

*** Problem16.1(c)**

Briefly describe the experimental design you would choose for each of the following situations, and explain why.

Rodent activity may be affected by photoperiod patterns. We wish to test this possibility by treating newly-weaned mouse pups with three different treatments. Treatment 1 is a control with the mice getting 14 hours of light and 10 hours of dark per day. Treatment 2 also has 14 hours of light, but the 10 hours of dark are replaced by 10 hours of a low light level. Treatment 3 has 24 hours of full light.

Mice will be housed in individual cages, and motion detectors connected to computers will record activity. We can use 24 cages, but the computer equipment must be shared and is only available to us for 1 month. Mice should be on a treatment for 3 days—one day to adjust and then 2 days to take measurements. We may use each mouse for more than one treatment, but if we do, there should be 7 days of standard photoperiod between treatments. We expect large subject-to-subject variation. There may or may not be a change in activity as the rat pups age; we don't know.

< Sol >

本題想要討論的議題對老鼠採取不同的處理後，是否有證據去證明處理之間存在著顯著差異。處理總共有 3 種，實驗個體為老鼠，共有 24 隻老鼠。題目的限制為實驗時間為一個月，而每個處理都要花 3 天的時間完成，每一隻老鼠都可以做一種以上的處理，但處理與處理之間要有 7 天的時間讓老鼠休息，消除前一個處理的影響，其餘並無限制。

經過簡單計算後，發現一個月的時間足夠對每一隻老鼠進行 3 種處理，花費天數約為 23 天的時間。因此我可能採取隨機區集實驗(Random block designment;RBD)，由於每一隻老鼠的獨特性可能會影響實驗結果，這個因素無法透過實驗控制的方式排除，為了減少這個因素的影響，希望透過 RBD 實驗設計，將老鼠之間的差異的消彌掉。

每一個老鼠的實驗處理順序為隨機排列，處理排列順序為以下 6 種，這 6 種處理排列順序隨機分配給每一隻老鼠進行實驗。

1	處理 1 →處理 2→處理 3
2	處理 1 →處理 3→處理 2
3	處理 2 →處理 1→處理 3
4	處理 2 →處理 3→處理 1
5	處理 3 →處理 2→處理 1
6	處理 3 →處理 1→處理 2

經過 23 天後，預期會得到以下表格的形式。

	處理 1	處理 2	處理 3
老鼠 1 號			
老鼠 2 號			
老鼠 3 號			
...			
老鼠 23 號			
老鼠 24 號			

最後在進行 RBD 的 ANOVA 分析，並解釋結果。

* Problem16.4

Most universities teach many sections of introductory calculus, and faculty are constantly looking for a method to evaluate students consistently across sections. Generally, all sections of intro-calculus take the final exam at the same time, so a single exam is used for all sections. An exam service claims that it can supply different exams that consistently evaluate students. Some faculty doubt this claim, in part because they believe that there may be an interaction between the text used and the exam used.

Three math departments (one each at Minnesota, Washington, and Berkeley) propose the following experiment. Three random final exams are obtained from the service: E1, E2, and E3. At Minnesota, the three exams will be used in random order in the fall, winter, and spring quarters. Randomization will also be done at Washington and Berkeley. The three schools all use the same two intro calculus texts. Sections of intro calculus at each school will be divided at random into two groups, with half of the sections using text A and the other half using text B. At the end of the year, the mean test scores are tallied with the following results.

School	Exam	Text	
		A	B
Wash	1	81	87
	2	79	85
	3	70	78
Minn	1	84	82
	2	81	81
	3	83	84
Berk	1	87	98
	2	82	93
	3	86	90

Analyze these data to determine if there is any evidence of variation between exams, text effect, or exam by text interaction. Be sure to include an explicit description of the model you used.

< Sol >

有三間大學分別為在 Minnesota, Washington, and Berkeley，每間學校有三次考試(Exam)，以及兩種不同版的課本(Text)，研究著想從這三個不同影響考試成績的因素找出是否有顯著的因素存在。而我們採取列區實驗去做分析。Block 為學校，其餘為因子為 Exam 以及 Text，以下為分析的 ANOVA table

H_0 : 該因子效應或區集效應不存在

H_1 : 該因子效應或區集效應存在

	Sum Sq	Mean Sq	NumDF	DenDF	F value	Pr(>F)
Block	194.564	97.282	2	4	6.8135	0.05149
Exam	46.615	23.308	2	4	1.6324	0.30316
Text	112.5	112.5	1	6	7.8794	0.03088
Exam:Text	1.333	0.667	2	6	0.0467	0.95472

分析

以顯著水準 α 為 0.05 下，先看考試與課本的交互作用項，發現其 p 值為 0.95 多，遠大於顯著水準 0.05，意味著沒有足夠的證據去拒絕 H_0 ，即沒有顯著的交互作用項。接下來個別看區集以及因素是否有顯著，只有課本 (Text) 的 p 值小於 0.05，有足夠的證據去拒絕 H_0 的假設，顯示使用不同的課本會讓成績有所不同，而我們仔細看實驗的樣本，會發現使用課本 B 考試的成績會比使用課本 A 的考試成績來得高，也間接證明使用不同的課本會有成績的差異。而學校的區集 p 值為 0.05149 很接近顯著水準 α ，我認為有足夠證據去拒絕 H_0 的假設，顯示不同的學校亦會造成成績有所不同，回頭查看三間學校的成績，會發現 Berkeley 的學生成績普遍表現較其他兩所學校好，Washington 的成績想對來講就比另外兩間差，也顯示了學校會影響成績。

*Appendix

```
library(lmerTest)

# Input data
Block <- rep(c("Wash", "Minn", "Berk"), each = 3, times = 2)
Exam <- rep(rep(c("1", "2", "3"), times = 3), times = 2)
Text <- rep(c("A", "B"), each = 9)
score <- c(81, 79, 70, 84, 81, 83, 87, 82, 86, 87, 85, 78, 82, 81, 84, 98, 93, 90)
data.score <- cbind.data.frame(Block, Exam, Text, score)

str(data.score)

## 'data.frame': 18 obs. of 4 variables:
## $ Block: Factor w/ 3 levels "Berk", "Minn", ...: 3 3 3 2 2 2 1 1 1 3 ...
## $ Exam : Factor w/ 3 levels "1", "2", "3": 1 2 3 1 2 3 1 2 3 1 ...
## $ Text : Factor w/ 2 levels "A", "B": 1 1 1 1 1 1 1 1 1 2 ...
## $ score: num 81 79 70 84 81 83 87 82 86 87 ...

temp <- lmerTest::lmer(score ~ Block + Exam * Text + (1 | Block:Exam), data = data.score)
anova(temp)

## Type III Analysis of Variance Table with Satterthwaite's method
##           Sum Sq Mean Sq NumDF DenDF F value Pr(>F)
## Block      194.564   97.282     2     4   6.8135 0.05149 .
## Exam        46.615   23.308     2     4   1.6324 0.30316
## Text       112.500  112.500     1     6   7.8794 0.03088 *
## Exam:Text    1.333    0.667     2     6   0.0467 0.95472
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```