# Design of experiment - Homework 06

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## ※ Exercise 7.1

Find the smallest sample size giving power of at least .7 when testing equality of six groups at the .05 level when $\zeta = 4n$.

$H_0$: 六組的品質皆一致

$H_1$: 六組的品質不一致

在$H_0$為真下的機率分配為 $F(g-1, N-g)$，$H_1$為真下的機率分配為 $F(g-1, N-g, \zeta)$，g 為總組數，$n_i$為每一組的樣本數( $i = 1\sim g$ )，N 為總樣本數( $N = \sum_i^g n_i$ )，$\zeta$ 為非中心參數 ( $\zeta = \frac{(\sum_i^g w_i \alpha_i)^2}{\sigma^2 \sum_i^g \frac{w_i^2}{n_i}}$ )，$\alpha_i$ 第 i 個處理效應(treatmeant effect)，$w_i$ 為第 i 個 contrast 係數，顯著水準為 0.05。

power 為在$H_1$為真下，拒絕$H_0$假設的機率。

拒絕域 : $F \geq F_{0.05}(g-1,\ N-g) \Rightarrow \text{power} = P(F > F_{0.05}(g-1, N-g) \mid F \sim F(g-1, N-g, \zeta))$

< Sol >

總組數 g = 6, 顯著水準為 0.05，$\zeta$ 為非中心參數為 4n，$w_i = 1$ ， $i = 1\sim g$ ，

拒絕域為$F \geq F_{0.05}(5, 6n-6)$，n = 2,3 …，

$$\text{power} = P(F > F_{0.05}(5, 6n-6) \mid F \sim F(5, 6n-6, 4n))$$

| n | power |
|---|---|
| 2 | 0.2715161 |
| 3 | 0.5529867 |
| 4 | 0.7640361 |
| 5 | 0.8888004 |
| 6 | 0.9520672 |

由上表得知，當每組樣本數 ≥ 4 即總樣本數 ≥ 24 時，則 power≥ 0.7。

## ※ Exercise 7.3

What is the probability of rejecting the null hypothesis when there are four groups, the sum of the squared treatment effects is 6, the error variance is 3, the group sample sizes are 4, and E is .01?

<sol>

$H_0$: 四組的特性皆一致

$H_1$: 四組的特性不一致

Power 公式同 exercise 7.1，分別帶入$g = 4$，處理效應平方和 $(\sum_i^g w_i\alpha_i)^2 = 6$，$\widehat{\sigma^2} = 3$，$n = 4$,顯著水準為 0.01，透過運算得知 $\zeta = 8$ ，拒絕域：$F \geq F_{0.01}(3, 12) = 5.9525$，使得

$$power = P(F > F_{0.01}(3, 12) | F \sim F(3, 12, 8)) = 0.2261。$$

## ※ Problem 7.2

Nondigestible carbohydrates can be used in diet foods, but they may have effects on colonic hydrogen production in humans. We want to test to see if inulin, fructooligosaccharide, and lactulose are equivalent in their hydrogen production. Preliminary data suggest that the treatment means could be about 45, 32, and 60 respectively, with the error variance conservatively estimated at 35. How many subjects do we need to have power .95 for this situation when testing at the $E_I = .01$ level?

$H_0$: Inulin, fructooligosaccharide 和 lactulose 效果一致

$H_1$: Inulin, fructooligosaccharide 和 lactulose 效果不一致

< Sol >

power 公式同 exercise 7.1，

處理效應平方和$\sum_{i=1}^3(\overline{Y_{i.}} - \overline{Y_{..}})^2 = (45 - 45.667)^2 + (32 - 45.667)^2 + (60 - 45.667)^2 = 392.667$，

$\widehat{\sigma^2} = 35$, 顯著水準為 0.01，組數 g = 3。推算得知 $\zeta = \frac{392.667n}{35} = 11.21906n$，$n \geq 2$。

拒絕域：$F \geq F_{0.01}(2, 3n - 3)$

$$power = P(F > F_{0.01}(2, 3n - 3) | F \sim F(2, 3n - 3, 11.21906n))$$

| n | power |
|---|---|
| 2 | 0.2424548 |
| 3 | 0.804055 |
| 4 | 0.9808418 |
| 5 | 0.9989454 |
| 6 | 0.9999594 |

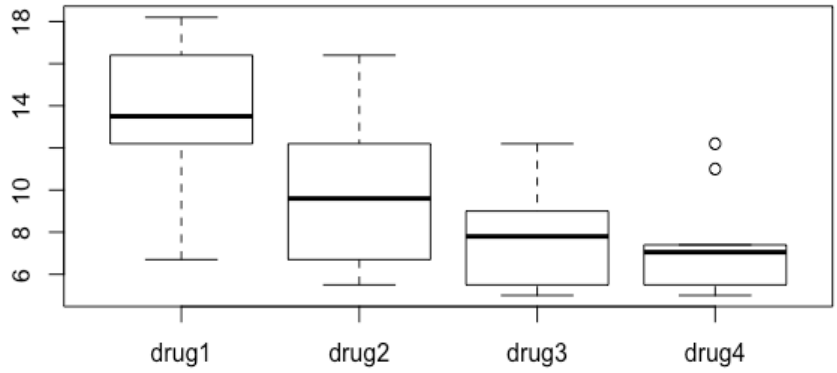
由上表得知，當每組樣本數 $\geq 4$ 即總樣本數 $\geq 12$ 時，則 power$\geq 0.95$。

## ※ Exercise 6.3

In order to determine the efficacy and lethal dosage of cardiac relaxants, anesthetized guinea pigs are infused with a drug (the treatment) till death occurs. The total dosage required for death is the response; smaller lethal doses are considered more effective. There are four drugs, and ten guinea pigs are chosen at random for each drug. Lethal dosages follow.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 第一種藥物 | 18.2 | 16.4 | 10.0 | 13.5 | 13.5 | 6.7 | 12.2 | 18.2 | 13.5 | 16.4 |
| 第二種藥物 | 5.5 | 12.2 | 11.0 | 6.7 | 16.4 | 8.2 | 7.4 | 12.2 | 6.7 | 11.0 |
| 第三種藥物 | 5.5 | 5.0 | 8.2 | 9.0 | 10.0 | 6.0 | 7.4 | 5.5 | 12.2 | 8.2 |
| 第四種藥物 | 6.0 | 7.4 | 12.2 | 11.0 | 5.0 | 7.4 | 7.4 | 5.5 | 6.7 | 5.5 |

初步畫出個別的 boxplot，判斷是否存在差異性：




顯示四種藥物的 Lethal dosages 用量可能存在差異性

檢定資料是否符合使用 ANOVA 分析的前置：殘差具有常態性、均齊性以及獨立性，並檢測是否有離群值存在。

**(1)**殘差常態性檢定

$H_0$: 殘差為常態性資料

$H_1$: 殘差不為常態性資料

在顯著水準為 0.05 下，使用 Shapiro-Wilk normality test 去檢測，P-value 為 0.6699 > 0.05。

不拒絕$H_0$的假設，即沒有足夠的證據顯示殘差不具有常態性。

**(2)**殘差均齊性檢定

$H_0$: 殘差為均齊性資料

$H_1$: 殘差不為均齊性資料

在顯著水準為 0.05 下，Non-constant Variance Score Test 去檢測，P-value 為 0.1106437 > 0.05。

不拒絕$H_0$的假設，即沒有足夠的證據顯示殘差不具有均齊性。

**(3)**殘差獨立性檢定

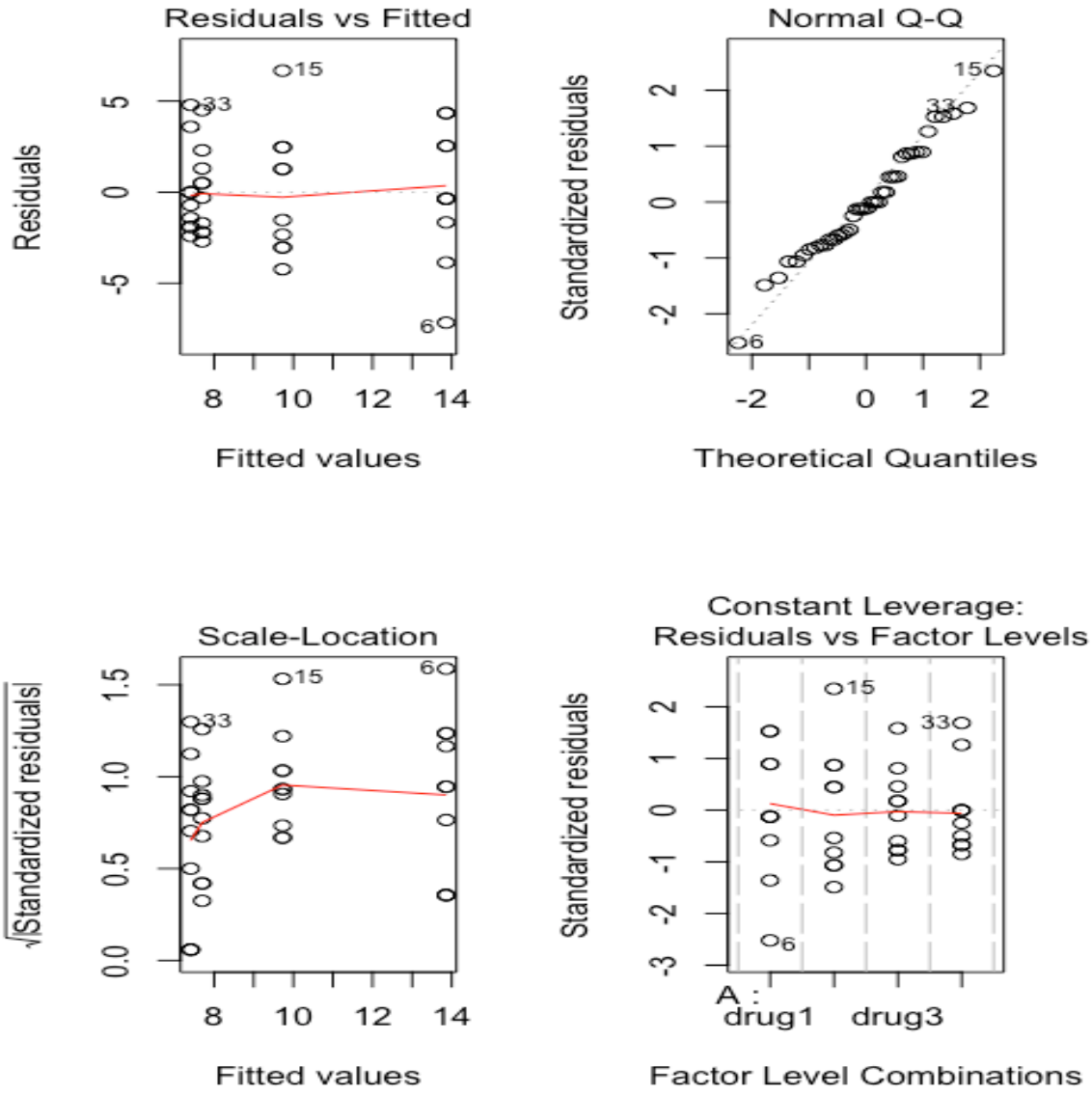
$H_0$: 殘差為獨立性資料

$H_1$: 殘差不為獨立性資料

在顯著水準為 0.05 下，使用 Durbin-Watson 檢定去檢測，P-value 為 0.724 > 0.05。

不拒絕$H_0$的假設，即沒有足夠的證據顯示殘差不具有獨立性。

以圖表示




綜合(1),(2),(3)的結果，顯示這組資料符合 ANOVA 的使用前置條件：殘差具有常態性、齊一性以及獨立性。

**(4)**檢測是否有離群值存在

$H_0$: 無離群值存在

$H_1$: 有離群值存在

在顯著水準為 0.05 下，使用 Bonferonni 離群值檢測， P-value 為 0.38626 > 0.05。

不拒絕$H_0$的假設，即沒有足夠的證據顯示有離群值存在。

**(5)**檢測四種藥物是否具有相同用量

$H_0$: 四種藥物對於 lethal dosage 用量相同

$H_1$: 四種藥物對於 lethal dosage 用量不盡相同

| | **SS** | **df** | **MS** | **F** | **P** |
|---|---|---|---|---|---|
| **Treatment** | 265.5 | 3 | 88.49 | 9.865 | 0.0000691 |
| **Error** | 322.9 | 36 | 8.97 | | |

在顯著水準為 0.05 下，P-value = 0.0000691 < 0.05 ，拒絕$H_0$的假設，有足夠的證據拒絕四種藥物效果相同的假設。

**(6)**兩兩檢測藥物是否具有相同效果

在顯著水準為 0.05 下，使用 Turkey 方法去檢測兩兩資料，檢定標準以 95%信賴區間為依據，判斷是否 0 包含在內。若是該信賴區間有包含 0 則不拒絕$H_0$假設，反之則拒絕。

以圖表方式呈現

| $H_0$ | Estimate |
|---|---|
| drug2 - drug1 == 0 | -4.13 |
| drug3 - drug1 == 0 | -6.16 |
| drug4 - drug1 == 0 | -6.45 |
| drug3 - drug2 == 0 | -2.03 |
| drug4 - drug2 == 0 | -2.32 |
| drug4 - drug3 == 0 | -0.29 |


由右圖可知只有$H_0$: drug3 用量 = drug2 用量、$H_0$: drug4 用量 = drug2 用量以及$H_0$: drug3 用量 = drug4 用量沒有被拒絕，其餘都拒絕$H_0$的假設。另外由右圖來看，看出 drug2 vs drug1, drug3 vs drug1 與 drug4 vs drug1 有顯著差異，相減的信來區間都小於 0，代表 drug1 用量相對其他的 drug 用量來得大。因此從致命量的用量效果來看，第一種藥物效果較差，需要用量較多。其餘三種的效果並沒有顯著的差異。

## ※ Appendeix

```r
# Exercise 01 ------------------------------------------------------------
# Exercise 7.1
# H0: 6 個組別的mean 都一樣
# H1: 6 個組別的mean 存在不一樣
# 假設每一組的樣本數都為 n，alpha = 0.05，ζ = 4n. 找個最小 n 使得 power >= 0.7
## Design a power function

E1 <- function(n){
tmp <- 1 - pf(qf(1-0.05, 6-1, 6*n - 6),6-1, 6*n-6, ncp = 4*n)
return(tmp)
}

## Show the power when giving different n
temp <- NULL
for( i in 2:8){
  temp <- c(temp, E1(i))
}
temp <- cbind(c(2:8),temp)
colnames(temp) <- c("n","Power")
temp

##      n     Power
## [1,] 2 0.2715161
## [2,] 3 0.5529867
## [3,] 4 0.7640361
## [4,] 5 0.8888004
## [5,] 6 0.9520672
## [6,] 7 0.9807802
## [7,] 8 0.9927432

# 由表可知 n 至少要 4，power 才會來到 0.7 以上，總樣本數為 24


# Exercise 02 ------------------------------------------------------------
# Exercise 7.3
#What is the probability of rejecting the null hypothesis when there are four groups,
#the sum of the squared treatment effects is 6,
#the error variance is 3, the group sample sizes are 4, and E is .01?
```

```r
g <- 4; n <- 4
SSTR <- 6
MSE <- 3
zeta = 6 * 4 / 3
beta <- pf(qf(1-0.01, g - 1, g*n - g), g - 1, g*n - g, ncp = zeta)
power <- 1 - beta
power
```

```
## [1] 0.2260942
```

```r
# power is 0.2261



# Exercise 03 ------------------------------------------------------------
# Problem 7.2
#Nondigestible carbohydrates can be used in diet foods,
#but they may have effects on colonic hydrogen production in humans.
#We want to test to see if inulin, fructooligosaccharide,
#and lactulose are equivalent in their hydrogen production.
#Preliminary data suggest that the treatment means could be about 45, 32, and 60 respec
tively,
#with the error variance conservatively estimated at 35.
#How many subjects do we need to have power .95 for this situation when testing at the E
I = .01 level?
g <- 3
x_bar <- c(45, 32, 60)
x_bar2 <- mean(x_bar)
SSTR <- sum((x_bar - x_bar2)^2)
MSE <- 35
P2 <- function(n){
  zeta = n * SSTR/35
  temp <- qf((1 - 0.01),(g-1),(n*g-g))
  power <- 1 - pf(temp,g-1,n*g-g,ncp = zeta)
  return(power)}
n = 2
while(P2(n)<0.95){
  n = n+1}
n
```

```
## [1] 4
```

```
P2(4)
```

```
## [1] 0.9808418
```

```
# If n = 4, then power will be 0.9808418, so total # of sample is 12.
```

```
# Exercise 04 ------------------------------------------------------------
# Exercise 6.3
library(MASS)
library(car)
library(multcomp)
```

```
## Loading required package: mvtnorm
```

```
## Loading required package: survival
```

```
## Loading required package: TH.data
```

```
##
## Attaching package: 'TH.data'
```

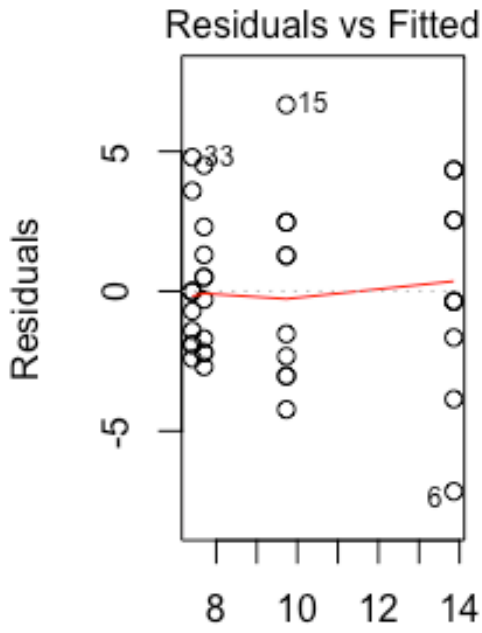
```
## The following object is masked from 'package:MASS':
##
##     geyser
```

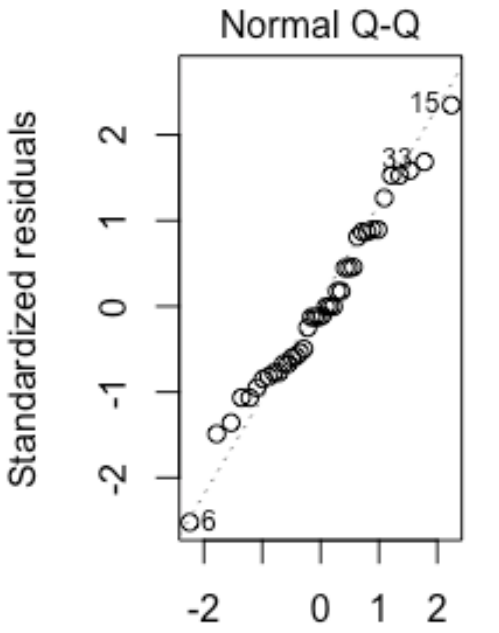
```
#In order to determine the efficacy and lethal dosage of cardiac relaxants,
#anesthetized guinea pigs are infused with a drug (the treatment) till death occurs.
#The total dosage required for death is the response; smaller lethal doses are consider
ed more effective.
#There are four drugs, and ten guinea pigs are chosen at random for each drug. Lethal d
osages follow.

#Determine which drugs are equivalent, which are more effective, and which less effecti
ve.
y1 <- c(18.2, 16.4, 10.0, 13.5, 13.5, 6.7, 12.2, 18.2, 13.5, 16.4)
y2 <- c(5.5, 12.2, 11.0, 6.7, 16.4, 8.2, 7.4, 12.2, 6.7, 11.0)
y3 <- c(5.5, 5.0, 8.2, 9.0, 10.0, 6.0, 7.4, 5.5, 12.2, 8.2)
y4 <- c(6.0, 7.4, 12.2, 11.0, 5.0, 7.4, 7.4, 5.5, 6.7, 5.5)
y <- c(y1, y2, y3, y4)
A <- as.factor(rep(c("drug1","drug2","drug3","drug4"), each = 10))
```

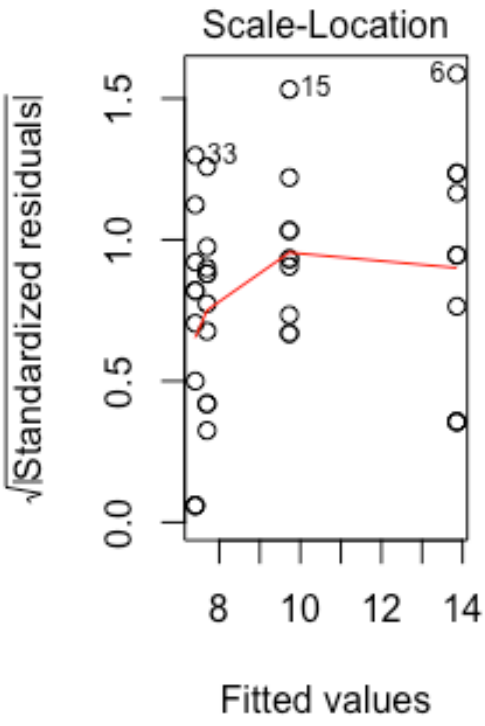
```r
temp <- lm(y~A)
summary(aov(temp))
```

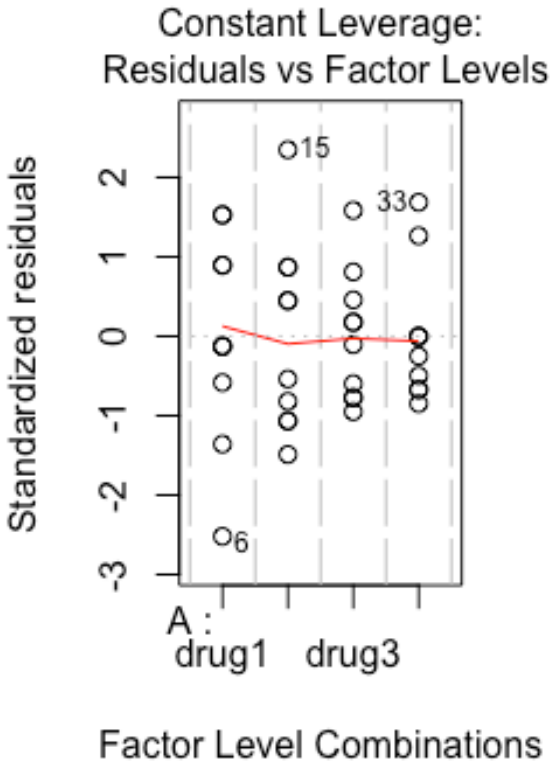
```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## A            3  265.5   88.49   9.865 6.91e-05 ***
## Residuals   36  322.9    8.97
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# 殘差的常態性、變異數均齊性以及獨立性
par(mfrow = c(1,2))
plot(temp)
```




```
resid <- (temp$residuals)
```

```
# 常態性検定
shapiro.test(resid )
```

```
##
##  Shapiro-Wilk normality test
##
## data:  resid
## W = 0.97947, p-value = 0.6699
```

```
# 均齊性
ncvTest(temp)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 2.545003    Df = 1     p = 0.1106437
```

```
# 殘差獨立性檢定
durbinWatsonTest(temp)
```

```
##  lag Autocorrelation D-W Statistic p-value
##    1       -0.163234      2.256845   0.698
##  Alternative hypothesis: rho != 0
```

```
# 離群值檢定
outlierTest(temp)
```

```
##
## No Studentized residuals with Bonferonni p < 0.05
## Largest |rstudent|:
##    rstudent unadjusted p-value Bonferonni p
## 6 -2.737837         0.0096564      0.38626
```

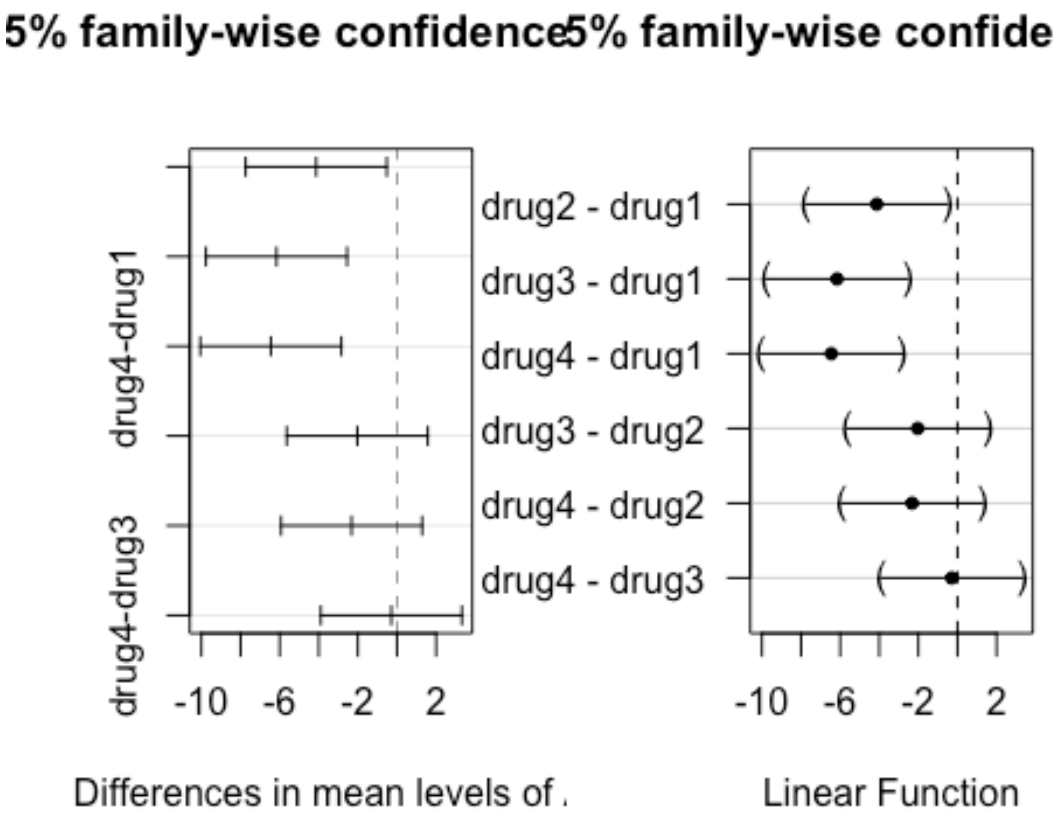
```
# ANOVA 檢定
aov(temp)
```

```
## Call:
##    aov(formula = temp)
##
## Terms:
##                       A Residuals
## Sum of Squares  265.481   322.934
## Deg. of Freedom       3        36
##
## Residual standard error: 2.995061
## Estimated effects may be unbalanced
```

```r
# 多重比較
TukeyHSD(aov(temp))
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = temp)
##
## $A
##              diff        lwr        upr     p adj
## drug2-drug1 -4.13  -7.737394 -0.5226057 0.0195604
## drug3-drug1 -6.16  -9.767394 -2.5526057 0.0002860
## drug4-drug1 -6.45 -10.057394 -2.8426057 0.0001496
## drug3-drug2 -2.03  -5.637394  1.5773943 0.4390060
## drug4-drug2 -2.32  -5.927394  1.2873943 0.3224748
## drug4-drug3 -0.29  -3.897394  3.3173943 0.9963493
```

```r
plot(TukeyHSD(aov(temp)))
library(multcomp)
temp1 <- glht(aov(temp), linfct = mcp(A = "Tukey"))
plot(temp1)
```



```r
# 檢測藥物對 Lethal dosages 的影響最大及最小
model.tables(aov(temp),type="effects")

## Tables of effects
##
##  A
## A
##  drug1  drug2  drug3  drug4
##  4.185  0.055 -1.975 -2.265
```