Structural Change

- Many economic and financial series undergo episodes of quite dramatic change.
- These changes are often not captured by any of the standard parametric models (that we have considered).
- See, for example, Hamilton (1994), p.678, dollar-denominated accounts held in Mexican banks.
- Similar dramatic breaks can result from wars, natural disasters, financial panics and changes in government policies.
- Two important issues:
- (1) Is such a time series stationary with a break, or nonstationary (or both)?
- (2) How should we model a change in the process followed by a time series?

Nonstationary or structural breaks?

- There is a “chicken or the egg” problem with testing for stationarity and structural breaks.
- Tests have been proposed that combine both, using dummy variables for the structural break(s). We will look at these after we address the second question above.
- Ultimately, we should look at “other information”, i.e. historical analysis of the data generating process, to determined what happened when there appears to be potential breaks.
- If there are clear breaks, we can also model these before testing for stationarity, then test the residuals (= the series with the break points removed) for stationarity.
- Also, a structural break is not the same thing as a shift in a series. If a RHS (explanatory) variable has a discrete change at the same time as (or leads) a change in the dependent variable, the parameters of the model may not change.

Detecting structural change

- One way to detect possible break points is by recursive least squares estimation.
- Suppose we have data for $t = 1, 2, \ldots, T$.
- We estimate the (presumably "best") model using only a subsample at the beginning of the time series (enough observations to be able to estimate the model), i.e. for $k < T$, we use only observations 1 to k.
- This gives parameter estimates $\hat{\beta}_k$.
- We then reestimate the model using observation 1 to $k + 1$ to obtain $\hat{\beta}_{k+1}$.
- Repeating this process up to $k = T$ gives a time series for the estimated coefficients.

Recursive least squares

- It can be shown that these estimates can be calculated recursively (using the Kalman filter).
- Let x_t = the value of the RHS variables in period t (not 1 to t), then:

$$\hat{\beta}_t = \hat{\beta}_{t-1} + A_t x_t f_t = (X_t' X_t)^{-1} X_t' Y_t,$$

$$A_t = A_{t-1} - \frac{1}{v_t} A_{t-1} x_t x_t' A_{t-1},$$

$$v_t = 1 + x_t' A_{t-1} x_t,$$

where, $y_t = x_t' \beta_t + e_t$, $f_t = y_t - x_t' \hat{\beta}_{t-1}$, $A_t = (X_t' X_t)^{-1}$, $Y_t = (y_1, y_2, \ldots y_t)$.

Recursive residuals

- If the model is valid (so that the parameters are all constant across the entire sample), then the forecast errors can be shown to be iid with variance,

$$\mathrm{var}(f_t) = \mathrm{var}(f_t) + \mathrm{var}(f_t) = \sigma^2(1 + x_t' A_{t-1} x_t) = \sigma^2 v_t.$$

- So, if the parameters of the model are constant, then

$$w_t = \frac{f_t}{\sqrt{v_t}} \sim N(0, \sigma^2), \quad t = k, k+1, \ldots, T.$$

- The values of w_t are called the **recursive residuals.**
- We can plot the recursive parameter estimates and recursive residuals to detect possible parameter shifts. See **strucch.R**, and the R package **strucchange** (see the help file: help(package=strucchange))

Rolling regression

- With recursive least squares, we are less likely to detect structural change towards the end of the sample, so instead we can use a moving window fixed sample size.
- We start by estimating the model with $t = 1, \ldots k$ observations, as before, but instead of just adding an observation at the end of the sample, we also substract one from the beginning, so we use

  $$t = 2, \ldots, k + 1,$$

  then

  $$t = 3, \ldots, k + 2,$$

  etc.
- The choice of window size is arbitrary, so we can try different sizes.
- We can plot the rolling parameter estimates and residuals to detect possible parameter shifts.

Testing for structural change

- The CUSUM test is based on the cumulative sums of the recursive residuals,

$$W_r = \sum_{t=k+1}^{r} \frac{w_t}{\hat{\sigma}}, \; r = k = 1, \ldots, n.$$

$$W_r \sim N(0, r-k).$$

- For a significance level of approx. 5%, model mispecification is indicated if there exists a point r for which

$$|W_r| > 0.948(1 + 2\frac{r-k}{n-k})\sqrt{n-k}.$$

Chow test for structural break point

- The Chow test splits the sample into two subgroups and tests for a statistically significant difference in the parameter estimates across the two subsamples.
- The restricted model is the model estimated on the entire sample. This is the unrestricted model is the models estimated on each subsample. The F-test is then,

$$F = \frac{S_0 - S_1 - S_2/k}{(S_1 + S_2)/(n_1 + n_2 - 2k)} \sim F(k, n_1 + n_2 - 2k),$$

  where S_0 is the RSS from the restricted model and $S_1 + S_2$ is the sum of the RSS from the restricted model(s).
- This is equivalent to (and is often performed by) including a dummy variable for the break point and interactions with the break point dummy for all RHS variables. A joint F-test of the dummy variable and all interaction terms is then carried out.
- We can try different break points to see which (if any) are indicated most strongly as existing.

Stationarity and structural breaks

- Suppose the true process is (see **strucch.R**)

$$y_t = \begin{cases} \alpha_0 + e_t & t \leq k \\ \alpha_1 + e_t & t > k \end{cases},$$

$$e_t \sim N(0, \sigma^2).$$

- If we perform a unit root test on this, we will likely find (if the difference between α_0 and α_1 is large enough) that y_t appears to not be stationary.
- In this case, we can include a dummy variable in the test equation to model the structural change.
- It is often not so obvious whether variation in a variable over time is nonstationarity or due to structural change, especially if there are several potential breaks.