GSERM 2017

Regression III

Polynomials, Interactions, Etc.

June 19, 2017 (afternoon session)

"Dummies"...

- ..."naturally" dichotomous, including
  - Structural breaks
  - Proper nouns
- "Factors":

$$\texttt{partyid} = \begin{cases} 0 = \text{Labor} \\ 1 = \text{Liberal} \\ 2 = \text{Conservative} \end{cases}$$

- Ordinal variables...
- Continuous variables...

Coding Dummies

"Dummy coding":

$$\texttt{female} = \begin{cases} 0 \text{ if male} \\ 1 \text{ if female} \end{cases}$$

vs. "Effect coding":

$$\texttt{female} = \begin{cases} -1 \text{ if male} \\ 1 \text{ if female} \end{cases}$$

TL;DR: Use the former.

For

$$Y_i = \beta_0 + \beta_1 D_i + u_i$$

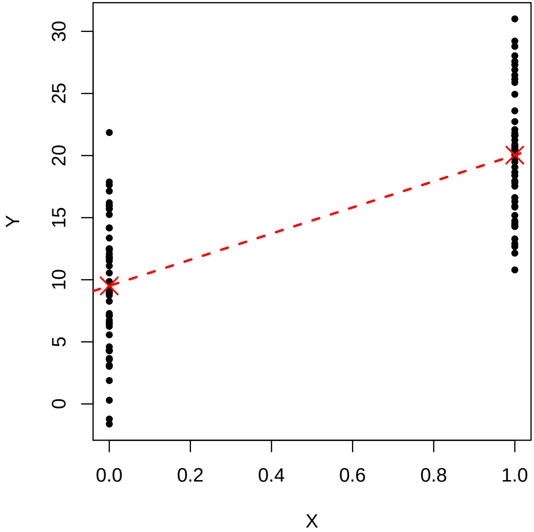
we have

$$\mathrm{E}(Y|D=0) = \beta_0$$

and

$$\mathrm{E}(Y|D=1) = \beta_0 + \beta_1.$$

Dichotomous X, Graphically



Many Dummies

For

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + ... + \beta_\ell D_{\ell i} + u_i$$

- $\mathrm{E}(Y|D_k = 0)\,\forall\, k \in \ell = \beta_0$,
- Otherwise, $\mathrm{E}(Y) = \beta_0 + \sum_{k=1}^{\ell} \beta_k \,\forall\, k\; s.t.\, D_k = 1$.

Note: where the D_ℓ are mutually exclusive and exhaustive:

- The expected values are the same as the within-group means.
- Identification requires we either
  - omit a "reference category," or
  - omit β_0.

Dummies and Ordinal Xs

Suppose we have:

$$\texttt{gopscale} = \begin{cases} -2 = \text{Strong Democrat} \\ -1 = \text{Weak Democrat} \\ 0 = \text{Independent} \\ 1 = \text{Weak Republican} \\ 2 = \text{Strong Republican} \end{cases}$$

Might estimate:

$$\texttt{closeness}_i = 46.0 + 17.5(\texttt{gopscale}_i) + u_i$$

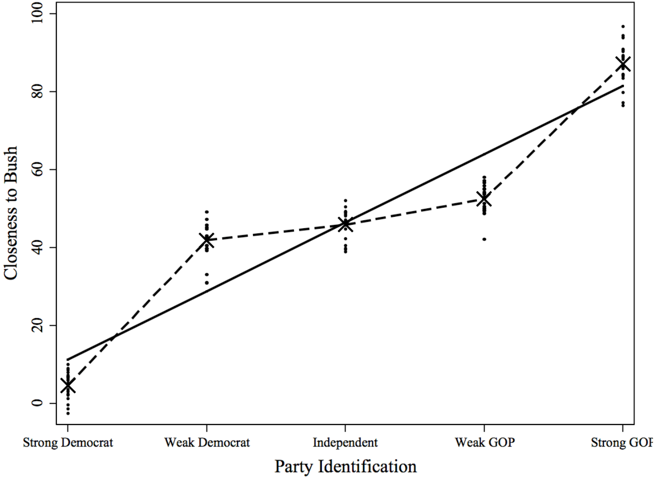
Alternative: "dummy out" `gopscale`:

$$\begin{aligned}\texttt{closeness}_i &= \beta_0 + \beta_1(\texttt{strongdem}_i) + \beta_2(\texttt{weakdem}_i) + \\ &\quad \beta_3(\texttt{weakgop}_i) + \beta_4(\texttt{stronggop}_i) + u_i\end{aligned}$$

yielding:

$$\begin{aligned}\texttt{closeness}_i &= 45.5 - 40(\texttt{strongdem}_i) - 6(\texttt{weakdem}_i) + \\ &\quad 7(\texttt{weakgop}_i) + 42(\texttt{stronggop}_i) + u_i\end{aligned}$$

Ordinal, Illustrated



Dichotomous + Continuous X

E.g.,

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 X_i + u_i$$

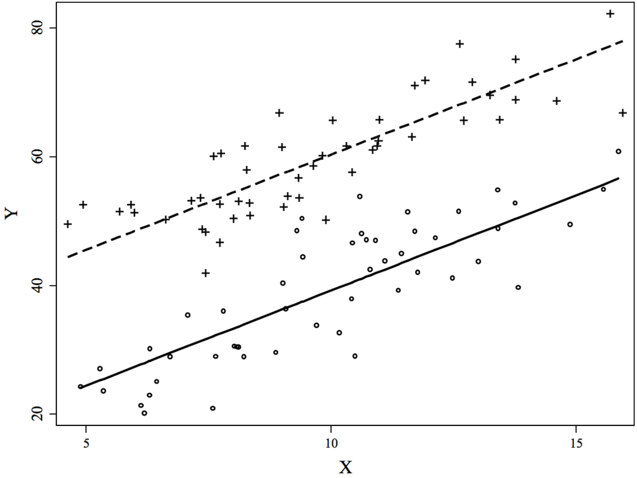
we have

$$\mathrm{E}(Y|X, D = 0) = \beta_0 + \beta_2 X_i$$

and

$$\mathrm{E}(Y|X, D = 1) = (\beta_0 + \beta_1) + \beta_2 X_i.$$

Dichotomous + Continuous X



Examples: SCOTUS

```
> library(RCurl)
> temp<-getURL("https://raw.githubusercontent.com/PrisonRodeo/GSERM-2017-git/master/
             Data/SCOTUS.csv")
> SCOTUS<-read.csv(text=temp, header=TRUE)
> summary(SCOTUS)
       id            term           Namici          lctdiss          multlaw
 Min.   :   1   Min.   :53.00   Min.   : 0.000   Min.   :0.0000   Min.   :0.0000
 1st Qu.:1791   1st Qu.:64.00   1st Qu.: 0.000   1st Qu.:0.0000   1st Qu.:0.0000
 Median :3581   Median :72.00   Median : 0.000   Median :0.0000   Median :0.0000
 Mean   :3581   Mean   :71.12   Mean   : 0.842   Mean   :0.1509   Mean   :0.1490
 3rd Qu.:5371   3rd Qu.:79.00   3rd Qu.: 1.000   3rd Qu.:0.0000   3rd Qu.:0.0000
 Max.   :7161   Max.   :85.00   Max.   :39.000   Max.   :1.0000   Max.   :1.0000
                NA's   : 4.00                    NA's   :4.0000   NA's   :5.0000
    civlibs           econs           constit           lctlib
 Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :  0.0000
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:  0.0000
 Median :1.0000   Median :0.0000   Median :0.0000   Median :  0.0000
 Mean   :0.5009   Mean   :0.1709   Mean   :0.2536   Mean   :  0.3742
 3rd Qu.:1.0000   3rd Qu.:0.0000   3rd Qu.:1.0000   3rd Qu.:  1.0000
 Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :  1.0000
                                                    NA's   :120.0000
```

Creating Dummies

All civil rights & economics cases:

```
> SCOTUS$civil.econ<-SCOTUS$civlibs + SCOTUS$econs
```

Factors:

```
> SCOTUS$termdummies<-factor(SCOTUS$term)
> is.factor(SCOTUS$termdummies)
[1] TRUE
> summary(SCOTUS$termdummies)
  53   54   55   56   57   58   59   60   61   62   63   64   65   66   67
 126  109  128  162  196  165  157  160  148  189  223  156  187  201  285

  68   69   70   71   72   73   74   75   76   77   78   79   80   81
 207  185  227  262  269  267  223  253  254  244  244  221  255  269

  82   83   84   85 NA's
 277  298  301  309    4
```

```
> fit1<-with(SCOTUS, lm(Namici~civlibs))
> summary(fit1)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.91774    0.03661  25.069  < 2e-16 ***
civlibs     -0.15136    0.05173  -2.926  0.00344 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.189 on 7159 degrees of freedom
Multiple R-squared:  0.001195,Adjusted R-squared:  0.001055
F-statistic: 8.563 on 1 and 7159 DF,  p-value: 0.003442
```

```
> with(SCOTUS, t.test(Namici~civlibs))

Welch Two Sample t-test

data:  Namici by civlibs
t = 2.9258, df = 7114.116, p-value = 0.003446
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.04995001 0.25277126
sample estimates:
mean in group 0 mean in group 1
      0.9177392       0.7663786
```

Effect Coding

```
> SCOTUS$civlibeffect<-SCOTUS$civlibs
> SCOTUS$civlibeffect[SCOTUS$civlibs==0]<-(-1)
> fit2<-with(SCOTUS, lm(Namici~SCOTUS$civlibeffect))
> summary(fit2)

Residuals:
   Min     1Q Median     3Q    Max
-0.918 -0.918 -0.766  0.082 38.234

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)          0.84206    0.02586  32.559  < 2e-16 ***
SCOTUS$civlibeffect -0.07568    0.02586  -2.926  0.00344 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.189 on 7159 degrees of freedom
Multiple R-squared:  0.001195,Adjusted R-squared:  0.001055
F-statistic: 8.563 on 1 and 7159 DF,  p-value: 0.003442
```

Many D_is

```
> fit3<-with(SCOTUS, lm(Namici~lctdiss+multlaw+civlibs+
+                         econs+constit+lctlib))
> summary(fit3)

Call:
lm(formula = Namici ~ lctdiss + multlaw + civlibs + econs + constit +
    lctlib)

Residuals:
   Min     1Q Median     3Q    Max
-2.582 -0.976 -0.472 -0.260 37.086

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.47245    0.05273   8.960  < 2e-16 ***
lctdiss      0.36760    0.07173   5.125 3.06e-07 ***
multlaw      0.61306    0.07445   8.235  < 2e-16 ***
civlibs     -0.21255    0.06022  -3.530 0.000419 ***
econs        0.08772    0.07652   1.146 0.251691
constit      0.53793    0.06372   8.442  < 2e-16 ***
lctlib       0.50309    0.05396   9.323  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.15 on 7033 degrees of freedom
  (121 observations deleted due to missingness)
Multiple R-squared:  0.05013,Adjusted R-squared:  0.04932
F-statistic: 61.86 on 6 and 7033 DF,  p-value: < 2.2e-16
```

Using factor

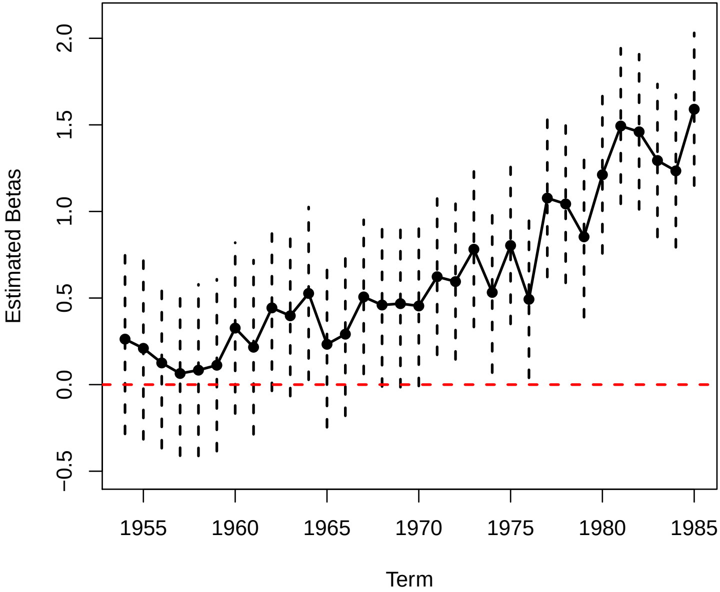
```
> fit4<-with(SCOTUS, lm(Namici~lctdiss+multlaw+civlibs+
+                         econs+constit+lctlib+term))
> summary(fit4)

Call:
lm(formula = Namici ~ lctdiss + multlaw + civlibs + econs + constit +
    lctlib + term)

Residuals:
   Min     1Q Median     3Q    Max
-2.968 -0.906 -0.428  0.143 36.958

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.726962   0.202367 -13.475  < 2e-16 ***
lctdiss      0.359494   0.070415   5.105 3.39e-07 ***
multlaw      0.649932   0.073109   8.890  < 2e-16 ***
civlibs     -0.289314   0.059295  -4.879 1.09e-06 ***
econs        0.199464   0.075419   2.645  0.00819 **
constit      0.515435   0.062559   8.239  < 2e-16 ***
lctlib       0.339891   0.053901   6.306 3.04e-10 ***
term         0.046142   0.002821  16.354  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.11 on 7032 degrees of freedom
  (121 observations deleted due to missingness)
Multiple R-squared:  0.08493,Adjusted R-squared:  0.08402
F-statistic: 93.24 on 7 and 7032 DF,  p-value: < 2.2e-16
```

Using factor

```
> fit5<-with(SCOTUS, lm(Namici~lctdiss+multlaw+civlibs+
+                       econs+constit+lctlib+as.factor(term)))
> summary(fit5)

Call:
lm(formula = Namici ~ lctdiss + multlaw + civlibs + econs + constit +
    lctlib + as.factor(term))

Residuals:
   Min     1Q Median     3Q    Max
-3.064 -0.920 -0.384  0.106 36.831

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       -0.16153    0.19530  -0.827 0.408200
lctdiss            0.34558    0.07067   4.890 1.03e-06 ***
multlaw            0.64348    0.07334   8.774  < 2e-16 ***
civlibs           -0.27137    0.05967  -4.548 5.51e-06 ***
econs              0.20039    0.07581   2.643 0.008232 **
constit            0.54280    0.06297   8.620  < 2e-16 ***
lctlib             0.33863    0.05458   6.205 5.80e-10 ***
.
.
.
```

```
as.factor(term)54  0.26276    0.27934   0.941 0.346918
as.factor(term)55  0.20958    0.26804   0.782 0.434309
as.factor(term)56  0.12536    0.25126   0.499 0.617859
as.factor(term)57  0.06432    0.24227   0.265 0.790654
as.factor(term)58  0.08353    0.25274   0.331 0.741025
.
.
.
as.factor(term)79  0.85363    0.23696   3.602 0.000318 ***
as.factor(term)80  1.21205    0.23183   5.228 1.76e-07 ***
as.factor(term)81  1.49347    0.22925   6.515 7.80e-11 ***
as.factor(term)82  1.46004    0.22858   6.388 1.79e-10 ***
as.factor(term)83  1.29417    0.22549   5.739 9.90e-09 ***
as.factor(term)84  1.23434    0.22517   5.482 4.36e-08 ***
as.factor(term)85  1.59037    0.22491   7.071 1.68e-12 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.108 on 7001 degrees of freedom
  (121 observations deleted due to missingness)
Multiple R-squared:  0.0914,Adjusted R-squared:  0.08647
F-statistic: 18.53 on 38 and 7001 DF,  p-value: < 2.2e-16
```

`factor` results, plotted



Multiplicative Interactions: Primitives

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + u_i$$

means:

$$E(Y|D_1 = 1, D_2 = 0) - E(Y|D_1 = 0, D_2 = 0) = E(Y|D_1 = 1, D_2 = 1) - E(Y|D_1 = 0, D_2 = 1)[\equiv \beta_1]$$

and

$$E(Y|D_1 = 0, D_2 = 1) - E(Y|D_1 = 0, D_2 = 0) = E(Y|D_1 = 1, D_2 = 1) - E(Y|D_1 = 1, D_2 = 0)[\equiv \beta_2].$$

Interaction Effects

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i$$

$$\begin{aligned} \mathrm{E}(Y_i) &= \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} \\ &= \beta_0 + \beta_2 X_{2i} + (\beta_1 + \beta_3 X_{2i}) X_{1i} \\ &= \beta_0 + \beta_2 X_{2i} + \psi_1 X_{1i} \end{aligned}$$

where $\psi_1 = \beta_1 + \beta_3 X_{2i}$. This means:

$$\frac{\partial \mathrm{E}(Y_i)}{\partial X_1} = \beta_1 + \beta_3 X_{2i}.$$

Similarly:

$$
\begin{aligned}
\mathrm{E}(Y_i) &= \beta_0 + \beta_1 X_{1i} + (\beta_2 + \beta_3 X_{1i}) X_{2i} \\
&= \beta_0 + \beta_1 X_{1i} + \psi_2 X_{2i}
\end{aligned}
$$

which implies:

$$
\frac{\partial \mathrm{E}(Y_i)}{\partial X_2} = \beta_2 + \beta_3 X_{1i}.
$$

"Direct Effects"

If $X_2 = 0$, then:

$$\begin{aligned} E(Y_i) &= \beta_0 + \beta_1 X_{1i} + \beta_2(0) + \beta_3 X_{1i}(0) \\ &= \beta_0 + \beta_1 X_{1i}. \end{aligned}$$

Similarly, for $X_1 = 0$:

$$\begin{aligned} E(Y_i) &= \beta_0 + \beta_1(0) + \beta_2 X_{2i} + \beta_3(0) X_{2i} \\ &= \beta_0 + \beta_2 X_{2i} \end{aligned}$$

In most instances, the quantities we care about are not β_1 and β_2, but rather ψ_1 and ψ_2.

Point estimates:

$$\hat{\psi}_1 = \hat{\beta}_1 + \hat{\beta}_3 X_2$$

and

$$\hat{\psi}_2 = \hat{\beta}_2 + \hat{\beta}_3 X_1.$$

For variance, recall that:

$$\mathrm{Var}(a + bZ) = \mathrm{Var}(a) + Z^2\mathrm{Var}(b) + 2Z\mathrm{Cov}(a, b)$$

Means that:

$$\widehat{\mathsf{Var}(\hat{\psi}_1)} = \widehat{\mathsf{Var}(\hat{\beta}_1)} + X_2^2\widehat{\mathsf{Var}(\hat{\beta}_3)} + 2X_2\widehat{\mathsf{Cov}(\hat{\beta}_1, \hat{\beta}_3)}.$$

and

$$\widehat{\mathsf{Var}(\hat{\psi}_2)} = \widehat{\mathsf{Var}(\hat{\beta}_2)} + X_1^2\widehat{\mathsf{Var}(\hat{\beta}_3)} + 2X_1\widehat{\mathsf{Cov}(\hat{\beta}_2, \hat{\beta}_3)}.$$

Types of Interactions: Dichotomous Xs

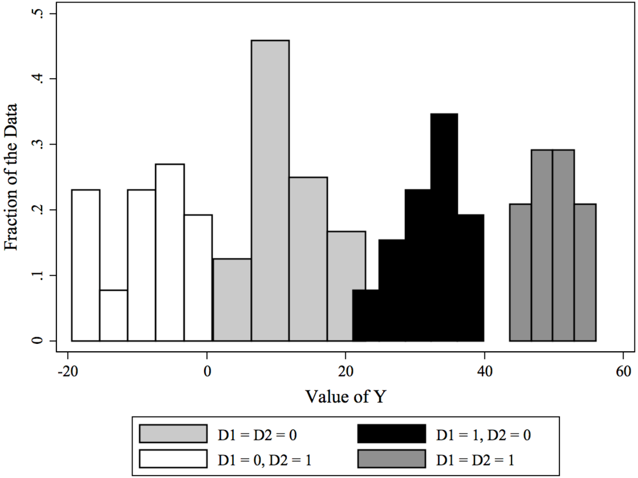
For

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 D_{1i} D_{2i} + u_i$$

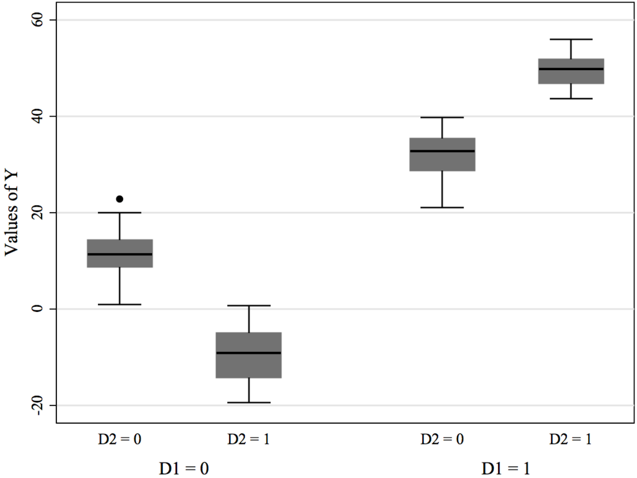
we have:

$$\begin{aligned}
\mathrm{E}(Y|D_1 = 0, D_2 = 0) &= \beta_0 \\
\mathrm{E}(Y|D_1 = 1, D_2 = 0) &= \beta_0 + \beta_1 \\
\mathrm{E}(Y|D_1 = 0, D_2 = 1) &= \beta_0 + \beta_2 \\
\mathrm{E}(Y|D_1 = 1, D_2 = 1) &= \beta_0 + \beta_1 + \beta_2 + \beta_3
\end{aligned}$$

Values of Y for Various Combinations of Values of D_1 and D_2



Values of Y for Various Combinations of Values of D_1 and D_2



Dichotomous and Continuous Xs

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i D_i + u_i$$

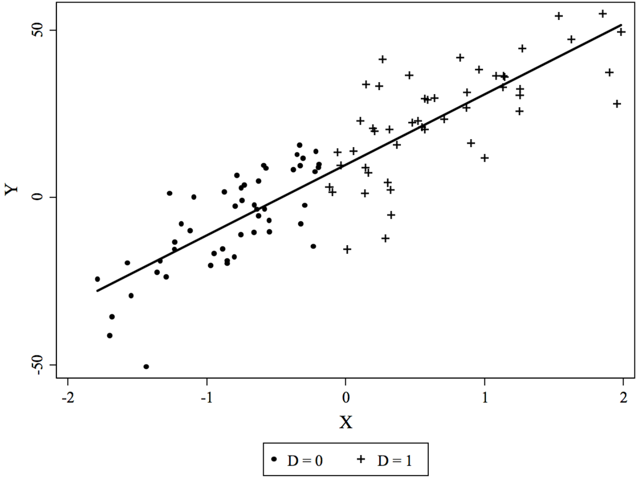
gives:

$$\begin{aligned} \mathrm{E}(Y|X, D=0) &= \beta_0 + \beta_1 X \\ \mathrm{E}(Y|X, D=1) &= (\beta_0 + \beta_2) + (\beta_1 + \beta_3) X \end{aligned}$$

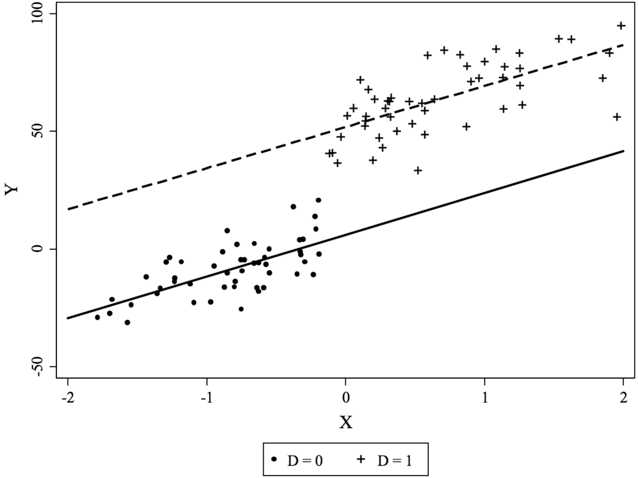
Four possibilities:

- $\beta_2 = \beta_3 = 0$
- $\beta_2 \neq 0$ and $\beta_3 = 0$
- $\beta_2 = 0$ and $\beta_3 \neq 0$
- $\beta_2 \neq 0$ and $\beta_3 \neq 0$

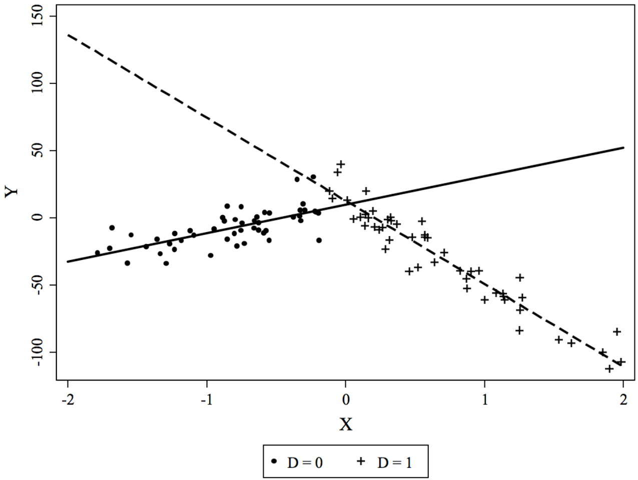
Scatterplot and Regression Lines of Y on X for $D = 0$ and $D = 1$: No Slope or Intercept Differences ($\beta_2 = \beta_3 = 0$)



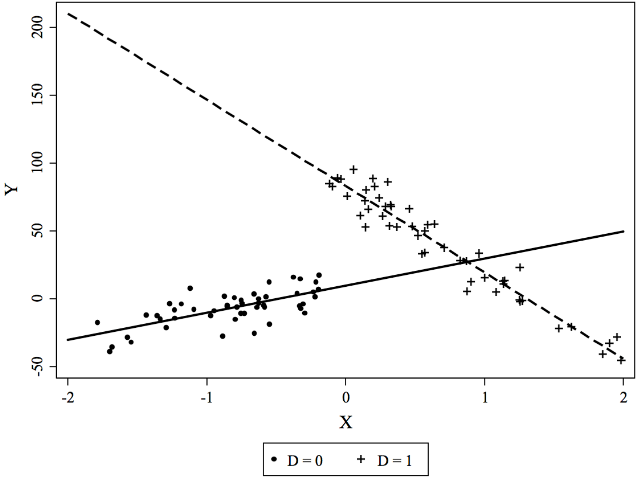
Scatterplot and Regression Lines of Y on X for $D = 0$ and $D = 1$: Intercept Shift ($\beta_2 \neq 0$, $\beta_3 = 0$)



Scatterplot and Regression Lines of Y on X for $D = 0$ and $D = 1$: Slope Change ($\beta_2 = 0$, $\beta_3 \neq 0$)



Scatterplot and Regression Lines of Y on X for $D = 0$ and $D = 1$: Slope and Intercept Change ($\beta_2 \neq 0$, $\beta_3 \neq 0$)



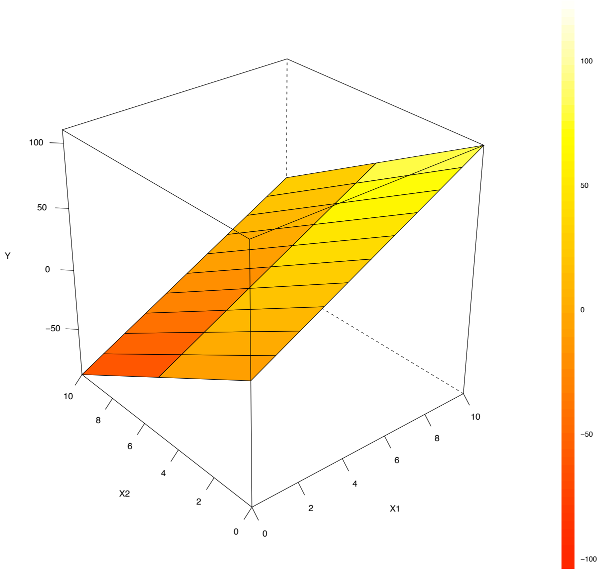
Two Continuous Xs

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i.$$

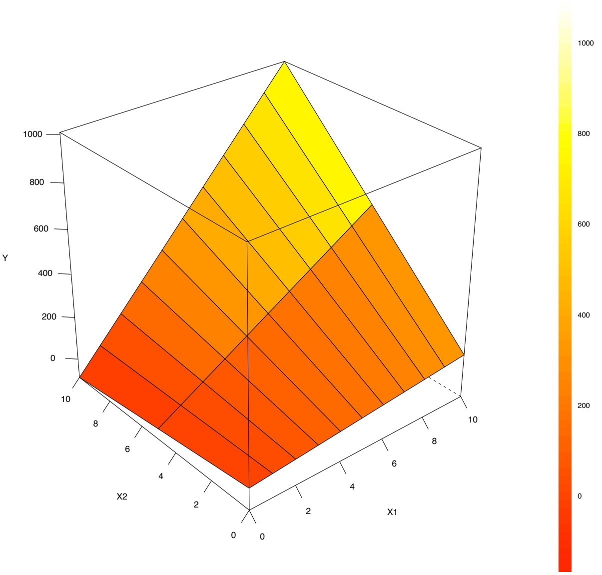
Implies

$$\beta_3 = 0 \;\rightarrow\; \frac{\partial E(Y)}{\partial X_1} = \beta_1 \,\forall\, X_2 \text{ and } \frac{\partial E(Y)}{\partial X_2} = \beta_2 \,\forall\, X_1$$

Two Continuous Variables: No Interactive Effects



Two Continuous Variables: Interaction Present



Quadratic, Cubic, and Other Polynomial Effects

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + ... + \beta_j X_i^j + u_i.$$

In general:

$$\frac{\partial \mathrm{E}(Y)}{\partial X} = \beta_1 + 2\beta_2 X + 3\beta_3 X^2 + ... + j\beta_j X^{j-1}$$

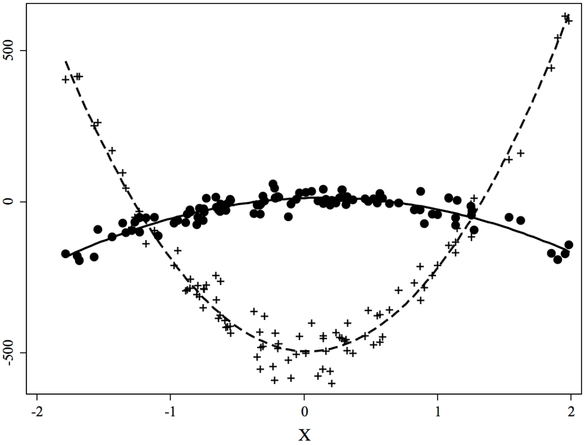
Quadratic case ($j = 2$):

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + u_i.$$

implies

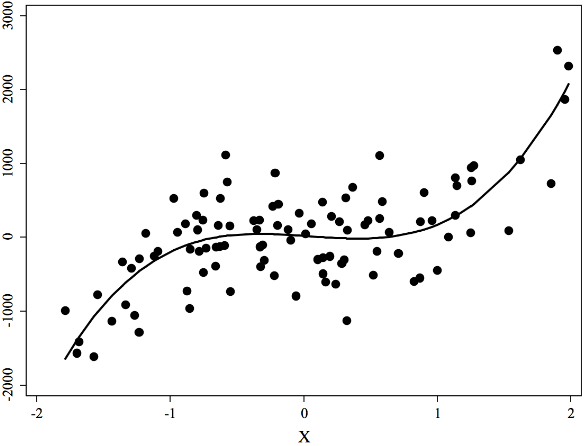
$$\frac{\partial \mathrm{E}(Y)}{\partial X} = \beta_1 + 2\beta_2 X$$

Examples of Two Quadratic Relationships



Note: Solid line is $Y_i = 10 + 10X_i - 50X_i^2 + u_i$; dashed line is $Y_i = -500 - 20X_i + 300X_i^2 + u_i$.

Example of a Cubic Relationship



Note: Solid line is $Y_i = 10 + 10X_i - 50X_i^2 + 300X_i^3 + u_i$.

Higher-Order Interactive Models

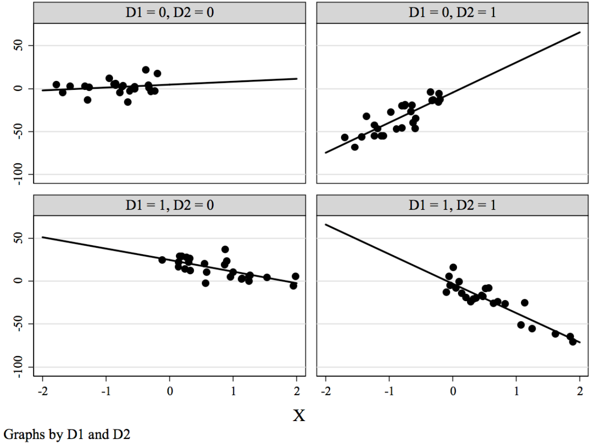
Three-way interaction:

$$\begin{aligned} Y_i &= \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \\ & \quad \beta_4 X_{1i} X_{2i} + \beta_4 X_{1i} X_{3i} u_i + \beta_5 X_{2i} X_{3i} + \beta_6 X_{1i} X_{2i} X_{3i} + u_i \end{aligned}$$

Special case of dichotomous X_1, X_2:

$$\begin{aligned} Y_i &= \beta_0 + \beta_1 X_i + \beta_2 D_{1i} + \beta_3 D_{2i} + \\ & \quad \beta_4 X_i D_{1i} + \beta_4 X_i D_{2i} u_i + \beta_5 D_{1i} D_{2i} + \beta_6 X_i D_{1i} D_{2i} + u_i \end{aligned}$$

Three-Way Interaction: Two Dummy and One Continuous Covariates



1996 NES Data

```
> library(RCurl)
> temp<-getURL("https://raw.githubusercontent.com/PrisonRodeo/
             GSERM-2017-git/master/Data/ClintonTherm.csv")
> ClintonTherm<-read.csv(text=temp, header=TRUE)
> rm(temp)
> summary(ClintonTherm)

    caseid      ClintonTherm     RConserv      ClintonConserv
 Min.   :1001   Min.   :  0   Min.   :1.000   Min.   :1.000
 1st Qu.:1440   1st Qu.: 30   1st Qu.:3.000   1st Qu.:2.000
 Median :1854   Median : 60   Median :4.000   Median :3.000
 Mean   :2001   Mean   : 57   Mean   :4.323   Mean   :2.985
 3rd Qu.:2262   3rd Qu.: 85   3rd Qu.:5.000   3rd Qu.:4.000
 Max.   :3403   Max.   :100   Max.   :7.000   Max.   :7.000
      PID             GOP
 Min.   :1.000   Min.   :0.0000
 1st Qu.:1.000   1st Qu.:0.0000
 Median :2.000   Median :0.0000
 Mean   :2.059   Mean   :0.3161
 3rd Qu.:3.000   3rd Qu.:1.0000
 Max.   :5.000   Max.   :1.0000
```

A Basic Regression

```
> summary(with(ClintonTherm, lm(ClintonTherm~RConserv+GOP)))

Call:
lm(formula = ClintonTherm ~ RConserv + GOP)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  93.4756     2.2278   41.96   <2e-16 ***
RConserv     -6.4866     0.5373  -12.07   <2e-16 ***
GOP         -26.6699     1.6056  -16.61   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 23.65 on 1294 degrees of freedom
Multiple R-squared: 0.3795,Adjusted R-squared: 0.3786
F-statistic: 395.7 on 2 and 1294 DF,  p-value: < 2.2e-16
```

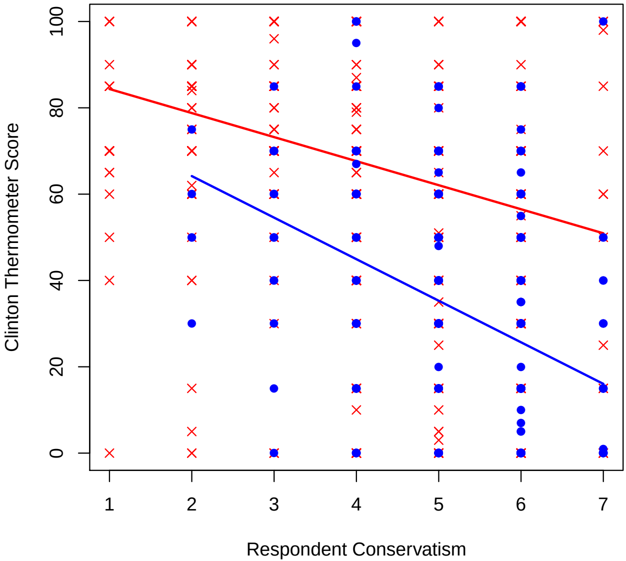
```
> fit1<-with(ClintonTherm, lm(ClintonTherm~RConserv+GOP+
             RConserv*GOP))
> summary(fit1)

Call:
lm(formula = ClintonTherm ~ RConserv + GOP + RConserv * GOP)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   89.9271     2.4866  36.165  < 2e-16 ***
RConserv      -5.5705     0.6085  -9.154  < 2e-16 ***
GOP           -6.4840     6.5690  -0.987  0.32379
RConserv:GOP  -4.0581     1.2808  -3.168  0.00157 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 23.57 on 1293 degrees of freedom
Multiple R-squared: 0.3843,Adjusted R-squared: 0.3829
F-statistic:   269 on 3 and 1293 DF,  p-value: < 2.2e-16
```

Two Regressions, Sort Of

$$
\begin{aligned}
\text{E(Thermometer} \mid \text{Non-GOP)}_i &= 89.9 - 6.5(0) - 5.6(\text{R's Conservatism}_i) \\
&\quad -4.0(0 \times \text{R's Conservatism}_i) \\
&= \mathbf{89.9} - \mathbf{5.6}(\text{R's Conservatism}_\mathbf{i})
\end{aligned}
$$

$$
\begin{aligned}
\text{E(Thermometer} \mid \text{GOP)}_i &= [89.9 - 6.5(1)] + [-5.6 - 4.0(1 \times \text{R's Conservatism}_i)] \\
&= \mathbf{83.4} - \mathbf{9.6}(\text{R's Conservatism}_\mathbf{i})
\end{aligned}
$$

Thermometer Scores by Conservatism, GOP and Non-GOP



Interactive Results are (Almost) Identical to Separate Regressions

```
> NonReps<-subset(ClintonTherm,GOP==0)
> summary(with(NonReps, lm(ClintonTherm~RConserv)))

Call:
lm(formula = ClintonTherm ~ RConserv, data = NonReps)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  89.9271     2.4695  36.416   <2e-16 ***
RConserv     -5.5705     0.6043  -9.217   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 23.41 on 885 degrees of freedom
Multiple R-squared: 0.08759,Adjusted R-squared: 0.08656
F-statistic: 84.96 on 1 and 885 DF,  p-value: < 2.2e-16
```

Interactive Results are (Almost) Identical to Separate Regressions

```
> Reps<-subset(ClintonTherm,GOP==1)
> summary(with(Reps, lm(ClintonTherm~RConserv)))

Call:
lm(formula = ClintonTherm ~ RConserv, data = Reps)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   83.443      6.170  13.524  < 2e-16 ***
RConserv      -9.629      1.144  -8.419 6.52e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 23.92 on 408 degrees of freedom
Multiple R-squared: 0.148,Adjusted R-squared: 0.1459
F-statistic: 70.88 on 1 and 408 DF,  p-value: 6.518e-16
```

Discovering $\hat{\psi}_1$ and $\hat{\psi}_2$

For `RConserv`:

$$\begin{aligned} \text{Clinton Thermometer}_i &= \beta_0 + (\beta_1 + \beta_3 \text{GOP}_i)\text{R's Conservatism}_i + \\ &\quad \beta_2 \text{GOP}_i + u_i \\ &= \beta_0 + \psi_{1i}\text{R's Conservatism}_i + \beta_2 \text{GOP}_i + u_i. \end{aligned}$$

So:

$$\hat{\psi}_{1i} = \hat{\beta}_1 + \hat{\beta}_3 \times \text{GOP}_i$$

and

$$\hat{\sigma}_{\psi_1} = \sqrt{\widehat{\text{Var}(\hat{\beta}_1)} + (\text{GOP})^2 \widehat{\text{Var}(\hat{\beta}_3)} + 2(\text{GOP})\widehat{\text{Cov}(\hat{\beta}_1, \hat{\beta}_3)}}.$$

Discovering $\hat{\psi}_1$ and $\hat{\psi}_2$

For `GOP`:

$$\begin{aligned}\text{Clinton Thermometer}_i &= \beta_0 + (\beta_2 + \beta_3 \times \text{R's Conservatism}_i)\text{GOP}_i + \\ &\quad \beta_1(\text{R's Conservatism}_i) + u_i \\ &= \beta_0 + \psi_{2i}\text{GOP}_i + \beta_1(\text{R's Conservatism}_i) + u_i.\end{aligned}$$

So:

$$\hat{\psi}_{2i} = \hat{\beta}_2 + \hat{\beta}_3 \times (\text{R's Conservatism}_i).$$

and

$$\hat{\sigma}_{\psi_2} = \sqrt{\widehat{\text{Var}(\hat{\beta}_2)} + (\text{R's Conservatism}_i)^2\widehat{\text{Var}(\hat{\beta}_3)} + 2k\widehat{\text{Cov}(\hat{\beta}_2, \hat{\beta}_3)}}.$$

Discovering $\hat{\psi}_1$ and $\hat{\psi}_2$

```
> Psi1<-fit1$coeff[2]+fit1$coeff[4]

> Psi1
 RConserv
-9.628577

> SPsi1<-sqrt(vcov(fit1)[2,2] + (1)^2*vcov(fit1)[4,4] + 2*1*vcov(fit1)[2,4])
> SPsi1
[1] 1.127016

> Psi1 / SPsi1 # <-- t-statistic
 RConserv
-8.543422
```

```
> # psi_2 | RConserv = 1
> fit1$coeff[3]+(1 * fit1$coeff[4])
      GOP
-10.54208

> sqrt(vcov(fit1)[3,3] + (1)^2*vcov(fit1)[4,4] + 2*1*vcov(fit1)[3,4])
[1] 5.335847

# Implies t is approximately 2
```

```
> # psi_2 | RConserv = 7
> fit1$coeff[3]+(7 * fit1$coeff[4])
      GOP
-34.89045

> sqrt(vcov(fit1)[3,3] + (7)^2*vcov(fit1)[4,4] + 2*7*vcov(fit1)[3,4])
[1] 3.048302

# t is approximately 11
```

An Easier Way: `linearHypothesis()`

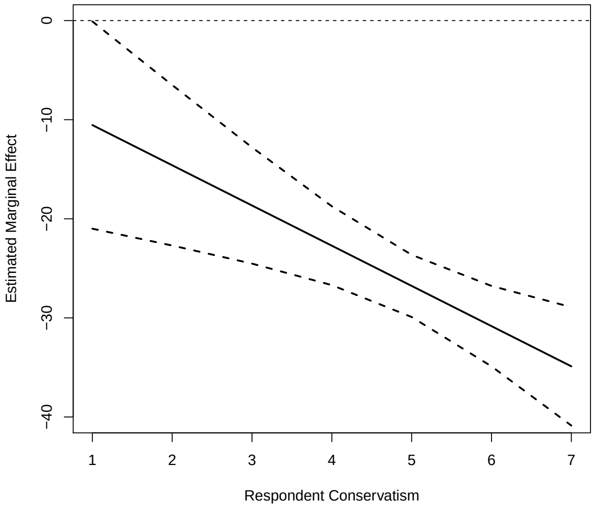
```
> library(car)
> linearHypothesis(fit1,"RConserv+RConserv:GOP")
Linear hypothesis test

Hypothesis:
RConserv  + RConserv:GOP = 0

Model 1: restricted model
Model 2: ClintonTherm ~ RConserv + GOP + RConserv * GOP

  Res.Df    RSS Df Sum of Sq     F    Pr(>F)
1   1294 758714
2   1293 718173  1     40541 72.99 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

> # Note: Same as t-test:
> sqrt(72.99)
[1] 8.543419
```

An Easier Way: `linearHypothesis()`

```
> # psi_2 | RConserv = 7:
> linearHypothesis(fit1,"GOP+7*RConserv:GOP")
Linear hypothesis test

Hypothesis:
GOP  + 7 RConserv:GOP = 0

Model 1: restricted model
Model 2: ClintonTherm ~ RConserv + GOP + RConserv * GOP

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   1294 790938
2   1293 718173  1     72766 131.01 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

Marginal Effects Plots, I

```
> ConsSim<-seq(1,7,1)
> psis<-fit1$coeff[3]+(ConsSim * fit1$coeff[4])
> psis.ses<-sqrt(vcov(fit1)[3,3] +
  (ConsSim)^2*vcov(fit1)[4,4] + 2*ConsSim*vcov(fit1)[3,4])

> plot(ConsSim,psis,t="l",lwd=2,xlab="Respondent Conservatism",
  ylab="Estimated Marginal Effect",ylim=c(-40,0))
> lines(ConsSim,psis+(1.96*psis.ses),lty=2,lwd=2)
> lines(ConsSim,psis-(1.96*psis.ses),lty=2,lwd=2)
> abline(h=0,lwd=1,lty=2)
```


0
−10
−20
−30
−40
Estimated Marginal Effect
1
2
3
4
5
6
7
Respondent Conservatism

Interacting Two Continuous Covariates

```
> fit2<-with(ClintonTherm,
+       lm(ClintonTherm~RConserv+ClintonConserv+RConserv*ClintonConserv))
> summary(fit2)

Call:
lm(formula = ClintonTherm ~ RConserv + ClintonConserv + RConserv *
    ClintonConserv)

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)             119.3515     5.1634  23.115  < 2e-16 ***
RConserv                -19.5673     1.0362 -18.884  < 2e-16 ***
ClintonConserv           -7.9311     1.6477  -4.813 1.66e-06 ***
RConserv:ClintonConserv   3.6293     0.3394  10.695  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 22.03 on 1293 degrees of freedom
Multiple R-squared: 0.4619,Adjusted R-squared: 0.4606
F-statistic:   370 on 3 and 1293 DF,  p-value: < 2.2e-16
```

```
> fit2$coef[2]+(1*fit2$coef[4])
 RConserv
-15.93803

> sqrt(vcov(fit2)[2,2] + (1)^2*vcov(fit2)[4,4] + 2*1*vcov(fit2)[2,4])
[1] 0.7439696
```

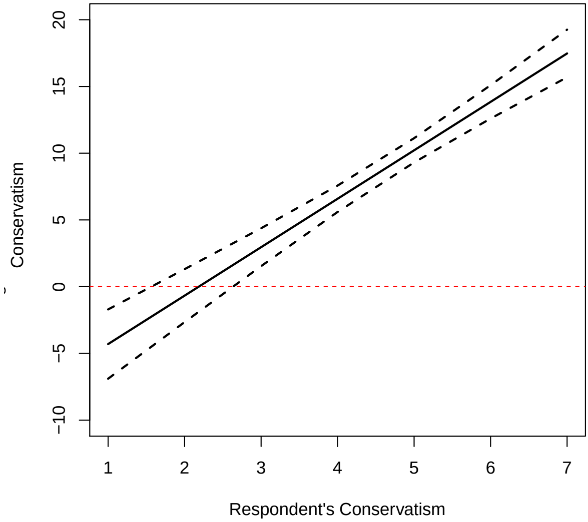
```
> linearHypothesis(fit2,"RConserv+1*RConserv:ClintonConserv")
Linear hypothesis test

Hypothesis:
RConserv  + RConserv:ClintonConserv = 0

Model 1: restricted model
Model 2: ClintonTherm ~ RConserv + ClintonConserv + RConserv * ClintonConserv

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   1294 850442
2   1293 627658  1    222784 458.94 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

More Hypothesis Tests

```
> # psi_1 | ClintonConserv = mean
> fit2$coef[2]+((mean(ClintonTherm$ClintonConserv))*fit2$coef[4])

 RConserv
-8.735424

> sqrt(vcov(fit2)[2,2] + (mean(ClintonTherm$ClintonConserv)^2*vcov(fit2)[4,4] +
+                            2*(mean(ClintonTherm$ClintonConserv))*vcov(fit2)[2,4]))

[1] 0.4507971

> pt(((fit2$coef[2]+(2.985*fit2$coef[4])) / sqrt(vcov(fit2)[2,2] +
+     (2.985)^2*vcov(fit2)[4,4] + 2*2.985*vcov(fit2)[2,4])),df=1293)

    RConserv
6.483788e-74

> # psi_2 | RConserv = 1
> fit2$coef[3]+(1*fit2$coef[4])

ClintonConserv
     -4.301803

> # psi_2 | RConserv = 6
> fit2$coef[3]+(6*fit2$coef[4])

ClintonConserv
      13.84463
```

Marginal Effect Plot, II

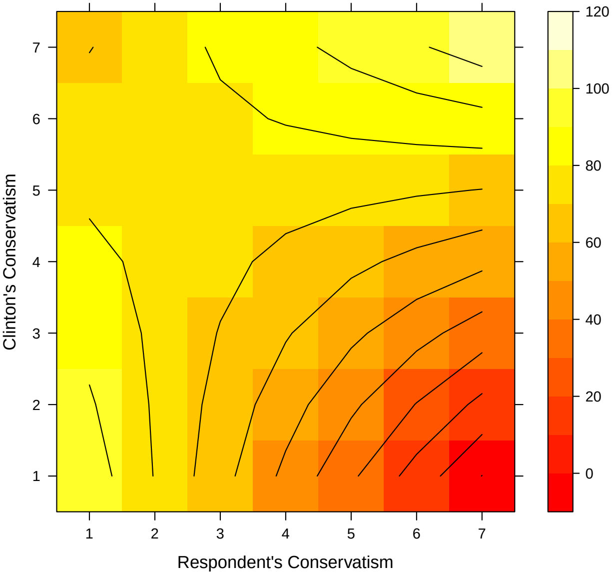
```
> psis2<-fit2$coef[3]+(ConsSim*fit2$coef[4])
> psis2.ses<-sqrt(vcov(fit2)[3,3] + (ConsSim)^2*vcov(fit2)[4,4]
  + 2*ConsSim*vcov(fit2)[3,4])

> plot(ConsSim,psis2,t="l",lwd=2,xlab="Respondent's
  Conservatism",ylab="Marginal Effect of Clinton's
  Conservatism",ylim=c(-10,20))
> lines(ConsSim,psis2+(1.96*psis2.ses),lty=2,lwd=2)
> lines(ConsSim,psis2-(1.96*psis2.ses),lty=2,lwd=2)
> abline(h=0,lty=2,lwd=1,col="red")
```



Conservatism
20
15
10
5
0
−5
−10
1
2
3
4
5
6
7
Respondent's Conservatism

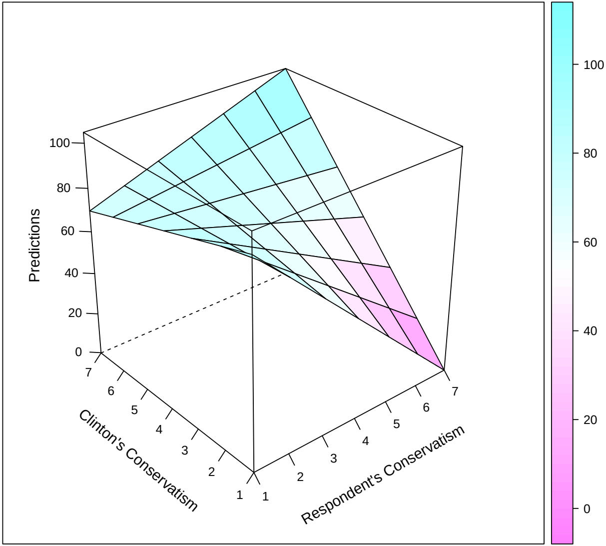
Predicted Values: A Contour Plot

```
> library(lattice)
> grid<-expand.grid(RConserv=seq(1,7,1),
  ClintonConserv=seq(1,7,1))
> hats<-predict(fit2,newdata=grid)

> levelplot(hats~grid$RConserv*grid$ClintonConserv,
  contour=TRUE,
  cuts=12,pretty=TRUE,xlab="Respondent's Conservatism",
  ylab="Clinton's Conservatism",
  col.regions=heat.colors)
```


Clinton's Conservatism
Respondent's Conservatism
1
2
3
4
5
6
7
1
2
3
4
5
6
7
0
20
40
60
80
100
120

Predicted Values: A Wireframe Plot

```
> trellis.par.set("axis.line",list(col="transparent"))

> wireframe(hats~grid$RConserv*grid$ClintonConserv,
  drape=TRUE,
  xlab=list("Respondent's Conservatism",rot=30),
  ylab=list("Clinton's Conservatism",
  rot=-40),zlab=list("Predictions",rot=90),
  scales=list(arrows=FALSE,col="black"),
  zoom=0.85,pretty=TRUE)
```



Predictions
100
80
60
40
20
0
7
6
5
4
3
2
1
Clinton's Conservatism
1
2
3
4
5
6
7
Respondent's Conservatism
100
80
60
40
20
0