GSERM 2017
Regression III
Varying Coefficients and Mixture Models, I: Unit Effects

June 21, 2017 (afternoon session)

Starting Points

- "Longitudinal" $\neq$ "Time Series"
- Terminology:
  - "Unit" / "Units" / "Units of observation" / "Panels" $=$ Things we observe repeatedly
  - "Observations" $=$ Each (one) measurement of a unit
  - "Time points" $=$ When each observation on a unit is made
  - $i \in \{1....N\}$ indexes units
  - $t \in \{1...T\}$ or $\{1...T_i\}$ indexes observations / time points
  - If $T_i = T \; \forall i$ then we have "balanced" panels / units
  - $NT =$ Total number of observations (if balanced)
- Averages:
  - Y_{it} indicates a variable that varies over both units and time,
  - $\bar{Y}_i = \frac{1}{T}\sum_{t=1}^{T} Y_{it} =$ the over-time mean of Y,
  - $\bar{Y}_t = \frac{1}{N}\sum_{i=1}^{N} Y_{it} =$ the across-unit mean of Y, and
  - $\bar{Y} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} Y_{it} =$ the grand mean of Y.

More Terminology

- $N >> T \rightarrow$ "panel" data
  - NES panel studies ($N = 2000$, $T = 3$)
  - Panel Study of Income Dynamics ($N =$ large, $T \approx 12$)
- $T >> N$ or $T \approx N \rightarrow$ "time-series cross-sectional" ("TSCS") data
- $N = 1 \rightarrow$ "time series" data

Panel/TSCS Data Structure

id	t	Y	X_1	...
1	1	250	3.4	...
1	2	290	3.3	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...
2	1	160	4.7	...
2	2	150	4.9	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...

Variation: A Tiny (Fake) Example

id	year	gender	pres	pid	approve
1	1998	female	clinton	dem	3
1	2000	female	clinton	dem	3
1	2002	female	bush	dem	5
1	2004	female	bush	dem	3
2	1998	male	clinton	gop	2
2	2000	male	clinton	gop	1
2	2002	male	bush	gop	4
2	2004	male	bush	gop	3
3	1998	male	clinton	gop	2
3	2000	male	clinton	gop	2
3	2002	male	bush	gop	4
3	2004	male	bush	dem	1

Aggregation: Cross-Sectional

id	gender	pres	pid	approve
1	female	?	dem	3.50
2	male	?	gop	2.50
3	male	?	?	2.25

Aggregation: Temporal

year	female	pres	pid	approve
1998	0.33	clinton	0.66(?)	2.33
2000	0.33	clinton	0.66(?)	2.00
2002	0.33	bush	0.66(?)	4.33
2004	0.33	bush	0.33(?)	2.33

Aggregation:

- Loses information
- Distorts relationships
- Forces arbitrary decisions

If you have variation in multiple dimensions, use it.

Within- and Between-Unit Variation

Define:

$$\bar{Y}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} Y_{it}$$

Then:

$$Y_{it} = \bar{Y}_i + (Y_{it} - \bar{Y}_i).$$

- The *total* variation in Y_{it} can be decomposed into
- The *between-unit* variation in the $\bar{Y}_i$s, and
- The *within-unit* variation around $\bar{Y}_i$ (that is, $Y_{it} - \bar{Y}_i$).

Variation (SCOTUS Tenure Remix)

"Total" Variation:

```
> with(scotus, describe(service))
   vars    n  mean   sd median trimmed mad min max range skew kurtosis
X1    1 1765 11.74 8.34     10   10.93 8.9   1  37    36 0.73    -0.28
    se
X1 0.2
```

"Between" Variation:

```
> scmeans <- ddply(scotus,.(justice),summarise,
+                  service = mean(service))
> with(scmeans, describe(service))
   vars   n mean   sd median trimmed  mad min max range skew kurtosis
X1    1 107 8.87 4.99    8.5    8.59 5.93 1.5  21  19.5  0.4    -0.92
     se
X1 0.48
```

"Within" Variation:

```
> scotus <- ddply(scotus, .(justice), mutate,
+                 servmean = mean(service))
> scotus$within <- with(scotus, service-servmean)
> with(scotus, describe(within))
   vars    n mean   sd median trimmed  mad min max range skew kurtosis
X1    1 1765    0 6.92      0       0 6.67 -18  18    36    0    -0.36
     se
X1 0.16
```

Model

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

assumes:

- All the usual OLS assumptions, plus
- $\beta_{0i} = \beta_0 \; \forall \; i$s
- $\beta_{1i} = \beta_1 \; \forall \; i$s

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

(same)

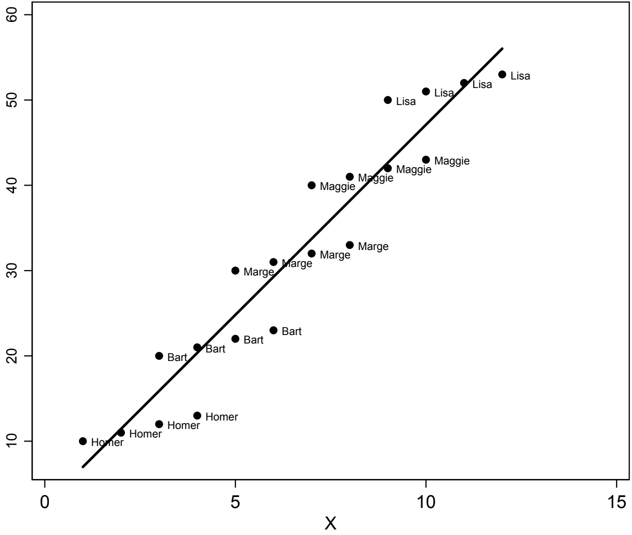
Variable Intercepts

$$Y_{it} = \beta_{0i} + \beta_1 X_{it} + u_{it}$$

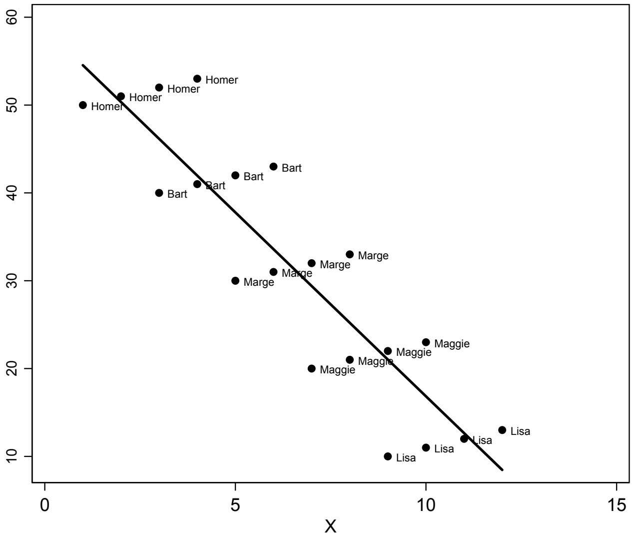
$$Y_{it} = \beta_{0t} + \beta_1 X_{it} + u_{it}$$

$$Y_{it} = \beta_{0it} + \beta_1 X_{it} + u_{it}$$

Varying Intercepts



Varying Intercepts



Varying Slopes (+ Intercepts)

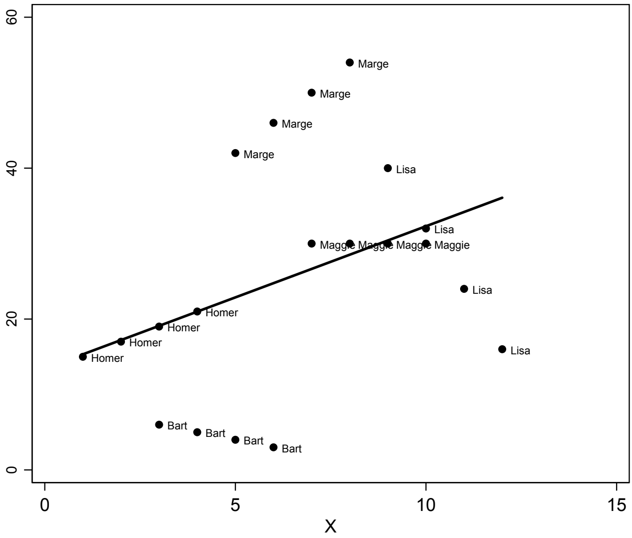
$$Y_{it} = \beta_0 + \beta_{1i} X_{it} + u_{it}$$

$$Y_{it} = \beta_{0i} + \beta_{1i} X_{it} + u_{it}$$

$$Y_{it} = \beta_{0t} + \beta_{1t} X_{it} + u_{it}$$

$$Y_{it} = \beta_{0it} + \beta_{1it} X_{it} + u_{it}$$

Varying Slopes + Intercepts



$$u_{it} \sim \text{i.i.d.} N(0, \sigma^2) \; \forall \; i, t$$

$$\begin{aligned}
\text{Var}(u_{it}) &= \text{Var}(u_{jt}) \; \forall \; i \neq j \text{ (i.e., no cross-unit heteroscedasticity)} \\
\text{Var}(u_{it}) &= \text{Var}(u_{is}) \; \forall \; t \neq s \text{ (i.e., no temporal heteroscedasticity)} \\
\text{Cov}(u_{it}, u_{js}) &= 0 \; \forall \; i \neq j, \; \forall \; t \neq s \text{ (i.e., no auto- or spatial correlation)}
\end{aligned}$$

Pooling

- Adds data
- Generalizability

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

Implies

- that the process governing the relationship between X and Y is exactly the same for each i,
- that the process governing the relationship between X and Y is the same for all t,
- that the process governing the us is the same $\forall$ i and t as well.

“Partial” Pooling

Two regimes:

$$Y_A = \beta_A' \mathbf{X}_A + u_A$$

$$Y_B = \beta_B' \mathbf{X}_B + u_B$$

with $\sigma_A^2 = \sigma_B^2$, and $\text{Cov}(u_A, u_B) = 0$.

Estimators:

$$\hat{\beta}_{A,B} = (\mathbf{X}_{A,B}' \mathbf{X}_{A,B})^{-1} \mathbf{X}_{A,B}' Y_{A,B}$$

and

$$\widehat{\text{Var}(\beta_{A,B})} = \hat{\sigma}_{A,B}^2 (\mathbf{X}_{A,B}' \mathbf{X}_{A,B})^{-1},$$

A Pooled Estimator

$$\begin{aligned}
\hat{\beta}_P &= (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}(\mathbf{X}_A' Y_A + \mathbf{X}_B' Y_B) \\
&= (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}[\beta_A(\mathbf{X}_A'\mathbf{X}_A) + \beta_B(\mathbf{X}_B'\mathbf{X}_B)],
\end{aligned}$$

$$\begin{aligned}
\mathsf{E}(\hat{\beta}_P) &= \beta_A + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_B'\mathbf{X}_B(\beta_B - \beta_A) \\
&= \beta_B + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_A'\mathbf{X}_A(\beta_A - \beta_B)
\end{aligned}$$

$$F = \frac{\frac{\hat{\mathbf{u}}_P'\hat{\mathbf{u}}_P - (\hat{\mathbf{u}}_A'\hat{\mathbf{u}}_A + \hat{\mathbf{u}}_B'\hat{\mathbf{u}}_B)}{K}}{\frac{(\hat{\mathbf{u}}_A'\hat{\mathbf{u}}_A + \hat{\mathbf{u}}_B'\hat{\mathbf{u}}_B)}{(N_A + N_B - 2K)}} \sim F_{[K,(N_A+N_B-2K)]}$$

$$\hat{\beta}_\lambda = (\lambda^2 \mathbf{X}_A' \mathbf{X}_A + \mathbf{X}_B' \mathbf{X}_B)^{-1} (\lambda^2 \mathbf{X}_A' Y_A + \mathbf{X}_B' Y_B)$$

with $\lambda \in [0, 1]$:

- $\lambda = 0 \;\rightarrow$ separate estimators for $\hat{\beta}_A$ and $\hat{\beta}_B$,
- $\lambda = 1 \;\rightarrow$ "fully pooled" estimator $\hat{\beta}_P$,
- $0 < \lambda < 1 \;\rightarrow$ a regression where data in regime A are given some "partial" weighting in their contribution towards an estimate of β.

Pooling, Summarized

"(R)oughly speaking, it makes sense to pool disparate observations if the underlying parameters governing those observations are sufficiently similar, but not otherwise."

"Unit Effects"

One- and Two-Way Unit Effects

Two-way variation:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \gamma V_i + \delta W_t + u_{it}$$

$\longrightarrow$ two-way effects:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + \eta_t + u_{it}$$

One-way effects:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \eta_t + u_{it} \quad \text{(time)}$$

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it} \quad \text{(units)}$$

“Brute force” model:

$$
\begin{aligned}
Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it} \\
&= \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_1 I(i=1)_i + \alpha_2 I(i=2)_i + ... + u_{it}
\end{aligned}
$$

Alternatively:

$$\bar{X}_i = \frac{\sum_{N_i} X_{it}}{N_i}$$

and

$$\tilde{X}_{it} = X_{it} - \bar{X}_i.$$

Yields:

$$Y_{it} = \bar{\mathbf{X}}_i\boldsymbol{\beta}_B + (\mathbf{X}_{it} - \bar{X}_i)\boldsymbol{\beta}_W + \alpha_i + u_{it}$$

Means that:

$$\begin{aligned} Y_{it}^* &= Y_{it} - \bar{Y}_i \\ \mathbf{X}_{it}^* &= \mathbf{X}_{it} - \bar{\mathbf{X}}_i \end{aligned}$$

$$Y_{it}^* = \boldsymbol{\beta}_{FE}\mathbf{X}_{it}^* + u_{it}.$$

$\equiv$ “Within-Effects” Model.

Standard F-test for

$$H_0 : \alpha_i = \alpha_j \forall i \neq j$$

versus

$$H_A : \alpha_i \neq \alpha_j \text{ for some } i \neq j$$

is $\sim F_{N-1, NT-(N-1)}$.

An Example: Refugee Flows in Africa, 1992-2001

Data:

- 50 African countries $\rightarrow$ $(50 \times 49 =)$ 2450 directed dyads
- Ten years
- i indexes directed dyads, t indexes years

Model:

$$\begin{aligned} ln(\text{Refugees})_{A \rightarrow Bt} &= \beta_0 + \beta_1 \text{Population Difference}_{ABt} + \beta_2 \text{Distance}_{AB} + \\ & \quad \beta_3 \text{POLITY Difference}_{ABt} + \beta_4 \text{War Difference}_{ABt} + u_{ABt} \end{aligned}$$

Data: Refugee Flows in Africa, 1992-2001

```
> summary(Refugees)
   dirdyadID           year        ln_ref_flow         pop_diff
 Min.   :404411   Min.   :1992   Min.   :-0.6931   Min.   :-0.117949
 1st Qu.:451461   1st Qu.:1994   1st Qu.:-0.6931   1st Qu.:-0.008848
 Median :510520   Median :1996   Median :-0.6931   Median : 0.000000
 Mean   :512160   Mean   :1996   Mean   :-0.6011   Mean   : 0.000000
 3rd Qu.:565553   3rd Qu.:1999   3rd Qu.:-0.6931   3rd Qu.: 0.008848
 Max.   :651625   Max.   :2001   Max.   :14.1343   Max.   : 0.117949
    distance       regimedif          wardiff    pop_between
 Min.   :0.000   Min.   :-1.00   Min.   :-4   Min.   :-0.109517
 1st Qu.:1.299   1st Qu.:-0.25   1st Qu.: 0   1st Qu.:-0.008833
 Median :2.169   Median : 0.00   Median : 0   Median : 0.000000
 Mean   :2.200   Mean   : 0.00   Mean   : 0   Mean   : 0.000000
 3rd Qu.:3.066   3rd Qu.: 0.25   3rd Qu.: 0   3rd Qu.: 0.008833
 Max.   :5.652   Max.   : 1.00   Max.   : 4   Max.   : 0.109517
   pop_within          regime_between   regime_within     war_between
 Min.   :-0.0088492   Min.   :-0.955   Min.   :-1.180   Min.   :-2.3
 1st Qu.:-0.0004707   1st Qu.:-0.225   1st Qu.:-0.085   1st Qu.:-0.4
 Median : 0.0000000   Median : 0.000   Median : 0.000   Median : 0.0
 Mean   : 0.0000000   Mean   : 0.000   Mean   : 0.000   Mean   : 0.0
 3rd Qu.: 0.0004707   3rd Qu.: 0.225   3rd Qu.: 0.085   3rd Qu.: 0.4
 Max.   : 0.0088492   Max.   : 0.955   Max.   : 1.180   Max.   : 2.3
   war_within
 Min.   :-2.5
 1st Qu.:-0.3
 Median : 0.0
 Mean   : 0.0
 3rd Qu.: 0.3
 Max.   : 2.5
```

An Example: Refugee Flows in Africa, 1992-2001

Pooled OLS:

```
> RefOLS<-lm(ln_ref_flow~pop_diff+distance+regimedif+wardiff, data=Refugees)
> summary(RefOLS)

Residuals:
    Min      1Q  Median      3Q     Max
-0.6114 -0.2109 -0.0857  0.0335 14.3756

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.3224073  0.0119195 -27.049   <2e-16 ***
pop_diff    -0.1732934  0.2166658  -0.800    0.424
distance    -0.1266528  0.0047016 -26.938   <2e-16 ***
regimedif   -0.0002476  0.0157962  -0.016    0.987
wardiff      0.0743220  0.0068169  10.903   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.9097 on 23613 degrees of freedom
Multiple R-squared: 0.03467,Adjusted R-squared: 0.03451
F-statistic:   212 on 4 and 23613 DF,  p-value: < 2.2e-16
```

An Example: Refugee Flows in Africa, 1992-2001

"Fixed" effects:

```
> library(plm)
> RefFE<-plm(ln_ref_flow~pop_diff+distance+regimedif+wardiff,
  data=Refugees, effect="individual", model="within")
> summary(RefFE)
Oneway (individual) effect Within Model

Unbalanced Panel: n=2450, T=1-10, N=23618

Residuals :
     Min.   1st Qu.    Median   3rd Qu.      Max.
-9.03e+00 -5.74e-03 -9.18e-06  5.72e-03  1.14e+01

Coefficients :
           Estimate Std. Error t-value Pr(>|t|)
pop_diff  6.8642028  2.5516636  2.6901 0.007149 **
regimedif 0.0050497  0.0223160  0.2263 0.820984
wardiff   0.0104144  0.0073673  1.4136 0.157493
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    8149.6
Residual Sum of Squares: 8146
R-Squared      :  0.00043949
      Adj. R-Squared :  0.00039385
F-statistic: 3.102 on 3 and 21165 DF, p-value: 0.025509
```

An Example: Refugee Flows in Africa, 1992-2001

Models of Refugees in Africa

Variable	OLS	Fixed Effects
Constant	-0.32	-
	(0.01)	
Population Difference	-0.17	6.86
	(0.22)	(2.55)
Distance	-0.13	(dropped)
	(0.005)	
POLITY Difference	-0.0002	0.005
	(0.016)	(0.022)
War Difference	0.074	0.010
	(0.007)	(0.007)
$\hat{\rho}$	-	0.61

Note: $NT = 23618$ ($N = 2450$, $\bar{T} = 9.6$)

Issues (?) with "Fixed" Effects

Pros:

- Specification Bias
- Intuitive
- Widely Used/Understood

Cons:

- Can't Estimate $\boldsymbol{\beta}_B$
- Slowly-Changing $\mathbf{X}$s
- (In)Efficiency / Inconsistency (Incidental Parameters)

“Between” Effects

From:

$$Y_{it} = \bar{\mathbf{X}}_i \boldsymbol{\beta}_B + (\mathbf{X}_{it} - \bar{\mathbf{X}}_i)\beta_W + \alpha_i + u_{it}.$$

“Between” effects:

$$\bar{Y}_i = \bar{\mathbf{X}}_i \boldsymbol{\beta}_B + u_{it}$$

- Essentially cross-sectional
- Based on N observations

Refugee Flows in Africa, 1992-2001

"Between" effects:

```
> RefBE<-plm(ln_ref_flow~pop_diff+distance+regimedif+wardiff, data=Refugees,
  effect="individual", model="between")
> summary(RefBE)
Oneway (individual) effect Between Model

Unbalanced Panel: n=2450, T=1-10, N=23618

Residuals :
   Min. 1st Qu.  Median 3rd Qu.    Max.
-0.5850 -0.2200 -0.0840  0.0534  9.6500

Coefficients :
             Estimate Std. Error  t-value  Pr(>|t|)
(Intercept) -0.299703   0.029741 -10.0771 < 2.2e-16 ***
pop_diff    -0.246861   0.525232  -0.4700    0.6384
distance    -0.134874   0.011755 -11.4742 < 2.2e-16 ***
regimedif    0.010709   0.045117   0.2374    0.8124
wardiff      0.124185   0.022004   5.6439 1.855e-08 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    1383.9
Residual Sum of Squares: 1296.7
R-Squared      :  0.063042
      Adj. R-Squared :  0.062913
F-statistic: 41.1269 on 4 and 2445 DF, p-value: < 2.22e-16
```

Refugee Example Redux

Variable	OLS	Fixed ("Within") Effects	Between Effects
Constant	-0.32	-	-0.30
	(0.01)		(0.03)
Population Difference	-0.17	6.86	-0.25
	(0.22)	(2.55)	(0.53)
Distance	-0.13	(dropped)	-0.13
	(0.005)		(0.01)
POLITY Difference	-0.0002	0.005	0.01
	(0.016)	(0.022)	(0.05)
War Difference	0.074	0.010	0.12
	(0.007)	(0.007)	(0.02)
$\hat{\rho}$	-	0.61	-

Note: $NT = 23618$ ($N = 2450$, $\bar{T} = 9.6$).

“Random” Effects

Model:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}$$

with:

$$u_{it} = \alpha_i + \lambda_t + \eta_{it}$$

and

$$\begin{aligned}
E(\alpha_i) = E(\lambda_t) = E(\eta_{it}) &= 0, \\
E(\alpha_i\lambda_t) = E(\alpha_i\eta_{it}) = E(\lambda_t\eta_{it}) &= 0, \\
E(\alpha_i\alpha_j) &= \sigma_\alpha^2 \text{ if } i = j, \text{ 0 otherwise}, \\
E(\lambda_t\lambda_s) &= \sigma_\lambda^2 \text{ if } t = s, \text{ 0 otherwise}, \\
E(\eta_{it}\eta_{js}) &= \sigma_\eta^2 \text{ if } i = j,\ t = s, \text{ 0 otherwise}, \\
E(\alpha_i\mathbf{X}_{it}) = E(\lambda_t\mathbf{X}_{it}) = E(\eta_{it}\mathbf{X}_{it}) &= 0.
\end{aligned}$$

"Variance Components":

$$Var(Y_{it}|\mathbf{X}_{it}) = \sigma_\alpha^2 + \sigma_\lambda^2 + \sigma_\eta^2$$

If we assume $\lambda_t = 0$, then we get a model like:

$$Y_{it} = \mathbf{X}_{it}\beta + \alpha_i + \eta_{it}$$

with total error variance:

$$\sigma_u^2 = \sigma_\alpha^2 + \sigma_\eta^2.$$

"Random" Effects: Estimation

$$
\begin{aligned}
E(\mathbf{u}_i\mathbf{u}_i') \equiv \mathbf{\Sigma}_i &= \sigma_\eta^2 \mathbf{I}_T + \sigma_\alpha^2 \mathbf{i}\mathbf{i}' \\
&= \begin{pmatrix} \sigma_\eta^2 + \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \sigma_\eta^2 + \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\eta^2 + \sigma_\alpha^2 \end{pmatrix}
\end{aligned}
$$

$$
\mathrm{Var}(\mathbf{u}) \equiv \mathbf{\Omega} = \begin{pmatrix} \mathbf{\Sigma}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{\Sigma}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{\Sigma}_N \end{pmatrix}
$$

"Random" Effects: Estimation

Can estimate:

$$\mathbf{\Sigma}^{-1/2} = \frac{1}{\sigma_\eta}\left[\mathbf{I}_T - \left(\frac{\theta}{T}\mathbf{i}\mathbf{i}'\right)\right]$$

where

$$\theta = 1 - \sqrt{\frac{\sigma_\eta^2}{T\sigma_\alpha^2 + \sigma_\eta^2}}.$$

With $\hat{\theta}$, calculate:

$$\begin{aligned} Y_{it}^* &= Y_{it} - \hat{\theta}\bar{Y}_i \\ X_{it}^* &= X_{it} - \hat{\theta}\bar{X}_i, \end{aligned}$$

estimate:

$$Y_{it}^* = (1-\hat{\theta})\alpha + X_{it}^*\beta_{RE} + [(1-\hat{\theta})\alpha_i + (\eta_{it} - \hat{\theta}\bar{\eta}_i)]$$

and iterate...

"Random" Effects: An Alternative View

Refugees Redux

```
> RefRE<-plm(ln_ref_flow~pop_diff+distance+regimedif+wardiff, data=Refugees,
   effect="individual", model="random")
> summary(RefRE)
Oneway (individual) effect Random Effect Model
   (Swamy-Arora's transformation)

Unbalanced Panel: n=2450, T=1-10, N=23618

Effects:
                 var std.dev share
idiosyncratic 0.3849  0.6204 0.466
individual    0.4416  0.6645 0.534
theta  :
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.3176  0.7168  0.7168  0.7141  0.7168  0.7168

Coefficients :
              Estimate Std. Error  t-value  Pr(>|t|)
(Intercept) -0.3063941  0.0285299 -10.7394 < 2.2e-16 ***
pop_diff     0.0638665  0.4974613   0.1284  0.897845
distance    -0.1324536  0.0112685 -11.7544 < 2.2e-16 ***
regimedif    0.0005633  0.0198580   0.0284  0.977370
wardiff      0.0228523  0.0069775   3.2751  0.001058 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    9216.6
Residual Sum of Squares: 9158.9
R-Squared      :  0.0062699
Adj. R-Squared :  0.0062686
F-statistic: 37.177 on 4 and 23613 DF, p-value: < 2.22e-16
```

Refugees Redux, Remix

```
> library(lme4)

> AltRefRE<-lmer(ln_ref_flow~pop_diff+distance+regimedif+wardiff+(1|dirdyadID), data=Refugees)
> summary(AltRefRE)
Linear mixed model fit by REML
Formula: ln_ref_flow ~ pop_diff + distance + regimedif + wardiff + (1 |      dirdyadID)
   Data: Refugees
   AIC   BIC logLik deviance REMLdev
 50733 50790 -25360    50692   50719
Random effects:
 Groups    Name        Variance Std.Dev.
 dirdyadID (Intercept) 0.46653  0.68303
 Residual              0.38592  0.62123
Number of obs: 23618, groups: dirdyadID, 2450

Fixed effects:
              Estimate Std. Error t value
(Intercept) -0.3061471  0.0291477 -10.503
pop_diff     0.0758989  0.5075942   0.150
distance    -0.1325429  0.0115127 -11.513
regimedif    0.0007138  0.0199078   0.036
wardiff      0.0223476  0.0069779   3.203

Correlation of Fixed Effects:
          (Intr) pp_dff distnc regmdf
pop_diff   0.000
distance  -0.869  0.000
regimedif  0.000  0.036  0.000
wardiff    0.000 -0.004  0.000  0.109
```

Refugees Redux

Variable	OLS	Fixed Effects	Between Effects	Random Effects
Constant	-0.32	-	-0.30	-0.31
	(0.01)		(0.03)	(0.03)
Population Difference	-0.17	6.86	-0.25	0.09
	(0.22)	(2.55)	(0.53)	(0.52)
Distance	-0.13	(dropped)	-0.13	-0.13
	(0.005)		(0.01)	(0.01)
POLITY Difference	-0.0002	0.005	0.01	0.0005
	(0.016)	(0.022)	(0.05)	(0.0199)
War Difference	0.074	0.010	0.12	0.023
	(0.007)	(0.007)	(0.02)	(0.007)
$\hat{\rho}$	-	0.61	-	0.56

Note: $NT = 23618$ ($N = 2450$, $\bar{T} = 9.6$).

Hausman test (FE vs. RE):

$$\hat{W} = (\hat{\beta}_{\mathbf{FE}} - \hat{\beta}_{\mathbf{RE}})'(\hat{\mathbf{V}}_{\mathbf{FE}} - \hat{\mathbf{V}}_{\mathbf{RE}})^{-1}(\hat{\beta}_{\mathbf{FE}} - \hat{\beta}_{\mathbf{RE}})$$

$$W \sim \chi^2_k$$

Issues:

- Asymptotic
- No guarantee $(\hat{\mathbf{V}}_{\mathbf{FE}} - \hat{\mathbf{V}}_{\mathbf{RE}})^{-1}$ is positive definite
- A <u>general</u> specification test...

Hausman test (FE vs. RE):

```
> phtest(RefFE, AltRefRE)

Hausman Test

data:  ln_ref_flow ~ pop_diff + distance + regimedif + wardiff
chisq = 34.712, df = 3, p-value = 0.0000001401
alternative hypothesis: one model is inconsistent
```

Practical “Fixed” vs. “Random” Effects

- “Panel” vs. “TSCS” Data
- Data-Generating Process
- Covariate Effects

Separating Within and Between Effects

$$Y_{it} = \bar{\mathbf{X}}_i \boldsymbol{\beta}_B + (\mathbf{X}_{it} - \bar{\mathbf{X}}_i)\boldsymbol{\beta}_W + u_{it}$$

- Simple...
- Easy interpretation
- Easty to test $\hat{\boldsymbol{\beta}}_B = \hat{\boldsymbol{\beta}}_W$

Again With The Refugees...

Variable	Estimate
Constant	-0.32
	(0.01)
Distance	-0.13
	(0.004)
Between (Mean) Population Difference	-0.22
	(0.22)
Within Population Difference	6.86
	(3.74)
Between (Mean) POLITY Difference	0.01
	(0.02)
Within POLITY Difference	0.005
	(0.032)
Between (Mean) War Difference	0.12
	(0.01)
Within War Difference	0.01
	(0.01)

Note: $NT = 23618$ ($N = 2450$, $\bar{T} = 9.6$).

Unit Effects Models: Software

R :

- the `lme4` package; command is `lmer`
- the `plm` package; `plm` command
- the `nlme` package; command `lme`

Stata : `xtreg`

- the `re` (the default) = random effects
- the `fe` = fixed (within) effects
- the `be` = between-effects