

**ECON501 2018**

lab06

# DYNAMIC INEFFICIENCY IN DIAMOND MODEL

- Equilibrium in Ramsey model was Pareto efficient
- Diamond model admits the possibility of inefficiency
  - It is possible that  $k^*$  is greater than Golden Rule  $k$
  - Why? Because survival in old age depends on saving lots of income regardless of rate of return.
  - It is possible that the saving rate that is optimal for an individual might be higher than the saving rate that leads to Golden Rule level of  $k^*$

Ref: <https://www.reed.edu/economics/parker/314/notes1.html>

# SUPPOSE THAT $K^* > KGR$

- o Suppose that there was another (than capital) way of transferring money from youth to old age (Social Security) so that saving could go down
- o Young would be better off because they could consume more
- o Future generations would be better off because  $k^* \downarrow$  means higher steady-state  $c^*$
- o Everyone is made better off and no one worse off, so original equilibrium must not have been Pareto optimal

# HOW CAN MODEL BE INEFFICIENT? WHERE IS THE MARKET FAILURE?

- o No externalities, but an absent market: no way for current generation to trade effectively with future generations
- o Only way to provide for retirement is through saving, even if rate of return is zero or negative
- o From social standpoint, it is desirable to avoid low- or negative-return investment, but for individual facing retirement this is only choice
- o If benevolent government were to establish transfer scheme from young to old (like Social Security), then they would not need to accumulate useless capital in order to eat in retirement
- o You will do a problem this week looking at the effect of alternative Social Security regimes in the Diamond model

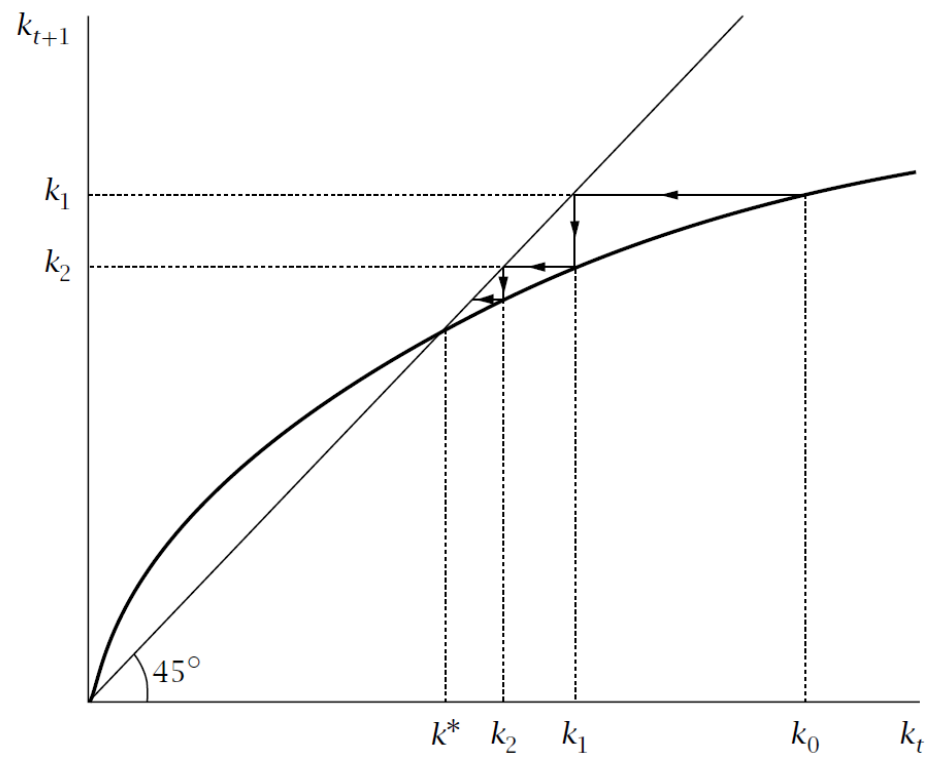


FIGURE 2.11 The dynamics of  $k$

## Logarithmic Utility and Cobb–Douglas Production

When  $\theta$  is 1, the fraction of labor income saved is  $1/(2 + \rho)$  (see equation [2.55]). And when production is Cobb–Douglas,  $f(k)$  is  $k^\alpha$  and  $f'(k)$  is  $\alpha k^{\alpha-1}$ . Equation (2.59) therefore becomes

$$k_{t+1} = \frac{1}{(1+n)(1+g)} \frac{1}{2+\rho} (1-\alpha) k_t^\alpha. \quad (2.60)$$

Figure 2.11 shows  $k_{t+1}$  as a function of  $k_t$ . A point where the  $k_{t+1}$  function intersects the 45-degree line is a point where  $k_{t+1}$  equals  $k_t$ . In the case we are considering,  $k_{t+1}$  equals  $k_t$  at  $k_t = 0$ ; it rises above  $k_t$  when  $k_t$  is small; and it then crosses the 45-degree line and remains below. There is thus a unique balanced-growth-path level of  $k$  (aside from  $k = 0$ ), which is denoted  $k^*$ .

**2.17. Social security in the Diamond model.** Consider a Diamond economy where  $g$  is zero, production is Cobb–Douglas, and utility is logarithmic.

- (a) **Pay-as-you-go social security.** Suppose the government taxes each young individual an amount  $T$  and uses the proceeds to pay benefits to old individuals; thus each old person receives  $(1 + n)T$ .
  - (i) How, if at all, does this change affect equation (2.60) giving  $k_{t+1}$  as a function of  $k_t$ ?
  - (ii) How, if at all, does this change affect the balanced-growth-path value of  $k$ ?
  - (iii) If the economy is initially on a balanced growth path that is dynamically efficient, how does a marginal increase in  $T$  affect the welfare of current and future generations? What happens if the initial balanced growth path is dynamically inefficient?
- (b) **Fully funded social security.** Suppose the government taxes each young person an amount  $T$  and uses the proceeds to purchase capital. Individuals born at  $t$  therefore receive  $(1 + r_{t+1})T$  when they are old.
  - (i) How, if at all, does this change affect equation (2.60) giving  $k_{t+1}$  as a function of  $k_t$ ?
  - (ii) How, if at all, does this change affect the balanced-growth-path value of  $k$ ?

<http://listinet.com/bibliografia-comuna/Cdu339-52C2.pdf>

Page 60, Romer, 2.16. Social Security

Since the budget constraint changes, we have to solve the Diamond model again.