Problem Set 5 -Estimation of Single-Agent Dynamic Discrete Choice Model-

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Remarks

1. Use A4 report papers, staple them in the upper left corner.
2. Type your answer.
3. You are allowed (and encouraged) to form a study group up to 3, and hand in one solution per group. If you do this, please put names and student IDs of all members.
4. Attach the print of your programming code for empirical exercise as an appendix.
5. When asked to report results present the answer in a table. Nothing fancy but don't simply attach a printout of the statistical program you used. You should attach the code you used to generate the results as an appendix.

Important

- I will give a preview of problem set 5 in make-up lecture posted at CourseN@vi. This will help you work on this problem set.

Question 1 Nested Fixed Point Algorithm

Download a dataset from the class website. The dataset is from a Rust-style optimal stopping problem for bus engines.

The data-generating process is as follows:

- Mileage for any bus at any time can take on 11 values, $x = 0, 1, \ldots, 10$
- At any time period, there is a probability λ that mileage will increase by 1, with a probability $1 - \lambda$ that mileage remains constant. If mileage is already 10, then it does not increase.

- The per-period utility of having a bus engine with mileage x is $u^0(x, q) = -(\theta_1 x + \theta_2 x^2) + \epsilon_0$
- The per-period utility of replacing a bus engine with mileage x is $u^1(x, q) = -RC + \epsilon_1$. Both the (ϵ_0, ϵ_1) are i.i.d. and distributed type 1 extreme value.
- Once you replace, the mileage is set to 0. In the next period, it will increase by 1 with probability λ, or remains the same with probability $1 - \lambda$.
- Zurcher discounts the future at the rate of β and chooses an optimal replacement policy given his objective function.

Since it is difficult to estimate , use $\beta = 0.95$ throughout this assignment. Also, while it is reasonably straightforward to estimate λ, use $\lambda = 0.7$ throughout this assignment.

The dataset includes information at the level of the bus-time period. Each observation contains the following information: beginning-of-period mileage, replacement decision and end-of-period mileage.

(a) Solve the dynamic programming problem for the model. Use $(\theta_1 = 0.3, \theta_2 = 0, RC = 4)$. Report the probability of replacement for any state.

(b) Estimate the model using a maximum likelihood fixed point algorithm. Report your parameter estimates.

Let $\theta = (\theta_1, \theta_2, RC)$. You will perform a nonlinear search over θ. For each θ, perform the following process:

1. Solve the dynamic programming problem
2. Using the value function, derive the contingent value of each action at each state (this can be done at the same time as solving the dynamic programming problem if you want)
3. Use the contingent valuation to calculate the probability of replacement at each state
4. Use the probability of replacement by state to derive a likelihood

(c) Calculate a variance/covariance matrix of the parameter estimates. We use the standard formula

$$Var(\hat{\theta}) = \frac{1}{N} \left[\sum_{i=1}^{N} \left(\frac{d \ln L_i(\theta)}{d\theta} \right) \left(\frac{d \ln L_i(\theta)}{d\theta} \right)' \right],$$

calculating the derivatives numerically. Report standard errors of the parameter estimates.

Question 2 Two-Step Estimator

(a) Estimate the probability of replacement at each state. Report this vector.

(b) Estimate the model parameter using the Hotz-Miller inversion. Please refer the lecture slide for implementation. Report the parameter estimates and the standard errors. Do not worry about the first-stage error in calculating the standard errors.