

ICPSR 2016 “Advanced Maximum Likelihood”: Survival Analysis

Day One

August 1, 2016

Preliminaries

- Instructor: Christopher Zorn (zorn@psu.edu)
- Teaching Assistants: Dominique Lewis (lewiso2@gmail.com) and George Williford (gwilliford@gmail.com)
- Class: 1:00-3:00 p.m. ET, in Room 296 Weiser (formerly Dennison) Hall
- Office Hours: 3:00 - ??? daily at Espresso Royale (State St.); also by appointment
- The syllabus is [here](#)
- Slides, readings, code, etc. are on the [github repo](#) (<https://github.com/PrisonRodeo/ICPSR-2016-git>)

< Code

Issues 0

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ICPSR Advanced MLE - Survival Analysis Repo (2016 remix) — Edit

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1 branch

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Zorn-Advanced-MLE-Survival-SU2016-Syllabus.pdf

Syllabus

3 months ago

README.md

ICPSR-2016-git

Github repo for the second two weeks of ICPSR's **Advanced Maximum Likelihood** course, August 1-12, 2016. The topic is survival analysis.

R

- All examples, plots, etc.
- Current version is 3.3.1
- Packages you'll use (see the [survival analysis task view](#) for more):
 - `survival` (nearly everything you need)
 - `eha`
 - `timereg`

Stata

- Current version is 14
- Mostly use the `-st-` series of commands (for “survival time”)

Survival Analysis

- Models for *time-to-event data*.
- Roots in biostats/epidemiology, plus engineering, sociology, economics.
- Examples...
 - Political careers, confirmation durations, position-taking, bill cosponsorship, campaign contributions, policy innovation/adoption, etc.
 - Cabinet/government durations, length of civil wars, coalition durability, etc.
 - War duration, peace duration, alliance longevity, length of trade agreements, etc.
 - Strike durations, work careers (including promotions, firings, etc.), criminal careers, marriage and child-bearing behavior, etc.

Characteristics of Time-To-Event Data

- Discrete events (i.e., not continuous),
- Take place over time,
- May not (or *never*) experience the event (i.e., possibility of censoring).

Survival Data Basics: Terminology

Y_i = the duration until the event occurs,

Z_i = the duration until the observation is “censored”

T_i = $\min\{Y_i, Z_i\}$,

C_i = 0 if observation i is censored, 1 if it is not.

Survival Data Basics: The Density

$$f(t) = \Pr(T_i = t)$$

Issues:

- $T_i = t$ iff $T_i > t - 1, t - 2$, etc.
- $C_i = 0$ (censoring)

Survival Data Basics: Survivor Function

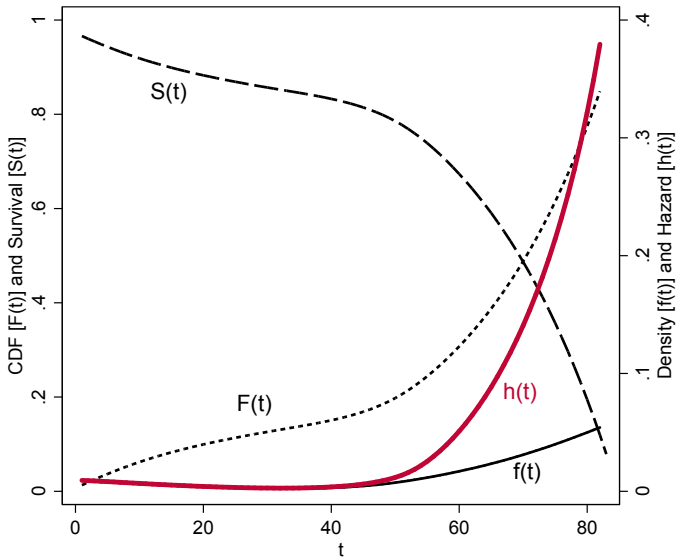
$$\Pr(T_i \leq t) \equiv F(t) = \int_0^t f(t) dt$$

$$\begin{aligned}\Pr(T_i \geq t) \equiv S(t) &= 1 - F(t) \\ &= 1 - \int_0^t f(t) dt\end{aligned}$$

Survival Data Basics: The Hazard

$$\begin{aligned}\Pr(T_i = t | T_i \geq t) \equiv h(t) &= \frac{f(t)}{S(t)} \\ &= \frac{f(t)}{1 - \int_0^t f(t) dt}\end{aligned}$$

Example: Human Mortality



Some Useful Equivalencies

$$f(t) = \frac{-\partial S(t)}{\partial t}$$

Implies

$$\begin{aligned} h(t) &= \frac{\frac{-\partial S(t)}{\partial t}}{S(t)} \\ &= \frac{-\partial \ln S(t)}{\partial t} \end{aligned}$$

More Useful Things: Integrated Hazard

Define

$$H(t) = \int_0^t h(t) dt.$$

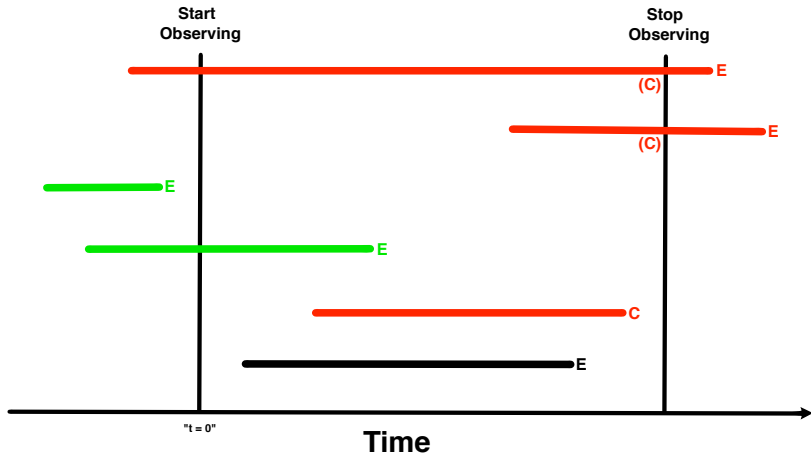
Implies

$$\begin{aligned} H(t) &= \int_0^t \frac{-\partial \ln S(t)}{\partial t} dt \\ &= -\ln[S(t)] \end{aligned}$$

and

$$S(t) = \exp[-H(t)]$$

Censoring and Truncation



- Defined by the researcher
- Conditionally independent of both T_i and \mathbf{X}_i
- Doesn't mean that the observation provides no information

Estimating $S(t)$

Assume N observations, *absorbing* events, and no ties. Then define

n_t = number of observations “at risk” for the event at t , and
 d_t = number of observations which experience the event at time t .

Then

$$\widehat{S(t_k)} = \prod_{t \leq t_k} \frac{n_t - d_t}{n_t}$$

Variance of $\widehat{S}(t)$

$$\text{Var}[\widehat{S}(t_k)] = \left[\widehat{S}(t_k) \right]^2 \sum_{t \leq t_k} \frac{d_t}{n_t(n_t - d_t)}$$

Note:

- $\text{Var}[\widehat{S}(t_k)]$ is increasing in $S(t)$,
- is also increasing in d_t , but
- is decreasing in n_t .

Estimating $H(t)$

“Nelson-Aalen”:

$$\widehat{H}(t_k) = \sum_{t \leq t_k} \frac{d_t}{n_t}$$

...which gives an alternative estimator for the survival function equal to:

$$\begin{aligned}\widehat{S}(t_k) &= \exp[-\widehat{H}(t_k)] \\ &= \exp\left[-\sum_{t \leq t_k} \frac{d_t}{n_t}\right]\end{aligned}$$

Bivariate Hypothesis Testing

	Treatment	Placebo	Total
Event	d_{1t}	d_{0t}	d_t
No Event	$n_{1t} - d_{1t}$	$n_{0t} - d_{0t}$	$n_t - d_t$
Total	n_{1t}	n_{0t}	n_t

Log-Rank Test:

$$Q = \frac{\left[\sum (d_{1t} - \frac{n_{1t}d_t}{n_t}) \right]^2}{\left[\frac{n_{1t}n_{0t}d_t(n_t - d_t)}{n_t^2(n_t - 1)} \right]}$$
$$\sim \chi_1^2$$

A Diversion: Survival Models and Counting Processes

Assume

- Event is *absorbing*,
- Y_i is duration to the event
- Z_i is duration to censoring
- Observe $T_i = \min(Y_i, Z_i)$, and
- C_i :
 - $C_i = 0$ if $T_i = Z_i$,
 - $C_i = 1$ if $T_i = Y_i$.
- $T_i \neq T_j \forall i \neq j$ (no “ties”)

Three Key Variables

1. *Counting Process* Indicator:

$$N_i(t) = I(T_i \leq t, C_i = 1)$$

2. *Risk* Indicator:

$$R_i(t) = I(T_i > t)$$

3. *Intensity Process*:

$$\lambda_i(t) dt = R_i(t)h(t)$$

Additional Things

With

$$\Lambda_i(t) = \int_0^t \lambda_i(t) dt$$

we can think of

$$N_i(t) = \Lambda_i(t) + M_i(t)$$

or

$$M_i(t) = N_i(t) - \Lambda_i(t).$$

Martingales!

$$E(X_{t+s} | X_0, X_1, \dots, X_i, \dots, X_t) = X_t \quad \forall s > 0$$

Data Structure and Organization: Non-Time-Varying

id	durat	censor	timein	timeout	X
1	4	0	30	34	0.12
2	2	1	12	14	0.19
3	5	1	5	10	0.09
...
N	10	1	21	31	0.22

Time-Varying Data

id	durat	censor	timein	timeout	X	Z
1	1	0	30	31	0.12	331
1	2	0	31	32	0.12	412
1	3	0	32	33	0.12	405
1	4	0	33	34	0.12	416
2	1	0	12	13	0.19	226
2	2	1	13	14	0.19	296
3	1	0	5	6	0.09	253
3	2	0	6	7	0.09	311
3	3	0	7	8	0.09	327
3	4	0	8	9	0.09	344
3	5	1	9	10	0.09	301
...

Analyzing Survival Data in R

survival object (non-time-varying):

```
library(survival)
NonTV<-read.csv(NonTVdata.csv)
NonTV.S<-Surv(NonTV$duration, NonTV$censor)
```

survival object (time-varying):

```
TV<-read.csv(TVdata.csv)
TV.S<-Surv(TV$starttime, TV$endtime, TV$censor)
```

An Example

OECD Cabinet survival [Strom (1985); King et al. (1990)],

$N = 314$ cabinets in 15 countries

Outcome: Duration of cabinet, in months

Covariates (all non-time varying):

- *Fractionalization*
- *Polarization*
- *Formation Attempts*
- **Investiture**
- *Numerical Status*
- *Post-Election*
- *Caretaker*

Also: Indicator for whether the cabinet ended within 12 months of the end of the “constitutional inter-election period” (→ censored)

KABL Data

```
> head(KABL)
```

	id	country	durat	ciep12	fract	polar	format	invest	numst2	eltime2	caret2
1	1		1	0.5	1	656	11	3	1	0	1
2	2		1	3.0	1	656	11	2	1	1	0
3	3		1	7.0	1	656	11	5	1	1	0
4	4		1	20.0	1	656	11	2	1	1	0
5	5		1	6.0	1	656	11	3	1	1	0
6	6		1	7.0	1	634	6	4	1	1	1

```
> KABL.S<-Surv(KABL$durat,KABL$ciep12)
```

```
> KABL.S[1:50,]
```

[1]	0.5	3.0	7.0	20.0	6.0	7.0	2.0	17.0	27.0	49.0+
[11]	4.0	29.0	49.0+	6.0	23.0	41.0+	10.0	12.0	2.0	33.0
[21]	1.0	16.0	2.0	9.0	3.0	5.0	5.0	6.0	45.0+	23.0
[31]	41.0	7.0	49.0+	46.0	9.0	51.0+	10.0	32.0	28.0	3.0
[41]	53.0+	17.0	59.0+	9.0	52.0+	3.0	23.0	33.0	1.0	30.0

Example survfit Object

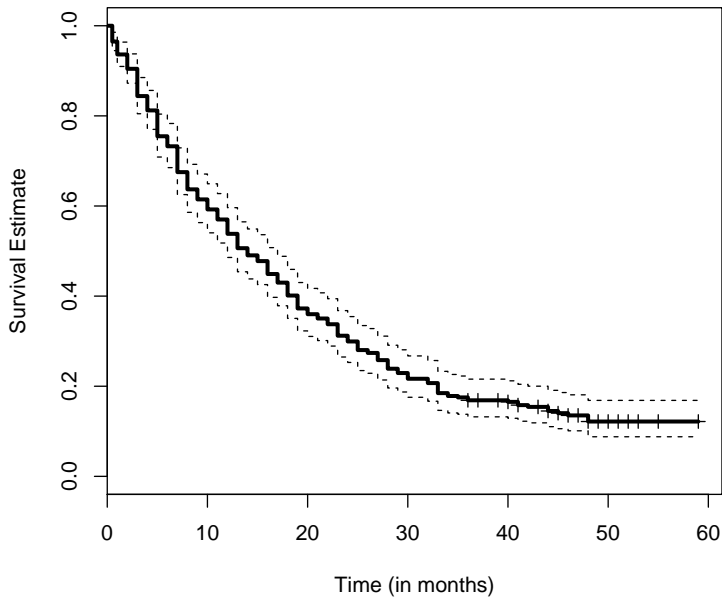
```
> KABL.fit<-survfit(KABL.S~1)
```

```
> str(KABL.fit)
```

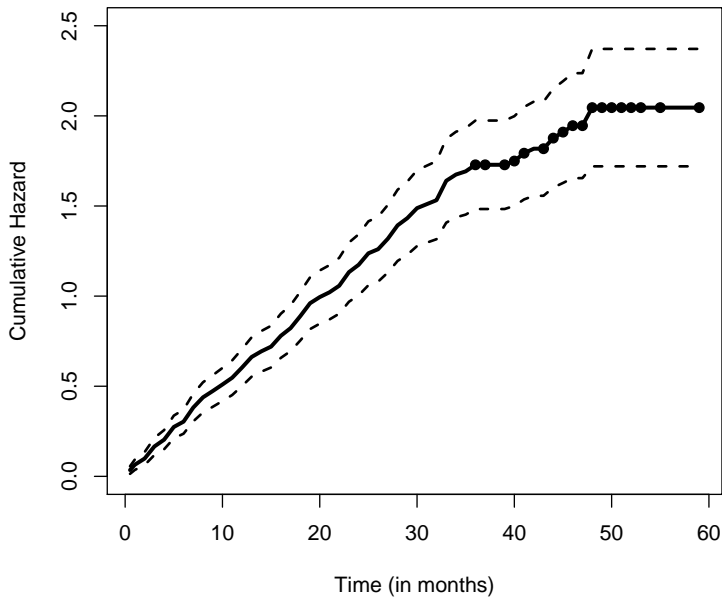
List of 13

```
$ n      : int 314
$ time   : num [1:54] 0.5 1 2 3 4 5 6 7 8 9 ...
$ n.risk : num [1:54] 314 303 294 284 265 255 237 230 212 200 ...
$ n.event : num [1:54] 11 9 10 19 10 18 7 18 12 7 ...
$ n.censor : num [1:54] 0 0 0 0 0 0 0 0 0 0 ...
$ surv    : num [1:54] 0.965 0.936 0.904 0.844 0.812 ...
$ type    : chr "right"
$ std.err : num [1:54] 0.0108 0.0147 0.0183 0.0243 0.0271 ...
$ upper   : num [1:54] 0.986 0.964 0.938 0.885 0.856 ...
$ lower   : num [1:54] 0.945 0.91 0.873 0.805 0.77 ...
$ conf.type: chr "log"
$ conf.int : num 0.95
$ call    : language survfit(formula = KABL.S ~ 1)
- attr(*, "class")= chr "survfit"
```

Plotting $\widehat{S}(t)$



Plotting $\widehat{H}(t)$



Comparing $\widehat{S}(t)$ s

Log-rank test:

```
> survdiff(KABL.S~invest,data=KABL,rho=0)
```

Call:

```
survdiff(formula = KABL.S ~ invest, data = KABL, rho = 0)
```

	N	Observed	Expected	(O-E) ² /E	(O-E) ² /V
invest=0	172	137	178.7	9.72	30.5
invest=1	142	134	92.3	18.81	30.5

Chisq= 30.5 on 1 degrees of freedom, p= 3.26e-08

Comparing $\widehat{S}(t)$ s

