = 5x + 5x^2$

   $dy / dx = 5 + 10x$

2. **If demand is given by**

   $P = 80 - Q$,

   what is marginal revenue?

   Total revenue is $PQ = 80Q – Q^2$. **So marginal revenue is** $MR = 80 – 2Q$.

3. **If a firm's profit function is**

   $\Pi = (80 – Q)Q – 10Q$,

   what quantity will maximize the firm's profit?

   Take the derivative of Π **with respect to** Q.

   $\Pi = 80Q – Q^2 – 10Q = 70Q – Q^2$

   $d\Pi / dQ = 70 – 2Q$

   Set $d\Pi / dQ$ **equal to zero and solve for** Q^*

   $70 – 2Q = 0$

   $Q^* = 35$

4. **Profit is**

$$\Pi = TR(Q) - TC(Q).$$

What is $d\Pi/dQ$?

$$d\Pi / dQ = (dTR(Q) / dQ) - (dTC(Q) / dQ)$$

5. **If the production function is of the Cobb-Douglas form, we write**

$$Q = AL^{\alpha}K^{1-\alpha}$$

where A and α are constants.

What are the marginal products of labor (L) and capital (K)?

$$MP_L = \partial Q / \partial L$$

$$MP_L = \alpha AL^{\alpha-1}K^{1-\alpha}$$

$$MP_K = \partial Q / \partial K$$

$$MP_K = AL^{\alpha}K^{1-\alpha-1} = (1-\alpha)AL^{\alpha}K^{-\alpha}$$

6. **If demand is $P = 50 - 2Q$ and total cost is $TC = Q^2 - 10Q + 5$, profit is**

$$\Pi = (50 - 2Q)Q - Q^2 + 10Q - 5$$

d. **What value of Q will maximize profit?**

$$\Pi = 50Q - 2Q^2 - Q^2 + 10Q - 5$$

$$\Pi = 60Q - 3Q^2 - 5$$

$$d\Pi / dQ = 60 - 6Q$$

$$60 - 6Q = 0$$

$$Q^* = 10$$

e. **What is the profit maximizing price?**

$$P^* = 50 - 2Q^* = 50 - 2(10)$$

$P^* = 30$

f. **What is the maximum profit?**

$\Pi^* = 60(10) – 3(10^2) – 5$

$\Pi^* = 600 – 300 – 5$

$\Pi^* = 295$

7. **If demand is $P = 100 – 0.10Q$, what value of Q will maximize total revenue? What will be the price and how much will total revenue be?**

$TR = 100Q – 0.10Q^2$

$MR = 100 – 0.20Q = 0$ **at the maximum of** TR**.**

$Q^* = 500$

$P^* = 100 – 0.10(500)$

$P^* = 50$

$TR^* = PQ = 50*500 = 25{,}000$ **or,**

$TR^* = 100(500) – 0.10(500^2) = 25{,}000$

8. **Suppose that the demand for football tickets to the Michigan-Ohio State game is**

$P = 50 – 0.00025Q$

a. **What price will maximize total revenue?**

$TR = 50Q – 0.00025Q^2$

$MR = 50 – 0.00050Q = 0$ **at the maximum of** TR**.**

$50 – 0.0005Q^* = 0$

$Q^* = 100{,}000$

$P^* = 50 – 0.00025(100{,}000)$

$P^* = 25$

b. **How many tickets will be sold?**

$Q^* = 100{,}000$

c. **How much will total revenue be?**

$TR = 25*100{,}000 = 2{,}500{,}000$

9. **Suppose that short-run profit is**

$\Pi = PQ(L) - wL$.

and output as a function of labor is given by $Q(L) = 2L^2$

Find the optimal value for L.

$d\Pi / dL = P(dQ / dL) - w = 0$

$w / P = (dQ / dL)$

$w / P = 4L$

$(1/4)(w / P) = L^*$